Numerical simulation of two-dimensional Langmuir collapse

A. I. D'yachenko, V. E. Zakharov, A. M. Rubenchik, R. Z. Sagdeev, and V. F. Shvets

Institute for Space Research, Academy of Sciences of the USSR
(Submitted 1 October 1987)
Zh. Eksp. Teor. Fiz. **94**, 144–155 (March 1988)

We give a detailed numerical simulation of the collapse of a separate Langmuir caviton in two-dimensional geometry. We propose and carry out a combined continuous calculation method in the framework of which the initial evolution of the caviton is described by averaged dynamical equations, and the results of these calculations are used as initial conditions for the simulation of the concluding stage of the collapse, using the particle method. We show that due to the specific properties of the two-dimensional geometry higher-order nonlinear effects (electron nonlinearities, change in the dispersion law, saturation of the nonlinearity, Landau damping, and so on) can appreciably affect the qualitative nature of the process. Depending on how far we are above the threshold for collapse, three regimes of the final stage of the evolution of the caviton are established: collapse, a quasi-stationary caviton, and delayed collapse.

1. INTRODUCTION

It has by now become clear that Langmuir collapse[1] (the self-accelerated compression of cavitons filled with Langmuir oscillations) is one of the fundamental ideas of plasma physics. The collapse of a separate caviton has been recorded experimentally.[2] This phenomenon has been described rather completely in a number of papers (see the reviews of Refs. 3–6) and the analytical results have been confirmed and supplemented by numerical simulations. A number of effects (e.g., the appearance of accelerated electrons) of powerful electromagnetic radiation or electron beams interact with a plasma can be naturally interpreted as a consequence of Langmuir collapse. The collapse of a Langmuir caviton is a basic structural element in the scenario of strong plasma turbulence and ensures an efficient mechanism for the dissipation of long-wavelength Langmuir oscillations. One should mention that collapse is not a phenomenon characteristic solely of Langmuir oscillations. Collapse in many respects physically similar to Langmuir collapse is also possible for electromagnetic waves,[7] lower-hybrid oscillations,[8] and so on.

The general scenario for collapse is now the following one. As the result of the development of the modulational instability cavitons are formed in a turbulent plasma which are filled with oscillations. The initial energy density W in the caviton is of the order of the average turbulent level W_0 and a characteristic size of a caviton is $l\sim r_D\ (nT/W)^{1/2}$. The process of the compression fo the caviton becomes rapidly self-similar and the shape of the caviton becomes a unifersal one which is noticeably flattened. During the collapse, the energy of the oscillations trapped in the caviton is conserved. When the minimum dimension of the caviton becomes comparable with the Debye radius, Landau damping becomes important and the oscillations which are trapped in the caviton "burn up" accelerating the plasma electrons. As a result the energy is transferred to a small group of accelerated particles.

Up to the final stage, the evolution of the caviton is described by a set of dynamical equations obtained in Ref. 1 in the framework of a hydrodynamical description of the plasma:

$$\Delta(2i\dot{\psi}+3\omega_p r_D^2\Delta\psi)=\frac{\omega_p}{n_0}\nabla(\delta n\nabla\psi), \tag{1a}$$

$$\delta\ddot{n}-c_s^2\Delta\delta n=\frac{\Delta|\nabla\psi|^2}{16\pi M}. \tag{1b}$$

Here ψ is the averaged potential of the high-frequency electrical field

$$\mathbf{E}=\nabla[\psi\exp(-i\omega_p t)+\text{c.c.}]/2,$$

and δn is the quasi-neutral variation in the plasma density. Basically a numerical simulation of the collapse has been carried out in the framework of Eqs. (1) (see the literature in Ref. 3). However, in the concluding stage of the caviton compression effects become important which are not taken into account in (1): electron nonlinearities, Landau damping, saturation of the nonlinearity, change in the dispersion law, and so on. It is impossible to write down sufficiently simple, generalized dynamical equations which describe adequately the final stage of the collapse. The only way to simulate it remains the particle method: simulation of a plasma from first principles by a large number of charged macroparticles. Only by using such a simulation can one find an answer to a number of problems of principal importance: what part of the energy is transferred to the electrons, after what length of time is the caviton burned up, what is the accelerated particle distribution, and so on.

The first attempt to simulate two-dimensional collapse by the particle method was made in Ref. 9. The simulation was carried out on a grid of $(45\times45)r_D^2$ with periodic boundary conditions. The initial density distribution was uniform. It was shown in Refs. 10 and 11 that in that case the conditions for collapse of a caviton are not satisfied on the whole. The collapse is possible only after the initial distribution is split into bunches with dimensions which turn out to be comparable to the Debye radius so that the collapse of these bunches is not observed. Because of this, in the calculations the energy of the oscillations remained practically unchanged, notwithstanding the occurrence of local maxima. A similar picture of the evolution of Langmuir oscillations was obtained also in the calculations of Ref. 12 which were carried out for an initially uniform ion density on a large grid. In Refs. 10 and 11 an initial distribution was chosen with a spatially modulated plasma density ensuring the necessary conditions for collapse. The resultant scenario was a growth of the electrical field energy and subsequent rapid

 0038-5646/88/030513-07$04.00

burning out of the Langmuir oscillations trapped in the caviton. However, in the calculations of Refs. 10 and 11 only one kind of initial conditions was considered, and the generality of the results remained unclear. Moreover, in those calculations the simplest periodic code was used in which the zeroth spatial harmonic of the electrical field vanished. In an isolated caviton, however, the average field along its minor axis is non-vanishing.[3] Therefore, in the calculations of Refs. 10 and 11 a rather complicated spatial picture was observed, with two peripheral cavitons close to a single central one and, thus, the collapse of a single caviton as a whole was not demonstrated.

