Trapping and entraining of electromagnetic-field packets by ion-sound waves into supercritical regions of an inhomogeneous plasma

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We discuss the mechanism for trapping and entraining of electromagnetic (EM) field packets by ion-sound waves into supercritical regions of an inhomogeneous plasma. The transfer is caused by the fact that there exist, in an inhomogeneous plasma with strong ion-sound waves, EM field states which are dragged along by these waves into dense plasma layers. We determine the efficiency of the excitation of the entrained states of the EM field by incidence of EM radiation of a given frequency upon the plasma and we find the penetration depth of the trapped field into the dense plasma layers. We show that it is possible that the EM field can penetrate into plasma layers having local density values several times larger than the density in the region of the turning point of the radiation incident upon the plasma.

1. INTRODUCTION

The problems of the penetration of electromagnetic (EM) waves into inhomogeneous plasma regions with local values of the plasma frequency larger than the frequency of the radiation incident upon the plasma (supercritical regions) are of considerable interest both for the theory of wave propagation and for applied studies of plasma physics. In the past the penetration of strong high-frequency waves has been studied; their action leads to a considerable nonlinear restructuring of the parameters of the medium.[1–8] In particular, the excitation of electro-acoustic waves by a modulated EM wave, incident from the vacuum, at the boundary between the vacuum and a uniform supercritical plasma has been studied in Ref. 8.

In the present paper the penetration of EM field bunches into supercritical regions of a smoothly inhomogeneous plasma is connected with their transport by strong ion-acoustic waves. This transport is caused by the existence in an inhomogeneous medium of quasi-localized states of the EM field, which are of variable frequency and are dragged along by the low-frequency wave of the parameter of the medium.[9] The transport mechanism can be demonstrated using as a model the problem of evolution of a one-dimensional wave field $\psi(z,t)$ described in dimensionless variables by a Schrödinger type of equation:

$$i\frac{\partial\psi}{\partial t}+\frac{1}{2}\frac{\partial^2\psi}{\partial z^2}-U(z,t)\psi=0$$

with a potential

$$U(z,t)=\frac{z}{2}+\tilde{V}(z-at),$$

where $\tilde{V}(z-at)$ is a low-frequency perturbation of the potential and moves with a velocity $a>0$ into a region with large values of the stationary potential $U_0(z)=z/2$. Equation (I) can be reduced through a change of the independent variables

$$\xi=z-at,\quad t'=t$$

and of the required function

$$\psi=\overline{\psi}(\xi,t)\exp(ia\xi-iat^2/4)$$

to the form

$$i\frac{\partial\psi}{\partial t}+\frac{1}{2}\frac{\partial^2\psi}{\partial\xi^2}+\left[\frac{a^2}{2}-\frac{\xi}{2}-\tilde{V}(\xi)\right]\psi=0,$$

from which it follows that for well- or hump-type perturbations $\tilde{V}(\xi)$ of the potential there can exist metastable states which are dragged into the region $z>0$,

$$\overline{\psi}_n=\overline{\psi}_n(\xi)\exp(-i\mu_n t-iat^2/4+ia\xi),$$

with a frequency $\omega_n=\mathrm{Re}\,\mu_n+at/2$ which increases monotonically with time. After the lifetime of the metastable state $t_n=|\mathrm{Im}\,\mu_n|^{-1}$, an upwards shift $\Delta\omega$ by an amount of the order $a|\mathrm{Im}\,\mu_n|^{-1}$ is reached. The penetration depth of the field is then $z_{\mathrm{pen}}=2\Delta\omega=2a|\mathrm{Im}\,\mu_n|^{-1}$. The magnitude of this shift and the penetration depth are larger for low-lying weakly damped states.

We consider in the present paper EM field states which are dragged along by ion-sound waves into dense plasma layers. We determine the coefficients for the excitation of such states. We find the lifetime and the corresponding penetration depth of the field dragged into the dense plasma layers. We show the possibility that the entrained field can be dragged into plasma layers with a local value of the density several times larger than the value of the plasma density in the trapping region.

