Zeeman solitons and simultons in quasiresonant nonlinear magnetooptics

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A theory is developed of magnetooptic solitons and simultons formed under normal Zeeman effect conditions due to the interaction of pulsed radiation with a quasiresonant medium. An analysis is made of the feasibility of controlling the profiles, polarization, and frequency properties of short light pulses by means of a magnetic field.

1. INTRODUCTION

Many striking achievements in nonlinear optics have been stimulated by the rapid progress in the development of methods for the generation of ultrashort light pulses and for controlling their parameters.[1-3] Nonlinear magnetooptical effects have been investigated initially in the processes of the interaction of resonant media with cw coherent radiation (see, for example, Refs. 4–9). However, recently some new physical results have been obtained using short pulses in nonlinear magnetooptics (this is particularly true of the technique of synchronized picosecond pulses used to record ultranarrow structures in nuclear magnetic resonance[10,11]). The importance of magnetic and polarization effects in the processes of the self-interaction of coherent pulsed radiation[10,12] is also due to the potential application of these effects in nonlinear optical data systems.

The present paper describes a theory of magnetooptical solitons and simultons formed as a result of a quasiresonant interaction of short light pulses with a Zeeman multiplet of radiative transitions. We consider the effects of a magnetic field on pulsed radiation as a result of the scattering of photons by split Zeeman sublevels. We use a system of coupled nonlinear Schrödinger equations for the amplitudes of opposite circular components of radiation to describe quasiresonant scattering of light in which a photon of one of the circular polarizations disappears and a photon with the opposite polarization appears.

2. NONLINEAR MAGNETOOPTICAL ACTIVITY OF QUASIRESONANT MEDIA

We assume that coherent pulsed radiation with a carrier frequency $\omega = kc$ acts on a transition between energy levels m and n with the Bohr frequency ω_{mn} and angular momenta $J_{m,n}$ and that this radiation propagates along the direction of the vector of an external magnetic field $\mathbf{H}$. Under the normal Zeeman effect conditions the splitting of the levels is described by the parameter $\Delta = \mu_0 g H/\hbar$, where μ_0 is the Bohr magneton and g is the Landé factor. If the parameter Δ or the absolute value of the frequency mismatch $\Omega = \omega - \omega_{mn}$ are large compared with the relaxation constants of the medium, the absorption is negligible and the self-interaction of the optical field is entirely due to the magnetooptical activity which results from the difference between the refractive indices n_σ for the opposite circular components of the radiation $\sigma = \pm 1$ with slow amplitudes $E_\sigma(t)$.

The main rules governing the magnetooptical activity in strong coherent fields were established by D'yakonov and Perel'[4] and were modified to describe resonant media in Refs. 7 and 13. It follows from the general relationships governing the nonlinear susceptibility[13] that the birefringence of the $\sigma = \pm 1$ components and the Faraday self-rotation of the polarization of a traveling optical wave in a quasiresonant medium with a cubic nonlinearity are determined by the following refractive indices

$$n_\sigma = n_\sigma^{(0)} + n_\sigma^{(2)} |E_\sigma|^2 + \tilde{n}_{-\sigma}^{(2)} |E_{-\sigma}|^2; \tag{1}$$

$$n_\sigma^{(0)} = 1 - \frac{\pi N |d_{mn}|^2}{\hbar(\Omega - \sigma\Delta)}, \quad n_\sigma^{(2)} = \frac{N |d_{mn}|^4 A_0 \Gamma}{\hbar^3 (\Omega - \sigma\Delta)^3} \left(\frac{1}{\Gamma_m} + \frac{1}{\Gamma_n} \right),$$
$$\tilde{n}_{-\sigma}^{(2)} = \frac{N |d_{mn}|^4}{\hbar^3 (\Omega^2 - \Delta^2)} \left[\frac{2\Gamma}{\Omega + \sigma\Delta} \left(\frac{A_{n2}}{\Gamma_m} + \frac{A_{m2}}{\Gamma_n} \right) + \frac{A_{m2} + A_{n2}}{\Omega - \sigma\Delta} \right]; \tag{2}$$

$$A_{m\varkappa} = \frac{9\pi}{4} \begin{Bmatrix} 1 & 1 & \varkappa \\ J_m & J_m & J_n \end{Bmatrix}^2, \quad A_{n\varkappa} = \frac{9\pi}{4} \begin{Bmatrix} 1 & 1 & \varkappa \\ J_n & J_n & J_m \end{Bmatrix}^2,$$
$$A_0 = (2A_{m0} + 3A_{m1} + A_{m2})/3. \tag{3}$$

