Kinetics of muon catalysis in a deuterium-tritium mixture (quasistationary state)

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Quasistationary muon catalysis in a $D_2 + T_2$ mixture, established for times t much longer than the lifetime of the mesic molecule $dt\mu$, has been investigated. The energy distribution functions for the $t\mu$-atoms in the $1s$-state have been calculated. The calculated spectrum of the mesic atoms $t\mu$ was found to differ from the Maxwellian distribution by less than 10%. The difference between the rate of resonant production of the $dt\mu$-molecule, averaged over the calculated and the Maxwellian functions, was found to be about 5%.

1. INTRODUCTION

The interpretation of experimental data,[1-5] and the planning of new experiments designed to produce information on the muon catalysis of fusion reactions between deuterium and tritium nuclei, require detailed investigation of the kinetics of processes occurring in the $D_2 + T_2$ mixture, which was begun in Ref. 6.

The main processes that determine the efficiency of muon catalysis are the production of the mesic molecule $dt\mu$ by the resonant mechanism[7,8] in binary collisions between $t\mu$-atoms and the molecules of the medium, i.e.,

$$(t\mu)_F + D_2 \to [(dt\mu)_F dee], \tag{1}$$

and the production of the mesic molecules $dt\mu$ by the quasiresonant mechanism[9] in ternary collisions:

$$(t\mu)_F + D_2 + D_2 \to [(dt\mu)_F dee] + D_2'. \tag{2}$$

The rates $\lambda^{(F)}$ of reactions (1) and (2) depend on the temperature of the $D_2 + T_2$ mixture. The character of the function $\lambda^{(F)}(T)$ is determined by the energy distribution $\Phi_F(E,T)$ of the $t\mu$-atoms.[7-9] At the same time, the rates of production of the $dt\mu$-molecules in reactions (1) and (2) are $\lambda^{(F)} \sim 10^8$–$10^9\ s^{-1}$ (Refs. 1–5, 8, 9), i.e., very high and comparable in order of magnitude with the rates of thermalization of $t\mu$-atoms[6,10,11] and the muon capture rate in the reactions

$$(d\mu)_{1s} + t \to (t\mu)_{1s} + d, \tag{3}$$

$$(t\mu)_{F=1} + t \leftrightarrow (t\mu)_{F=0} + t, \tag{4}$$

which occur in the $1s$-states of the mesic atoms with total spin F (Refs. 1–5, 12). It is therefore necessary to investigate the effect of processes (1)–(4) on the Maxwell distribution $f_M(E,T)$ of the mesic atoms $t\mu$.

In the absence of muon loss channels (muon decay or attachment to fusion byproduct nuclei), a stationary state characterized by a constant number of nuclear fusion reactions per unit time is established in a time $t_0 \gg \lambda_{\text{eff}}^{-1} \sim 10^{-12}$ s as the μ^--meson comes to rest in the $D_2 + T_2$ mixture, where $\lambda_{\text{eff}}^{-1}$ is the lifetime of the $dt\mu$ mesic molecule.[13] However, in reality, a quasistationary state is established in a time $t \gtrsim t_0$, in which the main components of muon catalysis reactions (neutrons, mesic atoms, and so on) decay exponentially with time, with decay constants $\lambda \sim \lambda_0$, where $\lambda_0 = 0.455 \times 10^6\ s^{-1}$ is the decay constant of the free muon. The great majority of nuclear fusion reactions occur in this regime.

In this paper, we derive and solve the equations describing the kinetics of muon catalysis in the quasistationary approximation. We also calculate the rates $\lambda^{(F)}(T)$ of the resonant production of the mesic molecules $dt\mu$, averaged over the distribution functions $\Phi_F(E) = \Phi_F(E,t)$ with allowance for the departure of the spectrum of $t\mu$-atoms from the equilibrium spectrum.

2. EQUATIONS FOR THE SPECTRUM OF THE $t\mu$ MESIC ATOMS

The equations describing the leading processes in the $D_2 + T_2$ mixture at temperature T, which determine the spectrum of the $t\mu$-atoms and are illustrated schematically in Fig. 1, are as follows:

$$\partial f_F(t,E)/\partial t = -f_F(t,E)\Big[\lambda_0 + \lambda^{(F)}(E) + \sum_{F'} \lambda_{FF'}(E)\Big] + \sum_{F'} \int_0^\infty f_{F'}(t,E')\Lambda_{F'F}(E',E)\,dE' + P(E)\Gamma_F N_{d\mu}^{(F)} + \eta_F \delta(E-E_0)\lambda_{dt} N_{d\mu} + \eta_F(1-qC_d)(1-\omega_s)K(E)\lambda_{eff} N_{d\mu}, \tag{5}$$

$$N_F(t) = \int_0^\infty f_F(t,E)\,dE, \tag{6}$$

$$dN_{d\mu}/dt = -(\lambda_0 + \lambda_{dt})N_{d\mu} + qC_d(1-\omega_s)\lambda_{eff} N_{dt\mu}, \tag{7}$$

$$dN_{dt\mu}/dt = -(\lambda_0 + \lambda_{eff} + \Gamma_F)N_{dt\mu} + Q_F, \tag{8}$$

$$dN_n/dt = \lambda_{eff} N_{dt\mu}, \tag{9}$$

$$\lambda_{F'F}(E) = \int_0^\infty \Lambda_{F'F}(E,E')\,dE', \qquad N_{dt\mu} = \sum_F N_{dt\mu}^{(F)}, \tag{10}$$

$$Q_F \equiv Q_F(t) = \int_0^\infty \lambda^{(F)}(E) f_F(t,E)\,dE, \tag{11}$$

where C_d and C_t are the concentrations of deuterium and tritium nuclei, $C_d + C_t = 1$, $N_F \equiv N_t^{(F)}$ and $N_{d\mu}$ is the number of $t\mu$ mesic atoms with spin F and the number of $d\mu$ mesic atoms, respectively, $N_{dt\mu}^{(F)}$ is the number of $dt\mu$ mesic molecules with spin F, N_n is the number of neutrons emitted in fusion reactions between the nuclei of the $dt\mu$ molecules, $f_F(t,E)$ is the distribution function of the $t\mu$-atoms in the $1s$ state with total spin F, and λ_{dt} is the rate of isotope exchange (3), whose value is $\lambda_{dt} \sim 2 \times 10^8\ c_t$ (it was calculated in Refs. 12 and 14 and measured in Refs. 1, 3, and 5). Equation

