Critical dynamics at the smectic-*A*–smectic-*C* phase transition

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(Submitted 8 December 1987)
Zh. Eksp. Teor. Fiz. **94**, 167–182 (April 1988)

Dynamical effects associated with fluctuations of the order parameter near the transition point of a second-order smectic-*A*–smectic-*C* transition are considered. A system of long-wavelength dynamical equations for the smectic, including an equation for the order parameter, is found. The fluctuation effects are studied by means of a diagram technique in which the vertices are determined by the nonlinearities in the dynamical equations. An effective action describing the critical mode is found by elimination of the weakly fluctuating degrees of freedom, and turns out to be renormalizable. This leads to scaling behavior of the correlators of the order parameter, and the corresponding indices can be determined from the first terms of the perturbation-theory series. The fluctuation contributions to the parameters of the low-frequency spectrum of the smectic are analyzed. The velocities of first and second sound tend to zero in the vicinity of the phase transition, and the bulk-viscosity coefficients determining their damping diverge. The corrections to the other kinetic coefficients (the shear viscosity, thermal conductivity, and permeation coefficient) do not contain divergences. The critical behavior of the parameters of the orientational mode in the *C* phase is also investigated.

1. INTRODUCTION

A large amount of experimental data pertaining to critical phenomena in liquid-crystalline phases has been accumulated.[1] However, an adequate theoretical description of the critical behavior of liquid crystals in largely lacking. This regrettable situation is due primarily to the strong anisotropy inherent in liquid crystals. Because of this, the usual universal models in which the components of the order parameter are defined in "isotopic" space are, at best, applicable to the description of phase transitions in liquid crystals only in an extremely narrow region (difficult to reach experimentally) near the transition. The actual critical phenomena observed in experiment, however, usually correspond to crossover behavior and therefore require a detailed analysis.

One of the simplest transitions for theoretical investigation is the second-order smectic-*A*–smectic-*C* phase transition. The region of strongly developed fluctuations in this transition is rather wide, and is sharply delineated in experiment. The thermodynamics of the smectic-*A*–smectic-*C* phase transition has been considered theoretically in Ref. 2. However, the dynamical phenomena near the transition point of this phase transition are considerably richer and more informative than the static phenomena, and this stimulated us to conduct a theoretical investigation of the critical dynamics in the smectic-*A*–smectic-*C* transition.

2. THE THERMODYNAMICS OF THE TRANSITION

To describe the layer structure of the smectic it is convenient to use a function $W(t,r)$ such that the equation $W=$ const specifies the position of a certain smectic layer. Correspondingly, ΔW gives the direction of the normal to the layer, and

$$\mathbf{v}=\nabla W/|\nabla W| \tag{1}$$

is the unit vector along this direction.

The average direction of the principal axes of the anisotropic molecules of which the liquid crystal consists is specified by the unit vector **n** (the director). In a smectic-*A* the director is perpendicular to the layer, and therefore the vectors **n** and **ν** coincide. In a smectic-*C* the principal axes of the molecules are tilted through a certain angle with respect to the normal **ν**. In this case the intermolecular forces fix only the component of the director along the normal to the layer.

To describe the smectic-*A*–smectic-*C* phase transition it is natural to introduce the following order parameter[2]:

$$\boldsymbol{\psi}=[\mathbf{n}\boldsymbol{\nu}]. \tag{2}$$

Its value is equal to zero in the *A* phase and is nonzero in the *C* phase. The modulus of the order parameter gives the tilt angle of the director with respect to the normal to the layer. According to the definition (2) the order parameter has nonzero components only in the plane of the smectic layer.

In the energy density the principal term associated with the nonuniform deformation of the director has the following form:

$$\tfrac{1}{2}K_1(\nabla\mathbf{n})^2+\tfrac{1}{2}K_2[\mathbf{n}[\nabla\mathbf{n}]]^2+\tfrac{1}{2}K_3[(\mathbf{n}\nabla)\mathbf{n}]^2. \tag{3}$$

Here K_1, K_2, and K_3 are the Frank constants.

The energy of deformation of the smectic layers is given by the well-known expression

$$\tfrac{1}{8}B[l^2(\nabla W)^2-1]^2. \tag{4}$$

The coefficient B is called the compression modulus of the smectic layers, and l is the equilibrium spacing between the smectic layers in the A phase. In the expansion of the energy density there are also terms specifying the relation between the orientation of the normal to the layer and the orientation of the director:

$$-\tfrac{1}{2}A[(\mathbf{n}\boldsymbol{\nu})^2-1]+\tfrac{1}{2}D[l^2(\nabla W)^2-1]\cdot[(\mathbf{n}\boldsymbol{\nu})^2-1]+\tfrac{1}{4}U'[(\mathbf{n}\boldsymbol{\nu})^2-1]^2. \tag{5}$$

The constants D and U' appearing in (5) are of the order of the modulus B. A phase transition occurs with respect to the parameter A: When $A<0$ the C phase is realized, and when $A>0$ the A phase is realized.

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It follows from the definition (2) that

$$\mathbf{n}=\boldsymbol{\nu}(1-\psi^2)^{1/2}+[\boldsymbol{\nu}\boldsymbol{\psi}]. \tag{6}$$

Substituting this expression into (3) and (5) we find the necessary terms of the expansion of the energy in the order parameter. The total energy density of the smectic is obtained by adding the expression for the internal energy $\varepsilon(\rho,\sigma)$, where ρ is the mass density and σ is the specific entropy. Thus, the energy density E of the smectic is a function of the following variables:

$$E=E(\rho,\ \sigma,\ \nabla W,\ \nabla\nabla W,\ \psi,\ \nabla\psi). \tag{7}$$

Near the smectic-A–smectic-C transition only the order parameter fluctuates strongly. The other variables appearing in (7) are weakly fluctuating, and we shall denote their deviations from equilibrium by the symbol φ_μ:

$$\varphi_\rho=\rho-\rho_0,\quad \varphi_\sigma=\sigma-\sigma_0,\quad \varphi_W=\nabla(W-W_0). \tag{8}$$

Here ρ_0, σ_0, and W_0 are the equilibrium values.

To investigate the distinctive features of the phase transition under consideration, in the energy density (7) it is sufficient to retain the terms of lowest order in ψ. In the terms coupling the order parameter with the weakly fluctuating variables we can confine ourselves to the approximation linear in the latter. In that part of the energy which does not contain ψ it is sufficient to confine ourselves to the term quadratic in φ_μ. As a result, the energy density acquires the following form:

$$E=E_1(\psi,\ \nabla\psi)+E_2(\psi,\ \varphi_\mu)+E_3(\varphi_\mu). \tag{9}$$

Here,

$$E_1=-{}^1/_2A\psi^2+{}^1/_2\{K_1(\boldsymbol{\nu}[\nabla\boldsymbol{\psi}])^2+K_2(\nabla\boldsymbol{\psi})^2 +K_3((\boldsymbol{\nu}\nabla)\boldsymbol{\psi})^2\}+{}^1/_4U'\psi^4, \tag{10}$$

$$E_2=D_\mu\varphi_\mu\psi^2, \tag{11}$$

$$E_3=\varepsilon(\rho,\ \sigma)+{}^1/_8B[l^2(\nabla W)^2-1]+{}^1/_2Kl^2(\nabla^2W)^2, \tag{12}$$

where $K=K_1$, and in the formula (11) the repeated indices μ are summed over and the following notation has been introduced:

$$D_\sigma=\frac{1}{2}\frac{\partial A}{\partial\sigma}-\frac{D}{\rho_0}\left(\frac{\partial\ln l}{\partial\sigma}\right)_\rho, \tag{13}$$

$$D_\rho=\frac{1}{2}\frac{\partial A}{\partial\rho}-\frac{D}{\rho_0}\left(\frac{\partial\ln l}{\partial\rho}\right)_\sigma, \tag{14}$$

$$D_W=-Dl\boldsymbol{\nu}. \tag{15}$$

In the expression (10) the normal vector $\boldsymbol{\nu}$ appears. Allowance for the fluctuations of $\boldsymbol{\nu}$ leads to the appearance in the energy density of terms of higher order than the terms that we have kept. Therefore $\boldsymbol{\nu}$ in (10) must be set equal to its equilibrium value (we shall assume it to be directed along the z axis). Correspondingly, by virtue of the identity $\boldsymbol{\nu}\cdot\boldsymbol{\psi}=0$ [which follows from the definition (2)], it follows that the order parameter has nonzero components only along the x and y axes. Henceforth, we shall indicate vector components along these axes by Greek indices α, β,..., so that the components of the order parameter are written in the form ψ_α.

