Higher-order edge magnetoplasma oscillations in a two-dimensional electron channel

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For the first time, we present an investigation of higher-order edge magnetoplasma oscillations in the two-dimensional electron channel of a GaAs–AlGaAs heterostructure. We determine ratios between the eigenfrequencies for the first three oscillations in samples of various shapes. We have found that the frequencies of the edge magnetoplasma oscillation spectrum are not equally spaced.

1. INTRODUCTION

Edge magnetoplasma oscillations (MOs) are the characteristic oscillations of the Hall current in a finite 2D channel. The Hall current for such oscillations is caused by an electric field, which in turn arises from charges at the boundary of the 2D channel. The MOs consist of rotation of the corresponding charge distributions and currents around the direction of the external magnetic field (which is perpendicular to the 2D channel). The MO eigenfrequencies are determined by the dimensions of the sample and the magnitude of the external magnetic field. These oscillations were recently observed experimentally in 2D electron channels at the surface of liquid He[1–3] and in GaAs–AlGaAs heterostructures.[4–6] The MOs of a 2D channel which is located in a solid are particularly interesting because they can be observed in the regime of the quantum Hall effect. In Ref. 7, it was shown that the attenuation of MOs is a minimum in this regime. The MO modes can be enumerated by using the azimuthal quantum number m (MOs with $1 \leqslant m \leqslant 10$ have been observed experimentally in 2D electron channels at the surface of liquid He.[1–3] In solids (specifically in GaAs–AlGaAs heterostructures) only MOs with $m = 1$ have been observed.[4–7] In order to excite MOs with $m > 1$, the sample must be located in an inhomogeneous external high-frequency field, which makes observation of such oscillations difficult. Meanwhile, the MOs with $m > 1$ are of considerable interest. Comparison of the experimental MO spectrum with theoretical calculations makes it possible to obtain information about dynamic processes in 2D channels. In addition, the presence of certain characteristic oscillations makes possible investigations of various nonlinear processes which are accompanied by "pumping" of energy from one oscillation to another, and which may be of interest from a practical standpoint as well.[1] It should be noted that the MO spectrum of a 2D channel at the surface of liquid He can differ significantly from that of a heterostructure 2D channel because of the presence in the former case of a metallic field electrode. The goal of the present article is to observe higher-order MOs (with $m = 2$ and 3) in the 2D channel of a GaAs–AlGaAs heterostructure.

2. EXPERIMENT

The samples (a 3 mm × 3 mm film and a 3 mm diameter disk, both of whose concentrations and mobilities at 4.2 K were 2×10^{11} cm^{-2} and 10^5 cm^2/V·sec, respectively) were placed in a straight-through tunable (from 200 to 700 MHz) coaxial resonator. The resonator was connected to a UHF generator and a superheterodyne unit by way of a coaxial cable. In order to excite the MO with $m = 1$, the sample was placed between the conductors of the resonator in a plane passing through the end of the inner conductor (Fig. 1a). There is an antinode of the high frequency electric field located in this plane; in addition, the field is fairly homogeneous in the region between the conductors of the resonator. In order to excite the MOs with $m = 2$ (Fig. 1b) and with $m = 3$ (Fig. 1c), a resonator was used whose inner conductor was terminated by stubs (two for the $m = 2$ mode and three for $m = 3$, Figs. 1b, 1c), which were positioned parallel and symmetrically relative to the resonator axis. In these cases the sample was located at the center of the resonator in the plane passing through the ends of the stubs. It is clear from Figs. 1b, 1c that the symmetry of the high-frequency field which penetrates the sample is optimal for exciting the MOs with $m = 2$ (Fig. 1b) and with $m = 3$ (Fig. 1c). The constant magnetic field (B) was directed along the axis of the resonator, and the measurement temperature held at 4.2 K. We measured the amplitude (A) of the UHF wave passing through the resonator as a function of magnetic field.