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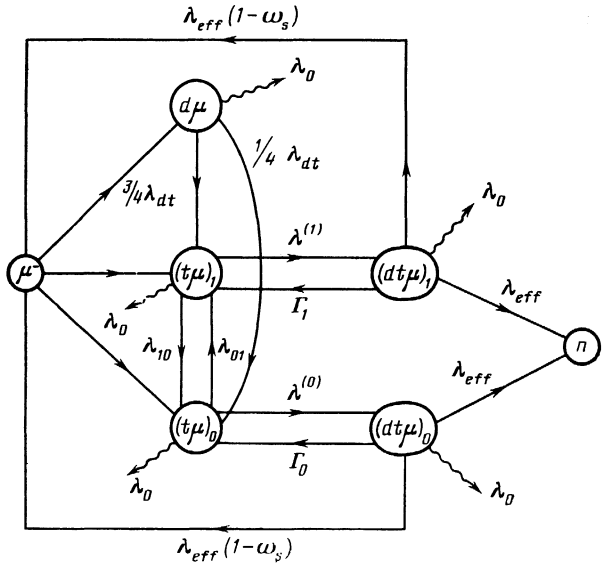

FIG. 1. Scheme describing the leading processes that determine the spectrum of the $t\mu$-atoms. Subscripts 0 and 1 denote the spin F of the $t\mu$ and $dt\mu$ states, respectively.

(5) takes into account the fact that the reactions (3) produce the $t\mu$ mesic atom with energy $E_0 \simeq 19$ eV and spin $F=0$ or 1 with probabilities proportional to the statistical weights $\eta_0 = 1/4$ and $\eta_1 = 3/4$, respectively. Next, $\varphi = N/N_0$, where N and $N_0 = 4.25\times 10^{22}$ cm^{-3} are, respectively, the densities of the mixture and of liquid hydrogen, and $\Lambda_{FF'}(E,E')dE'$ is the rate of transition of the mesic atom from the state with spin F and kinetic energy E to the state with spin F' and energy $E'+dE'$ in collisions with the molecules of the D_2+T_2 mixture. The expressions for the rates $\Lambda_{F'F}$ are given in Appendix A and take into account the data on the scattering of $t\mu$-atoms.[14-20] The rates $\lambda^{(F)}(E)$ in (5)–(11) are given by the following expression when the two different mechanisms for the production of the $dt\mu$ mesic molecules are taken into account:

$$\lambda^{(F)}(E)=\lambda_r^{(F)}(E)+\lambda_{qr}^{(F)}(E), \tag{12}$$

where $\lambda_r^{(F)}$ and $\lambda_{qr}^{(F)}$ are the rates of reactions (1) and (2), respectively. The dependence of $\lambda^{(F)}(E)$ on the kinetic energy E of the $t\mu$ mesic atoms is calculated in Refs. 9 and 21 (Fig. 2 shows the rates $\lambda_r^{(F)}(E)$ taken from Ref. 21). Next, Γ_F represents the rates of decay of the molecular mesic complexes:

$$[(dt\mu)^F dee]\rightarrow (t\mu)_F+D_2, \tag{13}$$

where $\Gamma_F \simeq 10^{10}$ s^{-1} (Ref. 21) and $\lambda_{\text{eff}} = \lambda_{dex}+\lambda_f$ is the total rate of stabilization of complexes of the form $[(dt\mu)dee]$ in the deexcitation of $dt\mu$ mesic molecules [$\lambda_{dex} = 1.27\times 10^{12}$ s^{-1} (Ref. 22)] and in the fusion reactions

$$dt\mu \xrightarrow{\omega_s\lambda_f} \mu^4\text{He}+n \tag{14a}$$

$$\downarrow$$

$$\xrightarrow{(1-\omega_s)\lambda_f} {}^4\text{He}+n+\mu^- \tag{14b}$$

where $\lambda_f = 3.9\times 10^7$ s^{-1} is the rate of the nuclear reaction,[23] $\omega_s = 0.58\times 10^{-2}$ (Ref. 24) is the attachment coefficient of the μ^- meson to the α particle, and $P(E)$ is the energy distribution function of the $t\mu$ atoms produced in the decay of the molecular mesic complexes in reactions such as (13). According to Ref. 21, the decay reactions (13) modify the observed production rate $\lambda_{dt\mu}^{(F)}$ of the $dt\mu$ mesic molecules by only about 5%, which means that, to the accuracy of the present work, we need not specify the form of the function $P(E)$, but choose it on the basis of its physical meaning to be Maxwellian:

$$P(E)=f_M(E), \tag{15}$$

$$f_M(E)=2(E/\pi)^{1/2}T^{-3/2}\exp(-E/T). \tag{16}$$

Actually, the molecular mesic complexes, whose rotational states are populated in accordance with the Boltzmann distribution,[25] may be looked upon as systems that are in thermal equilibrium with the medium and emit the $t\mu$ mesic atoms. Finally, $K(E)$ in (5) is the energy distribution function of the $t\mu$-atoms in the 1s state, produced as a result of processes occurring in the excited states of the mesic atoms.[10,26-31] We have taken the function $K(E)$ in the form of the Maxwell distribution (16): $K(E)=f_M(E)$, which is valid when the density of the mixture is $\varphi \gtrsim 0.05$ (Ref. 10).

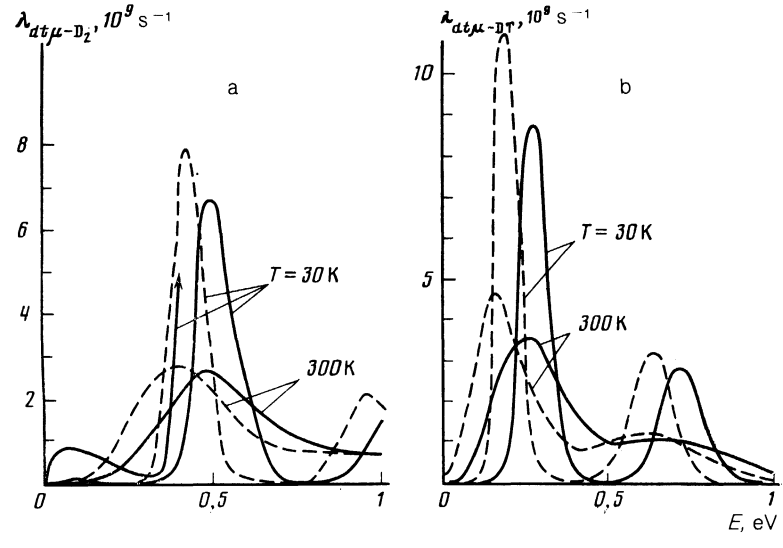

FIG. 2. Rate of resonant production of the $dt\mu$-molecules on D_2 (a) and DT (b) as a function of the energy of the $t\mu$-atom in the laboratory frame at $T=30$ and 300 K. Solid curves—$\lambda^{(F=0)}$, dashed curves—$\lambda^{(F=1)}$. The rate $\lambda_{dt\mu-D_2}^{(F=0)}, 10^8$ is also shown on a $\times 10$ scale (solid curve marked with the arrow).

