Josephson effect on a spin current

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The stationary Josephson effect on a spin current is considered. The stability region of the homogeneous superfluid state in the channel is found, and the dependence of the superfluid on the phase difference on the channel boundary is derived.

The order parameter of the B phase of ^{3}He is the rotation matrix $R(\alpha, \beta, \gamma)$ (α, β, γ, are the Euler angles). The system Hamiltonian does not depend on the angle α of rotation around the external magnetic field **H** which we direct along the z axis. Therefore the quantity $P = S_z - S_\zeta$, which is canonically conjugate to the angle α and corresponds to the spin current **I**, is conserved if dissipation is disregarded.[1] We have designated by S_z the spin projection on the $\hat{z}$ axis and by S_ζ the projection on the axis $\zeta = R(\alpha, \beta, \gamma)\hat{z}$. The properties of the spin current for a channel length L of the current **I** much longer than the correlation length ξ (to be defined below) were investigated by Fomin in Ref. 2. The critical current I_c, corresponding to the critical Ginzburg–Landau current in the case of ordinary superconductors, was calculated and found to be in satisfactory agreement with experiment.[3]

It is also of interest to consider the flow of spin current through the so-called weak links in the case $L \lesssim \xi$, when the boundary conditions must be taken into account. Weak links can be various constrictions in the current channel, or places in which the order parameter is suppressed (e.g., by local increase of the external magnetic field). Weak links in the case of a mass current were well investigated in Refs. 4 and 5.

1. We consider the stationary Josephson effect. If the order of magnitude of the condensate inhomogeneity energy is less than the dipole and Zeeman energies, the problem reduces to finding the exterma of a functional F with specified boundary conditions.[1] If the gradients are perpendicular to the external field **H**, the functional F is given by

$$F = \int \{F_\nabla - u\omega_p(\omega_L - \omega_p)\}dx, \tag{1}$$

$$F_\nabla = \frac{1}{2}(1-u)\{(1-u)c_\parallel^2 + (1+u)c_\perp^2\}\left(\frac{\partial\alpha}{\partial x_i}\right)^2 + \frac{1}{2}\left\{\frac{c_\perp^2}{1-u^2} + \frac{3c_\parallel^2}{(1+4u)(1+u)^2}\right\}\left(\frac{\partial u}{\partial x_i}\right)^2 - c_\parallel^2\frac{1-u}{1+u}\left(\frac{3}{1+4u}\right)^{1/2}\frac{\partial\alpha}{\partial x_i}\frac{\partial u}{\partial x_i}\,. \tag{2}$$

Here $u = \cos\beta$, $c_\parallel^2$ and $c_\perp^2$ are the squares of the spin-wave velocities, and F_∇ is the condensate inhomogeneity energy. The second term in the integrand (1) is the sum of the Zeeman energy and the quantity $\omega_p P$ that ensures precession with a frequency ω_p that differs from the Larmor frequency ω_L, and conservation of P in a closed volume. Later on we shall need an expression for the spin current **I** obtained by varying F_∇ with respect to α:

$$I_i = (1-u)\{(1-u)c_\parallel^2 + (1+u)c_\perp^2\} \times \frac{\partial\alpha}{\partial x_i} - c_\parallel^2\frac{1-u}{1+u}\left(\frac{3}{1+4u}\right)^{1/2}\frac{\partial u}{\partial x_i}. \tag{3}$$

Let the spin-current flow channel be directed perpendicular to **H** along the x axis and have a length $2L$ and a width a, with $a \ll L$. The last condition allows us to make the problem one-dimensional, so that the functional (1) is much simpler. We integrate (3) with respect to x and take into account the symmetry of the problem, $u(L) = u(-L)$. The second term in (3) is a total derivative and is cancelled out on integration. We obtain an isoperimetric variational problem with a specified phase difference $\Delta\alpha$ on the ends of the channels. We introduce the dimensionless variables

$$x = x' = Lx, \quad j = \frac{16}{5}\frac{IL}{c_\perp^2}, \quad \xi^2 = \frac{c_\perp^2}{\omega_p(\omega_p - \omega_L)},$$

where the length ξ plays a role similar to that of the correlation length in superconductors. We substitute (3) in (2) and discard the total derivatives $\partial u/\partial x$. We obtain ultimately the functional

