Magnetooptics of two-dimensional electrons under the conditions of the fractional quantum Hall effect. Coulomb gaps and a critical temperature of an incompressible Fermi liquid

I. V. Kukushkin and V. B. Timofeev

Institute of Solid-State Physics, Academy of Sciences of the USSR
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Anomalous behavior was observed of the spectral position of a $2D_e$ line representing luminescence formed as a result of recombination of two-dimensional ($2D$) electrons with photoholes in metal-insulator-semiconductor (silicon) structures under the conditions of the fractional quantum Hall effect. An abrupt change in the spectral position of the $2D_e$ line corresponding to an abrupt change in the chemical potential of a system of interacting $2D$ electrons, because of a change in their density near a fractional value of the occupancy factor ($\nu = 7/3, 8/3$), was used to find the gap in the energy spectrum of excitations of an incompressible Fermi liquid. The scales Δe and Δh of the Coulomb gap for quasielectron and quasihole excitations were different and for $\nu = 7/3$ they obeyed $\Delta_e > \Delta_h$. The condensation of a gas of interacting $2D$ electrons into an incompressible Fermi liquid has a temperature threshold. The critical condensation temperature T_c was determined. A study was made of the dependences of the activation energies W, of the Coulomb gaps Δ, and of the critical temperature T_c on the applied magnetic field and on the $2D$-electron mobility μ_e representing disorder in the system. The minimum electron mobility at which the scattering by defects destroyed the fractional quantum Hall effect was determined.

§1. INTRODUCTION

The fractional quantum Hall effect (QHE) is manifested as follows: at sufficiently low temperatures in the strong transverse magnetic field H the Hall conductivity σ_{xy} (or the resistivity ρ_{xy}) of ideal $2D$ electron (hole) systems, measured as a function of the occupancy factor $\nu = n_S h/eH$ (n_S is the density of $2D$ electrons), has a series of plateaus at fractional values of the factor $\nu = p/q$ (p is an integer and q is an odd integer). Every plateau of $\sigma_{xy}(\nu)$ corresponds to deep minima in the diagonal component of the conductivity tensor $\sigma_{xx}(\nu)$ (or the resistivity ρ_{xx}) at the same values of $\nu = p/q$ (Refs. 1–4). A microscopic theory put forward by Laughlin[5,6] attributes the fractional QHE to condensation of a gas of interacting electrons into a strongly correlated quantum liquid of a new type. The calculated plots of the total energy $E(N)$ of the ground state of a system of interacting $2D$ electrons, considered as a function of the total number of particles N (at $T = 0$ and for a fixed H), exhibit downward kinks at values $N = N_f$ corresponding to fractional values of $\nu = \nu_f = p/q$. A kink in the $E(N)$ plot means that the chemical potential of the system $\zeta \equiv dE/dN$ changes abruptly at $N = N_f$:

$$\delta\zeta = dE/dN|_- - dE/dN|_+. \tag{1}$$

This means also that the energy spectrum of interacting $2D$ electrons has a Coulomb gap Δ. The change in N near N_f results in creation (or absorption) of elementary excitations: quasielectrons if $N > N_f$ ($\nu > \nu_f$) and quasiholes if $N < N_f$ ($\nu < \nu_f$).

It is characteristic that these excitations are separated by a gap from the ground state and the gaps Δ_e for quasielectrons and Δ_h for quasiholes may differ.[6,7] The energy necessary for the creation of a quasielectron-quasihole pair is $\Delta = \Delta_e + \Delta_h$. The Coulomb gaps $\Delta_{e,h}$ are described theoretically by a single energy scale, which is the energy of the interaction of $2D$ electrons $e^2/\varepsilon l_H$ (l_H is the magnetic length and ε is the permittivity). Since there are no zero-gap excitations in the system, it follows that the ground state suggested by Laughlin[5] represents an incompressible Fermi liquid. According to the theory put forward in Refs. 5–7, elementary excitations in an incompressible Fermi liquid (quasielectrons and quasiholes) carry a fractional charge $e^* = e/q$ (for $\nu = p/q$). Introduction of an additional electron into a system with $\nu = p/q$ is equivalent to creation of q quasielectrons and a reduction in the number of electrons by unity is equivalent to creation of q quasiholes. Since elementary excitations have a fractional charge, the abrupt change in the chemical potential at $\nu = p/q$ is[7]