Here, N denotes the difference between the rates of excitation of the states m and n, and d_{mn} is the reduced dipole moment of a transition. The dependence of the refractive indices on the angular momenta of the levels is described by the coefficients $A_{m\varkappa, n\varkappa}$ ($\varkappa = 0, 1, 2$) expressed in terms of the Wigner $6J$ symbols. Equations (1)–(3) are derived assuming that the homogeneous line width Γ and the decay constants of the levels $\Gamma_{m,n}$ are small compared with the parameters Δ and $|\Omega|$. The nonlinear Faraday effect is due to an asymmetry of the dependence of the refractive index n_σ on the magnetic field.

This approximation is justified for gases if the Zeeman splitting of the levels satisfies the inequality $k\bar{v} \ll \Delta \ll \tau_c^{-1}(J_m + J_n)^{-1}$, where $\bar{v}$ is the average thermal velocity of the particles and τ_c is the duration of collisions between them. In this case we can ignore the Doppler line broadening and the dependence of the collisional relaxation frequencies on the magnetic field.

It follows from Eq. (2) that in the simplest model of the identical decay (relaxation) constants ($\Gamma = \Gamma_{m,n}$) the nonlinear parameters $n_\sigma^{(2)}$ and $\tilde{n}_{-\sigma}^{(2)}$ are independent of these constants:

$$n_\sigma^{(2)} = \frac{2N |d_{mn}|^4 A_0}{\hbar^3 (\Omega - \sigma\Delta)^3}, \quad \tilde{n}_{-\sigma}^{(2)} = \frac{N |d_{mn}|^4 A_0 A}{2\hbar^3 (\Omega^2 - \Delta^2)^2} (3\Omega - \sigma\Delta). \tag{2a}$$

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In a zero magnetic field the nonlinear coupling between the opposite circular components of the radiation is characterized simply by the coefficient $A = 2(A_{m2} + A_{n2})/A_0$, which has values ranging from 0 to 2:

$$A(J\leftrightarrow J)=\frac{(2J+3)(2J-1)}{2J^2+2J+1},\quad A(J-1\leftrightarrow J)=\frac{2J^3+3}{6J^2-1}. \tag{4}$$

This coupling is due to light scattering which alters the specific circular polarization of a photon to the opposite one, subject to the selection rules $\Delta M = \pm 2$ applied to the projection of the angular momentum M. The oppositely polarized circular components do not interact at all in the process of birefringence only for transitions of the $J = 1/2 \to 1/2$ type.

The propagation of light pulses in a quasiresonant magnetically active medium subject to dispersion of the group velocity in the case when $|n_\sigma^{(0)} - 1| \ll 1$ is described by a system of two coupled nonlinear Schrödinger equations for the amplitudes of the circular components of the radiation:

$$(\partial_z - i\theta_\sigma\partial_\tau^2)E_\sigma = i(\beta_\sigma|E_\sigma|^2 + \tilde\beta_{-\sigma}|E_{-\sigma}|^2)E_\sigma, \tag{5}$$

$$\tau = t - z/c,$$

$$\theta_\sigma = -\frac{k}{2}\frac{d^2n_\sigma^{(0)}}{d\omega^2} = \frac{\pi N|d_{mn}|^2k}{\hbar(\Omega-\sigma\Delta)^3},$$

$$\beta_\sigma = \frac{kn_\sigma^{(2)}}{2},\quad \tilde\beta_{-\sigma} = \frac{k\tilde n_{-\sigma}^{(2)}}{2}. \tag{6}$$

The system of equations (5) was derived using methods similar to those described in Ref. 3.