$$F(v) = \frac{5}{8}\frac{c_\perp^2}{L^2}\int_{-1}^{1} dx\left\{\frac{1}{2}m(\cos v)\left(\frac{dv}{dx}\right)^2 + \frac{L^2}{\xi^2}\cos v - \frac{j^2}{(1-\cos v)[5c^2(\cos v)+6]}\right\}, \tag{4}$$

$$\Delta\alpha = 4\int_{-1}^{1}\frac{j\,dx}{(1-\cos v)[5c^2(\cos v)+6]} = \text{const}, \tag{5}$$

$$c^2(\cos v) = (1+\cos v) + (c_\parallel/c_\perp)^2(1-\cos v),$$
$$m(\cos v) = 8c^2(\cos v)/[5c^2(\cos v)+6].$$

We have introduced here a new variable such that $u = (5\cos v + 3)/8$. The variable v has a range $0 < v < \pi$, and its value on the ends $x = \pm 1$ of the channel is $v = \pi$. The validity of the last statement follows from simple estimates. Indeed, the real deviation of the variable v from π, designated $\delta v = \pi - v$, is determined at the end of the channel by the competition between the kinetic energy $I^2a^2L/c^2(\delta v)^2 \sim c^2a^2/L(\delta v)^2$ of the current inside the channel [the third term in (4)] and the energy $(\delta v)^2c^2a^3/a^2 \sim (\delta v)^2c^2a$ due to the deviation of the order parameter from equilibrium in the shores themselves [first term of (4)] (we have used here the fact that v is restored over a length of order a from the channel boundary). Thus, the deviation of v from π is indeed small ($\delta v \sim (a/L)^{1/4} \ll 1$) in the considered geometry.

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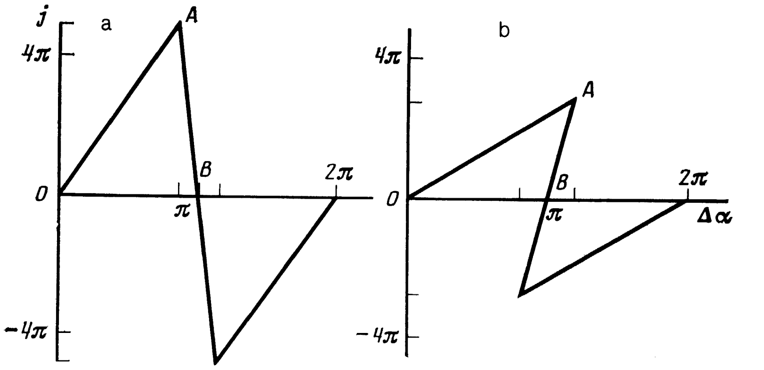

FIG. 1. Dependence of the dimensionless spin current j on the phase difference $\Delta\alpha$ at the ends of the channel. The case $L/\xi\to 0, c_\perp^2/c_\parallel^2 = 3/4$ (the hydrodynamic limit $T\to T_c$) is shown.

We now calculate in fact the $I(\Delta\alpha)$ dependence. The equation of motion is obtained by varying the functional (4) with respect to the variable v. In the region where $\Delta\alpha$ is small [see (8)] there exists a single solution that yields the minimum of the functional (4) and satisfies the boundary condition $v(-1) = v(1) = \pi$. The solution is represented by the line OA in Fig. 1a:

$$v=\pi,\quad j=[3+5(c_\parallel/c_\perp)^2]\Delta\alpha/2. \tag{6}$$

For this solution $\beta = \theta_L =$ const. From now on, when necessary, we shall expand the result for simplicity in powers of the small quantities $(L/\xi)^2$ and $(\delta c)^2 = (c_\parallel^2 - c_\perp^2)/c_\perp^2$. To investigate the stability of the solution (6), we linearize the functional $F(v)$ near $v = \pi$, retaining only the terms quadratic in $\delta v = \pi - v$:

$$F(\delta v)=\frac{5}{16}\frac{c_\perp^2}{L^2}\int dx\left\{\left[1+\frac{3}{8}(\delta c)^2\right]\left(\frac{dv}{dx}\right)^2 -\left[(4+5(\delta c)^2)\frac{\Delta\alpha^2}{16}-\frac{L^2}{\xi^2}\right](\delta v)^2\right\}. \tag{7}$$