2. BASIC EQUATIONS

We consider the field of EM waves in a plane-layered plasma without an external magnetic field in the presence of strong nonstationary density perturbations produced by ion-sound waves. To be specific, the wave vectors of the EM and ion-sound waves considered are assumed to be collinear with the direction of the density gradient of the unperturbed plasma, which coincides with the z axis. In that case, the equations describing the interaction of the EM waves with the strong ion-sound waves have the form

$$c^2\frac{\partial^2 E}{\partial z^2}-\frac{\partial^2 D}{\partial t^2}=0, \tag{1}$$

$$\frac{1}{c_s^2}\frac{\partial^2\rho_s}{\partial t^2}-\frac{\partial^2\rho_s}{\partial z^2}+\frac{\partial}{\partial z}\left\{\frac{\rho_s}{\rho_0}\frac{d\rho_0}{dz}\right\}=\frac{1}{16\pi c_s^2}\frac{\partial^2|E|^2}{\partial z^2}, \tag{2}$$

where E is the electric field strength of the EM wave, D the electric induction, c the velocity of light, ρ_s the perturbation

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of the plasma density in the ion-sound wave, $\rho_0 = \rho_0(z)$ the value of the plasma density when there are no ion-sound perturbations, and c_s the ion-sound wave velocity. The electrical induction D is connected with the field strength E through the integral relation[10]

$$D(z,t) = \int_0^\infty d\hat{t}\, \tilde{\varepsilon}(\hat{t},t,z) E(z,t-\hat{t}), \tag{3}$$

and the form of the kernel $\tilde{\varepsilon}(\hat{t},t,z)$ is determined by the properties of the medium. The $\hat{t}$-dependence of $\tilde{\varepsilon}(\hat{t},t,z)$ characterizes the temporal dispersion of the medium and the dependence on z and t corresponds to the inhomogeneity and non-stationarity of the parameters of the medium. The set of Eqs. (1), (2) for well- or hump-type profiles of the ion-sound density perturbations has solutions describing EM field states which are dragged along by such perturbations. We consider first of all entrained states of the field, neglecting the action of the EM waves on the ion-sound perturbations. Afterwards we take into account the change in the ion-sound wave parameters due to the action of the EM field.

3. ENTRAINED EM FIELD STATES IN AN INHOMOGENEOUS PLASMA

We consider the field of EM waves in a plane-layered plasma with a monotonic density profile in which a strong ion-sound perturbation propagates in the direction of the density gradient. We write the field of the EM waves $E(z,t)$ in (1) in the form of a wave packet with a frequency $\omega(t)$ which varies slowly with time:

$$E(z,t) = E_0(z,t) \exp\left\{ i \int \omega(t)\, dt \right\}, \tag{4}$$

where $E_0(z,t)$ is the envelope of the wave packet. Assuming the characteristic time T_0 for a change in the frequency ω and of the envelope field E_0 to be much larger than the wave period, $T_0\omega \ll 1$, we expand the amplitude $E_0(z,t-\hat{t})$ and the phase

$$\varphi(t-\hat{t}) = \int^{t-\hat{t}} d\tilde{t}\, \omega(\tilde{t})$$

in (3) in series in the vicinity of t up to terms of order $(T_0\omega)^{-1} \ll 1$. We then perform in (3) transformations similar to those in Ref. 11 and use the definition of the permittivity of a medium with temporal dispersion

$$\varepsilon(\omega,t,z) = \int_0^\infty d\hat{t}\, \tilde{\varepsilon}(\hat{t},t,z)\, e^{-i\omega\hat{t}},$$

to substitute the relation obtained for the electric induction $D(z,t)$ into (1). Retaining in the formula obtained terms of order $(T_0\omega)^{-1}$, we have the following equation for the field envelope E_0

$$c^2 \frac{\partial^2 E_0}{\partial z^2} - 2i\omega \frac{\partial E_0}{\partial t} + \left[i\frac{d\omega}{dt} - \omega^2\varepsilon_0 + i\left(\varepsilon' - \frac{1}{2}\frac{d\omega}{dt}\frac{\partial^2\varepsilon_0}{\partial\omega^2} \right)\omega^2 \right] E_0 = 0, \tag{5}$$

where

$$\varepsilon_0(\omega,t,z) = 1 - \omega_p^2(z,t)/\omega^2(t)$$

is the quasi-stationary value of the permittivity of an inhomogeneous and non-stationary plasma,

$$\omega_p(z,t) = [4\pi e^2 N(z,t)/m_e]^{1/2}$$

is the plasma frequency, $N(z,t)$ the plasma density; $\varepsilon' = \mathrm{Im}\,\varepsilon$ is the imaginary part of the permittivity and is connected with the non-stationarity of the parameters of a medium with temporal dispersion, and leads to an additional phase shift between D and E. If the wave packets (4) which we are considering are of finite size, the magnitude of the imaginary part ε' found from the condition that there exists for such packets the adiabatic invariant[11,12]

$$\frac{d}{dt}\left\{ \frac{1}{\omega} \int_{-\infty}^{\infty} dz\, |E_0(z,t)|^2 \right\} = 0,$$

is equal to $1/2(d\omega/dt)(\partial^2\varepsilon_0/\partial\omega^2)$. In that case Eq. (5) for the wave-packet envelope E_0 in an inhomogeneous and non-stationary plasma takes the form

$$c^2 \frac{\partial^2 E_0}{\partial z^2} - 2i\omega \frac{\partial E_0}{\partial t} + \left[i\frac{d\omega}{dt} + \omega^2\varepsilon_0(\omega,t,z) \right] E_0 = 0. \tag{6}$$

