Resonance effects in the spectroscopy of atomic hydrogen in a plasma with a quasimonochromatic electric field and located in a strong magnetic field

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The emission spectrum of a hydrogen atom acted upon by a field $\mathbf{H}+\mathbf{F}+\mathbf{E}_D(t)$ is investigated analytically ($\mathbf{H}$ is a quasistatic magnetic field, $\mathbf{F}$ a quasistatic electric field, and $\mathbf{E}_D(t)=\mathbf{E}_0\cos\omega t$ a high-frequency quasi-monochromatic electric field). It is shown that the most radical changes take place in the hydrogen-atom emission spectrum when the quasistatic field $\mathbf{H}+\mathbf{F}$ splits the energy levels of the atom into sets of equidistant sublevels (this corresponds to the condition $\mathbf{H}\cdot\mathbf{F}=0$), and the frequency of the field $\mathbf{E}_{D(t)}$ is at resonance with this splitting. In this case the hydrogen spectral lines undergo additional splitting. The additional resonant splitting of an arbitrary hydrogen spectral line is calculated in two cases: 1) $\mathbf{F}=0$, 2) $\mathbf{H}\perp\mathbf{F}$, $\mathbf{E}_D(t)\perp\mathbf{H}$, $\mathbf{E}_D(t)\perp\mathbf{F}$. Also analyzed is the spectrum of the $L\alpha$ line if the frequency of the field $\mathbf{E}_D(t)$, arbitrarily oriented relative to the vectors $\mathbf{H}$ and $\mathbf{F}$ ($\mathbf{H}\perp\mathbf{F}$), is at resonance to the splitting of the upper atomic level. It is shown that if the field $\mathbf{H}+\mathbf{F}$ shapes the quasistatic profile of the hydrogen spectral line in the plasma, characteristic reliefs can appear under the influence of the field $\mathbf{E}_D(t)$ in definite places on the quasistatic profile of the hydrogen line. Experimental plots of such reliefs would make it possible to determine the amplitude of the field $\mathbf{E}_D(t)$ in the plasma.

1. INTRODUCTION

Spectroscopic methods are widely used at present for diagnostics of high-frequency (hf) quasi-monchromatic electric fields (QEF)

$$E_D(t)=E_0\cos\omega t \tag{1}$$

in a plasma.[1–3] Strong fields (1) can exist in a plasma in which strong currents flow, the magnetic fields are recconnected, or the plasma is acted upon by electromagnetic radiation. A method of measuring hf QEF in a plasma, enveloped in Ref. 4, is based on recording the "reliefs" on the quasistatic Stark profiles of the spectral lines (SL) of hydrogen. These reliefs are due to the onset of resonance between the hf QEF frequency and the Stark splitting of the upper (or lower) atomic level under the influence of a quasistatic plasma electric field $\mathbf{F}_p$. (The field $\mathbf{F}_p$ can consist of individual microfields of the ions or low-frequency plasma-turbulence fields.) The relief procedure was used for the diagnostics of hf plasma turbulence in various experiments (see Refs. 4–7). In many experimental situations, however, the plasma is in a strong field that can radically transform the profiles of the hydrogen spectral lines (see, e.g., Ref. 8). A static magnetic field $\mathbf{H}$ can influence the radiation spectrum in two ways: first, via the Zeeman effect and second, via the Stark effect in the Lorentz electric field $\mathbf{F}_L=c^{-1}\mathbf{v}\times\mathbf{H}$ ($\mathbf{v}$ is the atom velocity).

The aim of the present paper is an examination of the resonant effects in the emission spectra of hydrogen atoms with participation of an hf QEF (1) and of two quasistatic fields, magnetic $\mathbf{H}$ and electric $\mathbf{F}=\mathbf{F}_L+\mathbf{F}_p$. In Sec. 2 we examine the physical meaning of the resonance effects. These effects are quantitatively investigated for different orientations of the vectors $\mathbf{H}$, $\mathbf{F}$, and $\mathbf{E}_0$. Analysis of the validity limits of the obtained solutions is the subject of Sec. 4. In Sec. 5 we discuss the results.

2. ESSENCE OF RESONANCE EFFECTS

Let a hydrogen atom moving with velocity $\mathbf{v}$ in a plasma be acted upon jointly by an hf QEF (1) and quasistatic fields: a magnetic field $\mathbf{H}$ and an electric plasma field $\mathbf{F}_p$. The Schrödinger equation for the hydrogen atom can then be represented in the form (we use here and below the atomic units $\hbar=m_e=e=1$)

$$\begin{gathered} i\partial\Phi/\partial t=(\mathscr{H}_a+\mathbf{r}\mathbf{E}_0\cos\omega t+V_H+V_L+V_p)\Phi, \\ V_H=\mu_0\mathbf{L}\mathbf{H},\quad V_L=\mathbf{r}\mathbf{F}_L=c^{-1}\mathbf{r}[\mathbf{v}\mathbf{H}],\quad V_p=\mathbf{r}\mathbf{F}_p, \end{gathered} \tag{2}$$

where $\mathscr{H}_a$ is the unperturbed Hamiltonian, $\mathbf{L}$ the orbital momentum of the electron, $\mathbf{r}$ the radius vector of the electron, and μ_0 the Bohr magneton. We assume that the frequency of the field (1) and the characteristic matrix elements of the interactions with the fields $\mathbf{H}$, $\mathbf{E}_0$, $\mathbf{F}_L$ and $\mathbf{F}_p$ are small compared with the distance between the considered level n (n is the principal quantum number) and the neighboring level $n+1$, but is on the other hand much larger than the spin-orbit splitting of the level n.

