New type of electromagnetic wave propagating at an interface

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A new type of electromagnetic wave is predicted. It is localized at an interface between two transparent media and travels along the interface. The presence of such waves is due to the difference between the symmetry of the two media in contact. Waves at an interface between an isotropic medium with a permittivity ε and a uniaxial crystal, characterized by $\varepsilon_\parallel$, and $\varepsilon_\perp$, are considered in the case when the optic axis is parallel to the interf^ce. It is shown that such surface waves exist if $\varepsilon_\parallel > \varepsilon > \varepsilon_\perp$ and that they can propagate in a certain range of angles with respect to the axis.

1. We shall consider a new type of electromagnetic waves localized at an interface between two transparent media and propagating along this interface. The existence of such waves is due to differences between the symmetry of the media in contact and, in contrast to the familiar cases, these waves do not appear because the permittivity is negative or has spatial dispersion.

We shall consider only the case of an interface between an isotropic medium with a permittivity ε and a uniaxial crystal for which the principal values of the permittivity tensor are $\varepsilon_\parallel$, $\varepsilon_\perp$, and $\varepsilon_\perp$. The values of ε, $\varepsilon_\parallel$, and $\varepsilon_\perp$, are assumed to be real and positive. The properties of these surface waves and the very possibility of their propagation are governed by the relationships between ε, $\varepsilon_\parallel$, and $\varepsilon_\perp$, as well as by the relative orientations of the normal to the interface, optic axis of the crystal, and the wave vector. We can easily show that in the simplest case when these three directions lie in the same plane, and particularly if the optic axis is perpendicular to the interface, there are no surface waves of the new type. The need to consider the more general geometry explains why the existence of localized waves at such an interface has remained undetected in spite of the fact that the reflection and refraction of light at an interface between an isotropic medium and a crystal have been studied for a long time and very thoroughly (see, for example, Ref. 1).

We shall consider the case when the optic axis of a crystal is parallel to the interface. We shall show that surface waves exist if $\varepsilon_\parallel > \varepsilon > \varepsilon_\perp$ and that they can propagate in a certain range of angles relative to the axis. The boundaries of this range are governed by the values of ε, $\varepsilon_\parallel$, and $\varepsilon_\perp$.

2. We shall assume that the interface coincides with the $x = 0$ plane. The half-space $x > 0$ is occupied by the isotropic medium, whereas the crystal with its optic axis parallel to the interface is in the half-space $x < 0$. The direction of the phase velocity of a surface wave making an angle φ with the optic axis is assumed to be the z axis (Fig. 1).

We shall solve the Maxwell equations in the form of a monochromatic wave traveling along the z axis and characterized by a wave vector q; this wave decays along the x axis in either direction away from the interface. In the isotropic medium there are two independent solutions with different polarizations and with the wave vector $\mathbf{q}_1 = (ik_1, 0, q)$, where

$$q^2-k_1^2=\varepsilon. \tag{1}$$

Here and below all the wave vectors will be measured in units of ω/c, where ω is the wave frequency and c is the velocity of light in vacuum.

Two independent solutions in the crystal (extraordinary and ordinary waves) have the wave vectors $\mathbf{q}_2 = (-ik_2, 0, q)$ and $\mathbf{q}_3 = (-ik_3, 0, q)$, respectively, and are governed by the dispersion laws

$$\frac{q^2\sin^2\varphi-k_2^2}{\varepsilon_\parallel}+\frac{q^2\cos^2\varphi}{\varepsilon_\perp}=1, \tag{2}$$

$$q^2-k_3^2=\varepsilon_\perp. \tag{3}$$

The components of the wave vectors along the x axis are assumed to be purely imaginary so as to ensure decay away from the interface. Clearly, it is necessary to ensure that k_1, k_2, and k_3 are all positive.

Solutions of the Maxwell equations $[\mathbf{qE}] = \mathbf{H}$ and $[\mathbf{qH}] = -\hat{\varepsilon}\mathbf{E}$ have the following form in the half-space $x < 0$. For an extraordinary wave, we have

$$\mathbf{E}=(ik_2q\cos\varphi,\ -\varepsilon_\perp\sin\varphi,\ (\varepsilon_\perp-q^2)\cos\varphi),$$
$$\mathbf{H}=(q\varepsilon_\perp\sin\varphi,\ ik_2\varepsilon_\perp\cos\varphi,\ ik_2\varepsilon_\perp\sin\varphi). \tag{4}$$

For an ordinary wave, we find that

$$\mathbf{E}=(q\sin\varphi,\ ik_3\cos\varphi,\ ik_3\sin\varphi),$$
$$\mathbf{H}=(-iqk_3\cos\varphi,\ \varepsilon_\perp\sin\varphi,\ k_3^2\cos\varphi). \tag{5}$$