As we have already mentioned, the eigenfrequencies of the MOs are functions of the external magnetic field. The measurements taken in Refs. 5 and 7 showed that the MO eigenfrequency with $m = 1$ (ω_1) is inversely proportional to the magnitude of the magnetic field: $\omega_1 \sim B^{-1}$ (for $B > 1$ T). As we will show below, the same dependence on B is observed for the frequencies of the $m = 2$ and $m = 3$ MOs (henceforth referred to as ω_2 and ω_3, respectively). For

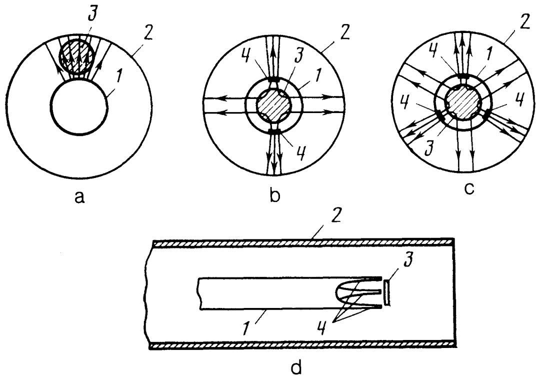

FIG. 1. (a), (b), (c) show the UHF electric field distributions in the sample plane for excitation of MOs with $m = 1,2,3$; (d) is a schematic illustration of part of the resonator used to excite the $m = 3$ MO. Notation: 1, 2—internal and external conductors, 3—sample, 4—stubs.

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those values of magnetic field which cause the eigenfrequency of some MO (ω_m) to coincide with the frequency of the UHF wave (ω), the resonance condition.

$$\omega_m(B)=\omega$$

is fulfilled. If the spatial distribution (symmetry) of the driving electric field of the UHF wave is such that a MO with index m is excited, then the electric field within the sample increases considerably at resonance. This causes the UHF power losses in the resonator to grow, and the amplitude of the UHF waves passing through the sample to decrease. Thus, the excitation of MOs in the sample should appear as a rather sharp minimum in the measured dependence $A(B)$.

When the magnetic field is varied, there is a change (due to the presence of the sample) not only in the resonator Q (attenuation) but also in the eigenfrequency of the UHF rsonator. In our experiments, the magnitude of this change was less than 1 MHz over the entire range of variation of B. So as to exclude errors associated with this fact, we maintained the generator frequency equal to the eigenfrequency of the resonator plus sample during the measurement. It should be noted that variation in the magnetic field also leads to oscillations in the conductivity (σ_{xx}) of the 2D channel (Shubnikov-de Haas oscillations). These oscillations also appear in the function $A(B)$; the minimum in the UHF power absorption corresponds to the minimum in σ_{xx}, and consequently to a maximum in the function $A(B)$.[2] In addition to the UHF measurements, we carried out standard measurements of the components of the magnetoresistance tensor ρ_{xx} and ρ_{xy} at constant current. The samples for these measurements were cut out from the same film as the one used for the UHF measurements; however, the carrier concentration in different samples could differ by 10 to 20%.

3. MEASUREMENT RESULTS

In Fig. 2 we show the measured results for the magnetoresistance tensor components, which allow us to evaluate the quality of the heterostructure. At $B\approx 2.3$ T and 6 T, the quantum plateaus are clearly visible in the function $\rho_{xy}(B)$. The minimum value in ρ_{xx} for $B\approx 6$ T comes to 1 ohm per square. The "slopes" in the quantum plateaus are apparently associated with edge effects due to the small length-to-width ratio of the sample (equal to two in our case). The influence of edge effects was discussed in Ref. 13.

In Fig. 3 we show the functions $A(B)$, corresponding to different symmetries of the exciting UHF field (whose frequency, however, was fixed at 600 MHz). The minima are clearly observable in the curves shown in Fig. 3 (marked by arrows). It is clear that the value of B corresponding to each minimum depends on the configuration of the exciting UHF field. Thus, curves 1,2,3 in Fig. 3 show evidence of excitation of MOs in the sample with $m = 1,2,3$.