In the case of moderately short pulses, when the dispersion of the group velocity can be ignored ($\theta_\sigma \to 0$), the solution of these equations

$$E_\sigma(\tau, z) = E_\sigma^{(0)}(\tau)e^{i\varphi_\sigma},$$

$$\varphi_\sigma(\tau, z) = z[\beta_\sigma|E_\sigma(\tau,0)|^2 + \tilde\beta_{-\sigma}|E_{-\sigma}(\tau,0)|^2] \tag{7}$$

describes phase self-modulation of the circular components of radiation. The nonlinear magnetooptical activity of the medium is manifested by splitting of the frequencies

$$\delta\omega_\sigma = -d\varphi_\sigma/d\tau, \tag{8}$$

which change not only because of the self-interaction but also because of the nonlinear interaction of the σ components. In the case of nonlinear coupling between components with opposite polarizations, these components contribute to modulating one another's phases.

We shall compare the characteristic nonlinear phase self-modulations L_σ of the σ components governed by the condition $\varphi_\sigma(0, L_\sigma) = 1$:

$$L_\sigma = [\beta_\sigma|E_\sigma^{(0)}|^2 + \tilde\beta_{-\sigma}|E_{-\sigma}^{(0)}|^2]^{-1},\quad E_\sigma^{(0)} = E_\sigma(0,0). \tag{9}$$

In the vicinity of a resonance[1)] of the component $\sigma = +1$ when $|\Omega - \Delta| \ll \Delta$, and $E_{+1}(\tau, 0) \approx E_{-1}(\tau, 0)$, we have

$$L_{+1} \approx 2[kn_{+1}^{(0)}|E_{+1}^{(0)}|^2]^{-1},\quad L_{-1} \approx 2[k\tilde n_{+1}^{(2)}|E_{+1}^{(0)}|^2]^{-1}. \tag{10}$$

Phase modulation of the component $\sigma = -1$ is in this case entirely induced and the lengths due to induced modulation and self-modulation are very different:

$$L_{-1}/L_{+1} \approx 2\Delta/A|\Omega-\Delta| \gg 1,\ \Gamma = \Gamma_{m,n}. \tag{11}$$

The characteristic frequency shifts $\delta\omega_\sigma$ for pulses with the Gaussian profile

$$|E_\sigma(\tau,0)| = |E_\sigma^{(0)}|\exp(-\tau^2/2\tau_\sigma^2) \tag{12}$$

and with widths τ_σ are described by the expression

$$\delta\omega_\sigma \approx {}^1/_2 kz[n_\sigma^{(2)}|E_\sigma^{(0)}|^2/\tau_\sigma + \tilde n_{-\sigma}^{(2)}|E_{-\sigma}^{(0)}|^2/\tau_{-\sigma}]. \tag{13}$$

This example illustrates the dependences of the frequency shifts $\delta\omega_\sigma$ on the initial durations and amplitudes of the pulses, on the selection rules governing the angular momentum, and on the relationship between the offset Ω and the Zeeman splitting parameter Δ. In the case of circular components with very different intensities the nonlinear magnetooptical activity effects are determined by the stronger component.

3. SOLITONS AND SIMULTONS NEAR A RESONANCE OF ONE OF THE COMPONENTS OF THE ZEEMAN STRUCTURE OF RADIATIVE TRANSITIONS

It is particularly interesting to consider steady-state propagation of multifrequency pulses with a common envelope in multilevel linear media. These pulses are called simultons.[14,15] We shall show later that under the Zeeman effect conditions both soliton and simulton propagation may arise because of the nonlinear magnetooptical activity of the medium.