To investigate the fluctuation effects associated with the self-interaction of the order parameter ψ it is convenient to effectively eliminate the weakly fluctuating variables φ_μ from the analysis. This elimination reduces to the calculation of the extremum of the energy density with respect to these variables. As a result of taking the extremum of the energy (11) with respect to φ_μ we obtain an effective energy that depends only on the order parameter:

$$E^{eff}={}^1/_2K_1(\varepsilon_{\alpha\beta}\nabla_\alpha\psi_\beta)^2+{}^1/_2K_2(\nabla_\alpha\psi_\alpha)^2 +{}^1/_2K_3(\nabla_z\psi_\alpha)^2+{}^1/_4U\psi^4+{}^1/_2A\psi^2. \tag{16}$$

Here $\varepsilon_{\alpha\beta}$ is the two-dimensional antisymmetric unit tensor, and

$$U=U'-{}^1/_2D_\mu g_{\mu\nu}{}^0D_\nu, \tag{17}$$

$$(g_{\mu\nu}{}^0)^{-1}=\partial^2E_3/\partial\varphi_\mu\partial\varphi_\nu. \tag{18}$$

The expansion (16) makes it possible to investigate the critical behavior of correlators of the order parameter. The critical behavior of such quantities as the specific heat and compressibility can be analyzed on the basis of the following expression [obtained from the form of the energy (9)] for the exact pair correlators of the weakly fluctuating quantities:

$$\langle\varphi_\mu(\mathbf{r}_1)\varphi_\nu(\mathbf{r}_2)\rangle=Tg_{\mu\nu}{}^0+g_{\mu\lambda}{}^0D_\lambda\langle\psi^2(\mathbf{r}_1)\psi^2(\mathbf{r}_2)\rangle D_\varkappa g_{\varkappa\nu}{}^0. \tag{19}$$

Here T is the temperature. The first term in the right-hand side of (19) is the bare contribution to the correlator and the second term is the fluctuation contribution.

The expression (16) corresponds to the energy density of a model with a two-component order parameter and interaction proportional to ψ^4. However, the order parameter is specified in real, not isotopic space. This circumstance leads to the result[2] that there exists a broad region of values of the parameter A in which is realized an intermediate asymptotic form described by nonuniversal critical indices that depend on the ratio of the bare Frank constants. Universal critical behavior is realized only in a very narrow region (not experimentally accessible, apparently) near the transition point.

3. THE DYNAMICAL EQUATIONS

To study the dynamical effects associated with the fluctuations of the order parameter it is necessary first of all to derive the system of nonlinear equations describing the dynamics of the long-wavelength degrees of freedom of the smectic near the transition from the A to the C phase. The complete set of long-wavelength degrees of freedom of the smectic-A consists of the momentum-density components j_k, the mass density ρ, the specific entropy σ, and the smectic variable W. Near the smectic-A–smectic-C transition it is necessary to take one further variable into account—the order parameter ψ introduced above. In the C phase outside the critical region the modulus of the order parameter is fixed, but not its phase, which must be included among the hydrodynamic variables.

The nondissipative equations of the system are constructed most simply using the method of Poisson brackets.[3] Dissipative terms are then added to the hydrodynamic equations in the standard manner.

The form of the Poisson brackets for the hydrodynamic variables of a smectic is well known[3]:

$$\{j_i(\mathbf{r}_1),\ \sigma(\mathbf{r}_2)\} = -\nabla_i\sigma\delta(\mathbf{r}_1-\mathbf{r}_2),$$

$$\{j_i(\mathbf{r}_1),\ \rho(\mathbf{r}_2)\} = -\nabla_i\rho\delta(\mathbf{r}_1-\mathbf{r}_2)+\nabla_i\delta(\mathbf{r}_1-\mathbf{r}_2)\rho(\mathbf{r}_2),$$

$$\{j_i(\mathbf{r}_1),\ j_k(\mathbf{r}_2)\} = -\nabla_i j_k\delta(\mathbf{r}_1-\mathbf{r}_2)+\nabla_i\delta(\mathbf{r}_1-\mathbf{r}_2)j_k(\mathbf{r}_2) + \nabla_k\delta(\mathbf{r}_1-\mathbf{r}_2)j_i(\mathbf{r}_2), \tag{20}$$

$$\{j_i(\mathbf{r}_1),\ \nabla_k W(\mathbf{r}_2)\} = -\nabla_i\nabla_k W\delta(\mathbf{r}_1-\mathbf{r}_2)+\nabla_k\delta(\mathbf{r}_1-\mathbf{r}_2)\nabla_i W. \tag{21}$$

Making use of the bracket (21), in accordance with the definition (1) we obtain

$$\{j_i(\mathbf{r}_1),\ \nu_k(\mathbf{r}_2)\} = -\nabla_i\nu_k\delta(\mathbf{r}_1-\mathbf{r}_2)+\nabla_m\delta(\mathbf{r}_1-\mathbf{r}_2)\cdot(\delta_{mk}-\nu_m\nu_k)\nu_i(\mathbf{r}_2). \tag{22}$$

To construct the Poisson bracket between the momentum density and the order parameter it is necessary to know the bracket $\{\mathbf{j},\mathbf{n}\}$. This bracket has the following form[4]:

$$\{j_i(\mathbf{r}_1),\ n_k(\mathbf{r}_2)\} = -\nabla_i n_k\delta(\mathbf{r}_1-\mathbf{r}_2)+\nabla_j\delta(\mathbf{r}_1-\mathbf{r}_2)\cdot{}^1/_2[(\delta_{jk}n_i-\delta_{ij}n_k)-\lambda(n_i\delta_{jk}+n_k\delta_{ij}-2n_in_kn_j)]. \tag{23}$$

Here λ is a dimensionless phenomenological parameter ~ 1. Using the expressions (23) and (22) and the definition (2) of the order parameter, we find

$$\{j_i(\mathbf{r}_1),\ \psi_k(\mathbf{r}_2)\} = -\nabla_i\psi_k\delta(\mathbf{r}_1-\mathbf{r}_2)+\nabla_m\delta(\mathbf{r}_1-\mathbf{r}_2)\cdot\cdot[\psi_k(\lambda n_in_m-\nu_i\nu_m)+\psi_n\varepsilon_{pni}\varepsilon_{pkm} + {}^1/_2(\lambda+1)(\varepsilon_{jik}n_m+n_i\varepsilon_{jmk})\nu_j]. \tag{24}$$

All the quantities in the right-hand sides of (23) and (24) are functions of $\mathbf{r}_2$.