$$\delta\zeta = q\Delta = q(\Delta_e + \Delta_h). \tag{2}$$

It would be desirable to measure the Coulomb gaps under the conditions of the fractional QHE and also to find the dependences of these gaps on an external magnetic field and on disorder in the system, which can be represented by the electron mobility. The gaps in the energy spectrum of an incompressible Fermi liquid have been determined experimentally only on the basis of investigations of magnetotransport characteristics,[8–12] namely from the activated dependences $\rho_{xx}(T)$ and $\sigma_{xx}(T)$ with minima corresponding to the fractional values of ν:

$$\rho_{xx} \propto \sigma_{xx} \sim \exp(-W/kT). \tag{3}$$

The temperature dependence of Eq. (3) is due to the presence of localized states in the gap and, therefore, the conductivity is governed by thermal excitation of a quasielectron-quasihole pair to the relevant mobility edges.[8] The activation

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energy W in the absence of disorder is

$$2W=\Delta=\delta\xi/q. \quad (4)$$

The precision of the method, based on a description of the magnetoconductivity in the context of a thermally activated mechanism, decreases rapidly on reduction of the scale $W(\nu)$, since cooling enhances the importance of variable-range hopping processes.[13] It is therefore obvious that other independent methods for the determination of the Coulomb gaps $\Delta_{e,h}$ are needed.

In a short letter[14] we described a spectroscopic method for determination of the Coulomb gap on the basis of the spectra of radiative recombination of $2D$ electrons with photoholes under the conditions of fractional QHE. We demonstrated that this method makes it possible, in contrast to the magnetotransport method, to determine separately the gaps for quasielectron and quasihole excitations. We report here a detailed study of the potentialities of this spectroscopic method (§3). We describe an investigation, carried out using this method, of the influence of temperature on the condensation of a gas of $2D$ electrons into an incompressible Fermi liquid (§4) and also analyze the behavior of the Coulomb gaps and of the critical condensation temperature as functions of the applied magnetic field and of the disorder in the system (§5).

§2. STRUCTURES AND EXPERIMENTAL METHOD

We investigated three metal-insulator-semiconductor (MIS) transistors with a ring geometry (Corbino disks) formed on a (001) surface of p-type silicon. These structures had a semitransparent gate of $\sim 1\ \text{mm}^2$ area. Nonequilibrium e-h pairs were generated using radiation from an LG-106 argon laser. All the investigated transistors (as well as structures demonstrating the fractional QHE and used by us earlier in Refs. 12 and 15–17) were made from the same plate under the same technological conditions, but they differed slightly in quality. The maximum $2D$-electron mobility μ_e^* at $T=0.35$ K was $41\times10^3\ \text{cm}^2\cdot\text{V}^{-1}\cdot\text{s}^{-1}$ (sample 1), $40\times10^3\ \text{cm}^2\cdot\text{V}^{-1}\cdot\text{s}^{-1}$ (sample 2), and $32\times10^3\ \text{cm}^2\cdot\text{V}^{-1}\cdot\text{s}^{-1}$ (sample 3), which corresponds to the following $2D$ electron densities $n_s=2.7\times10^{11}\ \text{cm}^{-2}$ (1), $3.7\times10^{11}\ \text{cm}^{-2}$ (2), and $4.0\times10^{11}\ \text{cm}^{-2}$ (3).