In the present paper we have attempted to study exhaustively, in two-dimensional calculations, the collapse of a single Langmuir caviton. We performed a combination simulation or, as we called it, an unintermitting calculation of the Langmuir collapse. A preliminary communication about the results was published in Ref. 13. As the initial condition we specified a large size caviton of hundreds of Debye radii. To describe it we used the averaged dynamical Eqs. (1). The caviton reached the self-similar compression regime rather rapidly and took on the universal shape independent of the initial conditions. When the size of the caviton became of the order of $30r_D$ the results of the calculation were used as the initial data for a further simulation by the particle method, with its aim the study of the final stage of the collapse and of the burning up of the caviton. In both stages of the simulation we used the symmetry properties of the caviton, which enabled us to introduce adequate boundary conditions and to broaden appreciably the possibilities for numerical simulation.

It is well known that in the two-dimensional case even a slight restructuring of the model (1), e.g., allowance for the saturation of the nonlinearity, leads to the possibility of the esixtence of stable cavitons (see, e.g., Ref. 13). The problem of interruption of the collapse and formation of cavitons (specific for the two-dimensional geometry) is also discussed in the present paper. We show that if the initial wave energy in the caviton ε is much larger (by a large factor) than its threshold value ε_0 collapse and a fast burning up of the caviton take place. When $\varepsilon - \varepsilon_0 \ll \varepsilon_0$ we observe the formation of quasi-cavitons–quasi-stationary formations which live tens of ion periods.

2. COLLAPSE CHARACTERISTICS IN TWO-DIMENSIONAL GEOMETRY

The behavior of the solutions of the set (1) differs considerably in the one-, two-, and three-dimensional geometries. Collapsing solutions occur only in two- and three-dimensional problems. One can indicate for arbitrary initial conditions the necessary condition for collapse. The system (1) conserves two integrals of motion, $N = \int |\nabla\psi|^2 d\mathbf{r}$ and the Hamiltonian of the system

$$H = \int \Big[\frac{3r_D^2}{16\pi} |\Delta\psi|^2 + \frac{\delta n}{16\pi n_0} |\nabla\psi|^2 + M\frac{n_0 v^2}{2} + \frac{Mc_s^2}{2n_0}(\delta n)^2 \Big] d\mathbf{r}. \qquad (2)$$

A sufficient condition for collapse is that the Hamiltonian is negative: $H < 0$. For quasi-stationary initial conditions this gives the estimate $W/nT > (kr_D)^2$, the usual criterion for the modulational instability.

In the three-dimensional case, when $W/nT > m/M$ we can neglect in the equation for the plasma density perturbation the fact that the sound velocity is finite (supersound regime):

$$\Delta(2i\dot\psi + 3\omega_p r_D^2 \Delta\psi) = \frac{\omega_p}{n_0}\nabla(\delta n \nabla\psi),$$
$$\delta\ddot n = \Delta|\nabla\psi|^2/16\pi M. \qquad (3)$$

Equations (3) have a self-similar solution which conserves the number of quanta in the caviton:

$$\psi = f(\xi), \quad \delta n = V(\xi), \quad \xi = r/(t_0 - t)^{2/3}. \qquad (4)$$

The properties of these equations have been studied in a number of papers and the reaching of a self-similar regime by the collapse has been shown. We note that for such solutions the supersound approximation becomes better with time and the kinetic energy of the ions, $n_0 v^2/2$, increases faster than the potential energy $c_s^2 n_0 (\delta n/n_0)^2$. This means that in the final stage the caviton shape is determined by the ion inertia. Therefore, even if the physical processes neglected in (1) must lead in the final stage to a stopping of the collapse, the inertia of the motion of the heavy ions leads to a compression of the caviton and to burning up of the energy trapped in it.

It is well known that the two-dimensional case is a borderline one: taking into account in Eqs. (1) small terms of a different physical nature (change in the dispersion law, saturation of the nonlinearity, and so on) can already stop the collapse. It has been shown in a number of papers (see, e.g., Ref. 3) that in the inertial range the compression of a caviton in the two-dimensional case also has a self-similar character:

$$\psi = f(\xi), \quad \delta n = V(\xi), \quad \xi = r/(t_0 - t). \qquad (5)$$

One shows easily that in that case the ratio of the kinetic and potential energies remains constant, that the role of the ion inertia is small, and that for cavitons only just above the critical size the effect of its inertial compression, described above, is unimportant.