Equations (5)–(7) allow for the fact that the $d\mu$ and $t\mu$ mesic atoms are produced in the 1s state with probabilities respectively equal to qC_d and $1-qC_d$, where q is the probability that the muon is not captured from an excited state of the $d\mu$ atoms in the isotope exchange reactions[32-33]

$$(d\mu)_n+t\to(t\mu)_n+d. \tag{17}$$

Moreover, conservation of the spin F of the $t\mu$ mesic atoms during deexcitation ensures that the hyperfine-structure sublevels of the 1s ground state are populated statistically with weights $\eta_F(1-qC_d)$.

The initial conditions for (5)–(11) are:

$$f_F(0,\ E)=\eta_F(1-qC_d)f_M(E), \tag{18a}$$

$$N_{d\mu}(0)=qC_d, \tag{18b}$$

$$N_{dt\mu}^{(F)}(0)=N_n(0)=0. \tag{18c}$$

3. QUASISTATIONARY MUON CATALYSIS

Let us now examine the kinetics of μ-mesic molecular processes for times $t>t_0$, where

$$t_0\gg(\lambda_{eff}+\Gamma_F)^{-1}\sim10^{-12}\text{ s}. \tag{19}$$

The quantity $N_{dt\mu}^{(F)}(t)$ varies relatively slowly for $t>t_0$, so that the solution of (8) is

$$N_{dt\mu}^{(F)}(t)=(\lambda_{eff}+\Gamma_F)^{-1}Q_F(t). \tag{20}$$

Taking this into account, we find that Eqs. (5)–(9) assume the form

$$\partial f_F(t,E)/\partial t=-f_F(t,E)\Big[\lambda_0+\lambda^{(F)}(E)+\sum_{F'}\lambda_{FF'}(E)\Big]+\sum_{F'}\int_0^\infty f_{F'}(t,E')\Lambda_{F'F}(E'E)\,dE'+P(E)(1-W_F)Q_F+\eta_F\lambda_{dt}\delta(E-E_0)N_{d\mu}+\eta_F(1-qC_d)K(E)(1-\omega_s)Q, \tag{21}$$

$$dN_{d\mu}/dt=-(\lambda_0+\lambda_{dt})N_{d\mu}+(1-\omega_s)qC_dQ, \tag{22}$$

$$dN_n/dt=Q, \tag{23}$$

where

$$Q=\sum_F W_FQ_F, \tag{24}$$

$$W_F=\lambda_{eff}/(\lambda_{eff}+\Gamma_F). \tag{25}$$

If we substitute $\lambda_0=\lambda^{(F)}=\lambda_{dt}=0$, we find from (21) that the $t\mu$ mesic atoms become thermalized in a time

$$t\gtrsim\tau_T=[\min\lambda_{FF'}(\varepsilon\sim T)]^{-1}, \tag{26}$$

where[12]

$$\lambda_{FF'}\sim10^9\varphi\,[\text{s}^{-1}] \tag{27}$$

(see Appendix B), i.e.,

$$f_F(E,\ t)=N_Ff_M(E), \tag{28}$$

and the Boltzmann equilibrium is established between the hyperfine-structure states of the $t\mu$ atoms:

$$N_0=3N_1\exp(-\Delta E/T) \tag{29}$$

where $\Delta E=\Delta E^{hfs}=0.241$ eV is the hyperfine splitting of the 1s state of the $t\mu$ mesic atoms.

In reality, $\lambda^{(F)}\neq0$, $\lambda_{dt}\neq0$ and a quasistationary state is established in which the function $f(E,t)$ can be written in the following form by analogy with (28) (see Appendix C):

$$f_F(t,E)=N_F(t)\bar\Phi_F(E),\quad\int_0^\infty\bar\Phi_F(E)\,dE=1. \tag{30}$$

Equations (21)–(23) for the time derivatives now assume the form[6]

$$dN_F/dt=-(\lambda_0+\tilde\lambda^{(F)}+\lambda_{FF'})N_F+\lambda_{F'F}N_{F'}+\eta_F(1-qC_d)(1-\omega_s)Q+\eta_F\lambda_{dt}N_{d\mu},\quad F'\neq F, \tag{31}$$

$$dN_{d\mu}/dt=-(\lambda_0+\lambda_{dt})N_{d\mu}+(1-\omega_s)qC_dQ, \tag{32}$$

$$dN_n/dt=Q, \tag{33}$$

where

$$\lambda^{(F)}=\frac{1}{I_F}\int_0^\infty\lambda^{(F)}(E)\bar\Phi_F(E)\,dE, \tag{34}$$

$$\lambda_{FF'}=\frac{1}{I_F}\int_0^\infty\lambda_{FF'}(E)\bar\Phi_F(E)\,dE, \tag{35}$$

$$I_F=\int_0^\infty\bar\Phi_F(E)\,dE, \tag{36}$$

$$\tilde\lambda^{(F)}=W_F\lambda^{(F)}. \tag{37}$$

We note that it follows from the definitions given in (12) and (24) that

$$Q_F(t)=\lambda^{(F)}N_F, \tag{38}$$

$$Q(t)=\sum_F\lambda^{(F)}N_F. \tag{39}$$

The initial conditions for (31)–(33) are defined by (18b), (18c), and the condition

$$N_F(0)=\eta_F(1-qC_d), \tag{18d}$$

which follows from (18a).

By analogy with the kinetics of processes occurring in pure deuterium,[34] the inclusion of the decay (13) of mesomolecular complexes in Eqs. (31)–(33), which describe reactions in the D_2+T_2 mixture, reduces to the replacement of the production rate $\lambda^{(F)}$ of the $dt\mu$ mesic molecules with the effective rates $\tilde\lambda^{(F)}$ given by (37).