This expression is equivalent to the Lagrangian of a harmonic oscillator. At small $\Delta\alpha$ the stiffness of the oscillator is small, so that only fluctuations with large wavelengths are possible. Instability sets in when the wavelenght of the fluctuations becomes smaller than the channel dimension $2L$. From (7) we find that the range of stable $\Delta\alpha$ is

$$0\leqslant\Delta\alpha\leqslant\pi-{}^7/_{16}\pi(\delta c)^2+2L^2/\pi\xi^2. \tag{8}$$

With further decrease of $\Delta\alpha$ a solution of (6) exists, but becomes unstable. Another solution with $v(x)\neq\pi$, shown by segment AB in Fig. 1a, turns out to be stable. To calculate the latter solution we need the first integral of the equation of motion, which is obtained from the functional (4). Note that the first term in (4) is the "kinetic" energy and the last two are the "potential" one. Then, since $v(x)$ is an even function, we get $\partial v(0)/\partial x = 0$ and $v(0) = v_0$ is the minimum of $v(x)$. This yields

$$\frac{1}{2}m(\cos v)\left(\frac{dv}{dx}\right)^2-\frac{L^2}{\xi^2}\cos v+\frac{j^2}{(1-\cos v)[5c^2(\cos v)+6]} =-\frac{L^2}{\xi^2}\cos v_0+\frac{j^2}{(1-\cos v_0)[5c^2(\cos v_0)+6]}\ . \tag{9}$$

We are interested not in the actual form of the $v(x)$ dependence, but only in the function $I(\Delta\alpha)$. We express dv/dx of (9) as a function of j and v_0 and substitute in the following identities:

$$1=\int_{-1}^{0}dx=\int_{-1}^{w_0}\frac{dx}{dw}dw, \tag{10}$$

$$\Delta\alpha=8\int_{-1}^{w_0}\frac{j}{(1-w)[5c^2(w)+6]}\frac{dx}{dw}dw. \tag{11}$$

We have introduced here a variable $w = \cos v$ with limits $[-1;1]$. Expansion in terms of the small $(\delta c)^2$ and L^2/ξ^2 yields $c^2(w) = 2 + (1-w)(\delta c)^2, m(w) = 1 + \frac{3}{16}(1-w)(\delta c)^2$. We obtain next from (9)

$$\frac{dx}{dw}=\frac{1}{j}\left\{\frac{(1-w_0)[16+3(1-w_0)](\delta c)^2}{2(1+w)(w_0-w)}\times\left[1-\frac{16L^2}{\xi^2}\frac{(1-w_0)(1-w)}{j^2}\right]^{-1}\right\}^{1/2}. \tag{12}$$

From (10) and (12) we obtain

$$j=2^{3/2}\pi(1-w_0)^{1/2}+O[(\delta c)^2,\ L^2/\xi^2], \tag{13}$$

where $O[(\delta c)^2, L^2/\xi^2]$ denotes terms of order $(\delta c)^2, L^2/\xi^2$ and higher. They are immaterial in (13), since they lead, when (13) is substituted in (11), to terms of even higher order in $(\delta c)^2$ and L^2/ξ^2. For the phase difference $\Delta\alpha$ we obtain the integral

$$\Delta\alpha=2^{9/2}\int_{-1}^{w_0}\frac{dx}{dw}\frac{j\,dw}{(1-w)[16+3(1-w)(\delta c)^2]}=\pi -\frac{7\sqrt{2}}{32}(\delta c)^2\int_{-1}^{w_0}\frac{(1-w_0)^{1/2}dw}{(1+w)(w_0-w)}+\frac{\sqrt{2}}{\pi}\frac{L^2}{\xi^2}\int_{-1}^{w_0}\frac{(1-w_0)^{1/2}dw}{(1+w)(w_0-w)}. \tag{14}$$

For the AB curve we obtain ultimately, calculating the integral (14),

$$\Delta\alpha=\pi-\frac{7}{64}(\delta c)^2 j+\frac{1}{2\pi^2}\frac{L^2}{\xi^2}j. \tag{15}$$

The solutions (6) and (15) intersect, as they should, in a point A with coordinates

$$j=4\pi+\frac{5\pi}{2}(\delta c)^2,\qquad \alpha=\pi-\frac{7}{16}\pi(\delta c)^2+\frac{2L^2}{\pi\xi^2}. \tag{16}$$

The stability region of the solution (15) is then

$$\pi-\frac{7}{16}\pi(\delta c)^2+\frac{2L^2}{\pi\xi^2}\leqslant\Delta\alpha\leqslant\pi. \tag{17}$$

The inequality (17) allows us to conclude that for a sufficiently long channel $L^2 \geqslant 7\pi^2(\delta c)^2\xi^2/32$ the function $I(\Delta\alpha)$ becomes multiply valued, and the solution (15) becomes unstable.