We now describe in more detail the properties of quasi-stationary cavitons which can be formed when the collapse is stopped. We consider only the effect of the saturation of the nonlinearity. Close to the stationary state, low-frequency motions can be considered to be adiabatic and the electron distribution to be a Boltzmann one. Assuming for the sake of simplicity that the ion temperature is zero, we have

$$\delta n = n_0[\exp(-\Phi/T) - 1]. \qquad (6)$$

Here Φ is the ponderomotive force potential: $\Phi = e^2|\mathbf{E}|^2/4m\omega_p^2$. Expanding the exponential and substituting (6) into (1a) we get in dimensionless variables

$$\Delta(i\dot\psi + \Delta\psi) + \nabla[\nabla\psi(|\nabla\psi|^2 - |\nabla\psi|^4)] = 0. \qquad (7)$$

The time is here normalized by ω_p^{-1}, the spatial dimensions by $(\frac{3}{2})^{1/2} r_D$, and the electrical field by $(32\pi n_0 T)^{1/2}$.

Equation (7) is a Hamiltonian one:

$$i\Delta\psi_t = -\frac{\delta H}{\delta\psi^*},$$
$$H = \int \Big[|\Delta\psi|^2 - \frac{1}{2}|\nabla\psi|^4 + \frac{1}{3}|\nabla\psi|^6 \Big] d\mathbf{r}. \qquad (8)$$

Its stationary solution of the form

$$\psi=\exp(i\lambda^2 t)\varphi$$

is described by the equation

$$-\lambda^2\Delta\varphi+\Delta^2\varphi+\nabla[\nabla\varphi|\nabla\varphi|^2(1-|\nabla\varphi|^2)]=0, \quad (9)$$

where λ^2 is the nonlinear frequency shift in the caviton, and λ has the meaning of its reciprocal size. These solutions realize a minimum of H for a fixed number of waves in the caviton, $N=\int|\nabla\varphi|^2 d\mathbf{r}$. Multiplying (9) by φ^* and integrating we get

$$\lambda^2 N+\int|\Delta\varphi|^2 d\mathbf{r}-\int|\nabla\varphi|^4 d\mathbf{r}+\int|\nabla\varphi|^6 d\mathbf{r}=0. \quad (10)$$

We consider a scale transformation that conserves N in the two-dimensional case, $\varphi\to\varphi(\lambda\mathbf{r})$. In that case

$$H(\lambda)=\int\left[\lambda^2\left(|\Delta\varphi|^2-\frac{1}{2}|\nabla\varphi|^4\right)+\frac{1}{3}\lambda^4|\nabla\varphi|^6\right]d\mathbf{r}. \quad (11)$$

In the caviton, $H(\lambda)$ must reach a minimum, whence $(\partial H/\partial\lambda^2)_{\lambda^2=1}=0$ for localized stationary solutions. This gives

$$H+\frac{1}{3}\int|\nabla\varphi|^6 d\mathbf{r}=0. \quad (12)$$

It is well known that if we neglect the saturation of the nonlinearity in the two-dimensional case the size of the caviton λ^{-1} is arbitrary. This is clear, e.g., from the fact that after the substitution $\mathbf{r}\to\lambda\mathbf{r}$ the stationary equation describing the caviton, neglecting the saturation of the nonlinearity, is independent of λ:

$$-\Delta\varphi_0+\Delta^2\varphi_0+\nabla(\nabla\varphi_0|\nabla\varphi_0|^2)=0. \quad (13)$$

It is clear from (12) that H is zero for such solutions, while the caviton energy

$$\omega_p N=\omega_p\int|\nabla\varphi_0|^2 d\mathbf{r}=\omega_p N_0$$

is also independent of its size. If the initial caviton energy is larger than the critical value $\omega_p N_0$, the caviton collapses.

When we take the saturation of the nonlinearity into account the degeneracy is lifted and the equilibrium size of the caviton is uniquely determined by its energy. We determine this connection assuming that the saturation of the nonlinearity is small and proceeding as in Ref. 14. We look for a solution in the form

$$\varphi=\varphi_0+\delta\varphi, \quad \delta\varphi\ll\varphi_0,$$

where $\varphi_0(\mathbf{r})$ is a solution of (13). We may assume that the function φ is real. We introduce

$$\delta N=N-N_0=2\int(\nabla\varphi_0,\nabla\delta\varphi)\,d\mathbf{r}.$$

Linearizing Eqs. (10) and (12) we get then

$$\lambda^2=3(N-N_0)\left(2\int|\nabla\varphi_0|^6 d\mathbf{r}\right)^{-1}. \quad (14)$$

We see that if the energy enclosed in the caviton is considerably larger than the critical one, $N\gtrsim N_0$, the equilibrium size λ^{-1} of the caviton in dimensional variables is of the order of a Debye radius. It is clear that such cavitons cannot exist, owing toLandau demping. If we are just above criticality, the size of the caviton increases,

$$l\sim r_D[N_0/(N-N_0)]^{1/2}, \quad (15)$$

and the role of the Landau damping decreases rapidly.