We solve the Schrödinger equation (2) using as the basis the states obtained by diagonalizing the static magnetic and electric interactions.[9] According to Ref. 9, the splitting of level n of a hydrogen atom in a field $\mathbf{H}+\mathbf{F}$ is given by

$$\begin{gathered} \varepsilon_{nn'n''}=|\varkappa_1|n'+|\varkappa_2|n'', \\ n',\ n''=-j,\ -j+1,\ldots,j,\quad j=(n-1)/2, \end{gathered} \tag{3}$$

where the vectors $\varkappa_1$ and $\varkappa_2$ are of the form

$$\varkappa_1=\mu_0\mathbf{H}-{}^3/_2n\mathbf{F},\quad \varkappa_2=\mu_0\mathbf{H}+{}^3/_2n\mathbf{F}. \tag{4}$$

The wave functions (WF) corresponding to the frequencies (3) can be written in the form

$$\varphi_{nn'n''}(\mathbf{r})=\sum_{i_1,i_2=-j}^{j} D^j_{n'i_1}(0,\alpha_1,0)D_{n''i_2}(0,-\alpha_2,0)\varphi_{ni_1i_2}(\mathbf{r}), \tag{5}$$

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where α and α_2 are the angles between the vector $\mathbf{H}$ and the vectors $\varkappa$, and $\varkappa_2$, $D^j_{m,m'}$ are Wigner functions, $\varphi_{ni_1i_2}(\mathbf{r})$ are WF in parabolic coordinates with quantization axis along $\mathbf{H}$ and coinciding with those defined in Ref. 10 (the choice of the signs of the arguments of the functions $D^j_{n'i_1}$ and $D^j_{n''i_2}$ accords with the choice of phases $\varphi_{ni_1i_2(\mathbf{r})}$ of the WF), and the quantum numbers i_1 and i_2 are uniquely related with the parabolic quantum numbers n_1 and n_2: $i_1 = 2^{-1}(m + n_2 - n_1)$, $i_2 = 2^{-1}(m - n_2 + n_1)$.

It is easy to show that the only nonzero coordinate matrix elements are

$$\langle\varphi_{nn'n''}|\mathbf{r}|\varphi_{nn'n''}\rangle, \quad \langle\varphi_{nn'n''}|\mathbf{r}|\varphi_{n,n'\pm1,n''}\rangle,$$
$$\langle\varphi_{nn'n''}|\mathbf{r}|\varphi_{n,n',n''\pm1}\rangle.$$

Therefore one-photon resonance transitions can be induced by the field (1) between the sublevels $\varepsilon_{nn'n''}$ (3) only if the frequency ω is close to either $\varkappa_1$ or $\varkappa_2$. Resonance effects will be most noticeable under the condition $\varkappa_1 = \varkappa_2$. In this case the sublevels (3) (which are, generally speaking, degenerate) are equidistant and all the sublevels become involved simultaneously in the resonance $\omega \approx \varkappa_1$. The equality $\varkappa_1 = \varkappa_2$ corresponsds to the situation $\mathbf{H}\cdot\mathbf{F} = 0$, of which the particular case $H = 0$ was considered in Refs. 11–15. The present paper is devoted to resonance effects as functions of the relation between the intensities of the fields $\mathbf{H}$ and $\mathbf{F}$ that satisfy the condition $\mathbf{H}\cdot\mathbf{F}$ ($\mathbf{F} = \mathbf{F}_L + \mathbf{F}_p$) acting on different hydrogen atoms in the plasma can be made orthogonal, for example in the case when $\mathbf{F}_L \gg \mathbf{F}_p$ or in the case when intense low-frequency eletrostatic noise develops in a plane perpendicular to the magnetic field (see, e.g., Ref. 16).

3. CALCULATION OF HYDROGEN SPECTRAL-LINE SPLITTING UNDER RESONANCE CONDITIONS

Let the vectors $\mathbf{H}$, $\mathbf{F}$, and $\mathbf{E}_0$ have in an xyz coordinate frame the form

$$\mathbf{H}=H\mathbf{e}_z, \quad \mathbf{F}=F\mathbf{e}_x, \quad \mathbf{E}_0=E_{0x}\mathbf{e}_x+E_{0y}\mathbf{e}_y+E_{0z}\mathbf{e}_z$$