Near a resonance of the radiation with one of the components of the normal Zeeman doublet of a line ($|\Omega - \Delta| \ll \Delta$) and for $|E_{+1}| \sim |E_{-1}|$ we can ignore the nonlinear contribution of the nonresonant circular component of the radiation to the refractive index n_σ. In this approximation the amplitude of the component of the radiation E_{+1} close to a resonance obeys the usual nonlinear Schrödinger equation

$$(\partial_z - i\theta_{+1}\partial_\tau^2)E_{+1} = i\beta_{+1}|E_{+1}|^2E_{+1}, \tag{14}$$

which was solved by Zakharov and Shabat by the inverse scattering problem approach.[16] Propagation of the opposite nonresonant component $\sigma = -1$ of a pulse is described by the linear Schrödinger equation

$$(\partial_z - i\theta_{-1}\partial_\tau^2)E_{-1} = i\tilde\beta_{+1}|E_{+1}|^2E_{-1}, \tag{15}$$

the potential of which is proportional to $|E_{+1}|^2$.

The steady-state profile of the σ components of a pulse can be found by substituting in Eqs. (14) and (15) the expressions

$$E_\sigma = A_\sigma(\tau')\exp[i(\xi_\sigma z + \tau'/2\theta_\sigma v)],\quad \tau' = \tau - z/v. \tag{16}$$

It is known[1] that the one-soliton solution of Eq. (14) for the case when $\theta_{+1}, \beta_{+1} > 0$ is

$$A_{+1} = A_{+1}^{(0)}\operatorname{sech}\eta,\quad \eta = \tau'/\tau_{+1},\quad \Omega > \Delta. \tag{17}$$

The duration τ_{+1} and amplitude $A_{+1}^{(0)}$ of a soliton are related by

$$2\xi_{+1} = \theta_{+1}/\tau_{+1}^2 = \beta_{+1}A_{+1}^{(0)2}. \tag{18}$$

Substitution of Eq. (17) into Eq. (15) gives

$$\partial_\eta^2 A_{-1}+(E+U_0\,\mathrm{sech}^2\,\eta)A_{-1}=0, \tag{19}$$

$$E=-\xi_{-1}\tau_{+1}^2/\theta_{-1},\quad U_0=\beta_{+1}\theta_{+1}/\beta_{+1}\theta_{-1}>0. \tag{20}$$

In this formulation the problem reduces to determination of a discrete spectrum of negative energies for a particle moving in a field with the potential $U=-U_0\mathrm{sech}^2\eta$.

The Schrödinger equation (19) can be modified by the standard substitution $\alpha=\tanh\eta$ into the equation for the generalized Legendre functions[17]

$$\frac{d}{d\alpha}\left[(1-\alpha^2)\frac{dA_{-1}}{d\alpha}\right]+\left[p(p+1)-\frac{\varepsilon_2}{1-\alpha^2}\right]A_{-1}=0, \tag{21}$$

where

$$\varepsilon=(-E)^{1/2},\quad U_0=p(p+1),\quad p=[-1+(1+4U_0)^{1/2}]/2. \tag{22}$$

The solutions of the above equation which are damped out in the limits $\eta=\pm\infty$, can be expressed in terms of the Euler gamma functions $\Gamma(x)$ and the Gegenbauer ultraspherical polynomials $C_l^{(\varepsilon+1/2)}(\alpha)$ (see Ref. 18):

$$A_{-1}=A_{-1}^{(0)}\frac{l!\ \Gamma(2\varepsilon+1)}{\Gamma(2\varepsilon+l+1)}\mathrm{sech}^{p-l}\,\eta C_l^{(\varepsilon+1/2)}\ (\mathrm{th}\,\eta). \tag{23}$$

The condition $p-\varepsilon=l=0, 1, 2,\ldots$ determines the energy levels

$$E_l=-[-1-2l+(1+4U_0)^{1/2}]^2/4 \tag{24}$$

as well as the suitably quantized values of the parameter ξ_{-1}:

$$\xi_{-1}^{(l)}=E_l\theta_{-1}/\tau_{+1}^2, \tag{25}$$

where $l<p$.