We have already noted that as the size of the caviton decreases many effects, neglected in (1), become important. We mention only electron nonlinearities with characteristic times $\tau^{-1}\sim(kr_D)^2\omega_p E^2/8\pi nT$, corrections to the dispersion law with $\tau^{-1}\sim(kr_D)^4\omega_p$, and the saturation of the nonlinearity with $\tau^{-1}\sim\omega_p(E^2/8\pi nT)^2$. Since $(kr_D)^2\sim E^2/8\pi nT$ for a caviton, all these effects must be considered at the same time. Therefore, the calculations given above are only qualitative and show that the formation of caviton structures can be expected only in a regime just above criticality. We note that in that case the aforementioned effect of the inertial compression of a caviton is also anomalously small. However, in final reckoning the problem of the existence of cavitons can be solved only through numerical simulation.

3. ORGANIZATION OF THE NUMERICAL SIMULATION

In the initial stage the compression of the caviton is well described by Eqs. (1). It is well known that the caviton has an asymmetric flattened shape. One usually speaks of a dipole charge distribution. The structure of the potential satisfies then the following symmetry conditions:

$$\psi(x,y)=-\psi(x,-y)=\psi(-x,y) \quad (16)$$

(we have assumed that the caviton is flattened along the y-axis). Conditions (16) allow us to consider only one-quarter of the caviton in the numerical simulation. The following boundary conditions for the potential follow then from (16):

$$\psi|_{y=0}=0, \quad (\partial\psi/\partial n)_{x=0}=0.$$

On the two other boundaries ($x=L_x, y=L_y$) we shall also use the condition $\partial\psi/\partial n=0$.[15]

We must note that a jump in the potential along the dipole axis y takes place in the caviton, so that the boundary conditions for ψ in an individual caviton cannot be periodic, as was assumed in Refs. 9–12.

We chose as the initial condition for the set (1) a function ψ such that

$$\Delta\psi=\rho_0\sin k_y y(1+\cos k_x x), \quad k_y=\frac{\pi}{2L_y}, \quad k_x=\frac{\pi}{L_x}, \quad (17)$$

and for the low-frequency plasma density variation

$$\left.\frac{\delta n}{n_0}\right|_{t=0}=\frac{|\nabla\psi|^2}{16\pi n_0 T_e}, \quad \delta\dot{n}|_{t=0}=0.$$

The threshold value ρ_0 determined by the condition $H=0$ was in the most interesting case $k_x=k_y=k$ equal to $\rho_0^2=\frac{384}{181}k^4$, and the number of waves was

$$N_0=\int|\nabla\psi|^2 d\mathbf{r}=2\pi^2/3.$$

We have here introduced dimensionless variables for the set (1):

$$\Delta(i\dot{\psi}+\Delta\psi)=\nabla(n\nabla\psi), \quad \ddot{n}-\Delta n=\Delta|\nabla\psi|^2,$$

$$|\nabla\psi|^2\to\frac{64\pi}{3}nT\frac{m}{M}|\nabla\psi|^2, \quad \mathbf{r}\to\frac{3}{2}r_D\left(\frac{M}{m}\right)^{1/2}\mathbf{r},$$

$$t\to\frac{3}{2}\frac{M}{m}\omega_p^{-1}t, \quad \delta n\to\frac{4}{3}n_0\frac{m}{M}n.$$

The Hamiltonian of the system takes the form

$$H=\int\left[\ |\Delta\psi|^2+n|\nabla\psi|^2+\frac{n^2}{2}+\frac{(\nabla\Phi)^2}{2}\right]d\mathbf{r},$$

where Φ is the hydrodynamic potential of the low-frequency motions, $\partial n/\partial t=-\Delta\Phi$.

Calculations show that the collapsed caviton rapidly enters into a self-similar compression regime and its shape does no longer depend on the details of the initial distribution. To increase the inertial range we used the following method which we called an excision. When the size of the caviton became small compared with the calculated region we separated its central part $0<x<L_x/2$, $0<y<L_y/2$ and stretched it over the whole of the calculational grid. The first stage of the simulation was finished when the size of the caviton had diminished to $(20–30)r_D$ or the energy density of the field in the center has increased to $W_{\max}/nT\sim0.2$.

In the final stage of the calculations we used the particle method. The method of the calculations is similar to the one described in Ref. 16 and used in Refs. 10 and 11, but with modifications for the boundary conditions applied here. The distinctive features connected with the modification of the boundary conditions, and the method of organizing the unintermitting calculations and the architecture of the computational system will be described elsewhere. We note here that we used the computing center of the Institute for Space Research of the Academy of Sciences of the USSR, consisting of an ES-1037 control computer and an ES-2706 peripheral vector processor.

As initial data for the calculations by the particle method we used the results of the calculations in the framework of the averaged dynamical equations. As in the particle method the initial data are the ion and electron distribution functions $f_i(\mathbf{r},\mathbf{v})$ and $f_e(\mathbf{r},\mathbf{v})$ in phase space, it is necessary to reconstruct them from the complex envelope of the high-frequency potential $\psi(\mathbf{r},t)$ and the low-frequency plasma density variation $\delta n(\mathbf{r},t)$. In agreement with the applicability of the set (1) at the moment of transition to the kinetic description the particle distribution was assumed to be locally Maxwellian:

$$f_{e,i}=\frac{n_{e,i}(\mathbf{r})m_{e,i}}{2\pi T_{e,i}}\exp\frac{|\mathbf{v}-\mathbf{v}_{e,i}(\mathbf{r})|^2}{2v^2_{T_{e,i}}}. \quad (18)$$

Because of their large mass, the ions participate only in the low-frequency motions: $n_i=n_0+\delta n$, where δn is determined from Eq. (1b). The macroscopic ion velocity $\mathbf{v}$ is found from the linearized continuity equation $n_0\,\mathrm{div}\,\mathbf{v}_i+\partial\delta n/\partial t=0$.

