A supersymmetric theory of grand unification with automatic hierarchy and low-energy physics

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A mechanism is proposed which allows one to solve naturally the problem of doublet-triplet mass hierarchy in the $SU(5)$ supersymmetric theory. The mechanism consists in making the Higgs doublets be pseudo-Goldstone bosons of a spontaneously broken $SU(6)$ symmetry of the superpotential symmetry which contains the $SU(5)$ gauge symmetry as a subgroup. The doublets remain massless even after the radiative corrections are taken into account, as long as supersymmetry is not violated, but they acquire, in general, a mass of the order of the scale of supersymmetry breaking. The proposed mechanism is investigated in detail for the case of soft supersymmetry breaking by supergravity. The proposed model allows one to calculate the parameters of the low-energy Lagrangian which are usually considered to be phenomenological constants. The breaking of electroweak symmetry is related to radiative corrections. The mass of the lightest scalar Higgs boson also is of a radiative nature and its magnitude turns out to be close to 2.2 GeV. The Yukawa interaction of the t-quark is essential for the spontaneous symmetry breaking. The theory sets an upper bound of 52 GeV for the mass of the t-quark.

1. INTRODUCTION

Any grand unified theory runs into the well-known "hierarchy problem," i.e., the problem of existence of at least two mass scales differing by many orders of magnitude. For the $SU(5)$ theory the masses of the Higgs bosons (a colorless weak doublet and a color triplet which is a weak singlet) belonging to one quintuplet of the group $SU(5)$ must differ by 13–14 orders of magnitude, since the triplet exchange induces proton decay, whereas the doublet is the usual Glashow-Weinberg-Salam doublet. The most difficult part of this problem is naturally solved within the supersymmetric version of grand unification: The radiative corrections do not violate the hierarchy contained in the tree approximation.[1–3] Nevertheless, the desired hierarchy is realized by means of "fine tuning" of the parameters of the initial Lagrangian in the tree approximation.

Consider, for instance, a minimal supersymmetric $SU(5)$ theory. Such a theory contains the following Higgs fields: a $\underline{24}$-uplet Φ, a 5-plet H_1, and a $\underline{5}^*$-plet H_2, which are the scalar components of the chiral superfields $\hat{\Phi}$, $\hat{H}_1$, $\hat{H}_2$. The most general form of the superpotential in such a theory is

$$W=\frac{1}{2}M\,\mathrm{Sp}\,\hat{\Phi}^2+\frac{\lambda}{3}\,\mathrm{Sp}\,\hat{\Phi}^3+f(\hat{H}_2\hat{\Phi}\hat{H}_1)+m\hat{H}_2\hat{H}_1, \quad (1.1)$$

from which we immediately find the following expression for the Higgs potential:

$$V=\mathrm{Sp}|M\Phi+\lambda\Phi^2-{}^1/_5\mathrm{Sp}\,\Phi^2+fH_2\times H_1|^2$$
$$+H_2(m+f\Phi)(m+f\Phi^+)H_2^+ + H_1^+(m+f\Phi^+)(m+f\Phi)H_1. \quad (1.2)$$

The supersymmetric minimum of $V(V=0)$

$$\langle\Phi\rangle=\frac{M}{\lambda}\begin{bmatrix}2&&&&\\&2&&&\\&&2&&\\&&&-3&\\&&&&-3\end{bmatrix},\quad \langle H_1\rangle=\langle H_2\rangle=0 \quad (1.3)$$

corresponds to spontaneous breaking $SU(5)\to SU(3)\times SU(2)\times U(1)$ of the symmetry. It follows from Eq. (1.1) that the masses of the H_1 and H_2 are equal to $m+f\langle\Phi\rangle$, i.e., the mass of the triplet is $m+3fM/\lambda$ and that of the doublet is $m-3fM/\lambda$. Thus, in order to have massless doublets it is necessary to impose the relation

$$m=3fM/\lambda. \quad (1.4)$$

Eq. (1.4) guarantees the masslessness of the doublet Higgs fields in the tree approximation. The radiative corrections lead, in general, to some renormalization of the parameters M, λ, f, m which appear in the superpotential (1.1), but only on account of wave-function renormalizations. This means that the particles whose masses vanish in the tree approximation remain massless also when the radiative corrections are taken into account (the mass terms in the Lagrangian acquire only Dyson factors Z related to the renormalization of the appropriate wave functions). Thus, the masslessness of the doublets is not violated by radiative corrections and the doublets acquire a mass only at a scale where supersymmetry is violated. Nevertheless, the relation (1.4) in itself seems extremely artificial.

Earlier a series of authors have attempted to explain the doublet-triplet hierarchy in a natural manner. In Ref. 4 this hierarchy was guaranteed by a special group structure of the Higgs sector (the "omitted-partner mechanism"). Unfortunately, this leads to a very complicated Higgs content of the theory. In Ref. 5 the hierarchy was the result of a "sliding singlet" mechanism.[6] Later some criticism was expressed with regard to this mechanism, in particular, arguments which indicated that the theory must be unstable with respect to a violation of supersymmetry in the hidden sector at an energy $\sim 10^{10}$ GeV (Ref. 7).

A relatively simple method was proposed in Refs. 8, 9. It implies, however, either explicit,[9] or less explicit[8] fine tuning. In implicit form this amounted to assuming the equality of the coupling constants of the singlet and 24-uplet of Higgs fields. This equality is not related to any group symmetry and represents a variety of fine tuning. The exact masslessness of the doublets was guaranteed only by supersymmetry and, like the above-mentioned relation (1.4), it does not seem natural.

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In Refs. 10 a local $SU(6)$ symmetry was considered. Here again one must deal with an extremely complicated Higgs sector.

Finally, in connection with superstring models, the group E_6 was discussed in Ref. 11.

We restrict our attention to the simplest $SU(5)$-supersymmetric theory with an almost minimal Higgs content. We shall propose a very simple mechanism which automatically guarantees the masslessness of doublets at the grand unification scale M. In the proposed model the doublets are pseudo-Goldstone bosons of a definite broken global symmetry of the superpotential which is not a symmetry of the whole theory. More precisely: one of the two Higgs doublets is a pseudo-Goldstone boson, whereas the second remains massless as the superpartner of the first doublet. When the explicit soft supersymmetry breaking on account of supergravity[12] is included, the first doublet remains massless. This occurs because the supersymmetry-violating terms also exhibit the putative global symmetry. Therefore the spectrum contains, as before, a pseudo-Goldstone particle. The second Higgs boson acquires a mass of the order $m_{3/2}$, the gravitino mass. Finally, the radiative corrections from the gauge and Yukawa interactions lead to a nonzero mass also for the first Higgs doublet, since these interactions do not exhibit the global symmetry under discussion and supersymmetry is broken. Thus, the mass of the lightest Higgs boson, just like the breaking of the standard group $SU(5)\to SU(3)\times SU(2)\times U(1)$, is due to radiative corrections.