We set the plasma density $N(z,t)$ in (6) equal to

$$N(z,t) = N_0(1+z/L) + N_s(z-c_s t, \nu t),$$

where $N(z) = N_0(1+z/L)$ is the stationary value of the density, $N_s(z-c_s t,\nu t)$ is the deviation of the plasma density, caused by the propagation of the ion-sound wave into the dense plasma layers, from its equilibrium value $N(z)$; $\nu \ll 1$ is a small parameter corresponding to the change in the parameters of the ion-sound perturbation in the inhomogeneous plasma. If we change the variables

$$\xi = z - c_s t, \quad t' = t$$

and the required function

$$E_0 = \psi(\xi,\nu t)\left[\frac{\omega(t)}{\omega(0)} \right]^{1/2} \exp\left[i\frac{c_s\omega(t)}{c^2}\xi \right] \tag{7}$$

and neglect in the relation obtained terms of order $(c_s/c)^2$, Eq. (6) takes the form

$$c^2 \frac{\partial^2\psi}{\partial\xi^2} - 2i\omega\nu \frac{\partial\psi}{\partial t} + \left[\omega^2(t) - \omega_p^2(t) - \omega_p^2(0)\frac{\xi}{L} - \tilde{\omega}_p^2(\xi,\nu t) \right]\psi = 0. \tag{8}$$

where

$$\omega_p^2(t) = \omega_p^2(0)\left(1 + \frac{c_s t}{L}\right), \quad \omega_p^2(0) = \frac{4\pi e^2 N_0}{m_e},$$

$$\tilde{\omega}_p^2(\xi,\nu t) = \frac{4\pi e^2 N_s(\xi,\nu t)}{m_e}.$$

Equation (7) together with (8) describes the envelope of the EM wave packet. The frequency $\omega(t)$ of this packet is determined from the solution of the boundary-value problem for Eq. (8) for the eigenvalues of the field states ψ described by Eq. (8) for a given profile $\tilde{\omega}_p^2(\xi,\nu t)$:

$$\omega^2(t) = \omega_p^2(t) + \mathrm{Re}\,\mu. \tag{9}$$

Hence it follows that in an inhomogeneous plasma and when there are strong ion-sound waves present it is possible that there exist EM field states which are dragged along into dense plasma layers and have a frequency which increases monotonically with time.

4. TRAPPING OF EM FIELD PACKETS IN ENTRAINED STATES

To determine the efficiency of the excitation of the entrained EM field states we assume that an EM wave of frequency ω_0 is incident from the vacuum onto a plane-layered plasma normal to its surface, so that in the absence of ion-sound density perturbations a field distribution

$$E(z,t)=E_i(z)\exp(i\omega_0 t)$$

with an amplitude which is constant in time and which is described by the equation

$$c^2\frac{d^2E_i}{dz^2}+\left[\omega_0{}^2-\omega_p{}^2(0)\left(1+\frac{z}{L}\right)\right]E_i=0$$

is established in the plasma. Furthermore, we assume that from the region, of incidence of the EM field upon the plasma in there propagates the direction of the density gradient an ion-sound wave in the form of a compression step:

$$N_s(\xi,vt)=\begin{cases}N_s(t), & z<c_st\\ 0, & z>c_st\end{cases}.$$

Assuming that the ion-sound wave speed and the scale of the plasma inhomogeneity L are sufficiently large quantities, we take it that an ion-sound perturbation does not affect the nature of the propagation of the EM field incident upon the plasma when the plasma frequency in that perturbation $[\omega_p^2(t)+\tilde{\omega}_p^2(\xi,vt)]^{1/2}$ is less than the critical value ω_0, and becomes opaque to the field incident upon the plasma at the time t^* when the quantity $[\omega_p^2(t)+\tilde{\omega}_p^2(\xi,vt)]^{1/2}$ reaches the critical value ω_0. If we write the EM wave field in the region $z>c_st^*$, for $t>t^*$, as a sum of the fields of the entrained states the envelope and frequency of which are described by Eqs. (7)–(9), and, to be specific, assuming that the field E_i in the region $z>c_st^*$ contains m oscillation combinations we get for that time at $z>c_st^*$, in terms of the variables $z'=z-c_st^*$, $t'=t-t^*$ (we drop the primes in what follows)

$$\psi_m(z)=\sum_n C_{m,n}\psi_n(z)\exp\left(i\frac{c_s\omega_n(0)}{c^2}z\right), \tag{10}$$