A soliton σ component of the radiation close to a resonance induces nonlinearly a whole set of soliton regimes for the opposite nonresonant component, labeled by the index l. In particular, the value $l=0$ corresponds to the solution

$$A_{-1}=A_{-1}^{(0)}\mathrm{sech}^p\,\eta. \tag{26}$$

In the model of practical decay (relaxation) constants of Eq. (2a), we have

$$p\approx U_0^{1/2}=(2A)^{1/2}\Delta/(\Omega-\Delta)\gg 1. \tag{27}$$

The solutions (17) and (23) taken together with the frequency splitting of Eq. (16) describe a series of magnetooptical simultons, representing elliptically polarized pulses with a rotating polarization axis. If $p-l\gg 1$, the induced solitons of Eq. (23) are strongly compressed compared with the case described by Eq. (17).

4. EXACT SOLITON AND SIMULTON SOLUTIONS

In the case of a focusing nonlinear medium when θ_σ, $\beta_\sigma, \tilde{\beta}_\sigma>0$ the system of wave equations (5) can be modified by the substitution

$$E_\sigma=(2/\tilde{\beta}_\sigma)^{1/2}u_\sigma \tag{28}$$

to the form

$$(\partial_z-i\theta_\sigma\partial_\tau^2)u_\sigma=2i(\alpha_\sigma|u_\sigma|^2+|u_{-\sigma}|^2)u_\sigma,\quad \alpha_\sigma=n_\sigma^{(2)}/\tilde{n}_\sigma^{(2)}. \tag{29}$$

This system of equations has the integrals of motion

$$N_\sigma=\int_{-\infty}^{\infty}|u_\sigma|^2\,d\tau, \tag{30}$$

$$P=i\sum_\sigma\int_{-\infty}^{\infty}u_\sigma\partial_\tau u_\sigma^*\,d\tau, \tag{31}$$

$$\mathscr{H}=\sum_\sigma\int_{-\infty}^{\infty}d\tau(\theta_\sigma|\partial_\tau u_\sigma|^2-\alpha_\sigma|u_\sigma|^4-|u_\sigma|^2|u_{-\sigma}|^2). \tag{32}$$

Here, N_σ and P represent the numbers of photons in the σ component and the total momentum of the wave packet, where $\mathscr{H}$ is the Hamiltonian of the system:

$$i\partial_z u_\sigma=\delta\mathscr{H}/\delta u_\sigma^*. \tag{33}$$

The steady-state solutions of the system of coupled nonlinear Schrödinger equations [Eq. (29)] with zero boundary conditions at $\tau=\pm\infty$ are sought in the form

$$u_\sigma=\theta_\sigma^{1/2}q_\sigma(\tau')\exp[i(k_\sigma z+\tau'/2v\theta_\sigma+\delta_\sigma)],$$

$$\tau'=\tau-z/v,\quad k_\sigma=\theta_\sigma/\tau_0^2+1/4\theta_\sigma v^2. \tag{34}$$

The amplitudes q_σ satisfy the equations of motion

$$\partial_{\tau'}^2 q_\sigma=-\partial U/\partial q_\sigma \tag{35}$$

in a field with a potential U, which in terms of the polar coordinates

$$q_{+1}=r\cos\varphi,\quad q_{-1}=r\sin\varphi \tag{36}$$

is

$$U=[-r^2/\tau_0^2+B(\varphi)r^4]/2,$$

$$B(\varphi)=B_{+1}\cos^4\varphi+B_{-1}\sin^4\varphi+2\cos^2\varphi\sin^2\varphi,\quad B_\sigma=\alpha_\sigma\theta_{-\sigma}/\theta_\sigma. \tag{37}$$