Obviously, this scenario consists of two distinct parts: the first part relates to a theory with unbroken supersymmetry, where strict masslessness of the doublets is assured. It seems that this mechanism has a relatively general character, we called it GIFT ("Goldstones Instead of Fine Tuning"). The second part is related to the explicit form of supersymmetry breaking; here it turns out to be possible to develop an effective low-energy theory and derive definite predictions for experiments. The discussion here is similar to that in Refs. 13–18. In fact, the GIFT mechanism allows one to calculate the values of the parameters of the effective low-energy Lagrangian which are usually assumed to be arbitrary phenomenological constants. As in Refs. 13–18, it turns out that the Yukawa coupling is essential for the violation of the electroweak group, particularly the Yukawa interaction of the heaviest t-quark. The t-quark mass is found to be of order 40–50 GeV and its exact value depends on an unknown numerical parameter A, the magnitude of which is determined by the properties of the theory in the hidden supergravity sector. There exists, however, an upper bound of 52 GeV on the mass of this quark, independent of A. The most interesting quantity which, it appears, can be computed almost exactly, is the mass of the lightest Higgs boson. It is equal to 2.25 GeV.

The paper is organized as follows. In Sec. 2 the GIFT mechanism is described in the context of the simplest supersymmetric $SU(5)$ theory. In Sec. 3 a consistent low-energy model is formulated on the basis of the assumption that supersymmetry is broken on account of supergravity. In Sec. 4 the renormalization of the parameters of the effective low-energy Lagrangian is calculated, the question of breaking of the electroweak group is considered, and the masses of all particles are estimated. In Sec. 5 a slightly different mechanism responsible for the electroweak breaking is considered—a mechanism of the Coleman-Weinberg type. Finally, Sec. 6 summarizes the results.

2. A SOLUTION TO THE HIERARCHY PROBLEM

The simplest possible Higgs content in a supersymmetric $SU(5)$ grand unified theory consists of two quintuplets, $H_1\sim\underline{5}$ and $H_2\sim\underline{5}^*$, and one 24-uplet, $\Phi\sim\underline{24}$. All these fields are scalar components of the corresponding chiral fields, which we denote by $\hat{H}_1$, $\hat{H}_2$, $\hat{\Phi}$. To these fields we add the $SU(5)$-singlet chiral field $\hat{\varphi}$. The 35 resulting fields can be assigned to the unique adjoint representation of the group $SU(6)$ which contains $SU(5)$ as a subgroup. If $\hat{\Sigma}\sim\underline{35}$ of the group $SU(6)$ then the reduction of $\hat{\Sigma}$ into $\hat{H}_1$, $\hat{H}_2$, $\hat{\Phi}$ and $\hat{\varphi}$ has the form

$$\hat{\Sigma}=\begin{vmatrix} -5\hat{\varphi}/30^{1/2}, & \hat{H}_1 \\ \hat{H}_2, & \hat{\Phi}_{ij}+\delta_{ij}\hat{\varphi}/30^{1/2} \end{vmatrix}, \tag{2.1}$$

where the "hypercharge" matrix of $SU(6)$, $Y_{35}=[-5/30, 1/30,..., 1/30]_{\rm diag}$ is normalized to unity, $\mathrm{Tr}\, Y_{35}^2=1$, as are the other generators.

We assume that the Higgs sector of the theory exhibits a higher symmetry than the $SU(5)$ gauge symmetry, namely the global $SU(6)$ symmetry. Then the superpotential which depends only on the one field $\hat{\Sigma}$ has the simplest form

$$W={}^1/_2 M\,\mathrm{Sp}\,\hat{\Sigma}^2+{}^1/_3\lambda\,\mathrm{Sp}\,\hat{\Sigma}^3. \tag{2.2}$$

In terms of the $SU(5)$ fields Eq. (2.2) implies, of course, a relation between the constants of the superpotential expressed in terms of Φ, H_1, H_2, φ. From (2.2) one obtains easily the following expression for W:

$$W=\frac{1}{2}M\,\mathrm{Sp}\,\hat{\Phi}^2+M(\hat{H}_2\hat{H}_1)+\frac{1}{2}M\hat{\varphi}^2+\frac{1}{3}\lambda\,\mathrm{Sp}\,\hat{\Phi}^3$$
$$+\lambda(\hat{H}_2\hat{\Phi}\hat{H}_1)-\frac{2}{3}\left(\frac{2}{15}\right)^{1/2}\lambda\hat{\varphi}^3+\frac{\lambda}{30^{1/2}}(\mathrm{Sp}\,\hat{\Phi}^2)\hat{\varphi}$$
$$-2\left(\frac{2}{15}\right)^{1/2}\hat{H}_2\hat{H}_1\hat{\varphi}. \tag{2.3}$$

For the general case of $SU(n)$ symmetry a superpotential of the type (2.2) ($\hat{\Sigma}\sim n^2-1$) leads to the potential

$$V=\mathrm{Sp}\left|M\Sigma+\lambda\Sigma^2-\frac{\lambda}{n}\mathrm{Sp}\,\Sigma^2\right|^2, \tag{2.4}$$

where $|M\Sigma+...|^2$ denotes the product of the matrix

$$\left(M\Sigma_\beta{}^\alpha+\lambda(\Sigma^2)_\beta{}^\alpha-\delta_\beta{}^\alpha\frac{\lambda}{n}\mathrm{Sp}\,\Sigma^2\right)$$

by its hermitean adjoint.

There exists a series of supersymmetric vacua with zero energy, corresponding to the symmetry breaking:

$$SU(n)\to SU(m)+SU(n-m)\times U(1) \tag{2.5}$$

($m=1, ..., n-1$). The vacuum expectation values $\langle\Sigma\rangle$ in these vacua are

$$\langle\hat{\Sigma}\rangle=\left|\begin{array}{c|c} \begin{matrix} x & & \\ & \ddots & \\ & & x\end{matrix} & \\ \hline & \begin{matrix} y & & \\ & \ddots & \\ & & y\end{matrix} \end{array}\right| \begin{matrix} m \\ \\ n-m \end{matrix},$$

$$x=\frac{M}{\lambda}\frac{n-m}{2m-n},\quad y=\frac{M}{\lambda}\frac{-m}{2m-n}. \tag{2.6}$$

Any of these zero-energy states can be chosen as the real physical ground state.

We assume that for the $SU(6)$-symmetric potential the ground state corresponds to an unbroken $SU(4)\times SU(2)\times U(1)$ symmetry, i.e.,

$$\langle\Sigma\rangle=\frac{M}{\lambda}\begin{vmatrix}1&&&&&\\&1&&&&\\&&1&&&\\&&&1&&\\&&&&-2&\\&&&&&-2\end{vmatrix}. \tag{2.7}$$

As can be seen from the decomposition (2.1), in terms of the $SU(5)$ classification this value of $\langle\Sigma\rangle$ corresponds to:

$$\langle\varphi\rangle=-\frac{M}{\lambda}\left(\frac{6}{5}\right)^{1/2},$$
$$\langle\Phi\rangle=\frac{M}{\lambda}\begin{vmatrix}6/5&&&&\\&6/5&&&\\&&6/5&&\\&&&-9/5&\\&&&&-9/5\end{vmatrix}. \tag{2.8}$$

The $SU(5)$ gauge symmetry is broken to $SU(3)\times SU(2)\times U(1)$, as required. On the other hand,, as can be seen from Eq. (2.7) there remains an additional unbroken symmetry between color and the $SU(5)$-singlet φ. The total number of spontaneously broken generators is $35-(15+3+1)=16$. This implies the appearance of 16 Goldstone bosons, of which 12 are eaten by the Higgs mechanism and produce the masses of the X and Y gauge bosons of the group $SU(5)$. The four remaining Goldstone states are weak $SU(2)$ doublets. This is easily seen from the structure of $\langle\Sigma\rangle$. The Goldstone bosons under discussion are related to broken generators which reduce the $SU(5)$ singlet into a $SU(2)$ doublet, and vice versa. Thus, we can expect in this sector at least one massless Higgs doublet (four real degrees of freedom). It is easy to see, however, that supersymmetry guarantees the masslessness of both doublets which occur in the quintuplets H_1 and H_2.