where $C_{m,n}$ is the coefficient for the excitation of the nth state

$$E_n=\psi_n\exp(ic_s\omega_nz/c^2)$$

by the field ψ_m. The functions $\psi_{m(n)}$ in (10) satisfy the equation

$$c^2\frac{d^2\psi_{m(n)}}{dz^2}+\left[\mu_{m(n)}-\omega_p{}^2(0)\frac{z}{L}\right]\psi_{m(n)}=0 \tag{11}$$

with boundary condition $\psi_{m(n)}(0)=0$. The solution of (11) is in that case the Airy function $\mathrm{Ai}(-\rho_{m(n)})$ with argument

$$\rho_{m(n)}=A^{1/3}(\bar{z}-\bar{\mu}_{m(n)}/A),$$

where

$$\bar{z}=zk_0,\quad k_0=\omega_p(0)/c,\quad A^{-1}=k_0L,$$
$$\bar{\mu}_{m(n)}=\mu_{m(n)}/\omega_p{}^2(0),\quad \omega_n{}^2(0)=\omega_p{}^2(0)(1+\bar{\mu}_n),$$

while the quantity $-\bar{\rho}_{m(n)}=-\bar{\mu}_{m(n)}A^{-2/3}$ is the $m(n)$th root of the Airy function $\mathrm{Ai}(-\bar{\rho}_{m(n)})=0$. Multiplying (10) by the complex conjugate $\psi_m^*(z)$ of the quantity $\psi_m(z)$ and integrating the relation obtained over z from 0 to $+\infty$, using (11), we get

$$C_{m,n}=\exp(iV\bar{\rho}_m)\int_{-\bar{\rho}_m}^{\infty}d(-\rho_m)\mathrm{Ai}(-\rho_m)\mathrm{Ai}(-\rho_n)$$
$$\times\exp(-iV\rho_m)\left[\int_{-\bar{\rho}_m}^{\infty}d(-\rho_m)\mathrm{Ai}^2(-\rho_m)\right]^{-1}, \tag{12}$$

where

$$V=(c_s/c)(k_0L)^{1/3}(1+\bar{\mu}_n).$$

To start with we analyze Eq. (12) for small values of the parameter V. We assume that in the region which gives the essential contribution to the integral (12), $0<\rho_m<\bar{\rho}_m$, the relation $V\rho_m\ll 1$ is satisfied. In that case, using in (12) the relation $\exp(-iV\rho_m)\approx 1-iV\rho_m$ and the condition that the Airy functions $\mathrm{Ai}(-\rho_m)$ and $\mathrm{Ai}(-\rho_n)$ are orthogonal when $m\neq n$, we get for the excitation coefficients

$$C_{m,n}\approx iVI_{m,n},$$

$$I_{m,n}=-\int_{-\bar{\rho}_m}^{\infty}d(-\rho_m)\rho_m\mathrm{Ai}(-\rho_m)\mathrm{Ai}(-\rho_n)$$
$$\times\left[\int_{-\bar{\rho}_m}^{\infty}d(-\rho_m)\mathrm{Ai}^2(-\rho_m)\right]^{-1}.$$

The excitation coefficient is in this case small and proportional to the parameter V. The way $C_{m,n}$ depends on the indexes m and n is described by the integral $I_{m,n}$ and is found by numerically integrating that relation. We give here the quantities $I_{m,n}$ for m-values 4 and 6 and n values 1 and 2:

$$I_{4,1}\approx 0.17,\quad I_{4,2}\approx 0.44,\quad I_{6,1}\approx 0.09,\quad I_{6,2}\approx 0.13.$$

It is clear that the amplitude of the excitation coefficient increases both when the number of the excited state n increases, and when the number m decreases.

We analyze Eq. (12) for parameter values $V\gg 1$,[1)] corresponding to large values of the inhomogeneity scale k_0L. We consider the case $m\gg n$. The argument of the function $\mathrm{Ai}(-\rho_m)$ in the region where $\mathrm{Ai}(-\rho_n)$ is localized is then a large negative quantity, $\rho_m\gg 1$, and we can write the function $\mathrm{Ai}(-\rho_m)$ in the form

$$\mathrm{Ai}(-\rho_m)\approx\frac{1}{2\pi^{1/2}\rho_m{}^{1/4}}\left\{\exp\left[i\left(\frac{2}{3}\rho_m{}^{3/2}-\frac{\pi}{4}\right)\right]\right.$$
$$\left.+\exp\left[-i\left(\frac{2}{3}\rho_m{}^{3/2}-\frac{\pi}{4}\right)\right]\right\}. \tag{13}$$