In the half-space $x > 0$ the two independent solutions will be assumed to represent TM and TE waves:

$$TM:\ \mathbf{E}=(q,\ 0,\ -ik_1),\quad \mathbf{H}=(0,\ \varepsilon,\ 0), \tag{6}$$

$$TE:\ \mathbf{E}=(0,\ 1,\ 0),\quad \mathbf{H}=(-q,\ 0,\ ik_1). \tag{7}$$

We shall now find linear combinations of the solutions (4) and (5) for $x < 0$ and of the solutions (6) and (7) for $x > 0$, and we shall postulate that the tangential components of the fields $\mathbf{E}$ and $\mathbf{H}$ are continuous at the interface. This gives the equation

$$k_3(k_1+k_3)(\varepsilon_\perp k_1k_2+\varepsilon k_3^2)\cos^2\varphi$$
$$-\varepsilon_\perp(k_1+k_2)(\varepsilon_\perp k_1+\varepsilon k_3)\sin^2\varphi=0. \tag{8}$$

The system of equations (1)–(3) and (8) yields four unknowns k_1, k_2, k_3, and q if the angle φ is given.

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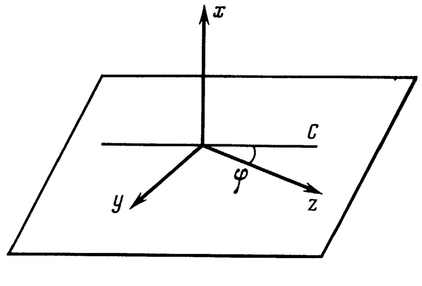

FIG. 1. Coordinate system. The half-space $x < 0$ is occupied by an isotropic medium, and the half-space $x < 0$ is occupied by a crystal; C is the optic axis of the crystal. The phase velocity of a surface wave is directed along the z axis.

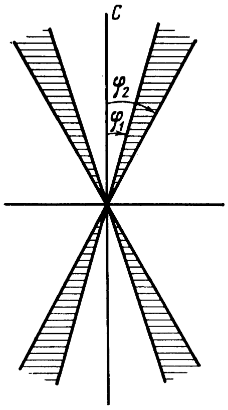

FIG. 2. Range of angles (shown shaded), relative to the optic axis C, where the phase velocity of a surface wave may be located. The boundaries of the range φ_1 and φ_2 are described by Eqs. (10) and (11).

3. Using Eqs. (1)–(3), we find that after fairly tedious algebraic transformations we can reduce Eq. (8) to a more convenient form:

$$(k_1+k_2)(k_1+k_3)(\varepsilon k_3+\varepsilon_\perp k_2)=(\varepsilon_\parallel-\varepsilon)(\varepsilon-\varepsilon_\perp)k_3. \tag{9}$$

Hence, it is clear that solutions with k_1, k_2, $k_3>0$ corresponding to a wave localized at the interface exist if either $\varepsilon_\parallel>\varepsilon>\varepsilon_\perp$ or $\varepsilon_\perp>\varepsilon>\varepsilon_\parallel$. However, we can readily show that in the latter case Eqs. (1)–(3) and (9) have no solutions. In fact, it follows from Eq. (8) that for $\varphi=0$ and $\varphi=\pi/2$ there are no solutions characterized by $k_1, k_2, k_3>0$. Consequently, such a solution can exist only in a certain range of angles φ. Then, at the limit, of this range one of the quantities k_1, k_2, or k_3 should vanish and the other two should be positive. For $k_3=0$ the conditions $k_1>0$, $k_2>0$ cannot be satisfied, as can be seen from Eq. (9). However, if we have $k_1=0$ or $k_2=0$, then for $\varepsilon_\perp>\varepsilon>\varepsilon_\parallel$ we have in both cases $k_3^2<0$, which follows from Eqs. (1)–(3). Therefore, we need to consider only the case when $\varepsilon_\parallel>\varepsilon>\varepsilon_\perp$.