In order to understand this better we imagine for a minute that the whole $SU(6)$ symmetry is a gauge symmetry. Then for a real vacuum expectation $\langle\Sigma\rangle$ and purely imaginary generators acting in the adjoint representation $\underline{35}$, only the hermitean part of $\Sigma+\Sigma^+$ is mixed with the gauge bosons. The hermitean part of $\Sigma+\Sigma^+$ contains the quintuplet $H_1+H_2^+$, the doublet part of which is, consequently, a Goldstone boson. The mass of the doublet which occurs in $H_1-H_2^+$ also vanishes, but only on account of the supersymmetry. This is nothing but the degeneracy of the two real scalar degrees of freedom of any chiral field.

These field-theory arguments can be verified directly. For this it suffices to make the substitution:

$$\hat\Phi=\langle\Phi\rangle+\hat\Phi',\quad \hat\varphi=\langle\varphi\rangle+\hat\varphi' \tag{2.9}$$

and to express the superpotentials in terms of $\hat\Phi'$ and $\hat\varphi'$. The legitimacy of such a procedure is based on the fact that supersymmetry is not violated. This means that no F-terms develop any nonzero vacuum expectation values. The occurrence of $\langle\hat\Phi\rangle\neq0$ and $\langle\hat\varphi\rangle\neq0$ can then be understood as the appearance of nonzero vacuum expectation values $\langle\hat\Phi\rangle\neq0$ and $\langle\hat\varphi\rangle\neq0$ with nonzero A-components. Subtracting $\langle\hat\Phi\rangle$ and $\langle\hat\varphi\rangle$ from $\hat\Phi$ and $\hat\varphi$, respectively, we arrive at (2.9). If the supersymmetry were broken, we could not have made use of the superpotential and would be forced to discuss the potential V itself (cf. infra).

From Eqs. (2.9) and (2.3) we obtain

$$\begin{aligned}W=&\hat H_2 m\hat H_1+\frac{3}{2}M\,\mathrm{Sp}(\hat A^2-\hat D^2)\\&+\frac{13}{10}M\hat\varphi'^2+\left(\frac{6}{5}\right)^{1/2}M\,\mathrm{Sp}\,\hat A\varphi'\\&+\frac{1}{3}\lambda\,\mathrm{Sp}\,\hat\Phi'^3-\frac{2}{3}\left(\frac{2}{15}\right)^{1/2}\lambda\varphi'^3+\lambda(\hat H_2\hat\Phi'\hat H_1)\\&+\frac{\lambda}{30^{1/2}}\hat\varphi'\,\mathrm{Sp}\,\hat\Phi'^2\\&-2\left(\frac{2}{15}\right)^{1/2}\lambda\hat H_2\hat H_1\hat\varphi',\end{aligned} \tag{2.10}$$

$$m=M\begin{vmatrix}3,&0\\0,&0\end{vmatrix},\quad \hat\Phi'=\begin{vmatrix}A,&B\\C,&D\end{vmatrix},\quad \mathrm{Sp}\,\hat A=-\mathrm{Sp}\,\hat D.$$

Here we have partially used notation (for m and $\hat\Phi'$) where the color triplet and weak doublet parts are explicitly separated. Accordingly, A is a 3×3 matrix, D is a 2×2 matrix, and B and C are 2×3 and 3×2 matrices. Similar notations are understood for the matrix M. We see that the doublets in H_1 and H_2 remain massless. With the help of Eq. (2.10) we can describe the whole spectrum of particles in the theory. In addition to the massless Higgs doublets and their fermionic superpartners there are:

1) massive color triplets of chiral superfields (scalar and fermionic components) of mass $3M$, belonging to the quintuplets $\hat H_1$ and $\hat H_2$;

2) twelve massive gauge bosons of $SU(5)$ (the X and Y bosons) which have absorbed by means of the Higgs mechanism the Goldstone bosons $(\Phi+\Phi^+)_{\mathrm{offdiag}}=B+C^+$. The mass of these bosons equals $M_X=M_Y=(3/2)^{1/2}g(M/\lambda)$. The scalar bosons $B-C^+$ acquire masses on account of the D-term:

$$\begin{aligned}D_a^2=&\frac{g^2}{8}(\Phi^+t^a\Phi)^2=\frac{1}{4}g^2\,\mathrm{Sp}[\Phi^+,\Phi]^2\to(\Phi=\langle\Phi\rangle+\Phi')\\&\to\frac{1}{4}g^2\,\mathrm{Sp}[\langle\Phi\rangle,\Phi'-\Phi'^+]^2\\&=\frac{9}{2}g^2\left(\frac{M}{\lambda}\right)^2\mathrm{Sp}(B-C^+)(B^+-C).\end{aligned} \tag{2.11}$$

These bosons are degenerate with the X and Y gauge bosons and represent together $12\times3+6\times2=48$ bosonic degrees of freedom. Simultaneously, the theory contains 12 Dirac fermions of the same mass ($12\times4=48$ fermionic degrees of freedom). The left-handed components of these fermions are the fermionic components of the chiral fields B and C, and their right-handed components are the X- and Y-gauginos;

3) There remain the $\hat A$ and $\hat D$ chiral superfields. The traceless part of $\hat A$ transforms like the $(\underline{8},\underline{1})$ representation under $SU(3)_C\times SU(2)_L$, and the traceless part of $\hat D$ transforms under $(\underline{1},\underline{3})$. The mass of these fields is $3M$. The fields $\hat A$ and $\hat D$ also contain a field $\hat\Phi_{24}$, related to the weak hypercharge $Y_{24}=\lambda_{24}/2^{1/2}$:

$$\frac{\lambda_{24}}{2^{1/2}}=\left(\frac{2}{15}\right)^{1/2}\begin{vmatrix}1&&&&\\&1&&&\\&&1&&\\&&&-3/2&\\&&&&-3/2\end{vmatrix}. \qquad (2.12)$$

The field Φ_{24} mixes with the $SU(5)$-singlet $\hat{\varphi}$. The two states:

$$\hat{X}_1=\frac{3}{10^{1/2}}\hat{\varphi}+\frac{1}{10^{1/2}}\hat{\Phi}_{24},\quad \hat{X}_2=-\frac{1}{10^{1/2}}\hat{\varphi}+\frac{3}{10^{1/2}}\hat{\Phi}_{24}, \qquad (2.13)$$

have the masses $3M$ and M, respectively.

The inclusion of the radiative corrections leads, in general, to some change in the particle masses. Since, however, for unbroken supersymmetry the radiative corrections reduce to a wave-function renormalization, only nonvanishing masses will change, whereas massless doublet fields remain massless.

3. SUPERGRAVITY AND THE LAGRANGIAN OF LIGHT FIELDS

In order to formulate a consistent low-energy theory it is necessary to indicate the mechanism which violates supersymmetry. We assume that a soft supersymmetry breaking occurs on account of supergravity, which seems to be the most popular current version.