When $V\gg 1$ we can use (13) to integrate the numerator of (12) by the stationary-phase method (see, e.g., Ref. 13). Assuming the stationary phase point, $\rho_m^{st}=V^2\gg 1$, to lie inside the integration interval of (12) we take the function $\mathrm{Ai}(-\rho_n)\rho_m^{-1/4}$ outside the integral sign with the value at that point. Expanding the exponent in the integrand in the vicinity of ρ_m^{st} in a series up to terms of order $(\rho_m-\rho_m^{st})^2$ and extending the lower integration limit in the obtained integral to $-\infty$, we get

$$C_{m,n}\approx-\frac{\mathrm{Ai}\{-(V^2+\bar{\rho}_n-\bar{\rho}_m)\}}{2^{1/2}P_m}\exp\left(-iV\rho_m-i\frac{V^3}{3}\right). \tag{14}$$

We evaluate the norm of the Airy functions

$$P_m=\int_{-\bar{\rho}_m}^{\infty} d(-\rho_m)\,\mathrm{Ai}^2(-\rho_m), \qquad (15)$$

for which we use (11) to integrate (15) by parts. Using the boundary condition $\mathrm{Ai}(-\bar{\rho}_m)=0$, we have

$$P_m=|d\mathrm{Ai}(-\rho_m)/d(-\rho_m)|^2_{\rho_m=\bar{\rho}_m}.$$

When $m\ll 1$ we see, using (13), that the norm of the Airy functions equals $\rho_m^{-1/2}/\pi$. As a result Eq. (14) takes the form

$$C_{m,n}\approx-\frac{\pi\mathrm{Ai}\{-(V^2+\bar{\rho}_n-\bar{\rho}_m)\}}{(2\bar{\rho}_m)^{1/2}}\exp\left(-iV\bar{\rho}_m-i\frac{V^3}{3}\right), \qquad (16)$$

$$\bar{\rho}_m=\left\{\frac{3\pi}{2}\left(m-\frac{1}{4}\right)\right\}^{2/3}.$$

It follows from (16) that the most efficient excitation of entrained states E_n is realized in the interval

$$0<V^2+\bar{\rho}_n-\bar{\rho}_m<\bar{\rho}_n,$$

while the maximum value of the amplitude of the excited states decreases with increasing m. When

$$V^2+\bar{\rho}_n-\bar{\rho}_m<0$$

the quantity $C_{m,n}$ is exponentially small and equal to zero when $V^2>\bar{\rho}_m$. What we have said is illustrated in Figs. 1 and 2. We show in Fig. 1 the function $|C_{5,n}|^2$ as function of the parameter V for various values of the number of the excited state n and in Fig. 2 the function $|C_{m,1}|^2$ as function of V for various values of m.

5. ENTRAINING OF EM FIELD PACKETS INTO A SUPERCRITICAL PLASMA

EM field packets trapped by entrained states penetrate in a supercritical plasma into dense plasma layers. The penetration depth and, correspondingly, the lifetime of these states in an inhomogeneous plasma are determined by three factors: 1) the nonlinear transformation of the ion-sound wave energy in the entrained field, 2) the tunnel de-excitation of the EM field from the localization region, and 3) thermal losses due to collisions of the plasma particles. We consider the effect of each of these factors separately.

1. Nonlinear transformation of ion-sound waves in an inhomogeneous plasma

The ion-sound waves perform work on the trapped EM field they entrain into dense plasma layers. This leads to a

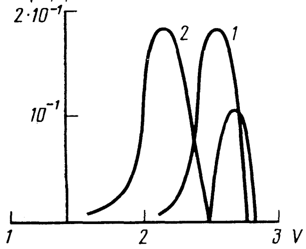

FIG. 1. Square of the modulus of the excitation coefficient $|C_{5,n}|^2$ as function of the parameter V for different values of the number of excited state: curve 1 corresponds to the value $n=1$, curve 2 to $n=2$.

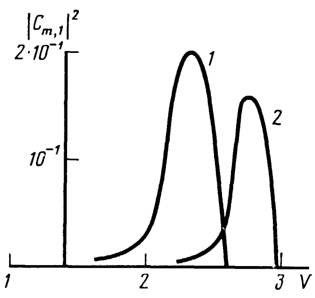

FIG. 2. Square of the modulus of the excitation coefficient $|C_{m,1}|^2$ as function of the parameter V for different values of m: curve 1 corresponds to the value $m=4$, curve 2 to $m=6$.

change in the ion-sound wave parameters, in this case to their damping caused by the nonlinear transformation into the entrained EM field.