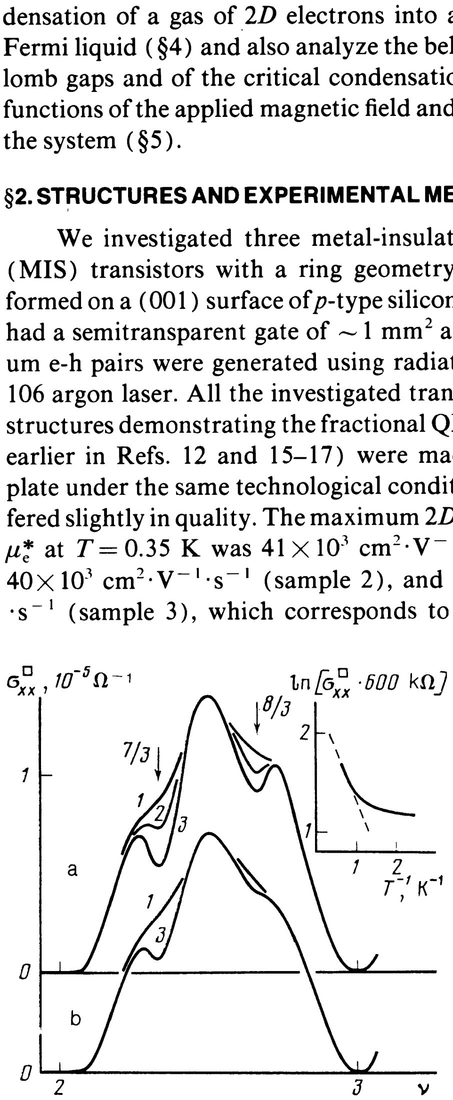

FIG. 1. Dependences of the magnetoconductivity σ_{xx} on the occupancy factor ν measured for two MIS structures (*a*—sample 2; *b*—sample 3) in a field $H=8$ T at different temperatures (K): 1) 1.5; 2) 1.2; 3) 0.4. The inset shows the temperature dependence $\sigma_{xx}(T)$ determined for $\nu=7/3$ in a field $H=8$ T and plotted using the coordinates $\ln\sigma_{xx}$ and T^{-1}. The slope of the dependence of $\ln\sigma_{xx}$ on T^{-1} at high temperatures is represented by a dashed line and corresponds to $W=1$ K.

Both magnetotransport and spectroscopic measurements could be made on these MIS transistors and we could compare the parameters obtained by different methods.

Our experiments were carried out in an optical cryostat with a solenoid in the Faraday geometry using fields up to 8 T. The spectra were analyzed with a DFS-12 double monochromator with a dispersion of 10 Å/mm in the working range of wavelengths. The recombination radiation (luminescence) was recorded by the photon counting method employing a cooled photomultiplier (with an S-1 cathode). The recorded signal was accumulated in a multichannel analyzer and the subsequent analysis of the spectra was made on a computer. The magnetotransport coefficients were measured using 20-Hz alternating current and the drain-source field did not exceed 10^{-2} V/cm, so that there was no heating of the $2D$-electron system. The other details of the experimental procedure can be found in Ref. 18.

§3. SPECTROSCOPIC DETERMINATION OF THE COULOMB GAP UNDER THE CONDITIONS OF THE FRACTIONAL QUANTUM HALL EFFECT

We consider first measurements of the Coulomb gap by the method of activated magnetoconductivity. Figure 1 shows typical dependences of the magnetoconductivity σ_{xx} on the occupancy factor ν measured for two samples in a field $H=8$ T at various temperatures. We can clearly see the minima of σ_{xx} corresponding to the fractional values of the factor $\nu=7/3$ and 8/3 (moreover, for the same samples, in a field $H=8$ T and at a temperature $T=0.35$ K, we observed minima of σ_{xx} corresponding to $\nu=4/3$, 5/3, and 10/3—see Ref. 14) and due to condensation of the $2D$-electron gas into an incompressible Fermi liquid. The depths of the minima of σ_{xx} increased as a result of cooling. The temperature dependence of σ_{xx} at the minima corresponding to the fractional values of ν made it possible to find the energy for the activation of excitations from the ground state W and to estimate the total gap in the energy spectrum [see Eqs. (3) and (4)]. The temperature dependence of $\sigma_{xx}(T)$ determined for $\nu=7/3$ in a field $H=8$ T was determined in the form $\ln\sigma_{xx}$-T^{-1} (inset in Fig. 1a). When these coordinates were used, it was found that the $\sigma_{xx}(T)$ plot had typically two slopes. The activation energy deduced from the slope in the high-temperature range[9] was $W=1.0\pm0.05$ K. Clearly, at lower temperatures $\sigma_{xx}(T)$ was not described by the simple Arrhenius law (3), because at these temperatures the activated processes were strongly masked by the variable-range hopping. Therefore, it was quite clear that the limit set on the precision of the determination of the Coulomb gap by the activated magnetoconductivity method were of fundamental nature, and that the precision and reliability of the method fell rapidly with increase of the magnetic field and of the $2D$-electron mobility.