Then, in place of the potential (2.4) (for $n = 6$) we have[21]

$$V=\mathrm{Sp}\left|M\Sigma+\lambda\Sigma^2-\frac{\lambda}{6}\mathrm{Sp}\,\Sigma^2+m_{3/2}\Sigma^+\right|^2$$
$$+(A-3)m_{3/2}\mathrm{Sp}\left(\frac{1}{2}M\Sigma^2+\frac{1}{2}M\Sigma^{+2}+\frac{1}{3}\lambda\Sigma^3+\frac{1}{3}\lambda\Sigma^{+3}\right), \qquad (3.1)$$

where $m_{3/2}$ is the gravitino mass ($m_{3/2}\sim 100$ GeV) and A is a numerical parameter depending on the hidden sector of the theory (in general $A\sim 1$).

Since $m_{3/2}\ll M$ we search for a minimum of V in the form (2.7) with an extra multiplier,

$$\langle\Sigma\rangle=x\frac{M}{\lambda}\begin{vmatrix}1&&&&&\\&1&&&&\\&&1&&&\\&&&1&&\\&&&&-2&\\&&&&&-2\end{vmatrix} \qquad (3.2)$$

and expand x in powers of $m_{3/2}/M$. A direct computation yields

$$x=1+m_{3/2}/M+(A-3)m_{3/2}^2/M^2+O(m_{3/2}^3/M^3). \qquad (3.3)$$

In order to obtain the masses of the scalar doublets H_1 and H_2 (since in the sequel we shall no longer be interested in triplets we retain for the doublets the notations H_1 and H_2 which was used before for the full quintuplets) it is necessary to turn to the potential which follows from the superpotential (2.3). As explained already, in the case when the supersymmetry is broken we can no longer use the superpotential. We shall not write out the whole complicated expression which arises from the substitution $\Sigma=\langle\Sigma\rangle+\Sigma'$ ($\Phi=\langle\Phi\rangle+\Phi'$, $\varphi=\langle\varphi\rangle+\varphi'$) in the potential containing H_1 and H_2. We list only the final result and limit ourselves to a few remarks. There are two sources for the appearance of masses for the scalars H_1 and H_2: a) the deviation of x in Eq. (3.2) from unity and b) a "direct contribution of supergravity" of the type (3.1). A straightforward calculation shows that these two contributions cancel each other in the leading approximation for the squared masses of H_1 and H_2, i.e., to order $M^2(m_{3/2}/M)=Mm_{3/2}$. It is known that this property is generic.[21] In the next order one can obtain an expression for the mass terms:

$$V=2m_{3/2}(H_1^+H_1+H_2^+H_2-H_1H_2-H_1^+H_2^+)$$
$$=4m_{3/2}^2\left(\frac{H_1^+-H_2}{2^{1/2}}\right)\left(\frac{H_1-H_2^+}{2^{1/2}}\right). \qquad (3.4)$$

Thus, the field $(H_1-H_2^+)/2^{1/2}$ acquires the mass $2m_{2/3}$, whereas the field $(H_1+H_2^+)/2^{1/2}$ remains massless. This is not surprising since the potential (3.1) does not break $SU(6)$ explicitly and the state $(H_1+H_2^+)/2^{1/2}$ remains a Goldstone boson. It is curious that the mass of the boson $(H_1-H_2^+)/2^{1/2}$ turns out to be independent of the parameter A.

Another mass term which appears as a result of the supersymmetry breaking is the mass of the spinor components H_1 and H_2, i.e., the higgsino mass. It is easy to see that this mass appears in first order in $(m_{3/2}/M)$ for the quantity $x-1$. A simple calculation leads to the following result for the corresponding mass term

$$-m_{3/2}(\tilde{H}_1C\tilde{H}_2)+\text{h.c.}, \qquad (3.5)$$

where H_1 and H_2 are the left-handed higgsino fields and C is the charge conjugation matrix.

Let us compare these results with the "phenomenological" low-energy Lagranian of generic form.[22] It is obvious that the most general form for the superpotential containing the chiral superfields H_1 and H_2 can be written in the form

$$W=-\mu_0(\hat{H}_1\hat{H}_2). \qquad (3.6)$$

For supersymmetry breaking by supergravity we obtain for the Higgs potential of the scalar fields

$$V(H_1,H_2)=(\mu_0^2+m_{3/2}^2)(H_1^+H_1+H_2^+H_2)$$
$$-Bm_{3/2}\mu_0(H_1H_2+H_1^+H_2^+)$$
$$+{}^1/_8g'^2(H_1^+H_1-H_2^+H_2)^2+{}^1/_8g^2[H_1^+\tau H_1+H_2^+(-\tau^*)H_2]^2. \qquad (3.7)$$

The last terms represent D-terms, and B is an unknown parameter. A comparison of (3.7) with Eq. (3.4) yields

$$\mu_0=m_{3/2},\quad B=2. \qquad (3.8)$$

The value $\mu_0=m_{3/2}$ also agrees with the higgsino mass value (3.5). Thus, the GIFT mechanism allows one to calculate the phenomenological parameters of the low-energy Lagrangian.

4. RENORMALIZATION AND BREAKING OF THE ELECTROWEAK SYMMETRY

One can now analyze the question of breaking of the electroweak symmetry. Retaining in Eq. (3.7) only

$$\langle H_1^0\rangle=v_1/2^{1/2},\quad \langle H_2^0\rangle=v_2/2^{1/2}, \qquad (4.1)$$

we have

$$V=\frac{1}{2}\mu_1^2v_1^2+\frac{1}{2}\mu_2^2v_2^2-\mu_3^2v_1v_2+\frac{\bar{g}^2}{32}(v_1^2-v_2^2)^2,$$
$$\mu_1^2=\mu_2^2=\mu_0^2+m_{3/2}^2,\quad \mu_3^2=Bm_{3/2}\mu_0,\quad \bar{g}^2=g^2+g'^2, \qquad (4.2)$$

or, taking into account the relations (3.8),

$$\mu_1^2=\mu_2^2=\mu_3^2=2m_{3/2}^2. \tag{4.3}$$

In the general case, when $\mu_1^2 \neq \mu_2^2 \neq \mu_3^2$, there exist general conditions for the breaking of the symmetry. The following inequalities must hold

$$\tfrac{1}{2}(\mu_1^2+\mu_2^2) \geqslant \mu_3^2 \geqslant (\mu_1^2\mu_2^2)^{1/2}. \tag{4.4}$$

The first of these is necessary in order to guarantee the stability along the line $v_1 = v_2$. If it is not satisfied, $V \to -\infty$ when $|v_1| = |v_2| \to \infty$. The second condition provides the instability at the point $v_1 = v_2 = 0$, since at that point $\|\partial^2 V/\partial v_i \partial v_j\| < 0$.

The equation (4.3) shows that we are at the boundary of spontaneous symmetry breaking. Since, however, the $SU(6)$ symmetry is in fact a pseudosymmetry (it does not refer to gauge or Yukawa couplings), it is not indifferent for which normalization momentum one assumes the existence of this symmetry, and consequently the validity of the relations (4.3). Since (4.3) appears in the grand unified theory it seems reasonable to assume that (4.3), i.e, the $SU(6)$-symmetry of the superpotential, is valid at the grand unification scale (or even at the Planck mass scale). Then for energies ~ 100 GeV, $\mu_1^2 \neq \mu_2^2 \neq \mu_3^2$, owing to renormalization effects. The renormalization of the parameters μ_1, μ_2, μ_3, from the grand unification mass to low energies (~ 100 GeV) was discussed in the papers 13–18. We shall follow closely Refs. 17 and 18.