There are certain paths in the potential field U along which the coefficient $B(\varphi)$ is extremal and, consequently, we have $\partial U/\partial\varphi=0$. The motion along these paths corresponds to the solutions

$$r=\tau_0^{-1}B^{-1/2}(\varphi_0)\mathrm{sech}(\tau'/\tau_0), \tag{38}$$

where φ_0 represents the roots of the equation

$$\partial B(\varphi)/\partial\varphi=0. \tag{39}$$

In view of the symmetry of the function $B(\varphi)$, it is sufficient to analyze its changes in the interval $0\leqslant\varphi\leqslant\pi/2$. The roots $\varphi=0$ and $\pi/2$ correspond to solitons with opposite circular polarizations ($B=B_\sigma$ for $u_\sigma\neq 0, u_{-\sigma}=0$). Moreover, there is a third root

$$\varphi_0=\mathrm{arctg}\,[(B_{+1}-1)/(B_{-1}-1)]^{1/2}, \tag{40}$$

provided the parameters B_σ satisfy the inequality

$$(B_{+1}-1)/(B_{-1}-1)>0. \tag{41}$$

This root is characterized by

$$B(\varphi_0)=(1-B_{+1}B_{-1})/(2-B_{+1}-B_{-1}) \tag{42}$$

and it represents a coupled state of two opposite circular components with split frequencies, i.e., it corresponds to a magnetooptical simulton.

We can determine the conditions of stability of this solution against one-dimensional perturbations simply by deriving the Lyapunov functional which in this case is extremal. This problem was solved earlier by Zakharov and Kuznetsov.[19] We shall take the Lyapunov functional to be the quantity

$$\tilde{\mathscr{H}}=\mathscr{H}-P/2v-N/4v^2, \quad N=\sum_\sigma N_\sigma/\theta_\sigma. \tag{43}$$

These solutions of the system (29) are stationary points of the Hamiltonian $\mathscr{H}$ for fixed values of N and P, because these equations appear when we vary the corresponding functional

$$\delta\left(\mathscr{H}-P/2v-\sum_\sigma N_\sigma\theta_{-\sigma}/\tau_0^2\right)=0. \tag{44}$$

The functional $\tilde{\mathscr{H}}$ represents transformation of the Hamiltonian $\mathscr{H}$ to a moving reference system. The transformation of the amplitudes of the circular components of the radiation u_σ is then similar to the Galilean transformation for the wave functions:

$$u_\sigma(\tau,z)=\tilde{u}_\sigma(\tau-z/v,z)\exp(i\tau/2v\theta_\sigma), \tag{45}$$

so that $\tilde{\mathscr{H}}=\mathscr{H}(\tilde{u}_\sigma)$. We have to find the extrema of $\tilde{\mathscr{H}}$ and the conditions of their existence.

Introducing new variables

$$\tilde{u}_{+1}=\theta_{-1}^{1/2}\, r\cos\varphi\, e^{i\Phi_{+1}}, \quad \tilde{u}_{-1}=\theta_{+1}^{1/2}\, r\sin\varphi\, e^{i\Phi_{-1}}, \tag{46}$$

we find that the lower limit of the functional

$$\tilde{\mathscr{H}}=\theta_{+1}\theta_{-1}\int_{-\infty}^{\infty}\{r_\tau^2+r^2[\varphi_\tau^2+\cos^2\varphi(\partial_\tau\Phi_{+1})^2 + \sin^2\varphi(\partial_\tau\Phi_{-1})^2]-r^4B(\varphi)\}d\tau \tag{47}$$

is determined by the inequality

$$\tilde{\mathscr{H}}\geqslant\theta_{+1}\theta_{-1}\int_{-\infty}^{\infty}[r_\tau^2-r^4\max B(\varphi)]d\tau. \tag{48}$$

A minimum of the functional $\tilde{\mathscr{H}}$ corresponds to $\varphi_\tau=\partial_\tau\Phi_\sigma=0$ and $B=B(\tilde{\varphi})$, where $\tilde{\varphi}$ is the root of Eq. (39) for which the coefficient B is maximal.