4. SOLUTION OF EQUATIONS (31)–(33)

To first order in λ_0 and ω_s, the solution of (31)–(33) is

$$N_F(t)=N_F^{(0)}e^{-\lambda t}, \tag{40}$$

$$N_{d\mu}(t)=N_{d\mu}^{(0)}e^{-\lambda t}, \tag{41}$$

$$dN_n/dt=\lambda_ce^{-\lambda t}, \tag{42}$$

$$\lambda=\lambda_0+\omega_s\lambda_c, \tag{43}$$

where

$$\lambda_c=(qC_d/\lambda_{dt}+B/A)^{-1} \tag{44}$$

is the rate of the cycle and

$$A=\tilde{\lambda}^{(1)}\tilde{\lambda}^{(0)}+\tilde{\lambda}^{(1)}\lambda_{01}+\tilde{\lambda}^{(0)}\lambda_{10}, \quad (45)$$

$$B=\eta_0\tilde{\lambda}^{(1)}+\eta_1\tilde{\lambda}^{(0)}+\lambda_{01}+\lambda_{10}, \quad (46)$$

$$N_F^{(0)}=(\lambda_c/A)(\eta_F\tilde{\lambda}^{(F')}+\lambda_{F'F}), \quad F'\neq F, \quad (47)$$

$$N_{d\mu}^{(0)}=qC_d\lambda_c/\lambda_{dt}. \quad (48)$$

The expression for the total number X_c of μ-catalysis cycles per muon stopping in the mixture is well known[6]:

$$X_c^{-1}=\omega_s+\lambda_0/\lambda_c. \quad (49)$$

Physically, the quantity λ_c given by (44) is the rate at which neutrons appear in reaction (14) (Ref. 2), and is formally identical with the result obtained in Ref. 35 by neglecting in (45) and (46) the rates λ_{01} of processes (4), which are significant at high temperatures $T>700$ K. However, we have taken into account the fact that the production rates $\tilde{\lambda}^{(F)}$ of the $dt\mu$ molecules in (45) and (46) are the effective rates according to (37), and are given by (34), which is valid only in the quasistationary state.

Using (40) and (41), and the expressions

$$N_F(t)=a_FN_{d\mu}(t), \quad (50)$$

$$a_F=N_F^{(0)}/N_{d\mu}^{(0)}, \quad (51)$$

that follow from them, and also condition (30), we find that (21) can be reduced to the form

$$-\Phi_F(E)\left[\lambda^{(F)}(E)+\sum_{F'}\lambda_{FF'}(E)\right]+\sum_{F'}\int_0^\infty\Phi_{F'}(E)\Lambda_{F'F}(E',E)\,dE'+\Pi_F(E)=0,$$

$$F=0,1. \quad (52)$$

where

$$\Phi_F(E)=a_F\tilde{\Phi}_F(E), \quad (53)$$

$$\Pi_F(E)=P(E)(1-W_F)\lambda^{(F)}a_F+K(E)\eta_F[(qC_d)^{-1}-1]\lambda_{dt}+\delta(E-E_0)\eta_F\lambda_{dt}. \quad (54)$$

In (52), we have discarded terms $\sim\omega_s$ and $\sim\lambda_0/\lambda$.

It is shown in Appendix B that inclusion of the charge-transfer process (3) in the calculation of the production rates $\lambda^{(F)}$ results in a small correction that applies in a wide range of temperature and concentration. If this is so, we can replace $\delta(E-E_0)$ with the Maxwell distribution (16) and, using (45), (51), and (53), we can write (54) in the form

$$\Pi_F(E)=f_M(E)\eta_FW_F\lambda^{(F)}\lambda^{(F')}+(1-\eta_{F'})W_F\lambda_{F'F}+\eta_FW_{F'}\lambda^{(F')}\lambda_{FF'}, \quad F\neq F'. \quad (55)$$

The values of the quantities $\lambda^{(F)}$ and $\lambda_{FF'}$ in (55) are determined by (34)–(36) and are unaffected by the replacement $\tilde{\Phi}_F\to\Phi_F$ when (53) is taken into account.

The rates $\lambda^{(F)}$ in (52) were determined in accordance with (12) in terms of the quantities $\lambda_r^{(F)}(E)$ and $\lambda_{qr}^{(F)}(E)$. The rates $\lambda_r^{(F)}$ of reaction (1) were calculated in Ref. 21 to within 1%. On the other hand, the estimated rates $\lambda_{qr}^{(F)}$ reported in Ref. 9 are

$$\lambda_{qr}^{(0)}=(2\text{–}4)\,10^8\varphi^2C_d^2\,[\mathrm{s}^{-1}], \quad \lambda_{qr}^{(1)}\ll\lambda_{qr}^{(0)},$$

and are subject to considerable uncertainty. Since there are no detailed calculations, we have assumed that

$$\lambda_{qr}^{(0)}(E)=\lambda_{qr}^{(0)}\equiv\lambda_{qr}, \quad \lambda_{qr}^{(1)}(E)=\lambda_{qr}^{(1)}=0, \quad (56)$$

which is in qualitative agreement with experiment.[3]

5. RESULTS

Muon catalysis reactions are governed by a number of different mechanisms, so that the solutions of (52) depend on the density, temperature, and component concentration of the D_2+T_2 mixture in a relatively complicated manner.

Figure 3 shows the solutions $\Phi_F(E)$ with and without taking (2) into account for $\varphi=1$, $C_t=0.3$ and $T=600$ K. It is clear from the figure that the functions $\Phi_F(E)$ (solid curves) and the Maxwell distributions (broken curves), normalized to the same number of mesic atoms, differ by less than 10%. This small difference is explained, on the one hand, by the high probability (17) of muon capture in the excited states of the mesic atoms[32,33] and, on the other hand, by the rapid slowing down of the $t\mu$ mesic atoms (Appendix B) as compared with the rate of production of the mesic molecules.

Table I lists the effective rates $\lambda^{(F)}$ of production of the $dt\mu$ mesic molecules for $\varphi=1$, $C_t=0.3$, and different temperatures. The rates were calculated from (34), using the calculated functions $\Phi_F(E)$ and the Maxwell distribution $f_M(E)$. Comparison shows that the Maxwellian spectrum is distorted by less than 5%. The table also lists the rates of the processes (4), which are important in the study of the kinetics of muon catalysis.