We first consider only gauge coupling and neglect the Yukawa interactions. Then the relation $\mu_1^2 = \mu_2^2$ remains valid, but $\mu_3^2 \neq \mu_1^2 = \mu_2^2$. The inequalities (4.4) are not satisfied and symmetry breaking is impossible. This means that it is necessary to include Yukawa coupling which is different for H_1 and H_2 and therefore leads to $\mu_1^2 \neq \mu_2^2$. From all these couplings we retain only the Yukawa interaction of the t-quark and the t-quarkino (also known as the t-squark):

$$L_Y=h_t[\hat{\bar{t}}_R(\hat{H}_1\varepsilon\hat{Q}_L)]_{F\text{-term}}=h_t\hat{\bar{t}}(\hat{H}_1{}^0\hat{t}_L-\hat{H}_1{}^+\hat{b}_L)_F. \tag{4.5}$$

The renormalization-group equations for the case we are interested in were first derived in Ref. 13. In the notations of Ref. 18 we have

$$\begin{aligned}
\mu_1^2(l_0)&=\mu_0^2q^2(l_0)+m_{3/2}^2a_1(l_0)=m_{3/2}^2[q_1^2(l_0)+a_1(l_0)],\\
\mu_2^2(l_0)&=\mu_0^2q^2(l_0)+m_{3/2}^2a_2(l_0)=m_{3/2}^2[q_1^2(l_0)+a_2(l_0)],\\
\mu_3^2(l_0)&=m_{3/2}\mu_0Bq(l_0)[\tilde{B}(l_0)/B]=2m_{3/2}^2[q(l_0)\tilde{B}(l_0)/B],\\
l_0&=\ln(M_{GUT}^2/M_W^2).
\end{aligned} \tag{4.6}$$

Making use of the equations listed in the Appendix[1)] to Ref. 18, we can determine the functions $q(l_0)$, $a_1(l_0)$, $a_2(l_0)$, and $\tilde{B}(l_0)$. For simplicity we assume that the mass of the gaugino vanishes: $M_{1/2} = 0$ (i.e., $\tilde{\gamma} = M_{1/2}/m_{3/2} = 0$), which although literally unacceptable from a phenomenological point of view, corresponds to "minimal coupling" to supergravity. We then obtain

$$\begin{aligned}
q(l_0)&=\frac{[\lambda_2(l_0)]^{3/2b_2}}{[\lambda_1(l_0)]^{1/2b_1}}\frac{1}{[D(l_0)]^{1/4}},\\
\lambda_j(l_0)&=1+b_j\frac{g_j^2(0)}{16\pi^2}\ln\frac{M_{GUT}^2}{M_W^2},\\
\frac{g_3^2(0)}{4\pi}&=\frac{g_2^2(0)}{4\pi}=\frac{5}{3}\frac{g_1^2(0)}{4\pi}=\frac{1}{24},\\
b_1&=11,\quad b_2=1,\quad b_3=-3,\quad M_{GUT}\approx 6\cdot 10^{15}\text{ GeV},\quad l_0\approx 64.
\end{aligned} \tag{4.7}$$

The gauge coupling constants $g_3(0)$, $g_2(0)$, $g_1(0)$ in (4.7) related to the group $SU(3)\times SU(2)\times U(1)$, are defined at the grand unification scale M_{GUT}. The function $D(l)$ describes the evolution of the Yukawa constant $h_t(l)$. The relation between the quantity $h_t(l_0)$, which determines directly the mass of the t-quark $m_t = h_t(l_0)v_1/2^{1/2}$, and the coupling constant $h_t(0)$ at M_{GUT} is given by the relations

$$\begin{aligned}
[h_t(0)]^2&=[h_t(l_0)]^2\frac{D(l_0)}{E(l_0)},\\
D(l_0)&=\left[1-\frac{6F(l_0)}{E(l_0)}\frac{[h_t(l_0)]^2}{16\pi^2}\right]^{-1},\\
E(l_0)&=\exp\int_0^{l_0}\left[\frac{16}{3}g_3^2(l)+3g_2^2(l)+\frac{13}{9}g_1^2(l)\right]\frac{dl}{16\pi^2},\\
F(l_0)&=\int_0^{l_0}E(l)\,dl.
\end{aligned} \tag{4.8}$$

Whereas the gauge couplings are known, the Yukawa constant $h_t(l_0)$ is not *a priori* fixed (as long as we don't know the mass of the t-quark). Substituting the numerical values of g_1, g_2, g_3 we obtain[18]

$$\begin{aligned}
&E(l_0)\approx 13,\quad F(l_0)\approx 290,\quad D(l_0)=\{1-0.848[h_t(l_0)]^2\}^{-1},\\
&[h_t(0)]^2=13^{-1}[h_t(l_0)]^2\{1-0.848[h_t(l_0)]^2\}^{-1}.
\end{aligned} \tag{4.9}$$

For the function $q(l_0)$ we obtain

$$q(l_0)=1.28(1-0.85h_t^2)^{1/4}\approx 1.28(1-0.21h_t^2-0.068h_t^4), \tag{4.10}$$

where $h_t = h_t(l_0) = 2^{1/2}m_t/v_1$. In the last equation (4.10) we have expanded in terms of h_t^2, since, as will be clear in the sequel, $h_t^2 \ll 1$.

We must now calculate $a_1(l_0)$, $a_2(l_0)$, and $\tilde{B}(l_0)$, which enter into Eq. (4.6). The renormalization of $a_2(l_0)$ is determined only by the Yukawa interaction of the b-quark, which we have neglected. Consequently

$$a_2(l_0)=1. \tag{4.11}$$

As regards the functions $a_1(l_0)$ and $\tilde{B}(l_0)$, they depend, unfortunately, on the parameter A introduced in the preceding section, which is determined by the vacuum expectation values in the hidden sector and represents the coefficient in the cubic term of the potential: $Am_{3/2}\times$ (scalar fields)^3. For $a_1(l_0)$ one can find

$$a_1(l_0)=1-0.424(A^2+3)h_t^2+0.360A^2h_t^4. \tag{4.12}$$

Finally, for $\tilde{B}(l_0)/B$ ($B = 2$) we have

$$\tilde{B}(l_0)/B=1+0.212Ah_t^2. \tag{4.13}$$

The equations (4.10)–(4.13) yield the following renormalized values of μ_1^2, μ_2^2, and μ_3^2:

$$\begin{aligned}
\mu_1^2&=m_{3/2}^2[2.64-(1.97+0.42A^2)h_t^2+(0.36A^2-0.15)h_t^4],\\
\mu_2^2&=m_{3/2}^2[2.64-0.70h_t^2-0.15h_t^4],\\
\mu_3^2&=m_{3/2}^2[2.56+0.54(A-1)h_t^2-(0.17+0.115A)h_t^4].
\end{aligned} \tag{4.14}$$

As already mentioned, without the Yukawa interaction ($h_t = 0$) we would have $\mu_1^2 = \mu_2^2 \neq \mu_3^2$. An interesting trait of the solution (4.14) consists in the fact that μ_3^2 is almost equal to $\mu_1^2 = \mu_2^2$, although the renormalization of each of these quantities is not that small. This means that the terms $\sim h_t^2, h_t^4$ must be small: $h_t^2 \ll 1$.