The expressions given in (19), (21), and (24) exhaust the nonzero Poisson brackets for the system under consideration. All these brackets have a structure of the same type:

$$\{j_i(\mathbf{r}_1),\ \varphi_a(\mathbf{r}_2)\} = -\nabla_i\varphi_a\delta(\mathbf{r}_1-\mathbf{r}_2)+\nabla_k\delta(\mathbf{r}_1-\mathbf{r}_2)R_{ik}{}^a(\mathbf{r}_2). \tag{25}$$

Here, by φ_a we mean the complete set of hydrodynamic variables $\mathbf{j}$, φ_μ, ψ_α. The concrete expressions for the functions R^{α}_{ik} are determined by the right-hand sides of the formulas (19), (21), and (24).

We can now write out the nondissipative hydrodynamic equations, which are Liouville equations with the Hamiltonian

$$H = \int d^3r\left[\frac{j^2}{2\rho}+E(\rho,\sigma,\nabla W,\nabla\nabla W,\psi)\right]. \tag{26}$$

The equation for the momentum density has the form

$$\frac{\partial j_i}{\partial t} = \{H, j_i\} = h_a\nabla_i\varphi_a-\nabla_k(R_{ik}{}^a h_a). \tag{27}$$

In Eq. (27) repeated indices are summed over, and we have introduced the notation

$$h_a = \delta H/\delta\varphi_a; \tag{28}$$

h_a is the molecular field conjugate to the variable φ_a.

For variables φ_a other than the momentum density (i.e., for ψ_a and φ_μ) the hydrodynamic equations, in accordance with (25), have the following form:

$$\partial\varphi_a/\partial t = \{H, \varphi_a\} = -v_i\nabla_i\varphi_a-\nabla_k v_i R_{ik}{}^a, \tag{29}$$

where $\mathbf{v} = \mathbf{j}/\rho$ is the velocity. The right-hand side of (27) reduces to an exact divergence $-\nabla_k T_{ik}$. Here the stress tensor is represented in the form of the sum

$$T_{ik} = T_{1ik}+T_{2ik}+T_{3ik}, \tag{30}$$

where

$$T_{1ik} = P\delta_{ik}+\left(\frac{\partial E}{\partial\psi_n}-\nabla_j\frac{\partial E}{\partial\nabla_j\psi_n}\right)\{\psi_n[(\delta_{ik}-\nu_i\nu_k)(1-\lambda)] + {}^1/_2(\lambda+1)(\varepsilon_{inj}n_k+n_i\varepsilon_{knj})\nu_j\},$$

$$T_{2ik} = \frac{\partial E}{\partial\nabla_k W}\nabla_i W+2\frac{\partial E}{\partial\nabla_k\nabla_m W}\nabla_m\nabla_i W+\frac{\partial E}{\partial\nabla_k\psi_n}\nabla_i\psi_n - \frac{\partial E}{\partial\nabla_n\psi_i}\nabla_n\psi_k-\frac{\partial E}{\partial\psi_i}\psi_k,$$

$$T_{3ik} = \nabla_n\left[\frac{\partial E}{\partial\nabla_n\psi_i}\psi_k-\frac{\partial E}{\partial\nabla_n\nabla_k W}\nabla_i W\right].$$

One can convince oneself that the first term in (30) is explicitly symmetric in the indices i, k ($P = \rho\partial E/\partial\rho - E$ is the pressure). The antisymmetric part of the second term is equal to zero by virtue of the rotational invariance of the energy density (7). The divergence of the third term in (30) can be reduced to the divergence of a tensor that is symmetric in the indices i, k. Thus, the right-hand side of (28) can be transformed to the divergence of a symmetric tensor. Therefore, irrespective of the explicit form of the energy (7), the angular momentum is conserved.

To the nondissipative hydrodynamic equations constructed by means of the method of Poisson brackets we must add kinetic terms. When these are taken into account the equations acquire the following form:

$$\partial\varphi_a/\partial t = \{H, \varphi_a\}+\hat{\Gamma}_{ab}h_b. \tag{31}$$

The differential operator $\hat{\Gamma}_{ab}$ in (31) is determined by the set of kinetic coefficients. Thus, for the order parameter and the momentum density Eq. (31) acquires the following explicit forms:

$$\partial\psi_\alpha/\partial t = -v_i\nabla_i\psi_\alpha-R_{ik}{}^\alpha\nabla_k v_i-\Gamma h_\alpha, \tag{32}$$

$$\partial j_i/\partial t = h_a\nabla_i\varphi_a-\nabla_k(R_{ik}{}^a h_a-\eta_{iklm}\nabla_l v_m). \tag{33}$$

We recall that here R^{α}_{ik} is the quantity defined by the right-hand side of the Poisson bracket (24) in accordance with Eq. (25), and h_α is the molecular field conjugate to ψ_α. The coefficient Γ^{-1} in (32) has the dimensions of viscosity and is analogous to the torsional viscosity of a nematic. The viscous dissipative contribution appearing in the right-hand side of Eq. (33) can be represented conveniently in the form

$$\eta_{iklm} = \overset{\parallel}{\eta}_{iklm}+\overset{\perp}{\eta}_{iklm}. \tag{34}$$

Here,

$$\overset{\parallel}{\eta}_{iklm} = \eta_1\delta_{il}{}^\perp\overset{\perp}{\delta}_{km}+(\eta_2-\eta_1)\delta_{ik}{}^\perp\overset{\perp}{\delta}_{lm}+{}^1/_2\eta_3(\nu_i\nu_k\overset{\perp}{\delta}_{lm} + \nu_i\nu_m\delta_{kl}{}^\perp+\nu_k\nu_m\delta_{il}{}^\perp)+\eta_4(\nu_i\nu_k\overset{\perp}{\delta}_{lm}+\nu_l\nu_m\delta_{ik}{}^\perp)+\eta_5\nu_i\nu_l\nu_k\nu_m,$$

$$\overset{\perp}{\eta}_{iklm} = \eta_1\delta_{kl}{}^\perp\overset{\perp}{\delta}_{im}+{}^1/_2\eta_3\nu_k\nu_l\overset{\perp}{\delta}_{im}, \quad \delta_{ik}{}^\perp = \delta_{ik}-\nu_i\nu_k.$$

For the notation of the viscosity coefficients we have followed Ref. 5.

The remaining part of the operator $\hat{\Gamma}_{ab}$ is constructed in the standard manner[6] with the use of the permeation and thermal-conductivity coefficients. For us, the important point in the following will be the fact that in smectics, as in all liquid crystals, we have a dimensionless small parameter

$$\rho K/\eta^2 \ll 1. \tag{35}$$

Here K is a quantity of the order of the Frank constants and $\eta \sim \Gamma^{-1}$ is the characteristic viscosity.