The electrons participate in both the low- and the high-frequency motions:

$$n_e=n_0+\delta n+\delta\tilde{n}.$$

To determine $\delta\tilde{n}$ and $\mathbf{v}_e$ we used the Poisson equation and the electron equation of motion:

$$\Delta\Phi=4\pi e\delta\tilde{n},\qquad \frac{\partial\mathbf{v}_e}{\partial t}+3v_{T_e}{}^2\nabla\frac{\delta\tilde{n}}{n_0}=\frac{e}{m}\nabla\Phi,$$

$$\Phi=[\psi(\mathbf{r},t)\exp(-i\omega_p t)+\text{c.c.}]/2.$$

In the simulation by the particle method in a finite region we must supply boundary conditions not only for the Maxwell equations, but also for the particles, the ions and the electrons. Because the electrons move in the field of the Langmuir oscillations we cannot restrict ourselves to simulating one fourth of the caviton. We performed the simulation of half of the caviton in the region $0\leqslant x\leqslant L_x$, $-L_y\leqslant y\leqslant L_y$ with reflection boundary conditions for the particles and with the condition $\partial\Phi/\partial n=0$ on the boundary for the potential. The adequacy of the simulation was verified by running a test variant with completely periodic boundary conditions in a region containing two complete cavitons in which the electrical fields where in counterphase.

The statement of the problem described here, which uses the symmetry of the caviton, leads to a lowering of the computer time for the solution of the dynamical equations by a factor 8 and for the simulation of the kinetic stage by a factor 4 as compared with the equivalently stated problem with periodic boundary conditions.

In the unintermitting computation of the collapse, calculations in the framework of the set (1) started in the region $L_x=512r_D$, $L_y=256r_D$. Different sizes in x and y were chosen because of the flattened shape of the caviton. In the kinetic calculations, a typical size of the grid was $(128\times128)r_D^2$. The number of particles of each kind in a Debye cell ranged from 16 to 64 for different variants and the total number of particles was $\sim8\cdot10^5$.

As already mentioned, when the initial energy of the caviton is not too far above its critical value caviton structures can occur. To study them we performed additionally two sets of calcualtions in other, simpler models. In the first of them the calculations were performed in the framework of Eq. (7). In the second one we considered a mixed description.[13] The high-frequency motions were described by the equation

$$\Delta(2i\dot{\psi}+3\omega_p r_D{}^2\Delta\psi)=\frac{\omega_p}{n_0}\nabla(\delta n_i\nabla\psi), \quad (19)$$

and the motion of the ions in the field of the low-frequency potential φ was described by the kinetic equation

$$\frac{\partial f_i}{\partial t}+\mathbf{v}\frac{\partial f_i}{\partial\mathbf{r}}-\frac{e}{M}\nabla\varphi\frac{\partial f_i}{\partial\mathbf{v}}=0 \quad (20)$$

and solved by the particle method. The electron distribution, on the other hand, was assumed to be Boltzmannian:

$$\delta n_e=n_0\left[\exp\left(-\frac{e\varphi-\Phi}{T_e}\right)-1\right]=\delta n_i,\quad \Phi=\frac{e^2|\mathbf{E}|^2}{4m_e\omega_p{}^2}, \quad (21)$$

and the charge separation in the l.f. motions was neglected. In the static limit this hybrid semi-kinetic description reduces to Eq. (7) but, beside the effect of the saturation of the nonlinearity, it describes the ion nonlinearities and the Landau damping of the ions.

4. DISCUSSION OF THE RESULTS

We have already noted that one of the features of the unintermitting calculation of the Langmuir collapse is the presence of a large inertial range. This enables us to assume that the nature of the final stage of the collapse is independent of the details of the initial electrical field distribution and of the density in the caviton, but is determined solely by the number N (or energy ε) of the plasmons trapped in it. The critical value $N_0(\varepsilon_0)$ is determined in the two-dimensional geometry from the condition that the Hamiltonian of the set of Eqs. (1) vanish.

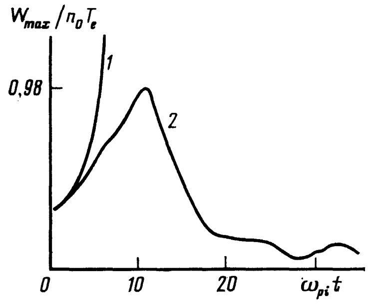

FIG. 1. Time dependence of the maximum energy density of the field in the caviton: 1: results of the solution of the dynamical equations; 2: results of the combined continuous calculation.