In order that the condition (4.4) be satisfied, μ_3^2 must lie between two quantities which are very close to each other; $(\mu_1^2 + \mu_2^2)/2$ and $(\mu_1^2 \mu_2^2)^{1/2}$. Indeed, although the quantity μ_1^2 differs from μ_2^2 by $O(h_t^2)$ the difference between $(\mu_1^2 + \mu_2^2)/2$ and $(\mu_1^2 \mu_2^2)^{1/2}$ is of order $O(h_t^4)$. This means that there must exist a narrow interval of values h_t^2 of width $\sim h_t^4$ in which the inequalities (4.4) are valid. The maximal value of h_t^2 is determined by the equation

$$\mu_3^2 = (\mu_1^2 + \mu_2^2)/2. \tag{4.15}$$

This yields, approximately,

$$h_{max}^2 = \frac{1}{10}\frac{1}{1+0.675A+0.263A^2} + \frac{1}{1000}\frac{0.25+1.44A+2.25A^2}{(1+0.675A+0.263A^2)^3}. \tag{4.16}$$

The minimal value $h_{\min}^2$ is determined by the condition

$$\mu_3^2 = (\mu_1^2 \mu_2^2)^{1/2}. \tag{4.17}$$

Expanding the difference $h_{\max}^2 - h_{\min}^2$ in terms of h_t^2 it is easy to obtain

$$h_{max}^2 - h_{min}^2 = \frac{(0.31+0.10A^2)^2}{1+0.675A+0.263A^2} h_{max}^4. \tag{4.18}$$

These limits in h_t determine the maximal and minimal values of the mass of the t-quark. Since $\mu_1^2 \approx \mu_2^2$, symmetry breaking occurs for $v_1 \approx v_2$. Consequently

$$m_t = h_t v_t / 2^{1/2} = h_t v/2, \quad v = (G_F \cdot 2^{1/2})^{-1/2} = 246 \text{ GeV}. \tag{4.19}$$

We consider two numerical examples. For $A = 0$, $h_{\max}^2 \approx 0.10$, $h_{\max}^2 - h_{\min}^2 \approx 0.001$, $m_t^{\max} = 39.36$ GeV, $m_t^{\max} - m_t^{\min} \approx 0.2$ GeV; For $A = 3$, $h_{\max}^2 \approx 0.02$, $m_t^{\max} = 16.82$ GeV, $m_t^{\max} - m_t^{\min} \approx 0.04$ GeV. It is interesting to note that there exists an absolute upper bound on the t-quark mass, independent of A (the maximum is attained for $A = -1.28$). This bound is

$$m_t < 51.7 \text{ GeV}. \tag{4.20}$$

If, however, A is considered as a free parameter, then there is no lower bound on m_t.

In the general case the solution of the equations $dV/dv_i = 0$ with V determined by the expression (4.2) is (for $v_1 < v_2$)

$$\frac{v_1}{v_2} = \left\{\frac{\mu_1^2+\mu_2^2-[(\mu_1^2+\mu_2^2)^2-4\mu_3^4]^{1/2}}{\mu_1^2+\mu_2^2+[(\mu_1^2+\mu_2^2)^2-4\mu_3^4]^{1/2}}\right\}^{1/2},$$
$$m_Z^2 = \frac{\bar{g}^2(v_1^2+v_2^2)}{4} = (\mu_1^2+\mu_2^2)\left\{\frac{|\mu_1^2-\mu_2^2|}{[(\mu_1^2+\mu_2^2)^2-4\mu_3^4]^{1/2}} - 1\right\}. \tag{4.21}$$

We see that for our case $v_1 \approx v_2$, whereas for the quantity $v(v^2 = v_1^2 + v_2^2)$ one can obtain from Eq. (4.21) an expression containing the quantities $h_{\max}^2$ and $h_{\min}^2$ determined by the equations (4.15)–(4.18):

$$m_Z^2 = \frac{\bar{g}^2 v^2}{4} = (\mu_1^2+\mu_2^2)\left[\left(\frac{h_{max}^2 - h_{min}^2}{h_{max}^2 - h_t^2}\right)^{1/2} - 1\right]. \tag{4.22}$$

When h_t varies from $h_{\min}$ to $h_{\max}$, v takes values from zero to infinity.

The mass of the neutral pseudoscalar particle which is in fact ($v_1 \operatorname{Im} H_2^0 - v_2 \operatorname{Im} H_1^0$), equals

$$m_P^2 = \mu_1^2 + \mu_2^2. \tag{4.23}$$

From (4.14) it follows that

$$m_P^2 \approx 5.28 m_{3/2}^2. \tag{4.24}$$

The mass of the charged Higgs boson is $m_{H^+}^2 = m_P^2 + m_W^2$.

For the neutral scalar particles the general expression for the mass is

$$m_{S_{H,L}}^2 = \tfrac{1}{2}(m_P^2+m_Z^2) \pm \tfrac{1}{2}[(m_P^2+m_Z^2)^2 - 4m_Z^2 m_P^2 \cos^2 2\theta]^{1/2}, \tag{4.25}$$

where $\sin 2\theta = 2\mu_3^2/(\mu_1^2 + \mu_2^2)$.

Since $\cos 2\theta \approx 0$, the heavy scalar is almost degenerate in mass with the charged Higgs boson, whereas the mass of the light scalar is

$$m_{S_L}^2 = \frac{m_P^2 m_Z^2}{m_P^2 + m_Z^2}\cos^2 2\theta. \tag{4.26}$$

This mass vanishes for $\mu_1^2 = \mu_2^2 = \mu_3^2$; $\cos^2 2\theta$ is proportional to $h_{\max}^2 - h_t^2$. The last difference can be expressed by means of Eq. (4.22) as

$$h_{max}^2 - h_t^2 = \left(\frac{m_P^2}{m_Z^2+m_Z^2}\right)^2 (h_{max}^2 - h_{min}^2)$$
$$= \left(\frac{m_P^2}{m_Z^2+m_P^2}\right)^2 \frac{0{,}31+0{,}1A^2}{1+0{,}675A+0{,}263A^2} h_{max}^4, \tag{4.27}$$

so that we finally obtain

$$m_{S_L}^2 = \frac{m_P^3 m_Z m_t^2}{(m_P^2+m_Z^2)^{3/2} v^2}(0.96+0.31A^2). \tag{4.28}$$

Since $m_P^3/(m_P^2 + m_Z^2)^{3/2} < 1$ we have, making use of (4.16),

$$m_{S_L} \leqslant m_Z \cdot \frac{1}{40}\frac{0.96+0.31A^2}{1+0.675A+0.263A^2} \approx \frac{m_Z}{40} = 2.25 \text{ GeV}. \tag{4.29}$$

If $m_P \gg m_Z$ then $m_{S_L} = 2.25$ GeV. In reality this limit is reached already for $m_P^2 \lesssim m_Z^2$. In fact, $m_{S_L} = 2.25$ GeV if $m_{3/2} \lesssim 40$–50 GeV [see Eq. (4.24)].

We now consider the masses of the superpartners of W and Z mixed with higgsinos. For the charged higgsino-wino we have for $v_1 = v_2 = v/2^{1/2}$ two massive Dirac spinors with masses

$$m_{H,L}^2(\tilde{W}) = \frac{1}{2}\mu'^2 + \frac{g^2v^2}{4} \pm \frac{1}{2}\mu'(\mu'^2+g^2v^2)^{1/2}, \tag{4.30}$$

where $\mu' = m_{3/2}\, q(l_0) = 1.28 m_{3/2}$ [cf. (4.10)]. If $\mu' \gg m_W$ (i.e., $m_{3/2} > 70$–80 GeV), the light $\tilde{W}_L$ has the mass $m(\tilde{W}_L) = m_W^2/\mu'$. Since there exists the experimental bound $m(\tilde{W}_L) > 23$ GeV, we find that $m_{3/2} < 220$ GeV.