4. THE EFFECTIVE ACTION

The dynamical effects associated with the fluctuations of the order parameter can be investigated conveniently by means of a diagram technique. The corresponding formalism was developed in Ref. 7, in which a generating functional for the determination of dynamical correlators of long-wavelength variables was constructed. For the system described by Eqs. (31), this generating functional has the following form:

$$Z(m_a, y_a) = \int D\varphi\, Dp \exp\left[i \int dt\, d^3r (\mathscr{L} + m_a \varphi_a + p_a y_a) \right]. \tag{36}$$

Here p_a are the auxiliary fields conjugate to the variables φ_a, and the Lagrangian density

$$\mathscr{L} = p_a \left(\frac{\partial \varphi_a}{\partial t} - \{H, \varphi_a\} - \hat{\Gamma}_{ab} h_b \right) + i p_a T \hat{\Gamma}_{ab} p_b. \tag{37}$$

By expanding the generating functional (36) in the fields m_a and y_a we obtain correlators, of different orders, of the long-wavelength variables φ_a and fields p_a. We introduce the following notation:

$$D_{ab}(t_1 - t_2, \mathbf{r}_1 - \mathbf{r}_2) = \langle \varphi_a(t_1, \mathbf{r}_1) \varphi_b(t_2, \mathbf{r}_2) \rangle = -\delta^2 Z / \delta m_a \delta m_b, \tag{38}$$

$$G_{ab}(t_1 - t_2, \mathbf{r}_1 - \mathbf{r}_2) = \langle \varphi_a(t_1, \mathbf{r}_1) p_b(t_2, \mathbf{r}_2) \rangle = -\delta^2 Z / \delta m_a \delta y_b. \tag{39}$$

The function D_{ab} is simply the pair correlator of the long-wavelength variables, and G_{ab} determines the linear susceptibility of the system; consequently, the singularities of the function G_{ab} $(\omega, \mathbf{k})$ give the spectrum of the normal modes.

The part of the Lagrangian quadratic in the variables φ_a and p_a determines the bare values of the correlators. The higher terms of the expansion, starting from the third, specify the vertices of the coupling of the fluctuations. Because of this coupling, fluctuation corrections to the bare correlators arise. For the case that we are considering the only important corrections are those associated with fluctuations of the order parameter ψ.

In view of this, in the analysis of the fluctuation effects we can substantially simplify the Lagrangian (37). In this Lagrangian it is sufficient to retain terms quadratic in the weakly fluctuating variables j_i and φ_μ listed in (8), and also the terms quadratic in the fields p_i and p_μ conjugate to them. In the terms describing the coupling of j_i, φ_μ, p_i, and p_μ with the order parameter it is sufficient to retain the terms linear in these variables. In this case the unit vector $\boldsymbol{\nu}$ that appears, in particular, in the viscosity tensor (34) must be assumed to be constant (and directed along the z axis). Then, in accordance with its definition (2), the order parameter will have only components along the x and y axes.

In the study of fluctuation effects in the dynamics, as in the statics, it is convenient to eliminate from the analysis the weakly fluctuating degrees of freedom. For this, in the functional integral (36) it is necessary to integrate over the variables j_i, φ_μ, p_i, and p_μ. After the simplifications of the Lagrangian that were described above, this integration reduces to Gaussian integrals and is performed explicitly. As a result, the generating functional is represented in the form of a functional integral, over ψ_α and the field p_α conjugate to it, of $\exp(iI^{\text{eff}})$, where I^{eff} is the extremum of the exponent in (36) with respect to the fields j_i, φ_μ, p_i, and p_μ. To calculate the extremum of the Lagrangian density (37) with respect to the fields j_i and p_i it is convenient to represent the velocity $\mathbf{v}$ in the form of a sum of two components:

$$\mathbf{v} = \mathbf{v}_\perp + \mathbf{v}_\parallel. \tag{40}$$

The symbols $\parallel$ and $\perp$ indicate the velocity components longitudinal and transverse to the wave vector in the plane of the smectic layer. For example, the transverse component is specified by the formula

$$v_{\perp i} = {}^1/_2 k_\perp^{-2} \nu_m \nu_j \varepsilon_{mni} \varepsilon_{jkl} k_n k_k v_l. \tag{41}$$

As a result of this calculation we obtain

$$p_\alpha(\partial \psi_\alpha / \partial t + \hat{\Gamma}^{eff}_{\alpha\beta} h_\beta) + i T p_\alpha \hat{\Gamma}^{eff}_{\alpha\beta} p_\beta$$
$$+ p_\mu(\partial \varphi_\mu / \partial t + \hat{\Gamma}^{eff}_{\mu\nu} h_\nu) + i T p_\mu \hat{\Gamma}^{eff}_{\mu\nu} p_\nu. \tag{42}$$

The molecular fields h_α and h_μ needed for the construction of the Lagrangian (42) should be calculated from the energy (9) in accordance with the definition (28).

The linear operators $\hat{\Gamma}^{\text{eff}}$ appearing in (41) have a rather complicated structure. They can be represented in the form $\hat{\Gamma}^{\text{eff}} = \Gamma^{\text{eff}}(i\partial/\partial t, -i\nabla)$. The explicit expressions have the following form:

$$\Gamma^{ff}_{\alpha\beta}(\omega, \mathbf{k}) = \Gamma \delta_{\alpha\beta} + i R_{mk}{}^{\alpha} g_{ki}{}^{\perp}(\omega, \mathbf{k}) k_p R_{pm}{}^{\beta}, \tag{43}$$

$$\Gamma^{eff}_{\mu\nu}(\omega, \mathbf{k}) = \Gamma_{\mu\nu}(\mathbf{k}) + i k_m R_{mk}{}^{\mu} g_{ki}{}'(\omega, \mathbf{k}) k_p R_{pi}{}^{\nu}, \tag{44}$$

where

$$(g_{km}{}^{\perp})^{-1} = \omega \rho \delta_{km} + i \eta^{\perp}_{iklm} k_i k_l, \tag{45}$$

$$(g_{km}{}')^{-1} = \omega \rho \delta_{km} + i \eta^{\parallel}_{iklm} k_i k_l. \tag{46}$$

In the expressions (43) and (44) the quantities R^{β}_{ik} and R^{ν}_{ik} [which, we recall, are determined from the Poisson brackets (19), (21), and (24) in accordance with (25)] are equal to their equilibrium values, and the quantities $\eta^{\perp}_{iklm}$ and $\eta^{\parallel}_{iklm}$ were defined in (34).

As a result of the elimination of the remaining weakly fluctuating variables we obtain a rather cumbersome expression for the generating functional. We give the leading terms of this expression, omitting the dependence on the external fields m_ν, y_ν, m_α, and y_α:

$$Z(m,y)=\int D\psi Dp\exp\left\{\int dt\,d^3r[i\mathscr{L}^{eff}+R_{im}{}^{\alpha}\nabla_m p_\alpha\hat{g}_{ik}{}^{\perp}y_k{}^{\perp}\right.$$
$$-m_k{}^{\perp}\hat{g}_{ki}{}^{\perp}R_{im}{}^{\alpha}\nabla_m h_\alpha^{eff}+2R_{im}{}^{\mu}D_\mu\nabla_m\psi_\beta\hat{\Gamma}_{\beta\alpha}^{eff}\,p_\alpha\hat{g}_{ik}{}^{\parallel}y_k{}^{\parallel}$$
$$\left.-m_k{}^{\parallel}\hat{g}_{ki}{}^{\parallel}R_{im}{}^{\mu}D_\mu\nabla_m\psi^2]\right\}. \tag{47}$$

Here, in analogy with (40) and (41) we have introduced the longitudinal and transverse components $\mathbf{m}^{\parallel}$, $\mathbf{m}^{\perp}$, $\mathbf{y}^{\parallel}$, and $\mathbf{y}^{\perp}$. In accordance with (38) and (39) the expansion of the generating functional (44) in the fields $\mathbf{m}^{\perp}$, $\mathbf{y}^{\perp}$, $\mathbf{m}^{\parallel}$, and $\mathbf{y}^{\parallel}$ determines the correlators $G_{ik}^{\perp}=\langle p_k v_{i\perp}\rangle$ and $g_{ik}=\langle p_k v_{i\parallel}\rangle$.