2. We consider now a channel of arbitrary shape. As already noted, in this case the complete solution of the problem becomes too complicated. We consider therefore only a situation in which u_0 becomes close to unity far from the aperture. Then

$$\nabla u \ll \nabla \alpha \tag{18}$$

and we can expand the functional (1), (2) in terms of the small quantity $\delta u = 1 - u$. The situation (18) can be realized in experiment in a domain wall, when the channel axis along which the Josephson current flows goes outside the limits of the processing domain. The equilibrium value of the order parameter is restored at a distance of order a from the neck (a is the characteristic dimension of the neck) and the domain wall size is of the order of ξ. Therefore, if the strong inequality $a \ll \xi$ is satisfied, it can be assumed that far from the aperture that joins the two vessels the order parameter assumes a constant value even if the channel axis passes through a domain wall. This allows us to vary the quantity u_0 and satisfy the inequality (18).

Taking the condition (18) into account, we substitute in the functional (2) $u = 5/8 \cos v + 3/8$ and consider only terms quadratic in v. In addition, we assume the condition $\xi \ll a$ to be exactly satisfied. We have then

$$F = \frac{5}{16} c_\perp^2 \int dx \left\{ \left(\frac{\partial v}{dx_i} \right)^2 + v^2 \left(\frac{\partial \alpha}{\partial x_i} \right)^2 \right\}. \tag{19}$$

Introducing the complex variables $\psi = ve^{i\alpha}$, we express the functional (19) in the form

$$F = \frac{5}{16} c_\perp^2 \int dx \frac{\partial}{\partial x_i} \psi^* \frac{\partial}{\partial x_i} \psi. \tag{20}$$

It is easy to relate to the functional (20) an equation of motion

$$\nabla^2 \psi = 0 \tag{21}$$

with a boundary condition $\partial\psi/\partial\mathbf{n} = 0$ on the boundaries of the vessels. Here $\mathbf{n}$ is the normal to the walls of the vessel. A solution of Eq. (21) with corresponding boundary condition is given in Ref. 4. It can be represented as a sum of two terms:

$$\psi = v_0[f(\mathbf{x}) e^{i\alpha_1} + (1 - f(\mathbf{x})) e^{i\alpha_2}], \tag{22}$$

where v_0, α_1, and α_2 are the values of the modulus and the phases of the order parameter far from the aperture in the right- and left-side vessels, $f(x)$ is the real solution of Eq. (21), which tends asymptotically to unity and zero with increasing distances from the channel towards the first and second vessels, respectively. We obtain a linearized equation for the current from (3):

$$I_i = \frac{5}{8} c_\perp^2 v^2 \frac{\partial \alpha}{\partial x_i} = \frac{5i}{8} c_\perp^2 (\psi \nabla \psi^* - \psi^* \nabla \psi), \tag{23}$$

and, substituting the solution (22) in (23), we get

$$I_i = {}^5/_{16} \nabla f(x) c_\perp^2 v_0^2 \sin(\alpha_2 - \alpha_1). \tag{24}$$

The quantity $\nabla f(x)$ depends on the specific geometry of the channel and its scale is a^{-1}. n the particular case of a flat screen with a round opening, of diameter a, the total critical current, i.e., the current integrated over the opening, is $I_c^{\text{tot}} = 5/16 a c_\perp^2 v_0^2$.