We shall determine the limits φ_1 and φ_2 of the range of angles φ in which surface waves of the new type can propagate, The values of φ_1 and φ_2 are found from the conditions $k_1=0$ and $k_2=0$, respectively. Equations (1)–(3) and (9) yield

$$\sin^2\varphi_1=\frac{\xi}{2}\{1-\eta\xi+[(1-\eta\xi)^2+4\eta]^{1/2}\}, \tag{10}$$

$$\sin^2\varphi_2=\frac{(1+\eta)^3\xi}{(1+\eta)^2(1+\eta\xi)-\eta^2(1-\xi)^2}, \tag{11}$$

where

$$\eta=\varepsilon_\parallel/\varepsilon_\perp-1,\qquad \xi=(\varepsilon-\varepsilon_\perp)/(\varepsilon_\parallel-\varepsilon_\perp). \tag{12}$$

In the case under discussion we have $\eta>0$, $0<\xi<1$. We can then demonstrate that $\sin^2\varphi_2>\sin^2\varphi_1$. If we consider the range $0<\varphi<\pi/2$, we find that surface waves exist for $\varphi_1<\varphi<\varphi_2$ (Fig. 2). If $\varphi=\varphi_1$, then surface waves transform into a plane wave in the isotropic medium, whereas for $\varphi=\varphi_2$ they transform into an extraordinary wave in the crystal. At the boundaries in the range (φ_1,φ_2) the phase velocity of the surface wave is equal to the phase velocity of the corresponding internal wave. Within this range the phase velocity of the surface wave is obviously less than the phase velocities of all the internal waves (propagating at the same angle φ). Figure 3 shows the dependences of the angles φ_1 and φ_2 on the value of ξ calculated from Eqs. (10) and (11) on the assumption that $\eta=4$.

By definition, the angle φ lies between the directions of the phase velocity of a surface wave and the optic axis of the crystal. The direction of the group velocity of this wave generally does not coincide with the direction of the phase velocity (as is true also of internal waves in an anisotropic medium). This direction can be found if we know the dependence $q(\varphi)$.

Figure 4 shows how the quantities k_1, k_2, k_3, and q depend on the angle φ, obtained by numerical solution of Eqs. (1)–(3) and (9) in the case when $\varepsilon_\parallel=5$ and $\varepsilon=1.8$, $\varepsilon_\perp=1$ ($\eta=4$, $\xi=0.2$).

4. In the case of real crystals the difference between $\varepsilon_\parallel$ and $\varepsilon_\perp$ is usually small, i.e., we have $\eta\ll1$. Then, Eqs. (1)–(3) and (9) can be solved analytically. When the necessary inequality $\varepsilon_\parallel>\varepsilon>\varepsilon_\perp$ is satisfied, the isotropic medium and the crystal under consideration differ in their optical properties. Therefore, it is clear a priori that if $\eta\ll1$, then the new surface waves have a phase velocity close to the phase velocities of internal waves and the decay lengths in both media should be long compared with the wavelength. We shall show below that the range of angles φ in which surface waves can propagate is very narrow.

It is clear from Eq. (9) that if $\eta\ll1$, then $k_1,k_2\ll k_3$. Bearing this point in mind, we find from Eq. (9) that

$$\varepsilon k_3(k_1+k_2)=(\varepsilon_\parallel-\varepsilon)(\varepsilon-\varepsilon_\perp), \tag{13}$$

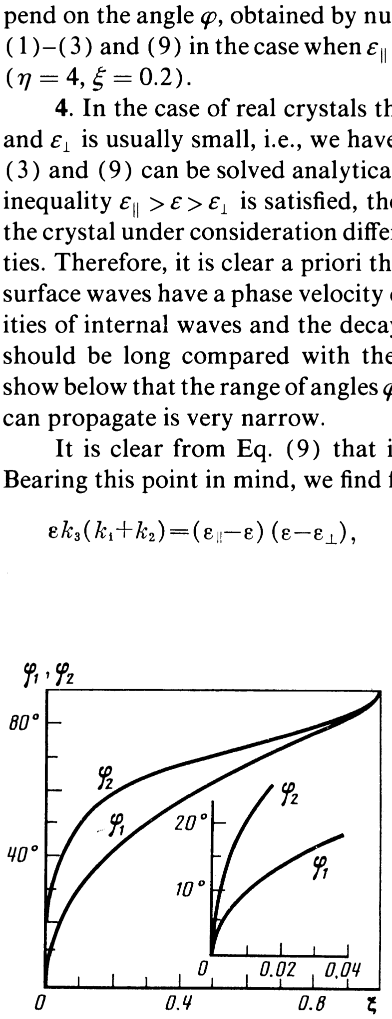

FIG. 3. Dependence of the angles φ_1 and φ_2 on $\xi=(\varepsilon-\varepsilon_\perp)/(\varepsilon_\parallel-\varepsilon_\perp)$, calculated for $\varepsilon_\parallel=5\varepsilon_\perp$. The inset shows the range of small values of ξ.