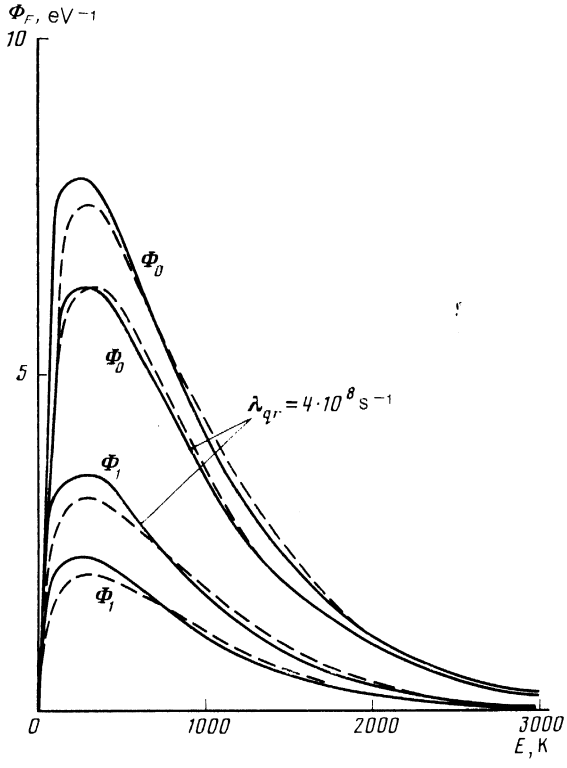

FIG. 3. Energy distribution of the $(t\mu)_F$-atoms for $\varphi=1$, $C=0.3$, and $T=600$ K (solid curves). Dashed curves represent the Maxwellian distribution.

TABLE I. Rate of production of $dt\mu$-molecules, $\lambda^{(F)}$, and the rates $\lambda_{FF'}$ of the processes $t\mu(F=1)+t\leftrightarrow t(F=0)+t$ in the D_2+T_2 mixture.

λ_T, $10^8\,s^{-1}$	λ_{qT}, 10^8, s^{-1}	T, K		
		300	600	900
$\lambda^{(0)}$	0	1.27 (1.27)	3.54 (3.62)	5.78 (5.81)
$\lambda^{(1)}$	0	1.49 (1.62)	5.77 (6.14)	8.12 (8.31)
λ_{10}	0	3.6	3.6	3.6
λ_{01}	0	0	0.04	0.16
$\lambda^{(0)}$	2	2.26 (2.24)	4.58 (4.6)	6.79 (6.79)
$\lambda^{(1)}$	2	1.49 (1.62)	5.79 (6.14)	8.12 (8.31)
λ_{10}	2	3.6	3.6	3.6
λ_{01}	2	0	0.04	0.16
$\lambda^{(0)}$	4	3.26 (3.23)	5.62 (5.58)	7.81 (7.77)
$\lambda^{(1)}$	4	1.49 (1.62)	5.81 (6.14)	8.13 (8.31)
λ_{10}	4	3.6	3.6	3.6
λ_{01}	4	0	0.04	0.16

Note. The numbers in parentheses are the rates calculated with the Maxwellian distribution. The rate values are listed for $\varphi=1$ and $C_t=0.3$.

In the derivation of (49) we have ignored, for simplicity, the production of $dd\mu$ and $dt\mu$ mesic molecules and muon capture by He nuclei. When these processes are taken into account, the quantity ω_s in (49) must be replaced with

$$W=\omega_s+\frac{\beta qC_d\omega_d\lambda_{dd\mu}}{\lambda_{dt}C_t}+\frac{\lambda_{tt\mu}\omega_t B}{A}+C_{\mathrm{He}}\left(\frac{qC_d\lambda_{d\mathrm{He}}}{\lambda_{dt}C_t}+\frac{\lambda_{t\mathrm{He}}B}{A}\right). \tag{57}$$

We note that (44) and (57) are valid when $\lambda_0+\lambda_{dd\mu}\ll\lambda_{dt}$, where $\lambda_{dd\mu}$ and $\lambda_{tt\mu}$ are the rates of production of the corresponding molecules, $\lambda_{d\,He}$ and $\lambda_{t\,He}$ are the rates of muon capture by He nuclei from the d and t nuclei, ω_d is the muon attachment coefficient in the dd fusion reaction, and $\beta=0.58$. We note that (41), (49), and (57) are valid in a wide range of density values, $\varphi\gtrsim 0.01$, in which condition (27) for the validity of the quasistationary approximation $\lambda\tau_b\ll 1$ is satisfied.

For $T\leqslant 500$ K, we can write

$$\lambda_c^{-1}=\left[\frac{qC_d}{\lambda_{dt}C_t}+\frac{0.75}{\lambda_{10}C_t}+\frac{1}{\lambda_{dt\mu}C_d}\right]\frac{1}{\varphi}. \tag{58}$$

In the analysis of the experiment reported in Ref. 3, the second term was not taken into account. The experimental results were therefore interpreted as a discrepancy between theoretical estimates and the measured values of q. Moreover, the dependence of the quantities q on the density and concentration of tritium was also different from that predicted theoretically.[32,33] However, the term $0.75/\lambda_{10}C_t$ is comparable with $qC_d/\lambda_{dt}C_t$ when $\varphi\gtrsim 0.5$ (Refs. 32 and 33). The discrepancy is removed when this is taken into account in the analysis of the experiment reported in Ref. 3.

6. CONCLUSIONS

Each muon undergoes approximately $X_c\gtrsim 100$ muon-catalysis cycles (Refs. 2, 3, 8, 36) in the time $\lambda_0^{-1}\gg\tau_b$. This enables us to use the quasistationary approximation, which means that the equations for the kinetics of the processes occurring in the D_2+T_2 mixture can be divided into two sets, of which one describes the time-domain distribution of the components participating in muon catalysis, and the other gives the energy distribution of the $t\mu$ atoms.

We note that, in accordance with the definition (27) of the quasistationary state, the kinetic equations derived here are not valid for processes in which an equilibrium is established over the hyperfine-structure states of the $t\mu$-atoms and which occur in time $t\lesssim\tau_b$ (see Appendices A and C). Studies of the muon-catalysis kinetics in this transition regime for times $t\lesssim\tau_b$ are of considerable interest because certain parameters of muon catalysis[4,5] can be measured in this regime for low mixture densities ($\varphi<0.01$). However, theoretical analysis of the transition regime is difficult in the case of low densities because the distribution function $K(E)$ is not well-known for the $t\mu$-atoms, and its form is determined by processes occurring in the excited states of the mesic atoms.