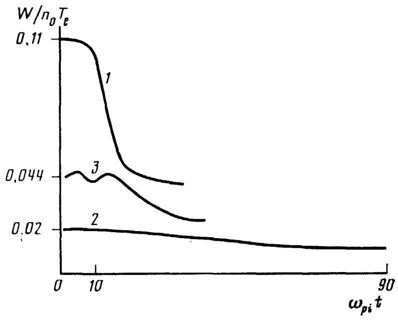

FIG. 2. Time-dependence of the average energy density of the field in the caviton: 1 $\varepsilon/\varepsilon_0 = 6.6$; 2: 1.25; 3: 2.7.

We now turn to an exposition of the results obtained in the numerical experiments and their analysis. First of all we note that in the framework of the averaged dynamical equations the cavitons reached rather rapidly the self-similar compression regime of (5). This fact was checked against the rate of change of the depth of the caviton and of the amplitude of the electrical field. The largest length of the inertial range was reached for variants with two excisions of the central part of the caviton. The size of the caviton up to the instant of transition to the kinetic stage of simulation had decreased by a factor of 10 to 15 as compared to its original size.

After changing to the kinetic description we performed parallel calculations by the particle method and using the dynamic equations. We show in Fig. 1 the evolution with time of the energy density of the oscillations for the variant $\varepsilon/\varepsilon_0 = 6.6$. It is clear that the dynamical equations satisfactorily describe the collapse up to a level of oscillations $W_{\max} \sim 0.4 n_0 T_e$

The calculations showed that, as expected, the evolution of the caviton depends significantly on the initial energy ε of the caviton. The calculations were performed for various values of the electron to ion mass ratio, $100 \leqslant M/m \leqslant 1836$. It was found that for all ε the evolution of the caviton depends on the ion mass ina self-similar way, while all characteristic times depend only on the product $\omega_{pi} t = \tau$.

When we exceed $\varepsilon/\varepsilon_0 \geqslant 6$ we observe a clear collapse picture (see Figs. 1–3). We show in Fig. 2 the evolution of the oscillation energy for some typical variants. It is clear that for $\varepsilon/\varepsilon_0 = 6.6$ there occurs a fast (after a time $\tau \sim 7$) burnup of an appreciable (65%) energy which was trapped in the caviton. We show in Fig. 3 the spatial distribution of the energy of the electrical field and of the plasma density at several successive times. The maximum energy density in this variant was $W_{max}/nT = 0.98$ and the depth of the ion well $\delta n_i/n_0 = -0.38$. It is then clear from Fig. 3 that, owing to the inertia of the ions, the caviton continues to deepen also after the burnup of the energy of the Langmuir oscillations. The size of the caviton at the time of the maximum compression is rather large: $\sim (10 \times 25) r_D^2$. The electron velocity distribution is also anisotropic (see Fig. 4). It is very clear that as a result of the collapse the energy of the h.f. field is transferred to a relatively small number of fast electrons and the latter are mainly accelerated in the direction of the average field in the caviton (along the y axis).

At a small excess above threshold, there was formed a long-lived ($\tau \sim 40$) caviton structure (Fig. 5). We note first of all its nonstationary nature. Such a behavior is completely natural in the framework of Eq. (7). Indeed, for a caviton the Hamiltonian has a completely well defined value which is, in general, different from the Hamiltonian of the initial distribution. Therefore, because of the conservation of the Hamiltonian, the caviton solution can be reached only if account is taken of small dissipative processes or of energy emission beyond the limits of the simulation region.

We performed additional calculations of caviton structures in the framework of (7) and of the hybrid semi-kinetic approach of (19)–(21). The calculations in those models gave a similar result. However, in this case the size of the caviton turned out to be one-and-a-half to two times smaller than in the unintermitting calculation. This indicates that effects such as electron nonlinearities, changes in the disper-

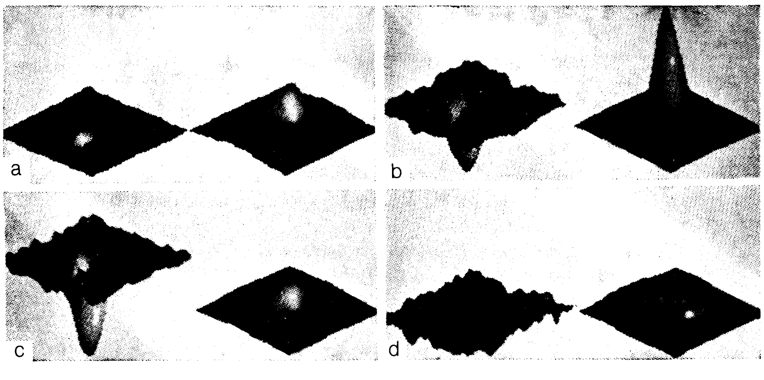

FIG. 3. Spatial distribution of the energy of the high-frequency field $E^2/16\pi n_0 T_e$ and of the plasma density n_i/n_0 for the variant with $\varepsilon/\varepsilon_0 = 6.6$: a: initial condition; b: time when the field in the caviton is a maximum ($t = 10.8\omega_{pi}^{-1}$); c: time when the depth of the caviton is a maximum ($t = 17.2\omega_{pi}^{-1}$); d: distribution after the burnup ($t = 35.1\omega_{pi}^{-1}$).