($\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$) are unit vectors along the axes x, y, and z). We seek the solution of Eq. (2) for the level n in the basis of the WF $\varphi_{nn'n''(\mathbf{r})}$ (5):

$$\Phi(\mathbf{r},t)=\sum_k b_k(t)\varphi_k(\mathbf{r})\exp[-i\mathscr{E}_nt-i\varepsilon_{nn'n''}t - i(\mathbf{r}_{kk}\mathbf{E}_0/\omega)\sin\omega t], \tag{6}$$

$$\mathbf{r}_{k_\alpha k_\beta}\equiv\langle\varphi_{k_\alpha}|\mathbf{r}|\varphi_{k_\beta}\rangle,$$

where $k\equiv(n, n', n'')$, $\mathscr{E}_n$ is the energy of the unperturbed level n, and $\varepsilon_{nn'n''}$ are, by virtue of (3) and (14), equal to

$$\varepsilon_{nn'n''}=(n'+n'')\varkappa, \quad \varkappa=(\mu_0^2H^2+9n^2F^2/4)^{1/2},$$
$$n', n''=-j, -j+1, \ldots, j, \quad j=(n-1)/2. \tag{7}$$

Substituting (6) in (2) we get

$$\dot b_{k_1}=-i\cos\omega t\sum_{p=2}^{5}\mathbf{r}_{k_1k_p}\mathbf{E}_0b_{k_p}\exp[iu_{k_1k_p}\sin\omega t+i(-1)^p\varkappa t], \tag{8}$$

where

$$k_1\equiv(n, n', n''), \quad k_2\equiv(n, n'-1, n''),$$
$$k_3\equiv(n, n'+1, n''), \quad k_4\equiv(n, n', n''-1),$$
$$k_5\equiv(n, n', n''+1), \quad u_{k_\alpha k_\beta}=\delta\mathbf{r}_{k_\alpha k_\beta}\mathbf{E}_0/\omega,$$
$$\delta\mathbf{r}_{k_\alpha k_\beta}=\mathbf{r}_{k_\alpha k_\alpha}-\mathbf{r}_{k_\beta k_\beta}.$$

We assume satisfaction of the multiphoton resonance condition

$$\varkappa=q\omega+\Delta, \quad q=1, 2, \ldots, \quad |\Delta|\ll\omega, \tag{9}$$

where Δ is the detuning. Using the resonance approximation, we neglect in (8) the terms containing rapidly oscillating coefficients (with frequency $\sim r\omega$, $r\neq0$). As a result we get

$$\dot b_{k_1}=i\sum_{p=2}^{5}(-1)^{qp+q+p}\mathbf{r}_{k_1k_p}\mathbf{E}_0b_{k_p}qJ_q(u_{k_1k_p}) \times u^{-1}_{k_1k_p}\exp[i(-1)^pt\Delta], \tag{10}$$

where $J_q(u)$ is a Bessel function.

We consider the solution of the system (10) for different relations between the vectors $\mathbf{H}$, $\mathbf{F}$, and $\mathbf{E}_0$. To this end, we note first of all some properties of the matrix elements $\mathbf{r}_{kp}$ contained in (10). The only nonzero diagonal matrix elements can be x_{kk} and $z_{kk'}$ while $y_{kk}\equiv0$. In addition, the following relations hold:

$$\lim_{H\to0}z_{kk}=\lim_{F\to0}x_{kk}=\lim_{H\to0}x_{kp}=\lim_{F\to0}z_{kp}=0 \quad (k\neq p). \tag{11}$$

Consider the case $F = 0$. In this case, without loss of generality, we put $E_{0y} = 0$. Taking (11) into account, it is easily seen that at $F = 0$ the system (10) describes the behavior of the hydrogen atom (in a state with principal quantum number n) in two effective static fields: a magnetic field $\mathbf{h} = \mu_0^{-1}\mathbf{e}_z\Delta$ and an electric field

$$\mathbf{f}=qE_{0x}\mathbf{e}_xJ_q(3nE_{0z}/2\omega)(3nE_{0z}/2\omega)^{-1}.$$

Therefore, according to (3) and (4), the additional splitting of the level n takes the form

$$\lambda^{(q)}_{nn'n''}=(n'+n'')[\Delta^2+{}^9/_4q^2n^2E_{0x}^2J_q^2(3nE_{0z}/2\omega)(3nE_{0z}/2\omega)^{-2}]^{1/2}, \tag{12}$$

where $n', n'' = -j, -j+1,\ldots, j$; $j = (n-1)/2$. In accordance with (5), (6), (9) and (12), we represent the solution of Eq. (2) in the form of the WF of a quasi-energy state (QES)

$$\Phi_{nn'n''}(\mathbf{r},t)=\exp(-i\mathscr{E}_nt-i\lambda^{(q)}_{nn'n''}t) \times\sum_{i_1,i_2=-j}^{+j}D^j_{n'i_1}(0,\beta_1,0)D^j_{n''i_2}(0,-\beta_2,0) \times(-1)^{i}\mathfrak{F}(n_1,n_2,m) \times\exp\{-im\omega t-i[3n(n_1-n_2)E_{0z}/2\omega]\sin\omega t\}\varphi_{n_1n_2m}(\mathbf{r}), \tag{13}$$

where $\varphi_{n_1n_2m}(\mathbf{r})$ is the WF in parabolic coordinates with quantization axis z; $i_1 = 2^{-1}(m + n_2 - n_1)$, $i_2 = 2^{-1}(m - n_2 + n_1)$; β_1, β_2 are the angles between the z axis and the vectors $\omega_1 = 3/2n\mathbf{f} + \mathbf{e}_z\Delta$. In Eq. (13) at $q = 2p + 1$ ($p = 0,1,2,\ldots$) all the $F(n_1,n_2,m)$ are zero, and at $q = 2p$ ($p = 1,2,\ldots$) we have $F(n_1,n_2,m = 0)$ for the states $|n_1n_2m\rangle$ for which