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APPENDIX A. EVALUATION OF THE PROBABILITIES $\Lambda_{FF'}(E,E')$ OF THE SCATTERING OF $t\mu$-ATOMS BY NUCLEI IN A D_2+T_2 MIXTURE

The cross sections for reactions such as

$$(t\mu)_F+a\leftrightarrow(t\mu)_{F'}+a, \tag{A1}$$

where $a=d,t$, can be calculated by methods developed in Refs. 12 and 14–17. When the scattering of the $t\mu$-atoms by the d and t nuclei in the D_2+T_2 mixture is investigated, it is essential to take into account the chemical bonding of nucleus a in the molecule AB (A,B = D,T) (Ref. 18) and the interaction between the $t\mu$-atoms and electrons in the molecule AB (Ref. 19). Figure 4 shows the calculated rates

$$\lambda^0_{FF'}(\varepsilon)=N_0Gv \tag{A2}$$

as functions of the energy ε of the collision between the mesic atom $t\mu$ and the nucleus a in their center-of-mass frame, taking into account the above effects. We note that the above molecular effects are unimportant in spin-flip processes (4).

The scattering rate $\Lambda_{FF'}(E,E')$ in the laboratory frame,

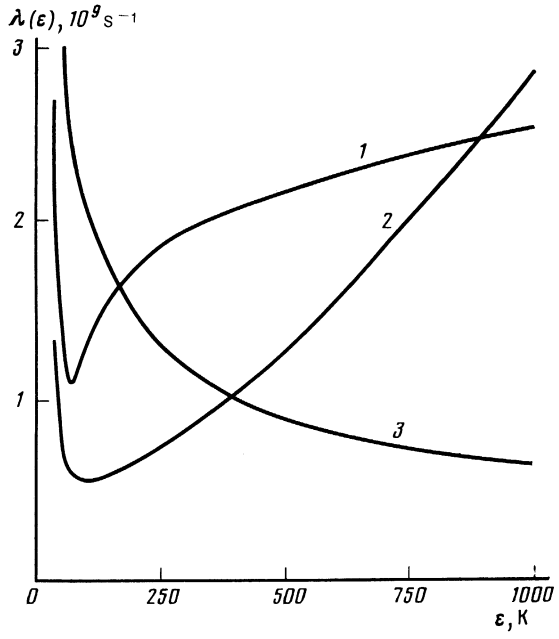

FIG. 4. The rates $\lambda_{FF'}(\varepsilon)$ as functions of the collision energy in the center-of-mass system, determined with allowance for the chemical bonding of nuclei a in the molecules and for the effect of the electronic shells of the molecules: curve 1—$t\mu(F=1)+t\to t\mu(F=1)+t$, curve 2—$t\mu+d\to t\mu+d$, curve 3—$t\mu(F=0)+t\to t\mu(F=0)+t$.

corresponding to $\lambda_{FF'}(\varepsilon)$, will be written in the form

$$\Lambda_{FF'}(E,E')=N_0\varphi[C_d\Sigma_{FF'}^{(d)}(E,E')+C_t\Sigma_{FF'}^{(t)}(E,E')]. \tag{A3}$$

The quantities $\Sigma_{FF'}^{(a)},(E,E')$ are determined by

$$\int\langle v\,d\sigma\rangle=\int\Sigma_{FF'}(E,E')\,dE'\equiv G(E), \tag{A4}$$

where v and $\sigma=\sigma_{FF'}^{(a)}$ are, respectively, the collision rate and the total cross section for the scattering of the $t\mu$-atoms in the center-of-mass system. The angle brackets denote averages over the momenta q_a of nuclei a with the Maxwellian distribution function. Only $\sigma_{FF'}^{(t)}$ and $\Sigma^{(t)}$ have nonzero values in the nonrelativistic approximation for $F\neq F'$.

For $t\mu$-atom energies $E\lesssim 1$ eV, we have the s-wave approximation

$$d\sigma=\sigma(p)\,d\Omega_{p'}/4\pi, \tag{A5}$$

where

$$\mathbf{p}=(M_a\mathbf{q}-M\mathbf{q}_a)/(M_a+M)=\mu_a\mathbf{v},$$
$$\mathbf{p}'=(M_a\mathbf{q}'-M\mathbf{q}_a')/(M_a+M)$$

are the center-of-mass momenta before and after the collision, q, q' and q_a, q_a' are the momenta of the $t\mu$-atoms and the nucleus a before and after collision in the laboratory frame, $\mu_a^{-1}=M^{-1}+M_a^{-1}$, and M and M_a are the masses of the $t\mu$-atoms and of the nucleus a, respectively. The expression for $G(E)$ given by (A4) can be reduced to

$$G(E)=\frac{1}{4\pi}\int v\sigma(p)\,d\Omega_{\mathbf{p}'}f(q_a)\,d^3q_a \tag{A6}$$
$$=\frac{1}{4\pi\mu_a^2}\int d^3p'\,d^3\mathscr{P}'\,\delta(E_f-E_i)\,\delta(\mathbf{q}'+\mathbf{q}_a'$$
$$-\mathbf{q}-\mathbf{q}_a)\,\frac{p}{p'}\sigma(p)f(q_a)\,d^3q_a,$$

where

$$\mathscr{P}'=\mathbf{q}'+\mathbf{q}_a',\quad f(q_a)=(2\pi M_aT)^{-3/2}\exp[-q_a^2/2M_aT],$$
$$E_f-E_i=\frac{p'^2}{2\mu_a}-\frac{p^2}{2\mu_a}$$
$$-\Delta E=\frac{q'^2}{2M}-\frac{q'^2}{2M_a}-\frac{q^2}{2M}-\frac{q_a^2}{2M_a}-\Delta E_{FF'},$$

$\Delta E_{FF'}=0$ for $F=F'$, $\Delta E_{FF'}=0.24$ eV for $F=1$, $F'=0$, and $\Delta E_{FF'}=-0.24$ eV for $F=0$ and $F'=1$.