In the expression (47) the linear operators $\hat{\Gamma}_{\alpha\beta}^{\text{eff}}$, $\hat{g}_{ik}^{\perp}$, and $\hat{g}_{ik}^{\parallel}$ appear. The first two of these are defined in (43) and (45), and the operator $\hat{g}_{ik}^{\parallel}$ in the Fourier representation has the form

$$[g_{ik}{}^{\parallel}(\omega,\mathbf{k})]^{-1}=(g_{ik}{}'+iR_{im}{}^{\mu}\Gamma_{\mu\nu}^{eff}R_{kl}{}^{\nu}k_m k_l)^{-1}$$
$$-\omega^{-1}R_{im}{}^{\mu}(g_{\mu\nu}{}^{0})^{-1}R_{kl}{}^{\nu}k_m k_l, \tag{48}$$

where g'_{ik} and $\Gamma_{\mu\nu}^{\text{eff}}$ are given by the expressions (44) and (46). The quantity h_α^{eff} appearing in (47) is determined as follows:

$$h_\alpha^{eff}=\delta E^{eff}/\delta\psi_\alpha-2D_\mu\psi^2[g_{\mu\nu}(i\partial/\partial t,-i\nabla)-g_{\mu\nu}(0,-i\nabla)]D_\nu\psi_\alpha. \tag{49}$$

Here,

$$g_{\mu\nu}{}^{-1}(\omega,\mathbf{k})=i[\omega\Gamma_{\mu\nu}^{eff}(\omega,\mathbf{k})+(g_{\mu\nu}{}^{0})^{-1}], \tag{50}$$

where $g_{\mu\nu}^{0}$ was defined in (18).

Now, finally, we can write out the expression for the effective Lagrangian density:

$$\mathscr{L}^{eff}=p_\alpha[\partial\psi_\alpha/\partial t+\Gamma_{\alpha\beta}^{eff}(i\partial/\partial t,-i\nabla)h_\beta^{eff}]$$
$$+iTp_\alpha\Gamma_{\alpha\beta}^{eff}(i\partial/\partial t,-i\nabla)p_\beta. \tag{51}$$

The correlator (50) determines the bare linear susceptibility of the weakly fluctuating variables φ_μ. The poles of this correlator describe the spectrum of the oscillations of φ_μ (in particular, the first-sound and second-sound spectra), calculated in the linear approximation.

5. RENORMALIZATION OF THE CRITICAL MODE

The effective Lagrangian (51) can be conveniently represented in the following form:

$$\mathscr{L}^{eff}(p,\ \psi)=\mathscr{L}_0+\mathscr{L}_1+\mathscr{L}_2, \tag{52}$$

$$\mathscr{L}_0=p_\alpha[\partial\psi_\alpha/\partial t+\Gamma_{\alpha\beta}^{eff}(0,-i\nabla)\delta E^{eff}/\delta\psi_\beta]$$
$$+ip_\alpha T\Gamma_{\alpha\beta}^{eff}(0,-i\nabla)p_\beta, \tag{53}$$

$$\mathscr{L}_1=p_\alpha[\Gamma_{\alpha\beta}^{eff}(i\partial/\partial t,-i\nabla)-\Gamma_{\alpha\beta}^{eff}(0,-i\nabla)]\delta E^{eff}/\delta\psi_\beta, \tag{54}$$

$$\mathscr{L}_2=p_\alpha\Gamma_{\alpha\beta}^{eff}(i\partial/\partial t,-i\nabla)(h_\beta^{eff}-\delta E^{eff}/\delta\psi_\beta). \tag{55}$$

The first term $\mathscr{L}_0$ in (52) corresponds to the case of purely dissipative dynamics of the system with energy density (16). In the study of the softest (critical) mode the terms $\mathscr{L}_1$ and $\mathscr{L}_2$ in the Lagrangian can be omitted. The corrections to the spectrum of the critical mode that are generated by the term $\mathscr{L}_1$ in (52) are small in proportion to the smallness of the ratio of the frequency of the critical mode (the slow director mode) to the frequency of the fast shear mode [see (35)]. As regards the corrections associated with $\mathscr{L}_2$, they are small in the hydrodynamic parameter. Thus, to investigate the self-interaction of the long-wavelength fluctuations of the order parameter, in the Lagrangian density (52) we must confine ourselves to just the term $\mathscr{L}_0$. (We note that $\mathscr{L}_2$ contains terms having a singularity at the frequency of the thermal-diffusion mode. However, the corresponding terms are smaller than $\mathscr{L}_0$ in the hydrodynamic parameter, since they originate from dissipative, rather than reactive interaction vertices.) Substituting into $\mathscr{L}_0$ the explicit expressions for the molecular field conjugate to the order parameter and for the kinetic coefficients (43), we obtain in the quadratic approximation an expression determining the bare correlator $G_{\alpha\beta}=\langle\varphi_\alpha p_\beta\rangle$:

$$-G_{\alpha\beta}{}^{-1}=[\omega+i\Gamma(K_1k_\perp{}^2+K_3k_z{}^2+{}^1/_2A)](\delta_{\alpha\beta}-k_\alpha k_\beta/k_\perp{}^2)$$
$$+[\omega+i\Gamma_1(K_2k_\perp{}^2+K_3k_z{}^2+{}^1/_2A)]k_\alpha k_\beta/k_\perp{}^2, \tag{56}$$

where

$$\Gamma_1=(\lambda+1)^2(2\eta_3+4\eta_1k_\perp{}^2/k_z{}^2)^{-1}+\Gamma. \tag{57}$$

By virtue of the fact that $\lambda\sim\Gamma\eta\sim1$, the two terms in (56) have the same order of magnitude. Therefore, the bare spectrum is strongly anisotropic, both on account of the anisotropy of the elastic constants and on account of the anisotropy of the kinetic coefficients.

Knowing $G_{\alpha\beta}$, we can use the fluctuation-dissipation theorem [a derivation of which is given in the Appendix—see Eq. (A6)] to calculate the pair correlator $D_{\alpha\beta}=\langle\varphi_\alpha\varphi_\beta\rangle$. The bare equal-time correlator $D_{\alpha\beta}$ found from formula (A7) with the use of (56) coincides, of course, with that obtained from the energy density (16) in the quadratic approximation.

In the Lagrangian density $\mathscr{L}_0$ the term

$$\mathscr{L}_{int}=U[p_\alpha\Gamma_{\alpha\beta}^{eff}(0,-i\nabla)\psi_\beta]\psi^2 \tag{58}$$

describes the self-interaction of the order parameter. Thus, the Lagrangian $\mathscr{L}_0$ generates the standard diagram technique, with the bare four-point vertex (58) (depicted on the diagrams by a point) and with the bare pair correlators $D_{\alpha\beta}$ and $G_{\alpha\beta}$ (depicted, respectively, by a solid line and by a solid line with an arrow that specifies the direction from ψ_α to p_β).

In four-dimensional space all the fluctuation corrections generated by (58) reduce to a logarithmic renormalization of the constants appearing in the expression for $\mathscr{L}_0$. This renormalization is described by the corresponding renormalization-group equations. In three-dimensional space the fluctuation effects in this case can be studied by the method of the ε-expansion from four-dimensional space. For us, however, it will be more convenient to apply the so-called real-space renormalization-group method,[2] since then the problem of the continuation to four-dimensional space of a Lagrangian $\mathscr{L}_0$ in which the order-parameter indices are coupled to the vector indices of spatial gradients does not arise.