If $B_\pm>1$, then the coefficient $B(\varphi)$ has two maxima at $\varphi=\tilde{\varphi}=0$ and at $\pi/2$, which correspond to stable soliton pulses with circular polarizations. In the model of equal decay (relaxation) constants of Eq. (2a), when

$$B_\sigma=2(\Omega-\sigma\Delta)^2/A(\Omega+\sigma\Delta)(3\Omega+\sigma\Delta), \tag{49}$$

these stability conditions become

$$\Omega>\Omega_0=\Delta\frac{2(A+1)}{|3A-2|}\left\{1+\left[1+\frac{(3A-2)(2-A)}{4(A+1)^2}\right]^{1/2}\right\},$$

$$A<2/3 \tag{50}$$

and—according to Eq. (4)—they are satisfied for transitions subject to the selection rules $\Delta J=\pm1$ applying to the angular momenta ($J>1$).

In the opposite case when $B_\pm<1$, the function $B(\varphi)$ has one maximum at $\varphi=\varphi_0$, where the root φ_0 is given by Eq. (40) and a simulton state of two nonlinearly coupled σ components of the radiation is stable. When the decay (relaxation) constants of the medium are equal the conditions for simulton stability are determined by the inequalities

$$\Omega>\Omega_0, \quad A>2/3, \tag{51}$$

which corresponds to transitions subject to the selection rule $\Delta J=0$ ($J\geqslant1$), and also to transitions of the $J=0\leftrightarrow1$ type.

In zero magnetic field, when

$$n_\sigma^{(0)}=1-\pi N|d_{mn}|^2/\hbar\Omega, \quad n_\sigma^{(2)}=2A_0N|d_{mn}|^4/\hbar^3\Omega^3=\tilde{n}_{-\sigma}^{(2)}/A, \tag{52}$$

the system of wave equations (5) reduces to a system of coupled nonlinear Schrödinger equations for a medium with a scalar nonlinearity, which was integrated by Manakov[20] using the inverse scattering method in the case when $A=1$. A mechanism similar to that described in Ref. 20 accounts for the interaction of steady-state pulses in an anisotropic magnetically active medium.

Thus, collisions of two solitons with opposite circular polarizations create a magnetooptical simulton of the type described by Eq. (40) if $B_\pm<1$. In fact, in this case there is no energetic restriction against two soliton σ components of the radiation coalescing into one simulton, because the simulton energy $\mathscr{H}_0$ is less than the total energy of the uncoupled circular components $\mathscr{H}_{+1}+\mathscr{H}_{-1}$. The condition of validity of the estimate

$$\mathscr{H}_0<\mathscr{H}_{+1}+\mathscr{H}_{-1} \tag{53}$$

is identical with the condition for stability of a simulton ($B_{\pm1}<1$).

In the case of scattering of two solitons with identical circular polarizations but different durations $\tau_{1,2}$ and centers moving with different velocities $v_1<v_2$, we find that $\Delta t_{1,2}$ are described by the expression[21]

$$\frac{\Delta t_2}{\tau_2}=-\frac{\Delta t_1}{\tau_1}=\ln\left\{\left[1+\left(2\theta_\sigma\frac{v_1v_2(\tau_1+\tau_2)}{(v_1-v_2)\tau_1\tau_2}\right)^2\right]\left[1+\left(2\theta_\sigma\frac{v_1v_2(\tau_1-\tau_2)}{(v_1-v_2)\tau_1\tau_2}\right)^2\right]^{-1}\right\}. \tag{54}$$

These shifts are particularly noticeable in the vicinity of a resonance of a given radiation component with the corresponding Zeeman component of a radiative transition ($|\Omega-\sigma\Delta|\ll\Delta$). Simulton regimes induced nonlinearly by a σ component close to a resonance and discussed in Sec. 3 reproduce the pattern of scattering of soliton pulses with identical polarizations.

We conclude by noting that we have exhibited the effects of formation and interaction of Zeeman solitons and simultons in quasiresonant magnetically active media on the basis of a one-dimensional model, i.e., ignoring the transverse structure of the pulses. Since the growth increment of a transverse instability of the pulses decreases away from a resonance,[22] it follows that this approximation is quite valid in the case of quasiresonant radiative processes.

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[1)]For these displacements $|\Omega| \sim 10^{12}\ s^{-1}$ the resonance condition is satisfied in a magnetic field of $H \sim 10^5$ Oe intensity.

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