We make now a few remarks concerning the $j(\Delta\alpha)$ dependence in the case when $u \to -\frac{1}{4}$ on going from the channel into the interior of the vessel. Clearly, when $\Delta\alpha = 0$ we also have $j = 0$. Next, it follows from (3) that the current vanishes when the pase trajectory $v(x)$, $\alpha(x)$ in a space with coordinates v and α passes through the point $u = 1$ or, equivalently through $v = 0$. For investigations near this point it suffices to have the functional (19) already expanded in terms of the small quantity v^2. The functional (19) reaches a minimum and a curve which is a geodesic for a space with coordinates (v, α). This space is flat, and the variable v parametrizes the distance along the radius, while the cyclic variable α is the angle measured from a certain specified direction. If terms nonlinear in v^2 are taken into account, the order-parameter space becomes curved, and has no singular points as before. It is therefore easy to assume that there exists a geodesic curve that will pass through the point $v = 0$ when the phase difference is $\Delta\alpha = \pi$. Thus, a plot of $j(\Delta\alpha)$ will pass through the points $j = 0$ and $\Delta\alpha = \pi k$, where k is an integer.

3. We consider now a case when the channel through which the spin current flows is parallel to the external magnetic field. The calculations must be carried out separately, since the functional F_∇ differs somewhat in form from (2) when the gradients are parallel to the external field, namely

$$F_\nabla = (1-u)[uc_\parallel^2 + (1-u)c_\perp^2] \left(\frac{\partial \alpha}{\partial x_i} \right)^2 + \frac{1}{2} \left\{ \frac{c_\parallel^2}{1-u^2} + \frac{3(2c_\perp^2 - c_\parallel^2)}{(1+4u)(1+u)^2} \right\} \left(\frac{\partial u}{\partial x_i} \right)^2 - (2c_\perp^2 - c_\parallel^2) \frac{1-u}{1+u} \left(\frac{3}{1+4u} \right)^{1/2} \frac{\partial \alpha}{\partial x_i} \frac{\partial u}{\partial x_i}. \tag{25}$$

We proceed next just as in Sec. 1. Without going into details (see Sec. 1), we obtain the following expressions. The spin current is

$$I_i = 2(1-u)(uc_\parallel^2 + (1-u)c_\perp^2) \times \frac{\partial \alpha}{\partial x_i} - (2c_\perp^2 - c_\parallel^2) \frac{1-u}{1+u} \left(\frac{3}{1+4u} \right)^{1/2} \frac{\partial u}{\partial x_i}. \tag{26}$$

We introduce the dimensionless variables

$$x = x' = Lx, \quad j = \frac{16}{5} \frac{IL}{c_\parallel^2}, \quad \xi^2 = \frac{c_\parallel^2}{\omega_p(\omega_p - \omega_L)}$$

in the functional (25). They differ from the case with the channel parallel to the field only by the substitution $c_\perp \to c_\parallel$. The functional (25) takes [after substitution $u = (5 \cos v + 3/8)$] the form

$$F(v) = \frac{5}{8} \frac{c_\parallel^2}{L^2} \int_{-1}^{1} dx \left\{ \frac{1}{2} m(\cos v) \left(\frac{dv}{dx} \right)^2 + \frac{L^2}{\xi^2} \cos v - \frac{1}{2} \frac{j^2}{(1 - \cos v)[5c(\cos v) + 3]} \right\}, \tag{27}$$

$$\Delta\alpha=2\int_{-1}^{1}\frac{j\,dx}{(1-\cos v)\,[5c(\cos v)+3]},\qquad (28)$$

$$c^2(\cos v)=(c_\perp/c_\parallel)^2(1-\cos v)+\cos v,$$
$$m(\cos v)=8c^2(\cos v)/[5c^2(\cos v)+3].$$

In the region where $\Delta\alpha$ is small (see (31) below) there exists a single solution that minimizes the functional (27). The solution is represented by the line OA in Fig. 1(b):

$$v=\pi,\quad j=\Delta\alpha[5(c_\perp/c_\parallel)^2-1].\qquad (29)$$

We expand the function in terms of the small quantities $(L/\xi)^2$ and $(\delta c)^2=(c_\perp^2-c_\parallel^2)/c_\parallel^2$. When testing the solution (29) for stability we need retain only in (27) the terms quadratic in $\delta v = v-\pi$. Then

$$F(\delta v)=\frac{5}{16}\frac{c_\parallel{}^2}{L^2}\int_{-1}^{1}dx\Big\{\Big[1-\frac{3}{4}(\delta c)^2\Big]\Big(\frac{dv}{dx}\Big)^2 -\Big[(2+5(\delta c)^2)\frac{(\Delta\alpha)^2}{8}-\frac{L^2}{\xi^2}\Big](\delta v)^2\Big\}.\qquad (30)$$