We now consider the measurement of the Coulomb gaps of an incompressible Fermi liquid by the spectroscopic method. This method is based on determination and subsequent analysis of the spectra of radiative recombination of $2D$ electrons with photoholes in p-type Si (001) MIS structures.[18,19] Under conditions of photoexcitation of nonequilibrium e-h pairs near the Si–SiO_2 interface when the gate voltage is positive and obeys $V_g > V_T$ (V_T is the threshold voltage), a $2D$-electron channel appears and screens almost completely the gate field, so that the valence and conduction energy bands are practically flat (the depletion layer disappears) at distances $\gtrsim 100$ Å behind the $2D$ channel inside the semiconductor.[18] The recombination spectrum ($2D_e$ line) is then a convolution of the distribution functions of $2D$ electrons and nonequilibrium holes. It is important to note that the width of the energy distribution of holes participating in the recombination process is small and amounts to $\lesssim 0.8$ meV (Refs. 20 and 21) and in the case under discussion it determines the width of the $2D_e$ line. The two-dimensional nature of the $2D_e$ line is demonstrated by the fact that in a transverse field under conditions of complete occupancy of the n Landau levels the radiative recombination spectrum has n lines and the splitting of these lines is equal to the cyclotron energy, whereas in an oblique magnetic field the spectrum changes greatly in accordance with the change in the normal component of **H** (Ref. 20). The method of optical spectroscopy has been used successfully to study the energy spectrum of 2D electrons under conditions of the integral QHE (Ref. 21). In the case of the fractional QHE it is important also to bear in mind that at thermal energies less than the paramagnetic splitting of the levels of holes at an acceptor, the recombination process involves participation only of electrons with the upward spin projection ($S_Z = +1/2$). Therefore, at low temperatures ($T \approx 1.5$ K) and in strong magnetic fields ($H \gtrsim 6$ T), only the electron states participate in the magnetooptic effects and these states must satisfy the condition $(2+4m) < \nu < (4+4m)$, where m is an integer.[20] Therefore, we shall be interested only in the Coulomb gaps characterized by $\nu = 7/3$ and $8/3$.

The Coulomb gaps can be determined by the spectroscopic method utilizing the relationship between the spectral position of the $2D_e$ line, determined under the conditions of the fractional QHE, and the chemical potential ζ of the interacting $2D$ electrons. Therefore, in full agreement with abrupt changes of $\zeta(\nu)$ at fractional values of ν, we can expect nonmonotonic behavior of the spectral position of this luminescence line. It is shown in Fig. 2 that if $T = 1.6$ K and $H = 8$ T an increase in the occupancy factor from 2.27 to 2.4 (near $\nu = 7/3$) left the profile of the $2D_e$ luminescence line

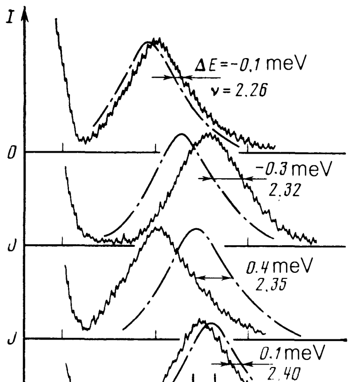

FIG. 2. Radiative recombination spectra of $2D$ electrons determined at $T = 1.6$ and 2.1 K ($> T_c$, dash-dot curves) for different values of ν ($H = 8$ T); ΔE is the difference between the spectral positions of the lines determined at different temperatures.