In order to determine the ratios between MO eigenfrequencies, it is convenient to select the excitation frequency for each MO in such a way that the resonance condition for all MOs is satisfied at one and the same value of magnetic field. In Fig. 4, we present results of these measurements; the resonance value of magnetic field corresponds to a quantum Hall conductivity plateau ($\sigma_{xy} = e^2h^{-1}n, n = 4$) and a minimum in σ_{xx}. The smallness of σ_{xx} leads to a decrease in the MO attenuation, and consequently to a sharper peak in the

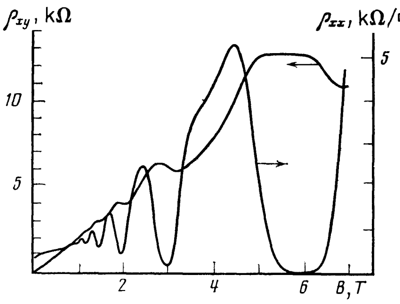

FIG. 2. Dependence of the components of the magnetoresistance tensor ρxx and ρxy on magnetic field for $T = 4.2$ K.

experimental functions. The data in Fig. 4 leads to ratios between the MO eigenfrequencies

$$\omega_3=(1.85\pm0.1)\,\omega_1,\quad \omega_2=(1.4\pm0.1)\,\omega_1. \tag{1}$$

Analogous measurements for the disk-shaped sample give

$$\omega_3=(2\pm0.1)\,\omega_1,\ \omega_2=(1.6\pm0.1)\,\omega_1. \tag{2}$$

We should note the presence in curve 1 of Fig. 4 of an additional peak at $B\sim 6$ T. It arises because of excitation of the $m = 3$ MO due to weak inhomogeneity of the UHF field in the space between the resonator conductors (Fig. 1a). In order to identify this peak, we carried out additional measurements; we tracked the position of the peak due to absorption by the $m = 3$ MO over the entire range of values of the excitation frequency in the interval 400 to 800 MHz (in this case the sample was placed in the location shown in Fig. 1c. It turns out that to good accuracy (10%) the value of the resonance magnetic field is inversely proportional to the excitation frequency. (We point out that the same regularity is also observed with the $m = 1$ and $m = 2$ MOs). In Fig. 5 (curve 1) we show the function $A(B)$ for a measurement frequency of 420 MHz. For convenience we reproduce curve 1 of this figure in Fig. 4 (labelled curve 2), which corresponds to the same measurement frequency but with the sample positioned as shown in Fig. 1a. On curve 1 (Fig. 5), besides the $m = 3$ MO absorption peak we can see a weak peak corresponding to absorption by the $m = 3$ MO at $B\sim 3.4$ T. The decrease in magnitude of this peak in curve 1 compared to curve 2 (Fig. 5) has two causes; when measuring curve 1 of Fig. 5, the sample was located in the region between the three stubs (Fig. 1c), where first of all the mag-

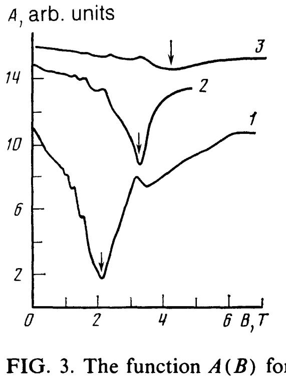

FIG. 3. The function $A(B)$ for a square sample: 1, 2,3—the sample is placed in the positions shown in Figs. 1(a), (b) and (c), respectively. The frequency of the UHF field was 600 MHz for all the curves. Curves 2, 3 are shifted vertically without change of scale; the amplitude of the wave passing through the resonator in zero magnetic field $A(B = 0)$ is the same for all the curves.

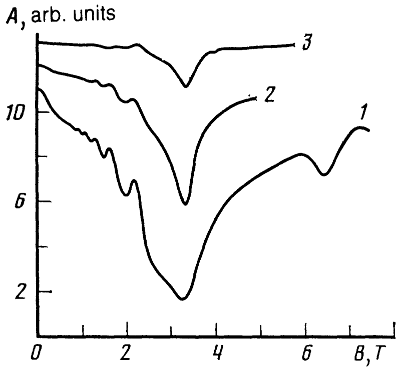

FIG. 4. The function $A(B)$ for a square sample. The numbers on the curves denote sample positions which are the same as in Fig. 3. The measurement frequencies are: 1—425 MHz, 2—600 MHz, and 3—780 MHz.

nitude of the UHF field is rather small, and secondly the symmetry of the field is such that excitation of the $m = 1$ MO is ineffective. In the case of the $m = 3$ MO, the decrease in the value of the UHF field in the region between the stubs is compensated by the symmetry of the field, which favors the excitation of the $m = 3$ MO.