The fluctuation correction to the parameter A in the one-loop approximation is depicted by the diagram given in Fig. 1. Corrections to the constants K arise only in the two-loop approximation (Fig. 2). The same two-loop diagram depicts fluctuation corrections to the kinetic coefficients Γ and Γ_1 in (56).

As already noted above, there exists in the vicinity of the transition a wide range of temperatures in which an intermediate asymptotic form, described by critical indices that depend on the ratio K_1/K_2, is realized. The kinetic coefficients Γ and Γ_1 and the Frank constants remain constant in this region, but the parameter A is renormalized in accordance with the law

$$A \propto \tau^{\gamma}, \quad \tau = |(T-T_c)/T_c|, \tag{59}$$

where T_c is the transition temperature. In the one-loop approximation the index γ was calculated in Ref. 2.

Universal dynamical behavior of the order parameter is realized only in a very narrow region near the transition, where the behavior is determined by the isotropic fixed point of the renormalization-group transformation. The renormalized elastic constants $\tilde{K}_1$ and $\tilde{K}_2$ in this region are equal, and the constant $\tilde{K}_3$, although different from $\tilde{K}_1$ and $\tilde{K}_2$, obeys the same scaling law. The kinetic coefficients are isotropized in an analogous manner. According to the results of Ref. 8, for an isotropic relaxation model with a four-point interaction the renormalized kinetic coefficients obey the following law:

$$\Gamma_1 = \Gamma \propto \tau^{(z-2)\nu}, \tag{60}$$

where z is the dynamical critical index and ν is the critical index describing the temperature dependence of the correlation length.

Thus, the renormalized spectrum of the critical mode in this region of temperatures is isotropic in the plane of the smectic layer:

$$\omega = i\Gamma(\tilde{K}_1 k_\perp^2 + \tilde{K}_3 k_z^2). \tag{61}$$

6. RENORMALIZATION OF THE DIFFUSION-MODE SPECTRUM

We shall consider the fluctuation corrections to the spectrum of the oscillations of the transverse component (41) of the velocity. The law of relaxation of this shear mode is determined by the poles of the corresponding linear susceptibility $G_{ik}^{\perp}$. In the calculation of this correlator we cannot neglect the term $\mathscr{L}_1$ in the Lagrangian (52):

$$\mathscr{L}_1 = ip_\alpha \nabla_m R_{mi}{}^{\alpha} [g_{ik}{}^{\perp}(i\partial/\partial t, -i\nabla) - g_{ik}{}^{\perp}(0, -i\nabla)] \nabla_p R_{pk}{}^{\beta} \delta E^{eff}/\delta\psi_\beta \tag{62}$$

[in obtaining this formula we used the explicit expressions (43) and (45)]. The point is that in the frequency region under consideration this term is singular.

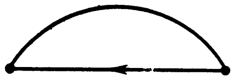

FIG. 1.

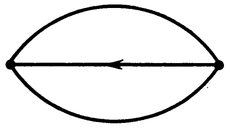

FIG. 2.

We shall consider the fluctuation corrections to the function $G_{ik}^{\perp}$ that arise from (62) (corrections of the type represented in Fig. 3). In Fig. 3 the wavy line depicts a difference of bare correlators

$$g_{ik}{}^{\perp}(\omega, \mathbf{k}) - g_{ik}{}^{\perp}(0, \mathbf{k}). \tag{63}$$

In this diagram this difference is integrated over the frequency. Since the pole of the function $g^{\perp}(\omega,\mathbf{k})$ corresponds to the frequency of the shear mode, on account of the integration this correction is small in the parameter (35). To the extent that (35) is small, all diagrams containing integration of the difference (63) over the frequency should be omitted. In the study of the shear mode, such a frequency integration always occurs in the fluctuation corrections originating from $\mathscr{L}_2$. Therefore, the contribution of $\mathscr{L}_2$ (55) to the Lagrangian density (52) can be omitted if the ratio of the frequency of the diffusive critical mode to the frequency of sound is small.

Thus, only diagrams in which the difference (63) is not integrated over the frequency are important for the renormalization of the spectrum of the shear mode. The sum of such diagrams is represented in Fig. 4, in which the rectangle denotes the quantity

$$Q^{\perp}(t,\mathbf{r}) = \Gamma_{\alpha\beta}^{eff}(0, -i\nabla)\langle h_\alpha(t,\mathbf{r}) p_\beta(0,0)\rangle.$$

These diagrams can be summed, since they form a geometrical series. As a result, we obtain (the technical details are given in the Appendix)

$$-[G_{ik}{}^{\perp}(\omega, \mathbf{k})]^{-1} = [g_{ik}{}^{\perp}(\omega, \mathbf{k})]^{-1} + (iR_{mi}{}^{\alpha} k_m R_{pn}{}^{\beta} k_n) \cdot [\delta_{pk} - g_{pm}{}^{\perp}(0, \mathbf{k})(g_{mk}{}^{\perp}(\omega, \mathbf{k}))^{-1}] Q^{\perp}(\omega, \mathbf{k}). \tag{64}$$

According to the formula (A10) in the Appendix,

$$Q^{\perp}(\omega,\mathbf{k})\Gamma_{\alpha\beta}^{eff}(0,\mathbf{k})[\delta_{\alpha\beta} + \omega G_{\beta\alpha}(\omega,\mathbf{k})].$$

Here all quantities except the exact linear susceptibility $G_{\beta\alpha}$ of the order parameter are bare quantities. Since in the hydrodynamic region we have $\omega G_{\alpha\beta} \ll \delta_{\alpha\beta}$, the position of the pole of $G^{\perp}(\omega,\mathbf{k})$ turns out to be independent of the distance from the transition point. Using the explicit expressions

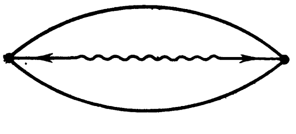

FIG. 3.

FIG. 4.

(23), (25), (43), and (45) for the quantities appearing in (64), we find the spectrum of the oscillations of the velocity component (41) transverse to the wave vector in the plane of the smectic layer:

$$\omega=-(\eta_1 k_\perp^2+{}^1/{}_2\eta_3 k_z^2)\Gamma_1\Gamma^{-1}, \tag{65}$$

where the coefficient Γ_1 is given by the formula (57).

To investigate the renormalization of the reactive parameter λ we must study the fluctuation corrections to the correlator

$$\langle v_{\perp i}p_\alpha\rangle. \tag{66}$$

We can convince ourselves that these corrections have the same structure as the corrections to the shear-mode spectrum. Therefore, in the hydrodynamic region there are only unimportant small corrections to the reactive parameter λ.