$$(n_2-n_1,\ m)=(1-n,\ 0),\ (2-n,\ 1),\dots,\ (0,\ n-1);$$
$$(3-n,\ -2),\ (4-n,\ -1),\dots,\ (2,\ n-3);\dots,$$

and $F(n_1,n_2,m) = 1$ for the states $|n_1n_2m\rangle$ for which

$$(n_2-n_1,\ m)=(2-n,\ -1),\ (3-n,\ 0),\dots,\ (1,\ n-2);$$
$$(4-n,\ -3),\ (5-n,\ -2),\dots,\ (3,\ n-4);\dots,$$

where n is the principal quantum number. We reduce the expression for the spectrum of radiation L_α with polarizations x, y, and $z(q = 1$ in (9) in the case $3E_0/\omega \ll 1)$, obtained with the aid of the WF (13):

$$I_x(\Delta\omega)=I_y(\Delta\omega)=4^{-1}\sum_{s=0}^{1}\Big\{2^{-1}(3E_{0x}/\Omega_\Delta)^2\delta[\Delta\omega-(-1)^s\omega]$$
$$+\sum_{p=0}^{1}[1-(-1)^s\Delta/\Omega_\Delta]^2\delta[\Delta\omega-(-1)^p(\Omega_\Delta-(-1)^s\omega)]\Big\},$$
$$I_z(\Delta\omega)=2\delta(\Delta\omega). \tag{14}$$

Here and below the frequency $\Delta\omega$ is reckoned from the unshifted position of the spectral line, and $\Omega_\Delta = (\Delta^2 + 9E^2_{0x}/4^{1/2}$. It follows from (14) that the most radical changes produced by the field (1) in the emission specrum occur in the case of exact resonance ($\Delta = 0$). In this case each Zeeman component in the spectra $I_x(\Delta\omega)$ and $I_y(\Delta\omega)$ undergoes a splitting that is linear in E_{0x}. At large offsets, $|\Delta| \gg 3|E_{0x}|/2$, the spectrum (14) goes over into the spectrum for the Zeeman effect, on which are superimposed two satellites of intensity $\propto E^2_{0x}$ at the frequencies $\Delta\omega = \pm\omega$.

Let now $H = 0$. We assume for the sake of argument, just as in the case $F = 0$, that the vector $\mathbf{E}_0$ is in the xz plane. It is easily seen that at $H = 0$ the system (10) describes the behavior of a hydrogen atom in an effective electric field

$$\mathbf{f}'=q(-1)^{q+1}E_{0z}\mathbf{e}_zJ_q(3nE_{0x}/2\omega)(3nE_{0x}/2\omega)^{-1}+(2\Delta/3n)\mathbf{e}_x, \tag{15}$$

and the additional splitting of the level n is therefore

$$\lambda^{(q)}_{nn_1'n_2'}={}^3/_2n(n_1'-n_2')f'$$
$$=(n_1'-n_2')[\Delta^2+q^2E_{0z}^2J_q^2(3nE_{0x}/2\omega)\omega^2/E_{0x}^2]^{1/2} \tag{16}$$

n_1' and n_2' are parabolic quantum numbers), which agrees (at $q = 1$ and $E_0/\omega \ll 1$) with the results of Refs. 12 and 13. The quantities $\lambda^{(q)}_{nn_1'n_2''}$ (16) correspond to the following solutions of Eq. (2):

$$\Phi_{ni_1'i_2'}(\mathbf{r},t)=\exp(-i\mathscr{E}_nt-i\lambda_{nn_1'n_2'}t)\sum_{i_1,i_2=-j}^{j}D^j_{i_1'i_1}(0,\theta,0)$$
$$\times D_{i_2'i_2}(0,-\theta,0)$$
$$\times\exp[-3in(n_1-n_2)E_{0z}\sin\omega t/2\omega-i(n_1-n_2)\omega t]\varphi_{ni_1i_2}(\mathbf{r}). \tag{17}$$

Here θ is the angle between the vector $\mathbf{F}$ and the vector $\mathbf{f}'$ (15), $j = (n - 1)/2$, $\varphi_{ni_1i_2(\mathbf{r})}$ are WF in parabolic coordinates with quantization axis along $\mathbf{F}$, and i_1, i_2, i_1', i_2' are expressed in terms of the parabolic quantum numbers as follows:

$$i_1=2^{-1}(m+n_2-n_1),\qquad i_2=2^{-1}(m-n_2+n_1),$$
$$i_1'=2^{-1}(m'+n_2'-n_1'),\qquad i_2'=2^{-1}(m'-n_2'+n_1').$$