To determine the efficiency of such a transformation we turn to Eq. (2). In that equation we change to a frame of reference moving with the ion-sound wave speed c_s. In the resultant equation we discard terms with second derivatives in time and use the relation

$$d\rho_0(z)/dz=c_s^{-1}\partial\rho_0(\xi+c_st)/\partial t,$$

to get

$$-2\frac{\partial^2\rho_s}{\partial\xi\partial t}+\frac{\partial}{\partial\xi}\left\{\rho_s\frac{\partial\rho_0(\xi+c_st)}{\partial t}\right\}=\frac{1}{16\pi c_s}\frac{\partial|E|^2}{\partial\xi}. \qquad (17)$$

We shall be interested in ion-sound perturbations of density ρ_s for which $\rho_s(\xi\to-\infty, vt)\to 0$. We neglect also the emission of EM field from the localization region $E(\xi\to-\infty, t)\to 0$. Integrating (17) over ξ and using in the relation obtained the assumptions made above we get

$$\frac{\partial}{\partial t}\left\{\frac{\rho_s}{\rho_0^{1/2}}\right\}=-\frac{1}{32\pi c_s\rho_0^{1/2}}\frac{\partial|E|^2}{\partial\xi}. \qquad (18)$$

We multiply (18) by $\rho_s/\rho_0^{1/2}$ and once again integrate the obtained relation over ξ from $-\infty$ to $+\infty$:

$$\frac{d}{dt}\left\{c_s\int_{-\infty}^{\infty}d\xi\frac{\rho_s^2}{\rho_0}\right\}+\frac{c_s}{16\pi}\int_{-\infty}^{\infty}d\xi\frac{\rho_s}{\rho_0}\frac{\partial|E|^2}{\partial\xi}=0. \qquad (19)$$

The expression in the braces in (19) corresponds to the ion-sound wave energy $W_s(t)$ in the inhomogeneous plasma. We show that the second term in (19) corresponds to the rate of change in the energy of the entrained EM field. In the approximation of real values of the eigenvalues μ_n which, as before, corresponds to neglecting the emission of EM field from the trapping region, the intensity of the EM field dragged along by the ion-sound wave can, if we use (7), be written in the form

$$|E(\xi,t)|^2=\sum_n\frac{\omega_n(t)}{\omega_n(0)}|\psi_n(\xi,vt)|^2, \qquad (20)$$

where ψ_n is given by Eq. (8). We assume that the characteristic time for a change in the function ψ_n is sufficiently large so that in (8) the term with the time derivative is small compared with the other terms in the equation and can be neglected. In that case we get for ψ_n the quasi-stationary equation

$$c^2\frac{\partial^2\psi_n}{\partial\xi^2}+\left[\mu_n-\omega_p{}^2(0)\frac{\xi}{L}-\tilde\omega_p{}^2(\xi,\nu t)\right]\psi_n=0. \quad (21)$$

We multiply (21) by $\partial\psi_n^*/\partial\xi$ and add it to the complex conjugate equation. Substituting the quantity $\rho_s\partial|\psi_n|^2/\partial\xi$ from the relation obtained into (19) and using (20) we get

$$\frac{dW_s}{dt}=-\frac{c_s}{16\pi}\sum_n\frac{\omega_n(t)}{\omega_n(0)}\int_{-\infty}^{\infty}\frac{d\xi}{\omega_p{}^2(\xi+c_st)}\times\left\{c^2\frac{\partial}{\partial\xi}\left|\frac{\partial\psi_n}{\partial\xi}\right|^2+\left[\mu_n-\omega_p{}^2(0)\frac{\xi}{L}\right]\frac{\partial|\psi_n|^2}{\partial\xi}\right\}. \quad (22)$$

Assuming that in (22) the scale L of the plasma inhomogeneity is much larger than the localization region of the functions ψ_n we take the quantity $\omega_p^2(\xi+c_sT)$ from under the integral sign at its value in the point ξ_n of the largest maximum of the field ψ_n. For instance, for compression-step ion-sound perturbations the point ξ_n is close to the turning point of the field ψ_n on the linear profile $\xi_n\approx L\bar\mu_n$. Performing then in (22) the integration and neglecting as before the emission of EM field from the localization region we get

$$\frac{dW_s}{dt}+\frac{d}{dt}\left\{\frac{1}{8\pi}\sum_n\frac{\omega_n(t)}{\omega_n(0)}\int_{-\infty}^{\infty}d\xi|\psi_n|^2\right\}=0. \quad (23)$$