It is easy to find the stability region. To this end it is necessary that $\Delta\alpha$ lies in the interval

$$0\leqslant\Delta\alpha\leqslant\pi-{}^7/_8\pi(\delta c)^2+2L^2/\pi\xi^2.$$

In this form, this region differs only by the coefficient 2 in the term with $(\delta c)^2$ from the region given by (8). We have used here, however, a different expression for $(\delta c)^2$, which is negative in the hydrodynamic region as $T\to T_c$. The solution (29) is thus stable even for $\Delta\alpha>\pi$. We shall show below that in this region $\Delta\alpha>\pi$ there exists also another solution that turns out, however, to be unstable in the hydrodynamic limit. To find it we turn to the functional (27) and obtain the first integral of the equation of motion

$$\frac{1}{2}m(w)\Big(\frac{dw}{dx}\Big)^2-w(1-w^2)\frac{L^2}{\xi^2}+\frac{1}{2}\frac{(1+w)j^2}{5c(w)+3} =-\frac{L^2}{\xi^2}w_0(1-w^2)+\frac{1}{2}\frac{(1-w^2)j^2}{2(1-w_0)[5c(w_0)+3]}.\qquad (31)$$

We have put here $w=\cos v$, with $w_0=w(0)$ the maximum of the function $w(x)$. To find the $I(\Delta\alpha)$ dependence we use identities similar to (10) and (11):

$$1=\int_{-1}^{0}dx=\int_{-1}^{w_0}\frac{dx}{dw}dw,\qquad (32)$$

$$\Delta\alpha=4\int_{-1}^{w_0}\frac{j}{(1-w)[5c(w)+3]}\frac{dx}{dw}dw.\qquad (33)$$

Expanding the velocity and mass in terms of small $(\delta c)^2$ and $(L/\xi)^2$, we find that

$$c(w)=1+(1-w)(\delta c)^2,\quad m(w)=1+{}^3/_8(1-w)(\delta c)^2,$$

and we get from (31)

$$\frac{dx}{dw}=\frac{1}{j}\Big\{\frac{(1-w_0)[8+3(1-w)(\delta c)^2]}{(1+w)(w_0-w)} \times\Big[1-\frac{L^2}{\xi^2}\frac{8(1-w_0)(1-w)}{j^2}\Big]^{-1}\Big\}^{1/2}.\qquad (34)$$

From (32) and (34) we obtain a relation identical to (13) between the current j and w_0, and for the phase difference $\Delta\alpha$ we have from (33)

$$\Delta\alpha=\pi-\frac{7\cdot2^{1/2}}{16}(\delta c)^2\pi(1-w_0)^{1/2}+\frac{2^{1/2}L^2}{\pi\xi^2}(1-w_0)^{1/2}.\qquad (35)$$

Substituting here expression (13), we obtain ultimately

$$\Delta\alpha=\pi-{}^7/_{32}(\delta c)^2j+L^2j/2\pi^2\xi^2.\qquad (36)$$

The solution (35) is represented in Fig. 1b by the line AB.

This solution is unstable in the hydrodynamic region as $T\to T_c$ (see Fig. 1b). The solutions (29) and (36) intersect at the point A whose coordinates are

$$j=4\pi+5\pi(\delta c)^2,\quad \Delta\alpha=\pi-{}^7/_8\pi(\delta c)^2+2L^2/\pi\xi^2.\qquad (37)$$

The results lead to the following conclusion. Provided that the thickness of channel a is much less than its length, in the hydrodynamic approximation $T\to T_c$ the channel parallel to the external field has no region with a unique $I(\Delta\alpha)$ dependence, i.e., with a pure Josephson effect. It appears, however, that this result depends strongly on the shape of the channel. In a region in which the conditions of Sec. 2 are met, the functional (25) reduces to (19) with the substitution $c_\perp\to c_\parallel$, and the usual Josephson relation (24) holds with a single-valued $I(\Delta\alpha)$ dependence. For a channel whose axis is perpendicular to the external magnetic field we always obtain in the case $L/\xi\ll1$ a single-valued $I(\Delta\alpha)$ dependence [see Eqs. (6), (15), and (24)].

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