In the case $H \neq 0$ and $F \neq 0$ the system has a simple analytic solution for arbitrary n if $\mathbf{E}_0 \| y$, $q = 1$. Since all the matrix elements $y_{kk} = 0$, the factors of form $J_1(u_{k_\alpha k_\beta})u^{-1}_{k_\alpha k_\beta}$ in (10) should be replaced by their limiting value, which equals 1/2 (as $u_{k_\alpha k_\beta} \to 0$). In this case the system (10) describes the behavior of a hydrogen atom in a superposition of two effective static fields, magnetic $(\Delta/\varkappa)H\mathbf{e}_z$ and electric $(\Delta/\varkappa)F\mathbf{e}_x + (E_0/2)\mathbf{e}_y$. The additional splitting of the level n can therefore be represented in the form

$$\lambda_{nk'k''}=(k'+k'')(\Delta^2+9n^2E_0^2/16)^{1/2},$$
$$k',\ k''=-j,\ -j+1,\dots,j;\quad j=(n-1)/2. \tag{18}$$

In the case of arbitrary orientation of the vector $\mathbf{E}_0$ relative to the vectors $\mathbf{F}$ and $\mathbf{H}$ (if $F \neq 0$ and $H \neq 0$), the solution of the system (10) for the level $n = 2$ leads to the following four QES WF:

$$\Phi_{s,p}(\mathbf{r},t)=\exp(-i\mathscr{E}_2t+i\Lambda_{sp}t)Q_{sp}^{-1}$$
$$\times\Big[-\frac{B_1B_2\exp(-iz_{11}E_{0z}\omega^{-1}\sin\omega t)}{B_1^*(pR_2+2^{-1}\Delta)}\varphi_1$$
$$+\frac{B_1B_2\exp(-iq\omega t-ix_{22}E_{0x}\omega^{-1}\sin\omega t)}{(sR_1+2^{-1}\Delta)(pR_2+2^{-1}\Delta)}\varphi_2$$
$$+\frac{B_1}{B_1^*}\exp(iq\omega t-ix_{33}E_{0x}\omega^{-1}\sin\omega t)\varphi_3$$
$$-\frac{B_1\exp(-iz_{44}E_{0z}\omega^{-1}\sin\omega t)}{sR_1+2^{-1}\Delta}\varphi_4\Big], \tag{19}$$

where $s = -1, 1$; $p = -1, 1$; B^*_k is the complex conjugate

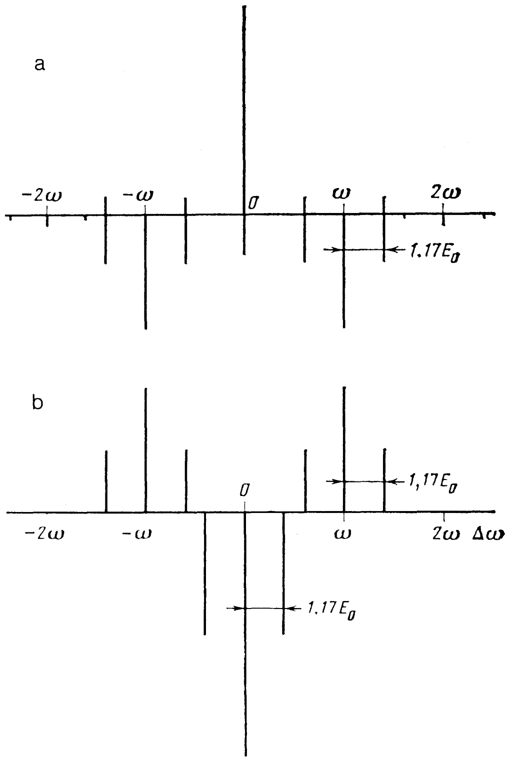

FIG. 1. Spectra of L_α line under conditions of one-photon resonance for the level $n = 1$ in fields $\mathbf{E}_0 \cos\omega t + H\mathbf{e}_z$, $\omega = \mu_0 H$ (above the abscissa axis) and $\mathbf{E}_0\cos\omega t + F\mathbf{e}_x$, $\omega = 3F$ (below the abscissa axis). The components of the vector $\mathbf{E}_0$ along the axes x, y, and z are equal to $E_{0x} = E_{0y} = E_{0z} = 3^{-3/2}\omega$. The following spectra are shown: a—$I_z(\Delta\omega)$, $\omega = \mu_0 H$; $I_x(\Delta\omega)$, $\omega = 3F$ (the intensity of the component at the frequency $\Delta\omega = 0$ in the $I_z(\Delta\omega)$ spectrum is shown reduced by a factor of two: b—$I_x(\Delta\omega)$, $I_y(\Delta\omega)$, $\omega = \mu_0 H$ $(I_x(\Delta\omega) = I_y(\Delta\omega)$; $I_y(\Delta\omega)$, $I_z(\Delta\omega)$, $\omega = 3F$ $(I_y(\Delta\omega) = I_z(\Delta\omega)$.)