One sees easily that the expression within the braces in (23) is the energy $W_\psi(t)$ of the EM field dragged along by the ion-sound wave in the inhomogeneous plasma and hence, Eq. (23) is the energy conservation law for the "ion-sound wave + entrained EM field" system in an inhomogeneous plasma when one neglects the emission of EM field from the localization region,

$$\frac{d}{dt}\{W_s(t)+W_\psi(t)\}=0. \quad (24)$$

The energy of the field entrained by the nth state increases with time in proportion to its frequency, $W_\psi\propto\omega_n(t)$. Such a change is caused by the work done by the ion-sound waves on the entrained field and it follows from (24) that it is accompanied by a nonlinear damping (transformation) of these waves in the inhomogeneous plasma into an entrained high-frequency field. The penetration of the EM field into the supercritical region is realized under the condition that the energy W_s of the ion-sound waves is much larger than the energy W_ψ of the entrained field. We determine the nonlinear transformation time t_{NL} from the condition $W_s(t_{NL})\sim W_\psi(t_{NL})$. Putting for estimates $\omega_n(t)\sim\omega_p(t)$ we find that in a plasma with a linear density profile $N(z)$ the nonlinear transformation time in dimensionless units $\tau=tc_s/L$ equals

$$\tau_{NL}=\frac{t_{NL}c_s}{L}\approx\left\{\frac{W_s(0)+W_\psi(0)}{2W_\psi(0)}\right\}^2-1. \quad (25)$$

2. Tunneling "radiation". Trapped states of an entrained field in an inhomogeneous plasma

The change in the number P of photons of the entrained EM field caused by their tunneling through a density barrier in the low-density region is

$$dP/dt=-PT/t_0, \quad (26)$$

where T is the coefficient for transmission of photons through the density barrier, t_0 is the flight time of the photons along a closed trajectory in the localization region neglecting de-excitation. As an example of ion-sound density perturbations we consider a compression step and use the fact that it follows from (18) that the amplitude N_s of the perturbations, when they propagate in a continuous-inhomogeneous medium and when we neglect the effect of the entrained EM field on these perturbations, changes in proportion to $N^{1/2}(t)$:

$$N_s(\xi,\nu t)=\begin{cases}N_s(0)\,[N(t)/N_0]^{1/2}, & \xi<0,\\ 0, & \xi>0.\end{cases} \quad (27)$$

In that case the quasi-localized states are those whose number, according to (21), satisfies the inequality

$$\frac{N_s(0)}{N_0}\left[\frac{N(t)}{N_0}\right]^{1/2}(k_0L)^{2/3}>\bar\mu_n.$$

The coefficient for the transmission of photons which are in one of these states through the density barrier takes, with allowance for (21), the form

$$T\approx\exp\left\{-\frac{4}{3}\left[\frac{N_s(0)}{N_0}\left[\frac{N(t)}{N_0}\right]^{1/2}(k_0L)^{2/3}-\bar\mu_n\right]^{3/2}\right\}. \quad (28)$$

We determine the flight time of the photons of the EM field along a closed trajectory in the $\xi>0$ region from the relation

$$t_0=2\int_0^{\xi_n}V_g{}^{-1}(\xi,t)\,d\xi,$$

where ξ_n is the turning point of the field ψ_n,

$$V_g(\xi,t)=\partial\omega_n(\xi,t)/\partial k=k(\xi)c^2/\omega_p(t)$$

is the group velocity of the wave packet ($k(\xi)$ is the wave number of the packet), which, with allowance for (21), is equal in the $\xi>0$ region to $k_0c^2(\bar\mu_n-\xi/L)^{1/2}$. Hence, the time t_0 is equal to

$$4\frac{\bar\mu_n{}^{1/2}L\omega_p(t)}{c\omega_p(0)}. \quad (29)$$

Using (28) and (29) to integrate (26) we get for the number of photons in the EM bunch

$$P(t)=P(0)\exp\left\{-\alpha\int_{\eta_0}^{\eta}dx\frac{e^{-x}}{x^{1/3}}\right\}, \quad (30)$$

where

$$\alpha=\frac{1}{4}\left(\frac{4}{3}\right)^{1/3}\frac{c}{c_s}\frac{N_0}{N_s(0)}\frac{1}{(k_0L)^{1/3}\bar\mu_n{}^{1/2}},$$

$$\eta=\frac{4}{3}\left\{\frac{N_s(0)}{N_0}\left[\frac{N(t)}{N_0}\right]^{1/2}(k_0L)^{2/3}-\bar\mu_n\right\}^{3/2},\quad \eta_0=\eta(t=0).$$

It follows from (30) that as $t\to\infty$ there remains a finite number of photons in the localization region

$$P(\infty)=P(0)\exp\{-\alpha\Gamma(2/3,\eta_0)\}, \quad (31)$$

which are trapped by the ion-sound perturbation. Here $\Gamma(2/3,\eta_0)$ is the incomplete gamma function.

When $\alpha_c=\alpha\Gamma(2/3,\eta_0)<1$ the number of trapped photons $P(\infty)$ differs from its initial value $P(0)$ by a factor less than e. Determining the lifetime of the entrained states

t_{rad} from the condition $P(t_{rad}) = P(0)/e$ we find that when $\alpha_c \ll 1$ the states of the EM field dragged into the supercritical plasma can be defined as non-emitting or trapped.