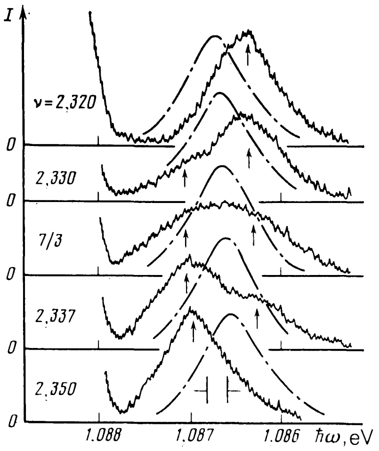

FIG. 3. Radiative recombination spectra of $2D$ electrons determined at $T = 1.5$ and 2.1 K ($> T_c$, chain curves) for different values of ν ($H = 8$ T).

practically unchanged, but the spectral position of the maximum of the line $2D_e$ ($\hbar\omega_{max}$) was a nonmonotonic function of ν. In the immediate vicinity of the fractional value $\nu = 7/3$ (from $\nu = 2.32$ to 2.35) the luminescence was of doublet nature (Fig. 3). When ν was increased within these limits the intensities of the components of this doublet changed so that initially the low-energy component was stronger and then its intensity began to fall, whereas the intensity of the high-energy component increased. This doublet nature of the luminescence spectrum obtained for $\nu = 7/3$ was a direct demonstration of the existence of a gap in the energy spectrum of the incompressible Fermi liquid. The energy splitting of the components of the doublet governed the value of this gap.

The dependence of the spectral position of the maximum of the recombination radiation line on the occupancy factor $\hbar\omega_{max}(\nu)$ is plotted in Fig. 4 demonstrating that the nonmonotonic dependence $\hbar\omega_{max}(\nu)$ was observed near $\nu = 7/3$ and only at low temperatures, and it was confined to structures with a high $2D$-electron mobility. It should be noted that the monotonic reduction in the energy $\hbar\omega_{max}$ on increase in ν, observed for all the investigated MIS structures in the mobility range $\mu_e < 20 \times 10^3$ cm^2·V^{-1}·s^{-1} at $T = 1.5$ K, was associated with the natural lowering of the

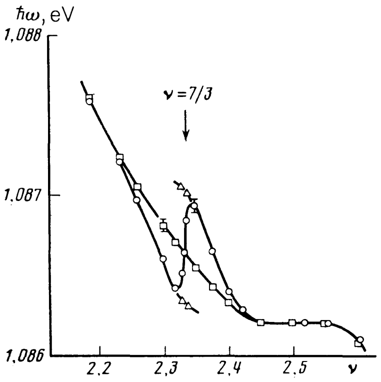

FIG. 4. Dependences of the spectral position of the maximum of the radiative recombination line of $2D$ electrons determined at $T = 1.5$ K (○) and $T = 2.1$ K (□). The triangles in the region of $\nu = 7/3$ correspond to splitting of the doublet luminescence line into two components. $H = 8$ T.

bottom of a size-quantized well.[21] The triangles in Fig. 4 near $\nu = 7/3$ represent separation of the recombination radiation into two lines with intensities governed by the deviation from 7/3. It is important that the doublet nature of the luminescence spectrum in the vicinity of $\nu = 7/3$ was due to an inhomogeneity of the system of $2D$ electrons. In an ideal case the discontinuity $\delta\zeta$ should occur in an infinitesimally narrow interval: $\Delta\nu \rightarrow 0$. In a real $2D$ system the presence of inhomogeneities has the effect that, for $\nu = 7/3$, part of the area of the structure is characterized by a value of ν slightly higher than 7/3 ($\nu = 7/3 + \Delta\nu$), whereas the other part is characterized by a slightly smaller value ($\nu = 7/3 - \Delta\nu$), which gives rise to luminescence lines in the range $(7/3 - \Delta\nu) < \nu < (7/3 + \Delta\nu)$. It is characteristic that in the experiments the value of $\Delta\nu$ decreased on improvement in the quality of the MIS structure.