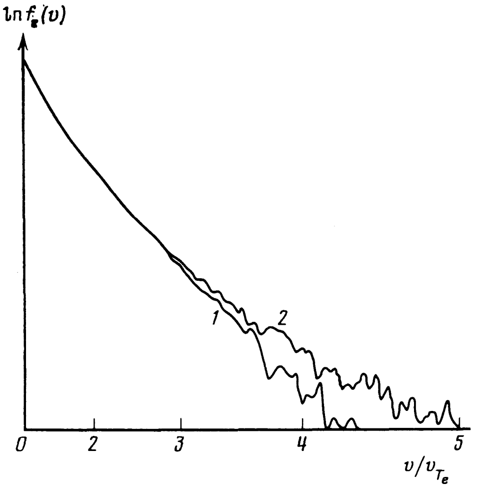

FIG. 4. The electron distribution function for the variant $\varepsilon/\varepsilon_0 = 6.6$, integrated over space and the velocities v_x (1) and v_y (2) at time $t = 35.1\omega_{pi}^{-1}$.

sion law of the Langmuir waves, and Landau damping make an appreciable contribution to the formation of the caviton.

We now discuss the cause of the damping of the caviton after a time $\tau \sim 40$. Passing through a caviton of size l, an electron gains an energy $\Delta\mathscr{E} = e\int E v dt$. The electrical field E changes in proportion to $\cos\omega_p t$. If the time for passing through the caviton is less than π/ω_p the electrical field does not change sign and the electron gains an energy $eEl \sim T$. Assuming that the characteristic electron velocities are of the order of $3v_T$ we find that the caviton starts to be strongly damped when $l = l_0 \sim 3\pi v_T/\omega_p \sim 10 r_D$ which corresponds to the minimum size of the caviton obtained in the calculations. When $l > l_0$ the quantity $\Delta\mathscr{E}$ is exponentially small: $\Delta\mathscr{E} \sim T\exp(-l/l_0)$, but for our calculations completely finite. Ultimately this nonadiabatic interaction with the electrons leads to the damping of the caviton.

To check this assumption we performed a one-dimensional calculation by the particle method in which we gave as the initial condition the soliton solution of the averaged dynamical equations. It turned out that a soliton with dimensions close to the caviton dimensions obtained in the unintermitting calculation also burns up after a time of the order $\tau \sim 40$.

We have described two opposite situations: pure collapse and the formation of quasistationary structures. Calculations for moderate supercriticality, $2 < \varepsilon/\varepsilon_0 < 6$, showed that, as one should expect, in that case an intermediate regime is realized which can naturally be called a delayed collapse (Fig. 6). We note that in all cases the minimum size of the caviton was $\sim (10\times 25) r_D^2$.

5. CONCLUSION

We have studied in the present paper the collapse of an individual Langmuir caviton. As initial conditions for the calculation by the particle method ww used the self-similar solution obtained as the result of the numerical simulation of the averaged dynamical Eqs. (1). We can thus assume that we have described the evolution of an elementary cell of strong turbulence—a collapsing caviton in a situation when the inertial range is rather long.

We have shown that if the initial energy ε of the oscillations in the caviton is appreciably larger than the critical ε_0, there occurs in the final stage of the collapse a burn-up of almost all the energy trapped in the caviton, and its minimum size is rather large and of the order of $10 r_D$. In that case we may expect that two-dimensional calculations simulate adequately three-dimensional turbulence. If ε is close to ε_0 in the final stage, a long-lived quasi-stationary state is formed. Its formation is connected with the two-dimensional nature of the calculations and one can, in general, not extrapolate these results to the three-dimensional situation. The results indicate additional difficulties arising in the interpretation of numerical calculations of two-dimensional strong turbulence.[17] In particular, it is interesting to elucidate the problem of the amount of energy trapped in the caviton when it is formed as the result of the modulational instability.

We note yet another fact. The results of the calculations of Refs. 17 and 18 show that fluctuations in the density excited by ponderomotive forces when the caviton collapses affect the turbulence properties strongly. We have shown in our calculations that the maximum amplitude of the density fluctuations which is reached already after the burn-up of the caviton in the stage of inertial compression is large, $\delta n/n_0 \sim 0.3$–0.4. Kinetic effects are already very important for such fluctuations and the problem of the level and the spectrum of the fluctuations remaining after the burnup of the caviton must be studied by means of the particle method. It is convenient to do this by the semi-classical model (19)–(21) described in the present paper.

The implementation of a three-dimensional simulation, by the particle method, of the final stage of the caviton evolu-

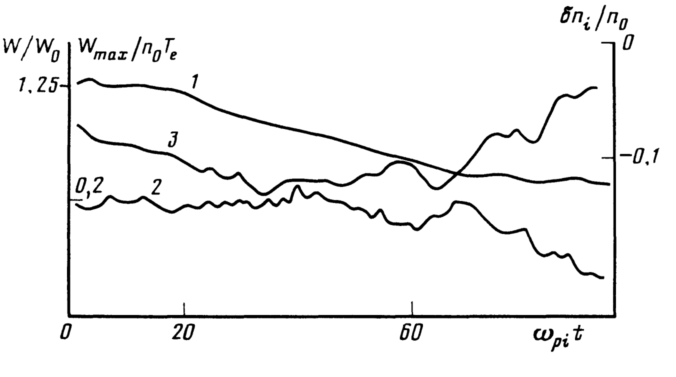

FIG. 5. The time-dependence of the caviton characteristics for the caviton variant $\varepsilon/\varepsilon_0 = 1.25$: 1: average energy density of the h.f. field W/n_0T_e; 2: maximum energy density W_{max}/n_0T_e; 3: density variation $\delta n/n_0$.