Assuming that $d^3p'd^3\mathscr{P}'=d^3q'd^3q_a'$, integrating with respect to q_a', and evaluating the integral with respect to the energy $\varepsilon=p^2/2\mu_a$ in (A6), i.e., the collision energy of the $t\mu$-atom in the center-of-mass system, we obtain

$$G(E)=\int_0^\infty d\varepsilon\,\theta(\varepsilon+\Delta E)\left(\frac{\varepsilon}{\varepsilon+\Delta E}\right)^{1/2}\sigma(\varepsilon)\,G(E,\varepsilon), \tag{A7}$$

$$G(E,\varepsilon)=\frac{1}{4\pi\mu_a^2}\int d^3q'\,d^3q_a\,f(q_a)\,\delta\left[\varepsilon-\frac{(M\mathbf{q}_a-M_a\mathbf{q})^2}{2\mu_a(M+M_a)^2}\right]$$
$$\cdot\,\delta\left(\Delta E_{FF'}+\frac{q^2}{2M}+\frac{q_a^2}{2M_a}-\frac{q'^2}{2M}-\frac{1}{2M}(\mathbf{q}+\mathbf{q}_a-\mathbf{q}')^2\right). \tag{A8}$$

Integrating with respect to Ω_q, and $\mathbf{k}$ in (A8), where $\mathbf{k}=\mathbf{q}_a-\mathbf{q}M_a/M$, we obtain

$$\Sigma_{FF'}^{(a)}(E,E')=\frac{\pi(M+M_a)^{5/2}}{(2\pi T)^{3/2}M^2M_aE^{1/2}}e^{E/T}\int_\Omega d\varepsilon\,dx\left(\frac{\varepsilon}{\varepsilon+\varepsilon_{FF'}}\right)^{1/2}$$
$$\cdot\,\sigma_{FF'}(\varepsilon)\exp\left(-\frac{\varepsilon}{T}-\frac{M_a}{MT}x^2\right), \tag{A9}$$

where Ω is the domain on the (x,ε) plane, defined by

$$x>0,\quad\left[x-\left(\frac{ME}{\mu_a}\right)^{1/2}\right]^2<\varepsilon<\left[x+\left(\frac{ME}{\mu_a}\right)^{1/2}\right]^2,$$
$$\left[x-\left(\frac{ME}{\mu_a}\right)^{1/2}\right]^2<\varepsilon+\varepsilon_{FF'}<\left[x+\left(\frac{ME}{\mu_a}\right)^{1/2}\right]^2.$$

The principle of detailed balance is valid for the quantities $\Sigma_{FF'}^{(a)}(E,E')$. In view of (A3), it is also valid for the total reaction rates (A1):

$$\Lambda_{FF'}(E,E')=\frac{2F'+1}{2F+1}\left(\frac{E'}{E}\right)^{1/2}\exp\left(\frac{\Delta E_{FF'}+E-E'}{T}\right)\Lambda_{F'F}(E',E). \tag{A10}$$

When the temperature of the mixture is $T\ll E$, the principal contribution to the rate $\Lambda_{10}(E,E')$ is provided by energies $\varepsilon\sim T$ and, consequently, the following approximate expressions are valid (here, and henceforth, $\lambda_{10}^0=\sigma_{10}vN_0|_{v\to 0}\approx 0.9\cdot 10^9\ \mathrm{s}^{-1}$; Ref. 12):

$$\sigma_{10}(\varepsilon)\approx\frac{\lambda_{10}}{2N_0}\left(\frac{M_t}{\varepsilon}\right)^{1/2},$$
$$\Lambda_{10}(E,E')\approx\lambda_{10}^0\varphi C_t\left(\frac{2}{\pi E\Delta E}\right)^{1/2}\exp\left(-\frac{E}{T}\right) \tag{A11}$$
$$\cdot\,F\left[\left(E'-\frac{\Delta E}{2}\right)\left(\frac{2}{T\Delta E}\right)^{1/2},\ \left(\frac{E}{T}\right)^{1/2}\right],$$

$$\lambda_{10}(E)=\int_0^\infty\lambda_{10}(E,E')\,dE'\approx\lambda_{10}^0\varphi C_t, \tag{A12}$$

$$\Lambda_{01}(E,E')\approx 3\lambda_{10}{}^{0}\varphi C_t\left(\frac{2}{\pi E\,\Delta E}\right)^{1/2}\exp\left(\frac{E-\Delta E-2E'}{T}\right)$$
$$\cdot F\left[\left(E-\frac{\Delta E}{2}\right)\left(\frac{2}{T\Delta E}\right)^{1/2},\left(\frac{E'}{T}\right)^{1/2}\right], \quad \text{(A13)}$$

$$\lambda_{01}(E)\approx {}^{3}/_{2}\lambda_{10}{}^{0}\varphi C_t\frac{T}{(E\Delta E)^{1/2}}\exp\left[\left(\frac{E-\Delta E}{T}\right)-\frac{(E-\Delta E/2)^2}{T\Delta E}\right]$$
$$F(y,z)=\int_{y}^{\infty}dx\exp(-x^2)\,\mathrm{sh}(2zx). \quad \text{(A14)}$$

Expressions (A11), (A13), and (A14) are accurate to order $(T/\Delta E)^{1/2}$.

It follows from (A13) and (A14) that, when $T\ll E$, the $(F=0)\to(F=1)$ transition occurs mostly in collisions between the $t\mu$-atom and the nucleus t, traveling in opposite directions with energies $\sim\Delta E/2$. We also note that the $(F=1)\to(F=0)$ transition is accompanied, according to (A11), by the appearance of the mesic atoms $(t\mu)_{F=0}=0$ with energy $E'\approx\Delta E/2=0.12$ eV and energy spread $|E'-\Delta E/2|\sim(T\Delta E)^{1/2}$.

APPENDIX B. SLOWING DOWN ON FAST $t\mu$-ATOMS

Muon-capture (3) and spin-flip (4) processes result in the appearance of fast $(t\mu)_{1s}$ mesic atoms with energies $E_0\sim 19$ and 0.12 eV, respectively. The slowing down of the $t\mu$ atoms from $E_0\sim 19$ eV to $E\sim 1$ eV occurs in the single charge-transfer event

$$t\mu(\sim 19\,\mathrm{eV})+t(\sim T)\to t+t\mu(\sim 1\,\mathrm{eV}).$$

The rate of this process is $\lambda_{ex}=C_t N\sigma_{ex}(E_0)v_0\sim 2\times 10^{10}\varphi C_t$ [s^{-1}], where σ_{ex} is the charge-transfer cross section, calculated in Ref. 12. The energy of the mesic atom falls by a factor of approximately two in each successive collision, and the time taken to slow down from the initial energy E_i to $E\sim T$ is

$$\tau\sim[\min\lambda_{FF}(\varepsilon)]^{-1},$$

where $T<\varepsilon<E$, and

$$\lambda_{FF}(\varepsilon)=Nv\sigma(\varepsilon)=C_t\lambda_{FF}^{(t)}(\varepsilon)+C_d\lambda^{(d)}(\varepsilon).$$

The partial rates $\lambda^{(d)}$ and $\lambda^{(t)}$ of the slowing down of the $t\mu$-atoms are shown in Fig. 4. When $0.1\leqslant C_t\leqslant 0.9$, the expression for $\lambda_{FF}(\varepsilon)$ has a minimum at $\varepsilon\sim 0.1$ eV. Hence,

$$\tau^{-1}\sim 10^{8}\varphi\,[\mathrm{s}^{-1}]. \quad \text{(B1)}$$

Thus, fast mesic atoms in the 1s state slow down in a time of order $\lambda_{ex}{}^{-1}+\tau$ for an initial energy $E_0\sim 19$ eV, and in a time of order τ for $E_0=0.12$ eV.