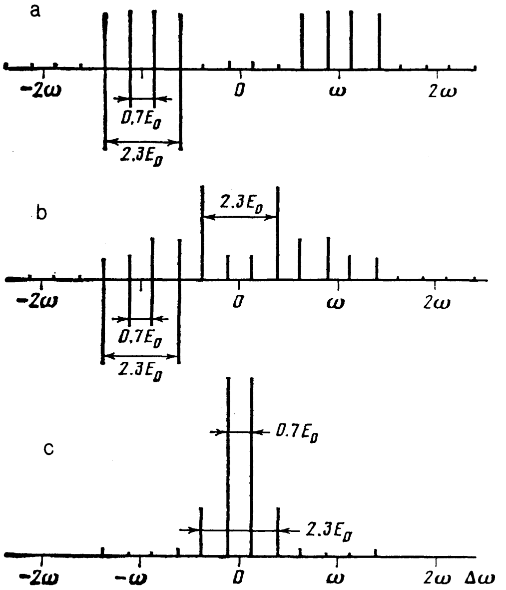

FIG. 2. Spectra of the L_α line in a field $\mathbf{E}_0\cos\omega t + H\mathbf{e}_z + F\mathbf{e}_x$ under conditions of one-photon resonance $\omega = (\mu_0^2H^2 + 9F^2)^{1/2}$ in the case $\mu_0 H = 3F$, $E_{0x} = E_{0y} = E_{0z} = 3^{-1/2}\omega$: a—$I_x(\Delta\omega)$; b—$I_y(\Delta\omega)$; c—$I_z(\Delta\omega)$.

of B_k; The function φ_k denotes the WF $\varphi_{nn'n''(\mathbf{r})}$ (5):

$$\varphi_1 \equiv \varphi_{2-\frac{1}{2}\frac{1}{2}}, \quad \varphi_2 \equiv \varphi_{2\frac{1}{2}\frac{1}{2}}, \quad \varphi_3 \equiv \varphi_{2-\frac{1}{2}-\frac{1}{2}}, \quad \varphi_4 \equiv \varphi_{2\frac{1}{2}-\frac{1}{2}},$$

and

$$\Lambda_{sp} = sR_1 + pR_2, \quad R_k = (\Delta^2/4 + |B_k|^2)^{1/2},$$
$$B_k = (-1)^{(k-1)(q+1)} q[x_{12}E_{0x} - (-1)^k (y_{12}E_{0y} + z_{12}E_{0z})] J_q(u_k) u_k^{-1},$$
$$u_k = \omega^{-1}[z_{11}E_{0z} + (-1)^k x_{22}E_{0x}], \quad k = 1, 2,$$
$$Q_{sp} = 2(R_1R_2)^{1/2}[(R_1 + s\Delta/2)(R_2 + p\Delta/2)]^{-1/2}. \tag{20}$$

Figures 1 and 2 show the spectra of L_α under conditions of one-photon resonance ($q = 1$) between the field frequency (1) and the splitting of the level $n = 2$ in a field $\mathbf{H} + \mathbf{F}$, calculated with the aid of the WF (19) for three values of $\mathbf{F}$ and $\mathbf{H}$ that satisfy the conditions

$$\mathbf{HF} = 0, \quad \omega = (\mu_0^2 H^2 + 9F^2)^{1/2}.$$

We emphasize that if $F \neq 0$ and $H \neq 0$ the inequality $|B_1| \neq |B_2|$ is satisfied for most possible orientations of the vector $\mathbf{E}_0$ relative to the vectors $\mathbf{F}$ and $\mathbf{H}$ (except for certain particular cases, for example $E_{0x} = 0$ or $E_0 = E_{0x}$). When the inequality $|B_1| \neq |B_2|$ is satisfied, the components at the frequencies $\Delta\omega = r\omega$ ($r = 0, \pm 1, \pm 2, \ldots$) in the L_α spectrum undergo complete splitting, since there is no emission in the spectral intervals

$$0 \leqslant |\Delta\omega - r\omega| < ||B_1| - |B_2|| \quad (r = 0, \pm 1, \pm 2 \ldots).$$

For comparison with Figs. 1 and 2, Fig. 3 shows the L_α spectrum under conditions of one-photon resonance $\omega = |\varkappa|$ for the level $n = 2$ in the case of non-orthogonal fields $\mathbf{F}$ and $\mathbf{H}$, which is calculated in the Appendix.

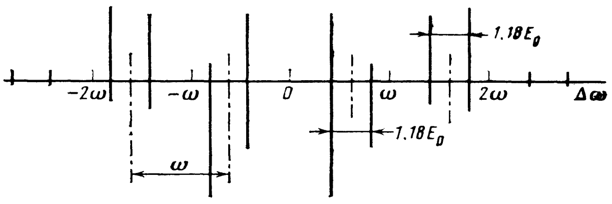

FIG. 3. Spectra of the L_α line in a field $\mathbf{E}_0\cos\omega t + F_x\mathbf{e}_x + (F_z + H)\mathbf{e}_z$ under one-photon resonance conditions $\omega = |\varkappa_1|$ in the case $\mu_0 H = 3F_x = 3F_z$, $E_{0x} = E_{0y} = E_{0z} = 3^{-3/2}\omega$. The $I_x(\Delta\omega)$ spectrum is shown above the abscissa axis, and below the abscissa axis are shown the blue half of the spectrum $I_y(\Delta\omega)$ (for $\Delta\omega > 0$) and the red half of the spectrum $I_z(\Delta\omega)$ (for $\Delta\omega < 0$). The spectra $I_x(\Delta\omega)$, $I_y(\Delta\omega)$, $I_z(\Delta\omega)$ are symmetric about the vertical axis $\Delta\omega = 0$. The dash-dot lines show the positions of the spectral components at $E_0 = 0$.