The effect of condensation of a gas of $2D$ electrons into an incompressible Fermi liquid on the energy position of the $2D_e$ line can be judged by comparing the observed changes in the spectral position of the lines shown in Figs. 2 and 3 with the spectra determined under such conditions that no Fermi liquid appears, i.e., when $T > \Delta$ (the corresponding spectra are represented by dash-dot curves in Figs. 2 and 3). Clearly, in the case when $T > \Delta$ the nonmonotonic dependence $\hbar\omega_{max}(\nu)$ at fractional values of ν disappeared (Figs. 2–4). The difference ΔE between the spectral positions of the $2D_e$ lines measured at $T = 1.5$ K and $T = 2.1$ K is greater than Δ and is a feature of the interaction of $2D$ electrons condensing into an incompressible Fermi liquid (Fig. 2). The $\Delta E(\nu)$ dependences recorded in a field $H = 8$ T at $\nu = 7/3$ and 8/3 for two MIS transistors (2 and 3) with the maximum μ_e^* values 40×10^3 and 32×10^3 $cm^2 \cdot V^{-1} \cdot s^{-1}$, respectively, are plotted in Fig. 5. Clearly, ΔE differed from zero only near $\nu = 7/3$ and 8/3 so that the anomalous behavior of $\hbar\omega_{max}(\nu)$ was related specifically to condensation of a gas of $2D$ electrons into an incompressible Fermi liquid. An important observation was that $\Delta E(\nu)$ behaved nonmonotonically: it was negative and minimal for $\nu < 7/3$ (8/3); then its sign changed and it reached its maximum at $\nu > 7/3$. Such a $\Delta E(\nu)$ dependence could be explained by the fact that each recombination event reduced the number of $2D$ electrons by unity. In the model of an incompressible Fermi liquid this was equivalent to creation of three quasihole excitations when $\nu \leqslant 7/3$ and to absorption of three quasielectron excitations when $\nu > 7/3$. Hence, it should be possible to determine separately the gaps for quasielectron (Δ_e) and quasihole (Δ_h) excitations. A similar analysis allowing for the electron-hole symmetry may be carried out in the ranges $\nu \leqslant 8/3$ and $\nu > 8/3$. The law of conservation of energy predicts that in the case of absorption of three quasielectrons the energy of the emitted photon increases by $3\Delta_e$ (compared with the case when $T > \Delta$ and an incompressible Fermi liquid does not form), whereas creation of three quasiholes reduces this energy by $3\Delta_h$. It follows from Fig. 5 that if $H = 8$ T and $\nu = 7/3$, then $3\Delta_e = 0.4 \pm 0.03$ meV and $3\Delta_h = 0.3 \pm 0.03$ meV for sample 2. Moreover, we can conclude from Fig. 5 that the scale of the Coulomb gaps decreases on transition to a structure with a lower $2D$-electron mobility (sample 3) and also as a result of a transition from $\nu = 7/3$ to $\nu = 8/3$. Another characteristic feature demonstrated clearly in Fig. 5 is the difference between the values of Δ_e and Δ_h. For

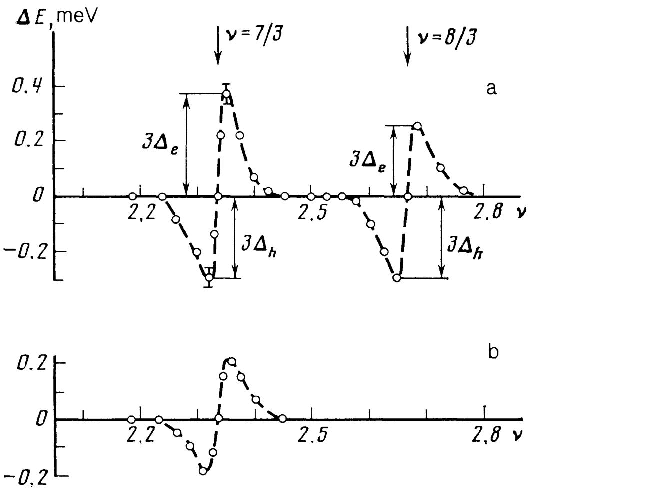

FIG. 5. Dependences of $\Delta E(\nu)$, i.e., of the difference between the spectral positions of the lines determined at $T = 1.5$ and 2.1 K ($> T_c$), on the occupancy factor ($H = 8$ T) of two structures: a) sample 2; b) sample 3.