Because the eigenfrequencies of all the MOs we investigated depend on the value of the magnetic field in the same way ($\omega_m \sim B^{-1}$), the ratios (1) and (2) are valid over the entire investigated range of magnetic fields. Finally, we note that Shubnikov-deHaas oscillations are visible in Figs. 3–5 in addition to the peaks connected with MO excitation. This circumstance does not hinder interpretation of the plots, because the position of peaks connected with the MOs depends on the UHF field frequency (as opposed to the Shubnikov-deHaas peaks); in addition, the MO peaks are significantly larger in magnitude than the Shubnikov-deHaas oscillations.

4. DISCUSSION OF RESULTS

The problem of determining the MO spectrum of a disk-shaped 2D channel was solved numerically in Ref. 14. Unfortunately, the MO spectrum presented there was for small values of magnetic field (so that the measurements could be carried out on heterostructures). The approximation method used to determine the MO eigenfrequencies was described in Ref. 6. It is based on the idea that the interactions which determine the MO propagation occur in a strip at the edge of the sample. Then we can take advantage of the dispersion law $\omega(k)$ for a semi-infinite 2D channel[15] and obtain the MO eigenfrequencies in a finite sample[6];

$$\omega_m = \omega(k = 2\pi m/p), \tag{3}$$

where p is the sample perimeter.

From Ref. 15 it follows that

$$\omega \sim \sigma_{xy} k \ln(2e/kl), \tag{4}$$

where e is the base of the natural logarithms and l is the width of the strip in which the MO charge is concentrated.

Equation (4) is valid if practically all the current flows near the edges of the sample in a strip whose width is much smaller than the sample dimensions. The same condition ensures that Eq. (3) is applicable. In order to compare our results [Eqs. (1) and (2)] with Eqs. (3) and (4), it is necessary to assume that $l \sim 0.1R \sim 10^{-2}$ (R is the sample size). However, the approach of Ref. 6 leads to the same MO spectrum, independent of the sample shape, for considerably smaller l.

Finally, let us consider the approach in which the real 2D channel is replaced by a very thin conducting ellipsoid of revolution. The corresponding equations can be obtained by using the results of Refs. 11, 12. In these references the MO frequencies were calculated for an anisotropic ellipsoid of revolution (i.e., in which the conductivity along the magnetic field equals zero). It turns out that in such a system the characteristic oscillations can be classified by using two indices n,m. The oscillations with equal indices ($n = m$) do not contain any dependence on the coordinate parallel to the external magnetic field, and therefore they are preserved even in the 2D limit. For thin ellipsoids of revolution (a,b are the axes of the ellipsoid, b is the axis of rotation which is directed along the magnetic field, and $b \ll a$), it follows from Refs. 11,12 that

$$\omega_1 = \pi^2 \sigma_{xy}^{(0)} b/a, \quad \omega_2 = 1.5\pi^2 \sigma_{xy}^{(0)} b/a, \quad \omega_3 = {}^{15}/_8 \pi^2 \sigma_{xy}^{(0)} b/a. \tag{5}$$

In (5) $\sigma_{xy}{}^{(0)}$ is the bulk Hall conductivity and ω_1, ω_2, ω_3 correspond to ω_{11}, ω_{22} and ω_{33} in the notation of Refs. 11, 12 Eq. (5) is exact in the limit $b/a \to 0(\sigma_{xy}{}^{(0)} \to b^{-1}\sigma_{xy}$, where σ_{xy} is the 2D channel conductivity); however, in this limit (5) gives the MO spectrum of a 2D disk in which σ_{xy} falls off from the center of the disk to the edge according to the law $\sigma_{xy} \propto [1 - (r/R)]^{1/2}$. Nevertheless, it is clear that (5) gives an excellent description (and without fitting parameters) of the observed experimental ratios between the MO frequencies. In addition, these equations are in good agreement ($\sim 30\%$) with the measured absolute values of the MO frequencies. The inverse proportionality of the MO eigenfrequencies to the magnitude of the magnetic field B which we observe in experiment also follows from Eq. (5), because $\sigma_{xy} \propto B^{-1}$ for $B > 1$T. However, a computation of the MO attenuation[7] in a model where the 2D channel is replaced by a thin ellipsoid of rotation leads to values of $\sigma_{xx}(\omega)$ which exceed by several orders of magnitude the value of σ_{xx} at constant current. It should be noted that in deriving Eqs. (4) and (5) we did not take into account the presence of the GaAs substrate near the 2D channel. The correction associated with the presence of the substrate (which cannot be calculated exactly as yet) apparently decreases as the ratio d/R decreases[6] (where d is the substrate thickness, which in our experiments was 0.3 mm). It is reasonable to assume