Let us now analyze the part played by the isotope-exchange reaction (3) in the kinetics of $dt\mu$ production. We shall divide the $t\mu$-atoms into two groups, namely, fast atoms with energies $E\simeq E_0=19$ eV and slow atoms with $E\sim T$. Neglecting processes in which the $dt\mu$-molecules are produced in collisions between fast $t\mu$-atoms ($\lambda_f\simeq 0$), we find that the ratio of the number of fast and slow mesic atoms, determined from the kinetic equations and corresponding to the processes of Fig. 5, can be estimated from the formula

$$\xi=N_{\mathrm{f}}/N_{\mathrm{s}}\approx qC_d\lambda_{\mathrm{s}}(\lambda_{\mathrm{ex}}^{-1}+\tau), \quad \text{(B2)}$$

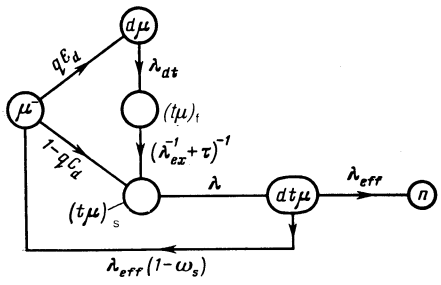

FIG. 5. Simplified schematic diagram of the slowing down of $t\mu$-atoms produced in the reaction $(d\mu)_{1s}+t\to d+(t\mu)_{1s}$.

where λ_s is the rate of production of mesic molecules from slow mesic atoms. The effective rate of production of the mesic molecule is

$$\lambda=(\lambda_{\mathrm{s}}N_{\mathrm{s}}+\lambda_{\mathrm{f}}N_{\mathrm{f}})(N_{\mathrm{s}}+N_{\mathrm{f}})^{-1}\approx\lambda_{\mathrm{s}}/(1+\xi), \quad \text{(B3)}$$

i.e., the parameter ζ is a correction to the rate λ. This correction is particularly significant for small φ and C_t. We then have $q\approx(1+160\varphi C_t)^{-1}$ (see Refs. 32 and 33) and, using (B1) and (B2), we obtain the estimate

$$\xi=0.003(0.05+C_t)\varphi^{-1}C_t^{-2}\ll 1. \quad \text{(B4)}$$

The correction to the rate of production of the mesic molecules that is due to the spin-flip process (4) is shown by estimates to be more substantial, and is therefore explicitly taken into account in (36) and (37). For low densities of the mixture, $\varphi\leqslant 0.05$, an additional source of fast $t\mu$-atoms becomes significant. It is due to the fact that an appreciable proportion of the mesic atoms does not become thermalized in excited states.[31] This case is discussed in Ref. 11, and requires detailed examination.

APPENDIX C. A MODEL FOR THE ESTABLISHMENT OF THE QUASISTATIONARY STATE

Let us divide the D_2+T_2 mixture into two groups of $t\mu$-atoms, namely, fast atoms with energies $E\sim 0.3$ eV and slow atoms with $E\sim T$ (the spin of the mesic atoms is ignored). Figure 6 shows the scheme for processes involving the participation of these mesic atoms, where N_f and N_s are, respectively, the numbers of fast and slow mesic atoms and λ_f and λ_s are the rates of production of the $dt\mu$-molecules in collisions between fast and slow $t\mu$-atoms with the molecules of the mixture.

For $t\gg\lambda_{\mathrm{eff}}^{-1}$, the numbers of the mesic atoms are, respectively, given by (see Fig. 6)

$$N_{\mathrm{f}}(t)=N_{\mathrm{f}}^{0}\left(1-\frac{\lambda_{\mathrm{s}}}{\lambda_{\mathrm{f}}}\right)\exp(-\Lambda_{\mathrm{f}}t)+\overline{N}_{\mathrm{f}}\exp(-\Lambda_{\mathrm{s}}t),$$
$$N_{\mathrm{s}}(t)=-N_{\mathrm{f}}^{0}\left(1-\frac{\lambda_{\mathrm{s}}}{\lambda}\right)\exp(-\Lambda_{\mathrm{f}}t)+\overline{N}_{\mathrm{s}}(-\Lambda_{\mathrm{s}}t), \quad \text{(C1)}$$

where

$$\Lambda_{\mathrm{f}}=\tau^{-1}+N_{\mathrm{s}}^{0}\lambda_{\mathrm{f}}+N_{\mathrm{f}}^{0}\lambda_{\mathrm{s}},\quad \Lambda_{\mathrm{s}}=\lambda_0+\omega_s\lambda_c,$$
$$\lambda_c=\lambda_{\mathrm{s}}(\lambda_{\mathrm{f}}+\tau^{-1})/\Lambda_{\mathrm{f}},\quad \overline{N}_{\mathrm{f}}=N_{\mathrm{f}}^{0}\lambda_{\mathrm{s}}/\Lambda_{\mathrm{f}},\quad \overline{N}_{\mathrm{s}}=1-\overline{N}_{\mathrm{f}}, \quad \text{(C2)}$$

N_f^0 and $N_s^0=1-N_f$ are the numbers of the $t\mu$-atoms in the 1s state for $t=0$, and τ is the time taken by the fast mesic atoms to slow down.

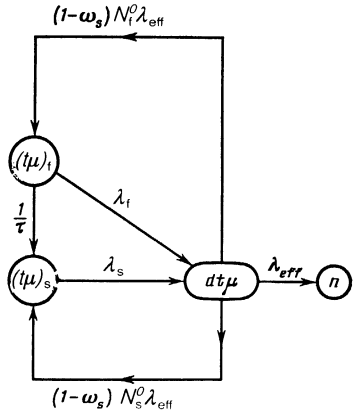

FIG. 6. Processes involving mesic atoms in the two-group approximation for the $t\mu$-atoms.

When $t \gg \Lambda_f^{-1}$, (C1) and (C2) assume the form

$$N_f(t) = N_f \exp(-\Lambda_s t), \quad N_s(t) = N_s \exp(-\Lambda_s t). \tag{C3}$$

Consequently, for $t \gg \Lambda_f^{-1}$, the dependence of the number of $t\mu$-atoms on time and energy can be split into separate factors.

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