The neutral higgsino-zino and photino are described by four Majorana spinors. Their masses are

$$m_{1,2}^2(\tilde{Z}) = \frac{1}{2}\mu'^2 + \frac{\bar{g}^2v^2}{4} \pm \frac{1}{2}\mu'(\mu'^2+\bar{g}^2v^2)^{1/2},$$

$$m_3(\tilde{Z})=\mu', \quad m_4(\tilde{\gamma})=0. \tag{4.31}$$

The masslessness of the photino is, of course, a consequence of the simplification, related to the assumption of the "minimal" character of supergravity.

Finally, it is interesting to estimate the masses of the superpartners of the quarks and leptons. To order $O(h_t^2)$ all these masses except that of the $\tilde{t}$-quarkino (squark) are equal to $m_{3/2}$ (the radiative corrections are of the order $h_t^2 = 4m_t^2/v^2$). For the $\tilde{t}$-quarkino one obtains easily

$$m^2_{\tilde{t}_n,\tilde{t}_l}=m_{3/2}^2\pm(A+1)m_{3/2}m_t. \tag{4.32}$$

This expression appears when one takes into account the mixing of the left-handed and right-handed quarkino-superpartners of the t_L- and t_R-quarks.

5. SYMMETRY BREAKING À LA COLEMAN-WEINBERG

The renormalization effects related to the large momentum interval between $M_{\rm GUT}$ and M_W violate the symmetry $\mu_1^2 = \mu_2^2 = \mu_3^2$ which reflects the GIFT mechanism. This "large-scale" renormalization, which looks quite natural in the real world, has no direct bearing on the GIFT mechanism. From a theoretical point of view it is interesting to raise the question: what happens if the "large-scale" renormalization is absent, i.e., the $SU(6)$ symmetry and the equalities $\mu_1^2 = \mu_2^2 = \mu_3^2$ hold for energies ~ 100 GeV? It is obvious that in this case a more accurate accounting of small momenta is needed, i.e., a calculation of the Coleman-Weinberg corrections to the effective potential.

At the tree level the potential (4.2) for $\mu_1^2 = \mu_2^2 = \mu_3^2 = 2m^2$ has the form

$$V_0=m^2(v_1-v_2)^2+{}^1/_{32}\bar{g}^2(v_1^2-v_2^2)^2=2\eta^2(m^2+{}^1/_{16}\bar{g}^2\xi^2),$$
$$\xi=(v_1+v_2)/2^{1/2}, \quad \eta=(v_1-v_2)/2^{1/2}. \tag{5.1}$$

(We have changed the notation: $m_{3/2}\to$ m.) The minimum of V_0 corresponds to $\langle\eta\rangle = 0$, (i.e., $v_1 = v_2$), whereas the value of $\langle\xi\rangle$ is undetermined. The quantity $\langle\xi\rangle$ must be determined from the one-loop Coleman-Weinberg potential. The latter is given by the well-known expression

$$64\pi^2V_1(\xi,\eta)=\sum_i(-1)^F m_i^4\ln m_i^2, \tag{5.2}$$

where m_i are the masses of all the particles in the external field (ξ, η) and the factor (-1) refers to all fermions.

It is easy to see that the solution $\langle\eta\rangle = 0$ remains valid also for $V_0\to V_0 + V_1$. (This is because V_1 depends in fact on η^2.) Therefore it suffices to have V_1 for $\eta = 0$, i.e., for $v_1 = v_2$ $(\xi = v)$.

We first neglect the Yukawa interactions and take into account the contributions of H_1, H_2, W, Z and their superpartners, leaving out the contribution of the t-quark/quarkino. Then it is easy to obtain

$$64\pi^2V_1(v)=2\left(\frac{g^2v^2}{4}+4m^2\right)^2\ln\frac{g^2v^2/4+m^2}{\Lambda^2}$$
$$+6\left(\frac{g^2v^2}{4}\right)^2\ln\frac{g^2v^2/4}{\Lambda^2}$$
$$-4\left[\frac{g^2v^2}{4}+\frac{1}{2}m^2+\frac{1}{2}m(m^2+g^2v^2)^{1/2}\right]^2$$
$$\cdot\ln\frac{g^2v^2/4+{}^1/_2m^2+{}^1/_2m(m^2+g^2v^2)^{1/2}}{\Lambda^2}-4\left[\frac{g^2v^2}{4}+\frac{1}{2}m^2\right.$$
$$\left.-\frac{1}{2}m(m^2+g^2v^2)^{1/2}\right]^2\ln\frac{g^2v^2/4+{}^1/_2m^2-{}^1/_2m(m^2+g^2v^2)^{1/2}}{\Lambda^2}$$
$$+\frac{1}{2}(g\to\bar{g}). \tag{5.3}$$

Here the first terms correspond to the contribution of H^+ and W, and the negative terms correspond to the higgsino-wino [cf. Eq. (4.30)]. The terms with $g\to\bar{g}$ correspond to the contribution of the neutral particles.

The potential (5.3) does not depend on the cut-off Λ, if one discards the inessential additive constant $(42m^4 \ln\Lambda^2)$. This circumstance is somewhat surprising. The cancellation of the terms $v^4 \ln \Lambda^2$ can be understood. For $m = 0$ and $v_1 = v_2$ supersymmetry is in fact not broken, so that the masses of the bosons in the external field v remain equal to the fermion masses, as a result of which terms proportional to $\ln \Lambda^2$ cancel in the sum (5.2). As regards the cancellation of the terms $m^2v^2 \ln \Lambda^2$, it has a somewhat "accidental" aspect. Thus, for instance, we shall see in the sequel that when the contributions of the t-quark and $\tilde{t}$-quarkino are taken into account, there is no such cancellation.

On account of what was said, the expression (5.3) can be rewritten in a form which does not contain Λ. Omitting the additive constant we obtain

$$\frac{64\pi^2V_1}{m^4}=f(x)+\frac{1}{2}f(\bar{x}), \quad x=\frac{g^2v^2}{4m^2}, \quad \bar{x}=\frac{\bar{g}^2v^2}{4m^2},$$

$$f(x)=28\ln x+2(x+4)^2\ln\frac{x+4}{x}$$
$$-4(2x+1)(4x+1)^{1/2}\ln\frac{1+2x+(4x+1)^{1/2}}{2x}; \tag{5.4}$$

the x-dependence of this expression yields the curvature of the potential along the valley $v_1 = v_2$. Unfortunately $f(x)$ has a maximum rather than a minimum for $x = 2.2$, and for $x\to\infty$ $f(x)\approx -8x\to -\infty$. Thus, in place of a self-consistent symmetry breaking we find instability for large x. The situation changes immediately as soon as one adds the contribution of the t-quark-quarkino to (5.3). This contribution equals

$$64\pi^2V_1^{(t)}=6\left[m^2+m(A+1)\frac{h_tv}{2}+\left(\frac{h_tv}{2}\right)^2\right]^2$$
$$\cdot\ln\frac{m^2+m(A+1)(h_tv/2)+(h_tv/2)^2}{\Lambda^2}+(m\to-m)$$
$$-12\left(\frac{h_tv}{4}\right)^4\ln\frac{(h_tv/2)^2}{\Lambda^2}. \tag{5.5}$$