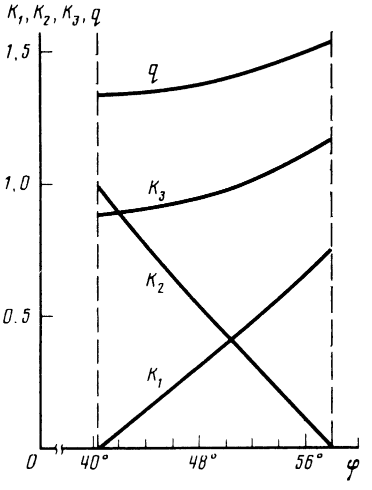

FIG. 4. Dependence of k_1, k_2, k_3, and q on the angle φ calculated for $\varepsilon_\parallel = 5$, $\varepsilon = 1.8$, and $\varepsilon_\perp = 1$.

whereas Eqs. (1)–(3) yield

$$k_3=(\varepsilon-\varepsilon_\perp)^{1/2}, \quad (14)$$

$$k_1^2-k_2^2=2\varepsilon\eta\alpha\xi^{1/2}(1-\xi)^{1/2}. \quad (15)$$

We have introduced here a new angle $\alpha \ll 1$ by adopting the definition $\alpha = \varphi - \varphi_0$, where apart from terms of the order of η, we have

$$\sin^2\varphi_0=\xi. \quad (16)$$

Solving Eqs. (13)–(15), we finally obtain

$$k_1=\varepsilon^{1/2}\frac{\delta/2+\alpha}{\eta^{1/2}(1-\xi)^{1/4}}, \quad k_2=\varepsilon^{1/2}\frac{\delta/2-\alpha}{\eta^{1/2}(1-\xi)^{1/4}}, \quad k_3=(\varepsilon\eta\xi)^{1/2}. \quad (17)$$

The quantity

$$\delta=\eta^2\xi^{1/2}(1-\xi)^{3/2} \quad (18)$$

represents the width of the range of angles which contains the direction of the phase velocity of a surface wave. The midpoint of this interval (angle φ_0) is defined by Eq. (16). Equations (16) and (18) can also be derived directly by expanding Eqs. (10) and (11) as a series in η to quadratic terms. We then obtain $\varphi_2 - \varphi_1 = \delta$ and $(\varphi_1 + \varphi_2)/2 = \varphi_0$.

It therefore follows that k_1 and k_2 vanish for $\alpha = -\delta/2$ and $\alpha = \delta/2$, respectively, and their maximum values are of the order of $\eta^{3/2}$. On the other hand, we have $k_3 \sim \eta^{1/2} \gg k_1, k_2$.

The dispersion law of the surface wave can easily be found using Eqs. (1) and (17). Adopting dimensionless units, we obtain

$$\omega=\frac{qc}{\varepsilon^{1/2}}F(\alpha), \quad F(\alpha)=1-\frac{1}{2}\frac{(\delta/2+\alpha)^2}{\eta(1-\xi)}. \quad (19)$$

We recall that α is the angle between the direction of the phase velocity and the optic axis, measured from the value of φ_0.

We shall now find the range of angles within which the group velocity direction may be located. The angle ψ between this direction and the optic axis can be represented in the form $\psi = \varphi_0 + \beta$, where $\beta \ll 1$. We then have

$$\cos(\alpha-\beta)=\frac{\mathbf{q}(\partial\omega/\partial\mathbf{q})}{q|\partial\omega/\partial\mathbf{q}|}=\frac{F}{[F^2+(\partial F/\partial\alpha)^2]^{1/2}}. \quad (20)$$

Hence, using Eq. (19), we readily obtain

$$\alpha-\beta=\frac{\alpha+\delta/2}{\eta(1-\xi)}. \quad (21)$$

Variation of the angle α within the permissible range $-\delta/2 < \alpha < \delta/2$ corresponds to variation of the angle ψ in an interval of width

$$\Delta=\eta\xi^{1/2}(1-\xi)^{1/2}. \quad (22)$$

The center of this interval is still given by the angle φ_0, apart from small terms. A comparison of Eqs. (18) and (22) shows that the range of permissible directions for the group velocity is considerably wider than for the phase velocity.

The polarization of the surface waves can be determined if we find the coefficients in the initial linear combination of the solutions (6) and (7). We can readily show that if $\eta \ll 1$, the wave is almost transverse and the vector $\mathbf{E}$ is practically parallel to the interface: $iE_x/E_y \propto \eta^{1/2}$ and $E_z/E_y \propto \eta^2$.

The existence of surface waves due to the differences between the anisotropy of the two media has been shown for a relatively simple case. Obviously, there should be a greater variety of waves of this type in more complex cases (when the optic axis is not parallel to the interface, biaxial crystals, gyrotropic media, etc.). We note also that surface waves may exhibit certain special features in the angular dependence of the reflection coefficient.

[1]F. I. Fedorov and V. V. Fillipov, *Reflection and Refraction of Light by Transparent Crystals* [in Russian], Nauka i Tekhnika, Minsk (1976).

Translated by A. Tybulewicz