This circumstance leads to an interesting result for the orientational mode in the C phase. Since in a smectic-C the angle of tilt of the director $\mathbf{n}$ to the layer normal $\mathbf{v}$ is fixed, the spectrum of the orientational mode in the low-frequency limit is fully coupled to the oscillations of the azimuthal angle φ. It can be specified conveniently by means of the variation

$$\delta\varphi=2[\mathbf{v}\boldsymbol{\psi}]\delta\psi/\psi^2. \tag{67}$$

From Eq. (32), by virtue of the definition (67), it follows that

$$\begin{aligned}\partial\varphi/\partial t+\mathbf{v}\nabla\varphi+\varepsilon_{jni}\nu_j\nabla_n v_i+4\Gamma'\delta E/\delta\varphi-\lambda_1\nabla_k v_i(\nu_k\psi_i\\ +\nu_i\psi_k)-\lambda_2\nabla_k v_i\{\psi_i[\boldsymbol{\psi}\mathbf{v}]_k+\psi_k[\boldsymbol{\psi}\mathbf{v}]_i\}=0.\end{aligned} \tag{68}$$

Equation (68) contains two reactive parameters λ_1 and λ_2, describing the dynamical anisotropy in the smectic-C layer:

$$\lambda_1=(\lambda+1)/|\psi|,\quad \lambda_2=\lambda,\quad \Gamma'=\Gamma/\psi^2. \tag{69}$$

Since λ remains a constant, the reactive coefficient λ_1 increases in accordance with the following scaling law as the transition point is approached:

$$\lambda_1\infty\tau^{-\beta},\quad \Gamma'\infty\tau^{-2\beta}. \tag{70}$$

Here β is the index of the order parameter. Taking into account the narrowness of the temperature range of existence of the smectic-C, we arrive at the conclusion that the coefficient λ_1 is anomalously large. Therefore, despite the smallness of the angle of tilt of the director with respect to the normal to the layers in smectics-C, the anisotropy of the smectic layers is important in the dynamical effects. We note that, by virtue of (68)–(70), the spectrum of the orientational mode in the C phase does not have critical singularities.

7. RENORMALIZATION OF THE SOUND SPECTRUM

We shall consider the fluctuation contribution to the equations determining the propagation of sound near the smectic-A–smectic-C transition. The spectra of both the sounds existing in smectics are determined by the poles of the linear susceptibility $G^{\parallel}$. This bare susceptibility calculated by means of the generating functional (47) in the quadratic approximation coincides with (46). However, in the calculation of fluctuation corrections to this correlator we must not neglect the contribution of (55) to the Lagrangian density (52):

$$\begin{aligned}\mathscr{L}_2=2D_\mu R_{im}{}^\mu\nabla_m[p_\alpha\Gamma_{\alpha\beta}^{eff}(i\partial/\partial t,-i\nabla)\psi_\beta]\\ \cdot[-g_{ik}{}^{\parallel}(i\partial/\partial t,-i\nabla)+g_{ik}{}^{\parallel}(0,-i\nabla)]D_\nu R_{km}{}^\nu\nabla_m\psi^2,\end{aligned} \tag{71}$$

since this term has a singularity at precisely the frequencies of the sound modes.

For the same reasons as were discussed in the preceding section, all diagrams in which the difference

$$g_{ik}{}^{\parallel}(\omega,\mathbf{k})-g_{ik}{}^{\parallel}(0,\mathbf{k}) \tag{72}$$

is integrated over the frequency are small, on the order of the ratio of the frequency of the soft critical mode to the sound frequency. An example of such a diagram is given in Fig. 5 (the difference (72) is depicted by the dashed line). All diagrams of this type must be omitted.

In diagrams describing fluctuation corrections to $G^{\parallel}$ the difference (63) appearing in the term $\mathscr{L}_1$ (54) of the Lagrangian density is always integrated over the frequency. Therefore, to the extent that the parameter (35) is small, $\mathscr{L}_1$ can be omitted in the investigation of the sound spectrum.

The remaining series of ladder diagrams can be summed explicitly. As a result, we obtain

$$-[G_{ik}{}^{\parallel}(\omega,\mathbf{k})]^{-1}=[g_{ik}{}^{\parallel}(\omega,\mathbf{k})]^{-1}+D_\mu R_{ip}{}^\mu k_p Q^{\parallel}(\omega,\mathbf{k})k_m R_{km}{}^\nu D_\nu, \tag{73}$$

where

$$Q^{\parallel}(t,\mathbf{r})=\langle\psi^2(0,0)p_\alpha\Gamma_{\alpha\beta}^{eff}(i\partial/\partial t,-i\nabla)\psi_\beta(\mathbf{r},t)\rangle.$$

In the hydrodynamic region the fluctuation corrections to the imaginary part of the spectrum, which determine the renormalization of the viscosity tensor, are much smaller than the real part. Taking this into account, with the aid of the formula (A7) we can show that the real part of the pole of $G^{\parallel}$ (the part which determines the fluctuation corrections to the sound velocity) reproduces the static limit (19). The imaginary part of the pole determines the renormalized viscosity tensor

$$\tilde{\eta}^{\parallel}_{iklm}=\eta^{\parallel}_{iklm}+R_{im}{}^\mu D_\mu R^\nu_{lk}D_\nu M, \tag{74}$$

where

$$M=\frac{1}{2T}\int dt\,d^3r\langle\psi^2(t,\mathbf{r})\psi^2(0,0)\rangle.$$

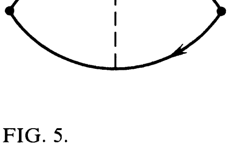

FIG. 5.

Using the explicit form of the functions (14), (15), (20), (25), and (45) appearing in (74), we find the following fluctuation contributions to the viscosity coefficients:

$$\Delta\eta_2=\frac{1}{2T}(D_\rho\rho)^2M,\quad \Delta\eta_4=\frac{1}{2T}\rho D_\rho(D+\rho D_\rho)M,$$

$$\Delta\eta_5=\frac{1}{2T}(\rho D_\rho+D)^2M. \tag{75}$$

In the vicinity of the smectic-*A*–smectic-*C* transition the quantity M appearing in (74) diverges. We shall denote the exponent of this singularity by x: $M\propto\tau^{-x}$. The critical index x can be related to the specific-heat index α, correlation-length index ν, and dynamical index z. In fact, according to the scaling relation,

$$\int\psi^2(t,\mathbf{r})\psi^2(0,0)e^{-i\omega t}\,dt\,d^3r\propto\tau^{-x}f(\omega\tau^{-z\nu}). \tag{76}$$

On the other hand, the fluctuation part of the specific heat is determined by the correlator

$$\int\psi^2(t,\mathbf{r})\psi^2(t,0)\,d^3r\propto\tau^{-\alpha}. \tag{77}$$

Comparing the expressions (76) and (77) we find that the critical index is equal to

$$x=\alpha+z\nu. \tag{78}$$

Up to now we have not been concerned with the renormalization of the spectrum of the thermal-diffusion mode. To calculate corrections to the spectrum of this mode it is necessary to follow a procedure analogous to that described above for the sound modes, since $\mathscr{L}_2$ contains a pole corresponding to the thermal-diffusion mode. As a result, the static renormalization of the specific heat is reproduced. As regards the thermal-conductivity and permeation coefficients, these have no divergent corrections. This is connected with the fact that the corresponding fluctuation terms contain dissipative vertices that are small in the hydrodynamic parameter in comparison with the reactive vertices.

8. CONCLUSION

Thus, we have shown that in a rather wide temperature region about the smectic-*A*–smectic-*C* transition the spectrum of the critical mode is anisotropic and has the form (56). The parameter A appearing in this formula obeys the law $A\propto\tau^{\gamma}$. The critical index γ of the susceptibility depends on the bare ratio K_1/K_2 of Frank constants and varies[2] in the range 1–1.25. Universal dynamics is realized only in a very narrow region near the transition. In this region the critical-mode spectrum, consisting of the oscillations of the director components transverse and longitudinal to the wave vector in the plane of the smectic layer becomes isotropic and is described by the formula (61). The kinetic coefficient $\tilde{\Gamma}$ obeys a scaling law: $\tilde{\Gamma}\propto\tau^{(z-2)\nu}$. However, the critical index of this kinetic coefficient is small: $(z-2)\nu\cong0.015$ (Ref. 8).