Here h_t is the Yukawa coupling constant, and A is the parameter we encountered before. The asymptotic behavior of the expression (5.5) for large v is

$$64\pi^2V_1^{(t)}\sim3m^2h_t^2v^2[(A+1)^2+2]\ln v^2, \tag{5.6}$$

whereas the asymptotic behavior of V_1 (5.4) is

$$64\pi^2V_1\sim-2m^2(g^2+{}^1/_2\bar{g}^2)v^2; \tag{5.7}$$

which means that for sufficiently large v the sum $V_1^{(t)} + V_1$ is positive. The function $V_1^{(t)} + V_1$ has a minimum, so that if one fixes the position of this minimum: $v = (2^{1/2}G_F)^{-1/2} = 246$ GeV, then one obtains for the mass of the Higgs particle an expression which does not depend on Λ. This is nothing else but the well-known Coleman-Weinberg mechanism. For $h_t v/2 \gg m$, for instance, it is easy to find

$$m_{s_L}^2 = \left(\frac{d^2(V_1^{(t)}+V_1)}{dv^2}\right)_{v=v_{min}} = \frac{3}{4\pi^2}\left(\frac{m^2 m_t^2}{v^2}\right)[(A+1)^2+2]. \tag{5.8}$$

The result (5.8) differs from the expression (4.28) obtained for the same mass in the preceding section.

6. CONCLUSION

Let us sum up briefly the results obtained in this paper. We have proposed a mechanism which explains in a natural manner the masslessness of the doublets which are part of the quintuplets of a supersymmetric $SU(5)$ grand unified theory. The reason for this is that the doublets are pseudo-Goldstone bosons of a globally spontaneously broken $SU(6)$ symmetry of the superpotential. Although this symmetry is not a symmetry of the whole theory, the doublets remain massless as long as the supersymmetry is intact. When the supersymmetry breaking is switched on, the doublets acquire, in general, a mass of the order of the scale of the supersymmetry breaking.

This general mechanism (GIFT) was then applied to treat the special case of supersymmetry breaking by supergravity. In this case even after the supersymmetry breaking one of the two doublets remains massless in the tree approximation, since it is again a Goldstone boson of the tree potential. However, since supersymmetry is now broken, it will acquire mass on account of radiative corrections. We have considered in detail the case when this occurs on account of renormalization of the parameters of the potential in a wide range of momenta, from the grand-unification mass to the mass of the W-boson. We have shown that in this case, for fixed values of the parameters determined by supergravity ($m_{3/2}$—the gravitino mass and A—a parameter related to the vacuum expectation values in the hidden sector of the theory), a correct spontaneous breaking of the electroweak symmetry occurs only in a narrow range of t-quark masses, where we obtain an absolute upper bound on the mass of the t-quark $m_t < 52$ GeV. The mass of the lightest scalar particle turns out to be, most likely, ~ 22 GeV. All the other masses are also fixed, but are numerically unknown, since the parameters $m_{3/2}$ and A are unknown.

In the last section of the paper we have considered a somewhat different mechanism for spontaneous breaking of the electroweak symmetry, à la Coleman-Weinberg. If the renormalization effects related to the wide range of momenta from $M_{\rm GUT}$ to M_W are absent, a more accurate accounting of small momenta becomes necessary, i.e., a calculation of the Coleman-Weinberg potential. (In the presence of "large-scale" renormalization, the Coleman-Weinberg corrections due to small momenta are only a few percent and can be neglected.) It turns out that in this case too the Yukawa coupling of the t-quark is essential for the electroweak symmetry breaking. The mass of the lightest scalar boson can be calculated, but its numerical value depends on A and $m_{3/2}$ and remains unknown.

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[1]Note that in the Appendix the quantities μ_1 and μ_2 are interchanged.

[1]E. Witten, Nucl. Phys. B **188**, 573 (1981).
[2]M. Sakai, Z. Phys. C **11**, 153 (1981).
[3]S. Dimopoulos and H. Georgi, Nucl. Phys. B **193**, 150 (1981).
[4]S. Dimopoulos and F. Wilczek, *Erice Summer Lectures*, Plenum Press, New York, 1981; Santa Barbara Preprint, VM-HE 81-71. B. Grinstein, Nucl. Phys. B **206**, 387 (1982). H. Georgi, Phys. Lett. **108B**, 283 (1982). A. Masiero, P. V. Nanopoulos, K. Tamvakis, and T. Yamagida, Phys. Lett. **115B**, 380 (1982).
[5]H. Tabata, I. Umemura, and Y. Yamamoto, Prog. Theor. Phys. (Kyoto) **71**, 615 (1984).
[6]E. Witten, Phys. Lett. **106B**, 267 (1981). P. V. Nanopoulos and K. Tamvakis, Phys. Lett. **113B**, 151 (1982).
[7]J. Polchinski and L. Susskind, Phys. Rev. D **26**, 3661 (1982). H. P. Niles, M. Srednicki, and D. Wyler, Phys. Lett. **124B**, 337 (1982).
[8]N. B. Krasnikov, Pis'ma v ZhETF **39**, 280 (1984) [JETP Lett. **39**, 280 (1984)].
[9]A. Yu. Smirnov and E. A. Taĭnov, Phys. Lett. **167B**, 195 (1986); Yad. Fiz. **43**, 496 (1986) [Sov. J. Nucl. Phys. **43**, 315 (1986)].
[10]A. Sen, Phys. Rev. D **31**, 900 (1985). Phys. Lett. **148B**, 65 (1984).
[11]A. Sen, Phys. Rev. Lett. **55**, 33 (1985). J. P. Breit, B. A. Ovrut, and G. C. Segre, Univ. Of Penna preprint UPR-0927-T, 1985.
[12]D. V. Volkov and V. A. Soroka, Pis'ma v ZhETF **78**, 529 (1973) [JETP Lett. **78** (1973)]. E. Cremmer, B. Julia, J. Scherk *et al.*, Nucl. Phys. **B147**, 65 (1979).
[13]H. Ynou, A. Kakuto, H. Komatsu, and S. Takashita, Prog. Theor. Phys. (Kyoto) **67**, 1889 (1982).
[14]R. A. Flores and M. Sher, Ann. Phys. (NY) **95**, 148 (1983).
[15]L. Alvarez-Gaumé, J. Polchinski, and M. Wise, Nucl. Phys. **B221**, 495 (1983).
[16]L. E. Ibañez and C. Lopez, Nucl. Phys. **B233**, 511 (1984).
[17]L. E. Ibañez, C. Lopez, and C. Muñoz, Nucl. Phys. **B563**, 218 (1985).
[18]M. I. Vysotskiĭ and K. A. Ter-Martirosyan, Zh. Eksp. Teor. Fiz. **90**, 838 (1986) [Sov. Phys. JETP **63**, 489 (1986)].
[19]A. A. Ansel'm and A. A. Ioganson, Pis'ma v ZhETF **44**, 488 (1986) [JETP Lett., **44**, 628 (1986)].
[20]A. A. Anselm, Preprint, Fermilab-pub-87/44-T, March, 1987.
[21]R. Barbieri, S. Ferrara, and C. A. Savoy, Phys. Lett. **119B**, 343 (1982). P. Nath, R. Arnowitt, and A. H. Chamseddine, Rev. Lett. **49**, 970 (1982).
[22]L. Hall, T. Lykken, and S. Weinberg, Phys. Rev. D **27**, 2359 (1983).

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