4. VALIDITY LIMITS

The quasi-energies and the hydrogen-atom QES WF corresponding to them were determined in the present paper by solving the system (8) in the resonance approximation. If follows hence, first, that the quasienergies obtained as a result of the solution of the system ($\lambda^{(q)}_{nn'n''}$ in (12), $\lambda^{(q)}_{nn_1'n_2'(\mathbf{r})}$ in (16), $\lambda_{nk'k''}$ in (18), and Λ_{sp} in (20)) should be substantially smaller in absolute value than the frequency ω. Second, the terms neglected on going from (8) to (10) should make a small contribution to the quasi-energy (e.g., contribution of the Bloch-Siegert type, see Ref. 17, §3.1) compared with the contributions from the terms containing $J_q(u_{k_1k_p})$. One of the sufficient conditions for this is the requirement

$$|J_{q+s}(u_{k_1k_p})| \leqslant |J_q(u_{k_1k_p})|, \quad p = 2, 3, 4, 5, \ s \neq 0.$$

Third, the contribution from terms neglected on going from (8) to (10) to the resulting QES WF should be small compared with the contribution from the terms containing $J_q(u_{k_1k_p})$. To this end it is necessary to stipulate

$$|(su_{k_1k_p}\omega)^{-1}\mathbf{r}_{k_1k_p}\mathbf{E}_0(q+s)J_{q+s}(u_{k_1k_p})| \ll 1, \quad p = 2, 3, 4, 5, \quad s \neq 0.$$

5. CONCLUSION

It follows from our results that the problem of a hydrogen atom in a field $\mathbf{H} + \mathbf{F} + \mathbf{E}_0\cos\omega t$ ($\mathbf{H}\cdot\mathbf{F} = 0$) under conditions of resonance between the field $\mathbf{E}_0\cos\omega t$ and the splitting of the level n in the field $\mathbf{H} + \mathbf{F}$, reduces in many cases to the known problem of a hydrogen atom in a superposition of effective static electric and magnetic fields. The onset of resonance for the upper (n_a) or lower (n_b) levels of the hydrogen atom leads to an additional splitting of the spectral lines (SL) corresponding to the $n_a \to n_b$ transition. (Such an SL splitting is similar to the component splitting in spectra of two-level systems interacting with a resonant electric field, see Ref. 18, and also Ref. 17, Chap. 9 and Ref. 19, Chap. 3.) As a result, resonance reliefs can appear in certain places on the resultant quasistatic profiles of the SL emitted by an ensemble of hydrogen atoms from a plasma. Let us analyze the locations of these reliefs in the case of one-photon resonance ($q = 1$), assuming that most hydrogen atoms in a plasma are located in a field $\mathbf{H} + \mathbf{F}$ satisfying the condition

$$\tfrac{3}{2}n\mu_0|\mathbf{HF}| \ll \max(\mu_0^2H^2, 9n^2F^2/4), \quad n = n_a, n_b.$$

The components of the emission spectrum of the hydrogen atoms that enter into resonance with the upper level n_a are then located in the vicinities of the following frequencies away from the line center:

$$\Delta\omega_{\alpha\beta}^{(a)}=n_a^{-1}\omega\{n_a(n'+n'')_\alpha-[(\mu_0H/\omega)^2(n_a^2-n_b^2)+n_b^2]^{1/2}\times(n'+n'')_\beta\},\tag{21}$$

$$n_\alpha',\ n_\alpha''=-j_a,\ -j_a+1,\ldots,j_a,\quad n_\beta',\ n_\beta''=-j_b,\ -j_b+1,\ldots,j_b,$$
$$j_\nu=(n_\nu-1)/2,\quad \nu=a,\ b.$$

We take into account here, besides the resonance condition $\varkappa^{(a)}=\omega$, the fact that in the field $\mathbf{H}+\mathbf{F}$ the positions of the spectral components are determined by the frequencies

$$(n'+n'')_\alpha\varkappa^{(a)}-(n'+n'')_\beta\varkappa^{(b)},\quad \varkappa^{(\nu)}=[(\mu_0H)^2+(3n_\nu F/2)^2]^{1/2},$$
$$\nu=a,\ b.$$

Similarly, in the case of resonance for the lower level n_b the components are located in the vicinities of the following frequencies away from the line center:

$$\Delta\omega_{\alpha\beta}^{(b)}=n_b^{-1}\omega\{[n_a^2-(\mu_0H/\omega)^2(n_a^2-n_b^2)]^{1/2}\times(n'+n'')_\alpha-n_b(n'+n'')_\beta\}.\tag{22}$$