As the smectic-*A*–smectic-*C* transition point is approached the velocity of the first sound and second sound decrease as $\tau^{\alpha/2}$, where α is the critical index of the specific heat. In the one-loop approximation, depending on the bare elastic anisotropy,[2] $\alpha=0.06$–0.14. In addition, the viscosity coefficients appearing in the dispersion laws for the sounds diverge in the neighborhood of this transition in accordance with the law $\tilde{\eta}_{2,4,5}\propto\tau^{-x}$, where the critical exponent $x=\alpha+z\nu\cong1.1$. The fluctuation corrections to the viscosity coefficients are related by $\Delta\eta_4^2=(\Delta\eta_2)(\Delta\eta_5)$, which can be checked experimentally.

The shear-viscosity coefficients η_1 and η_3 and the reactive parameter λ are not affected by the fluctuations. This result leads to a large magnitude of the dynamical anisotropy of the layers in the smectic-*C* phase. This circumstance implies, in particular, a strong coupling of the orientational mode with oscillations of the smectic layers.

Finally, fluctuation corrections to the permeation coefficients and thermal conductivity tensor are absent. The latter fact leads to the result that the dispersion law of the thermal-diffusion mode,

$$\omega=-i\hat{\varkappa}q^2/C_P$$

($\hat{\varkappa}$ is the thermal-conductivity tensor and C_P is the specific heat at constant pressure), becomes less "hard" because of the order-parameter fluctuations, which, according to Eq. (19), decrease the magnitude of the specific heat.

In our analysis we have not been concered with singularities of the smectic spectrum at small values of k_z or $k_\perp$. In this case, propagating second sound is absent in the spectrum, and in its place there appear two diffusion modes, associated, respectively, with the relaxation of the velocity component transverse to the wave vector and with the relaxation of the layer-displacement vector. The spectrum of these modes is determined by the moduli B and K introduced in (12), and also by the viscosity coefficients η_1 and η_3. Analysis shows that the corrections to η_1, η_3, and K associated with the fluctuations of ψ are unimportant, i.e., the renormalization of the spectrum of the above-mentioned modes is due entirely to the renormalization of the modulus B.

It is well known that the long-wavelength spectrum of a smectic is modified substantially by fluctuations of the displacement of the smectic layers.[9] Therefore, the question arises as to what role is played by fluctuations of the layers in the critical region. Analysis shows that fluctuations of the displacements of the layers in the critical region can be neglected. This is explained principally by the fact that, because of the fluctuations of the order parameter, the elastic modulus B in the critical region decreases substantially with increase of the scale.

Thus, we have both qualitative predictions (e.g., that the shear mode displays no particular critical behavior, etc.) and quantitative predictions pertaining to the specific law of softening of the sound velocities and to the growth of the bulk-viscosity coefficients. The presently available experimental data on the critical dynamics at the smectic-*A*–smectic-*C* phase transition[10] agree qualitatively with our proposed picture. However, for a quantitative comparison to be possible we need a more detailed knowledge of the material parameters of the systems under investigation than we have at our disposal.

APPENDIX

We shall derive the fluctuation-dissipation theorem for purely relaxational dynamics. Such a system is described by the Lagrangian

$$\mathscr{L}=p\left(\frac{\partial\psi}{\partial t}+\Gamma\frac{\delta E}{\delta\psi}\right)+iT\Gamma p^2, \tag{A1}$$

where Γ is a kinetic coefficient. We shall consider the correlators

$$\langle \psi^2(t_1),\ \psi^2(t_2)\rangle = D_2(t_1-t_2), \tag{A2}$$

$$\langle \psi^2(t_1),\ p\Gamma\psi(t_2)\rangle = G_2(t_1-t_2). \tag{A3}$$

In order to find the relationship between them we replace the auxiliary field p in the Lagrangian $\mathscr{L}$ by means of the formula

$$\tilde{p} = \Gamma^{-1}\left(p + \frac{i}{2T}\frac{\partial\psi}{\partial t}\right). \tag{A4}$$

In the new variables the Lagrangian is explicitly symmetric under time reversal $t \to -t$. Therefore,

$$\langle \psi^2(t_1)\tilde{p}\Gamma\psi(t_2)\rangle = \langle \psi^2(t_2)\tilde{p}\Gamma\psi(t_1)\rangle. \tag{A5}$$

Returning to the original variable p, for the Fourier components of the correlators (A2) and (A3) we obtain the following relation:

$$-D_2(\omega) = (2T/\omega)\,[G_2(\omega) - G_2(-\omega)]. \tag{A6}$$

Since the function $G_2(\omega)$ is analytic in the upper half-plane, and $G_2(-\omega)$ is analytic in the lower half-plane, after integrating (A6) over the frequency we obtain the fluctuation-dissipation theorem:

$$D(t,\ -t) = 2iTG(t,\ -t). \tag{A7}$$

This relation can be generalized to the many-component fields ψ and p and to correlators of all other orders. In addition, all the arguments remain valid when the kinetic coefficient Γ depends on the time as $\Gamma(t_1 - t_2)$.

In the case of purely dissipative dynamics the correlators $\langle p,h\rangle$ and $\langle \psi,p\rangle$ can be related in an analogous manner. For this we represent the molecular field h in the form

$$h = \Gamma^{-1}[\delta\mathscr{L}/\delta p - \partial\psi/\partial t - 2iT\Gamma p]. \tag{A8}$$

Taking into account that the correlator $\langle pp\rangle$ is equal to zero,[7] we obtain

$$\langle h,\ p\rangle = \Gamma^{-1}\langle \delta\mathscr{L}/\delta p,\ p\rangle - \langle \partial\psi/\partial t,\ p\rangle. \tag{A9}$$

Integrating the first term by parts, we find

$$\langle h,\ p\rangle = i\Gamma^{-1}(1 - \omega\langle\psi,\ p\rangle). \tag{A10}$$

This relation permits us to sum the series of diagrams depicted in Fig. 4.

[1] M. A. Anisimov, *Critical Phenomena in Liquids and Liquid Crystals* [in Russian], Nauka, Moscow (1987).

[2] E. I. Kats and V. V. Lebedev, Zh. Eksp. Teor. Fiz. **91**, 878 (1986) [Sov. Phys. JETP **64**, 518 (1986)].

[3] I. E. Dzyaloshinskii and G. E. Volovick, Ann. Phys. (N.Y.) **125**, 67 (1980).

[4] E. I. Kats and V. V. Lebedev, Zh. Eksp. Teor. Fiz. **88**, 823 (1985) [Sov. Phys. JETP **61**, 484 (1985)].

[5] L. D. Landau and E. M. Lifshitz, *Theory of Elasticity*, Pergamon Press, Oxford (1970) [Russ. original (3rd edition), Nauka, Moscow (1987)].

[6] G. E. Volovik and E. I. Kats, Zh. Eksp. Teor. Fiz. **81**, 240 (1981) [Sov. Phys. JETP **54**, 122 (1981)].

[7] V. V. Lebedev, A. I. Sukhorukhov, and I. M. Khalatnikov, Zh. Eksp. Teor. Fiz. **85**, 1590 (1983) [Sov. Phys. JETP **58**, 925 (1983)].

[8] P. C. Halperin and B. I. Hohenberg, Rev. Mod. Phys. **49**, 435 (1977).

[9] E. I. Kats and V. V. Lebedev, Zh. Eksp. Teor. Fiz. **85**, 2019 (1983) [Sov. Phys. JETP **58**, 1172 (1983)].

[10] A. P. Kapustin, *Experimental Investigations of Liquid Crystals* [in Russian], Nauka, Moscow (1978).

Translated by P. J. Shepherd