When $\alpha_c > 1$ the lifetime of the entrained states is finite and such states are metastable. We find the time t_{rad} from the relation

$$\alpha \int_{\eta_0}^{\eta_1} dx\, e^{-x} x^{-1/3} = 1, \qquad (32)$$

where $\eta_1 = \eta(t = t_{rad})$. We give an upper bound for the quantity t_{rad}. To do this we use in (32) the obvious inequality

$$\int_{\eta_0}^{\eta_1} dx\, e^{-x} x^{-1/3} < \frac{3}{2} e^{-\eta_0} (\eta_1^{2/3} - \eta_0^{2/3})$$

and we find that the time τ for the emission of the entrained field in dimensionless units does not exceed the value

$$\tau_{rad} = \left\{ 2 \frac{c_s}{c} \frac{e^{\eta_0} \bar{p}_n^{1/2}}{(k_0 L)^{1/3}} + 1 \right\}^2 - 1. \qquad (33)$$

After that time the EM field penetrates into a region with density

$$\frac{N(\tau_{rad})}{N_0} = \left\{ 2 \frac{c_s}{c} \frac{e^{\eta_0} \bar{p}_n^{1/2}}{(k_0 L)^{1/3}} + 1 \right\}^2.$$

3. Thermal losses

To estimate the effect of thermal losses due to collisions of the plasma particles we must recognize that the collision frequency is proportional to the plasma density $N(z)$. For a linear profile, $N(z) = N_0(1 + z/L)$, the number of photons in the EM bunch will change with time according to

$$P(t) = P(0) \exp\left\{ -\frac{[t + c_s t^2/2L]}{t_s} \right\},$$

where $t_s = \nu_{\text{eff}}^{-1}(z = 0)$ is the lifetime of the bunch in its initial position with $z = 0$, t_s (in s) $\approx 5 \cdot 10^{-2} T^{3/2}$ (in K)/N_0 (in cm^{-3}), c_s (in cm/s) $\approx 10^4 T^{1/2}$ (in K). After a time t_k, determined by the condition $t_k + c_s t_k^2/(2L) = t_s$, the number of EM field photons is reduced by a factor e and then the bunch has reached a region with density

$$N_k = N_0 (1 + c_s t_k / L) = N_0 (1 + 2 c_s t_s / L)^{1/2}.$$

Putting in this example the plasma parameters equal to $T_e \sim 5 \cdot 10^6$ K, and $N_0 \sim 10^{14}$ cm^{-3}, we get $c_s t_s \approx 125$ cm. When $L \sim 10$ cm we find that after its lifetime t_k the bunch has penetrated into a region with density $N_k / N_0 \sim 5.1$. The lifetime of the bunch itself,

$$t_k = \frac{L}{c_s} \left[\left(1 + 2 \frac{c_s t_s}{L} \right)^{1/2} - 1 \right]$$

is then of the order of $1.9 \cdot 10^{-6}$ s. The frequency of the bunch increases approximately in proportion to $(N_k / N_0)^{1/2} \sim 2.3$, i.e., by a factor 2.3.

We can estimate the time for tunneling radiation t_{rad} from Eq. (33). For instance, when an EM field in the ground state ($n = 1$) is dragged along, the radiation time t_{rad} for a plasma with the parameters given above is longer than t_k for a relative initial height of the step $N_s(0)/N_0 > 7.5 \cdot 10^{-2}$. We can estimate the time t_{NL} for the nonlinear transformation of the ion-sound wave energy into the entrained EM field from (25). In a plasma with the parameters given above the time t_{NL} is larger than t_k when $W_s(0)/W_\psi(0) > 3.6$.

The mechanism proposed here thus indicates the possibility that EM waves can penetrate into a supercritical plasma to an appreciable depth, which is undoubtedly of interest for transporting EM field energy into dense plasma layers.

One can similarly consider the trapping and entraining of Langmuir wave packets in an inhomogeneous plasma. It proves also possible to consider the entraining of high-frequency wave packets in a radially inhomogeneous plasma into central dense layers by spherically converging ion-sound waves.

[1)] For EM waves the condition $V \gg 1$ is reached for very large values of $k_0 L$. However, the consideration of this case is of methodological interest as in a study of the trapping of other kinds of hf waves into entrained states the relation $V \gtrsim 1$ can be reached for relatively smaller values of L. For instance, for Langmuir waves the parameter V corresponds to the quantity $(c_s/V_T)(L/r_D)^{1/3}$ (V_T is the thermal velocity and r_D the Debye radius in the trapping region) and reaches values of the order unity for $L \sim r_D (m_i/m_e)^{3/2}$.

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