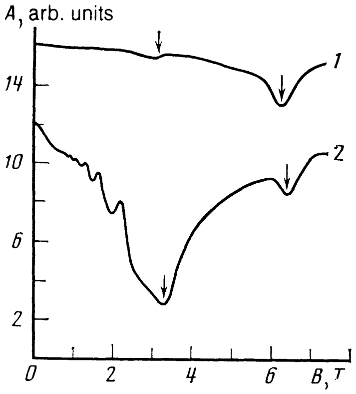

FIG. 5. The function $A(B)$ for a square sample at a frequency of 420 MHz; 1—the sample is located in the position shown in Fig. 1(c); 2—the sample is located in the position shown in Fig. 1(a).

that the ratios (1) and (2) depend on the presence of the substrate to a lesser degree than do their absolute values ω_m.

In conclusion, we can formulate our basic results as follows: we have observed higher-order MOs for the first time in a solid, and have found that the frequencies of the MO spectrum are not equally spaced.

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[1] In this connection, investigations have been carried out on characteristic oscillations of the magnetization in bounded ferromagnets (Walker modes[8,9]). The existence of a formal analogy between Walker modes and oscillations of the Hall current in bounded 2D systems was mentioned in Refs. 5 and 10–12.

[2] In Ref. 6 an alternate explanation was proposed which related these oscillations in the function $A(B)$ with oscillations in the frequency and attenuation of the MOs, which in turn arise from variations in the real and imaginary parts of σ_{xx}.

[1]D. B. Mast, A. J. Dahm, and A. L. Fetter, Phys. Rev. Lett. **54**, 1706 (1985)
[2]D. C. Giatti, E. Y. Andrei, G. Deville, J. Poitrenaud, and F. I. B. Williams, Phys. Rev. Lett. **54**, 1710 (1985).
[3]D. C. Glatti, E. Y. Andrei, G. Deville, and F. I. B. Williams, Surf. Sci. **170**, 70 (1986).
[4]S. J. Allen Jr., H. L. Störmer and J. C. M. Hwang, Phys. Rev. **B28**, 4875 (1983).
[5]S. A. Govorkov, M. I. Reznikov, A. I. Senichkin and V. I. Tal'yanski, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 96 (1986) [JETP Lett. **43**, 96 (1986)].
[6]V. A. Volkov, D. V. Galchenkov, L. A. Galchenkov, I. M. Grodnenski, O. R. Matob and S. A. Mikhailov, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 510 (1986) [JETP Lett. **44**, 655 (1986)].
[7]S. A. Govorkov, M. I. Reznikov, B. K. Medvedav *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 252 (1987) [JETP Lett. **45**, 316 (1987)].
[8]C. Kittel, Phys. Rev. **73**, 155 (1948).
[9]L. R. Walker, Phys. Rev. **105**, 390 (1957).
[10]V. I. Tal'yanski, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 96 (1986).
[11]V. I. Tal'yanski, Solid State Commun. **62**, 133 (1987).
[12]V. I. Tal'yanski, Zh. Eksp. Teor. Fiz. **92**, 1845 (1987) [Sov. Phys. JETP **65**, 1036 (1987)].
[13]K. V. Klitzing, Festkörperprobleme XXI, Vol. 1 (1981) [Translated in the collection *The Quantum Hall Effect*, Moscow: 1986], p. 25.
[14]A. L. Fetter, Phys. Rev. **B33**, 5221 (1986).
[15]V. A. Volkov and S. A. Mikhailov, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 450 (1986) [JETP Lett. **42**, 556 (1986)].

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