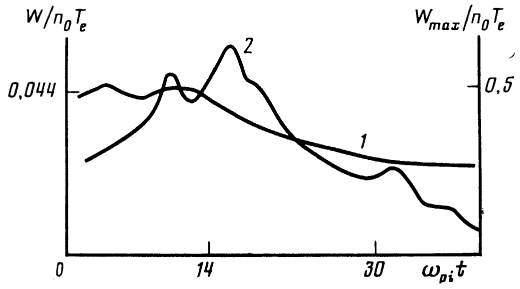

FIG. 6. Time dependences of the average (1) and the maximum (2) of the high-frequency field in the caviton for $\varepsilon/\varepsilon_0 = 2.7$, corresponding to a delayed collapse.

tion is extremely important, as is also a three-dimensional unintermitting calculation. These problems are at the limit of the possibilities of present-day computational techniques. The authors, nevertheless, hope that progress in the field of the development of many-processor assemblies[19,20] will enable us to realize the above program. The first steps in this direction have already been made.[20]

The authors are grateful to S. I. Anisimov, V. D. Shapiro, and V. I. Shevchenko for their interest in this work and for useful discussions, to A. Yu. Golovin and I. V. Petrov for their substantial participation in the development of the program for the calculations, and to E. M. Gavrilov for his calculation of the damping of a one-dimensional caviton.

[1]V. E. Zakharov, Zh. Eksp. Teor. Fiz. **62**, 1745 (1972) [Sov. Phys. JETP **35**, 908 (1972)].

[2]A. Y. Wong and P. Y. Cheung, Phys. Rev. Lett. **52**, 1744 (1984); **55**, 1880 (1985).

[3]V. E. Zakharov, in: Basic Plasma Physics, Elsevier, Vol. 2, p. 81 (1985).

[4]V. D. Shapiro and V. I. Shevchenko, *ibid.* Vol. 2, p. 123 (1985).

[5]M. V. Goldman, Rev. Mod. Phys. **56**, 709 (1984).

[6]A. M. Rubenchik, R. Z. Sagdeev, and V. E. Zakharov, Comm. Plasma Phys. Contr. Fusion **9**, 183 (1985).

[7]E. A. Kuznetsov, Zh. Eksp. Teor. Fiz. **66**, 2037 (1974) [Sov. Phys. JETP **39**, 1003 (1974)].

[8]S. L. Musher and B. I. Sturman, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 537 (1975) [JETP Lett. **22**, 265 (1975)].

[9]A. N. Polyudov, R. Z. Sagdeev, and Yu. S. Sigov, Preprint, Inst. Appl. Math., Acad. Sc. USSR, No 128, 1974.

[10]S. I. Anisimov, M. A. Berezovskii, M. F. Ivanov, *et al.*, Phys. Lett. **A92**, 32 (1982).

[11]S. I. Anisimov, M. A. Berezovskiĭ, V. E. Zakharov, I. V. Petrov, and A. M. Rubenchik, Zh. Eksp. Teor. Fiz. **84**, 2046 (1983) [Sov. Phys. JETP **57**, 1192 (1983)].

[12]T. Tajima, M. V. Goldman, J. N. Leboeuf, and J. M. Dawson, Phys. Fluids **24**, 182 (1981).

[13]A. I. D'yachenko, V. E. Zakharov, A. M. Rubenchik, R. Z. Sagdeev, and V. F. Shvets, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 504 (1986) [JETP Lett. **44**, 648 (1986)].

[14]V. E. Zakharov, V. V. Sobolev, and V. S. Synakh, Zh. Eksp. Teor. Fiz. **60**, 136 (1971) [Sov. Phys. JETP **33**, 77 (1971)].

[15]V. E. Zakharov, A. F. Mastryukov, and V. S. Synakh, Pis'ma Zh. Eksp. Teor. Fiz. **20**, 76 (1974) [JETP Lett. **20**, 32 (1974)].

[16]M. A. Berezovskiĭ, M. F. Ivanov, I. V. Petrov, and V. F. Shvets, Programmirovanie, No. 6, 37 (1980).

[17]L. M. Degtyarev, I. M. Ibragimov, R. Z. Sagdeev, *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **40**, 455 (1984) [JETP Lett. **40**, 1282 (1984)].

[18]G. D. Doolen, D. F. DuBois, and H. A. Rose, Phys. Rev. Lett. **54**, 804 (1985).

[19]E. P. Velikhov, Vl. Lazarov, St. Markov, *et al.*, Preprint Inst. Cosmic Res., Acad. Sc. USSR, No. 1169, 1986.

[20]A. Yu. Golovin, A. N. Pushkarev, R. Z. Sagdeev, V. F. Shvets, and V. I. Shevchenko, Preprint, Inst. Space Res., Acad. Sc. USSR, No. 1347 (1988).

Translated by D. ter Haar