$\nu = 7/3$ the Coulomb gap of quasielectron excitations was considerably greater than the quasihole gap, whereas for $\nu = 8/3$ the opposite was true: $\Delta_e < \Delta_h$; this was clearly associated with the electron-hole symmetry.

A comparison of the activation energy W measured by the magnetotransport method with the Coulomb gap Δ determined by the spectroscopic method demonstrated that they both satisfied Eq. (4) well. This could be regarded as an experimental confirmation of the fractional charge of quasiparticle excitations.

§4. INFLUENCE OF TEMPERATURE ON THE CONDENSATION OF A GAS OF 2D ELECTRONS

It is interesting to know how the Coulomb gap varies with temperature. This temperature dependence cannot be determined in principle by the method of thermally activated magnetoconductivity. However, the optical spectroscopy method makes it possible to find the required information experimentally. We need to determine how variation of temperature transforms the recombination radiation spectrum corresponding to the fractional value of the occupancy factor, such as $\nu = 7/3$. With this in mind we investigated the temperature dependences of the intensity of the $2D_e$ luminescence line in two fixed spectral intervals of its profile, identified by arrows in the spectra shown as an inset in Fig. 6. Clearly, the spectrum of the $2D_e$ line did not vary in a wide range of temperatures and then there was an abrupt transformation of the spectrum, followed by a region where the spectrum remained constant again (Fig. 6). The abrupt change in the spectrum in a field $H = 8$ T for $\nu = 7/3$ corresponded to a critical temperature $T_c = 1.96$ K. This experiment demonstrated directly that the condensation of a gas of $2D$ electrons into an incompressible Fermi liquid is a phase transition and is characterized by a critical temperature which, as found experimentally, depends on the applied magnetic field and on the $2D$-electron mobility (see §5).

We tried to find a hysteresis typical of first-order phase transitions in the temperature dependences of the radiation intensity $I(T)$. However, as demonstrated in Fig. 6, abrupt changes in the $I(T)$ dependences measured at different

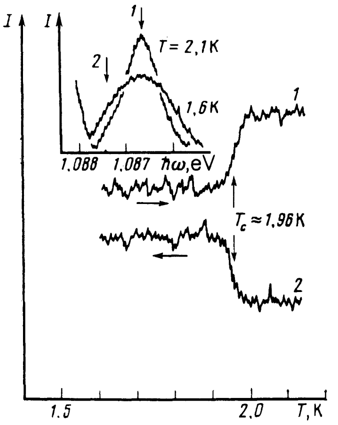

FIG. 6. Temperature dependences of the intensity of the $2D_e$ line for $\nu = 7/3$ and $H = 8$ T determined at two spectral points (1 and 2) identified by arrows in the inset showing the luminescence spectra at $T = 1.6$ and 2.1 K.

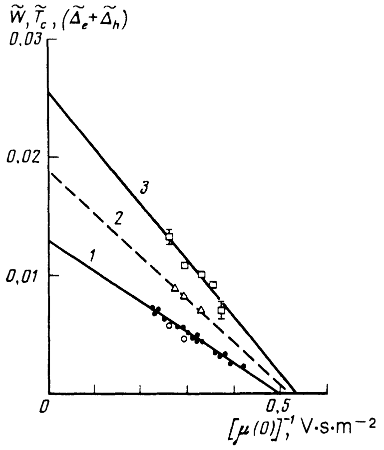

FIG. 7. Dependences of the activation energy W of Eq. (1), of the critical temperature T_c of Eq. (2), and of the Coulomb gap Δ of Eq. (3) expressed in the dimensionless form in units of the Coulomb energy $e^2/\varepsilon l_H$, on the reciprocal electron mobility. The figure includes the results obtained for all the MIS structures in different magnetic fields when $\nu = p/3$ ($p = 1, 2, 4, 5, 7, 8$).

wavelengths were all observed at the same temperature which was independent of whether temperature was increased or reduced during an experiment. Therefore, the condensation of a $2D$-electron gas into an incompressible Fermi liquid was most probably a second-order phase transition. The order parameter could be in this case the compressibility of a system of $2D$ electrons which changed abruptly $T = T_c$. It was also natural to expect that at $T = T_c$ there would be a loss of correlation in the distribution of $2D$ electrons (loss of the short-range order) typical of an incompressible Fermi liquid.[5,6]

§5. DEPENDENCES OF THE COULOMB GAPS AND CRITICAL TEMPERATURE ON THE APPLIED MAGNETIC FIELD AND ELECTRON MOBILITY

As pointed out already, the only energy scale in the theory of an incompressible Fermi liquid is the Coulomb energy $e^2/\varepsilon l_H$, so that naturally the theoretical values of the gaps for quasielectron and quasihole excitations are proportional to l_H^{-1}, i.e., $\Delta_{e,h} \propto H^{1/2}$. Precisely such a dependence of the activation energy W on the magnetic field was observed by us experimentally for MIS structures with a very high $2D$-electron mobility under the conditions such as $\mu_e = \text{const}$ (Ref. 12). We also found in the earlier study[12] that disorder in the $2D$-electron system destroyed the fractional QHE and reduced the measured values of W for structures with lower values of μ_e. It should be noted that the dependence of W on H and on μ_e can be factorized as follows:

$$W = W_0(1-\mu_0/\mu_e), \quad W_0 = Ae^2/l_H\varepsilon, \tag{5}$$

where W_0 is the limiting value of the activation energy for an ideal structure ($\mu_e \to \infty$), A is a constant, and μ_0 is the minimum $2D$ electron mobility at which the disorder destroys the fractional QHE.

We investigated how the values of the gap $(\Delta_e + \Delta_h)$ and the critical temperature T_c, measured by the spectroscopic method, varied with the magnetic field and disorder associated with fluctuations of the random potential. With this in mind we determined the values of $(\Delta_e + \Delta_h)$ and T_c in the vicinity of $\nu = 7/3$ and $\nu = 8/3$ when the magnetic field was varied. Since $n_s = \nu eH/h$, we found that variation of H generally altered n_S and, consequently, μ_e. We confirmed experimentally that in a constant magnetic field the values of $\Delta_{e,h}$ and T_c decreased for low values of μ_e (Fig. 5), whereas under the conditions when the $2D$-electron mobility was constant, the values of $\Delta_{e,h}$ and T_c varied in accordance with the law $\Delta \propto H^{1/2}$.

Figure 7 shows the dependences of the activation energy W, total Coulomb gap $\Delta = (\Delta_e + \Delta_h)$, and critical temperature T_c on the parameters H and μ_e, using the coordinates corresponding to the factorization given in Eq. (5). The values of the energy were reduced in this figure to zero dimensions in terms of the quantity $e^2/\varepsilon l_H$, and the $2D$-electron mobility $\mu(0)$ corresponded to the value found by extrapolation to $T \to 0$ from the temperature dependence $\mu_e(T)$ determined for the appropriate value of n_S. We included in Fig. 7 the results obtained for different MIS structures characterized by $\nu = p/3$ ($p = 1, 2, 4, 5, 7, 8$) and different values of H. Clearly, the nature of the dependences of the quantities W, Δ, and T_c on H and μ_e was in agreement with the factorization of Eq. (5). The condition $\delta\zeta = 6W = 3(\Delta_e + \Delta_h)$ was best obeyed at higher values of μ_e and the minimum mobility determined by optical and magnetotransport methods was similar (the value of μ_0 determined by an optical method was slightly less), in agreement with the estimated formula $\mu_0 = (h/e)^3(\varepsilon/m)^2$ (Ref. 12).

§6. CONCLUSIONS

It seems to us that the method of optical spectroscopy opens up fundamentally new opportunities for the investigation of the condensation of a gas of interacting $2D$ electrons into an incompressible Fermi liquid. This method can be used to tackle the problems associated with a hierarchy of states of incompressible liquids and, in particular, to answer the question how the scales of the Coulomb fields change upon variation of the fractional charge of quasiparticle excitations. In our opinion it is equally interesting to study the behavior of a system of interacting $2D$ electrons near the critical temperature of the condensation into an incompressible Fermi liquid.

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