For resonance reliefs to appear on the resultant quasistatic profiles of the SL it is necessary that the value of the frequency $\Delta\omega_{\alpha\beta}^{\nu}$ be one and the same for most emitting hydrogen atoms for which the resonance condition $\varkappa^{(\nu)}=\omega(\nu=a,b)$ is met. This is possible, for example, in the case when an ensemble of radiating hydrogen atoms is in a uniform magnetic field $H=\text{const}$ or when the magnetic field is not uniform, but on the average $\mu_0H \gg 3n_\nu F/2$, $\nu=a,b$ (in this case $\mu_0H/\omega\approx1$). In addition, in any case the resonance reliefs will exist at frequencies $\Delta\omega_{\alpha\beta}^{(a)}$ $V=\omega(n'+n'')_\alpha$ [i.e., at $(n'+n'')_\beta=0$ in (21)] and $\Delta\omega_{\alpha\beta}^{(b)}=-\omega(n'+n'')\beta$ [i.e., at $(n'+n''_\alpha=0$ in (22)]. Note that the resonance relief at the frequency $\Delta\omega_{\alpha\beta}^{(\nu)}$ $(\nu=a,b)$ will be more strongly pronounced the higher the probability of the corresponding dipole transition $\alpha\to\beta$ at $E_0=0$. Since the resonance reliefs on the resultant SL profiles are due to additional splitting of the spectral components of a definite group of hydrogen atoms, they should take, generally speaking, the form of dips on the profiles. In those cases when the additional splitting of the spectral components is not complete (e.g., if $F=0$ or $H=0$), a sharp peak should be located at the center of the dip. (The structure of the dips and on the resultant hydrogen-line profile at $H=0$ was investigated in detail in Refs. 12 and 13). Experimental recording of the dips on the resultant hydrogen SL profiles would yield the amplitude of the hf oscillating electric field in the plasma.

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APPENDIX

L_α spectrum under conditions of resonance with participation of non-orthogonal fields H and F

Let the vectors $\mathbf{H}$, $\mathbf{F}$, and $\mathbf{E}_0$ in an xyz frame be of the form

$$\mathbf{H}=H\mathbf{e}_z,\quad \mathbf{F}=F_x\mathbf{e}_x+F_z\mathbf{e}_z,\quad \mathbf{E}_0=E_{0x}\mathbf{e}_x+E_{0y}\mathbf{e}_y+E_{0z}\mathbf{e}_z.$$

According to (3), the field $\mathbf{H}+\mathbf{F}$ splits the $n=2$ level into four sublevels:

$$\varepsilon_1=-\varepsilon_4=2^{-1}(-\varkappa_1+\varkappa_2),\quad \varepsilon_2=-\varepsilon_3=2^{-1}(\varkappa_1+\varkappa_2),$$
$$\varkappa_k=[(\mu_0H+3(-1)^kF_z)^2+9F_x^2]^{1/2},\quad k=1,2.\tag{A.1}$$

Two types of resonance, $\varkappa_1\approx q\omega$ and $\varkappa_2\approx q\omega$, can be produced when the field $\mathbf{E}_0\cos\omega t$ interacts with the sublevels ϵ_p (A.1). Entering into resonance are the two sublevel pairs $\epsilon_1\leftrightarrow\epsilon_2$ and $\epsilon_3\leftrightarrow\epsilon_4$ separately in the first case, and the two sublevel pairs $\epsilon_1\leftrightarrow\epsilon_3$ and $\epsilon_2\leftrightarrow\epsilon_4$ in the second. We assume for the sake of argument that the following resonance condition is met:

$$\varkappa_1=q\omega+\Delta\quad (q=1,2,\ldots,|\Delta|\ll\omega).$$

The spectrum L_α can then be represented in the form

$$I^{(\mathbf{e})}(\Delta\omega)=S^{(\mathbf{e})}(\Delta\omega)+S^{(\mathbf{e})}(-\Delta\omega),$$

$$S^{(\mathbf{e})}(\Delta\omega)=2^{-1}\sum_{p=-\infty}^{+\infty}\sum_{s=0}^{1}|\mathbf{r}_{10}\mathbf{e}[1+(-1)^s\Delta/\rho_q]^{1/2}J_p(\mathbf{r}_{11}\mathbf{E}_0/\omega)$$
$$-(-1)^s(|V_{12}^{(q)}|/V_{12}^{(q)})\mathbf{r}_{20}\mathbf{e}[1-(-1)^s\Delta/\rho_q]^{1/2}\times J_{p-q}(\mathbf{r}_{22}\mathbf{E}_0/\omega)|^2\delta[\Delta\omega-p\omega-\varepsilon_1-\Delta/2+(-1)^s\rho_q/2],\tag{A.2}$$

where $\mathbf{e}$ is the unit vector of the polarization of the emitted photons, while the values of ρ_q and V_{12}^q are

$$\rho_q=(\Delta^2+4|V_{12}^{(q)}|^2)^{1/2},\quad V_{12}^{(q)}=\mathbf{r}_{12}\mathbf{E}_0qJ_q(u)u^{-1},$$
$$u=\omega^{-1}(\mathbf{r}_{11}-\mathbf{r}_{22})\mathbf{E}_0.\tag{A.3}$$

The coordinate matrix elements in (A.2) and (A.3) are calculated in (A.2) and (2.3) for the following states [see (5): $\varphi_0\equiv\varphi_{100}$, $\varphi_1\equiv\varphi_{2-1/21/2}$, $\varphi_2=\varphi_{21/21/2}$.

[1]H. R. Griem, *Spectral Line Broadening by Plasmas*, Academic, 1974.
[2]V. S. Lisitza and V. I. Kogan, Itogi nauk i techniki, Ser. Fizika plazmy **2**, 3, (1982).
[3]E. A. Oks, Izmerit. Tekh. No. 9, 9 (1986).
[4]A. I. Zhuzhunashvili and E. A. Oks, Zh. Eksp. Teor. Fiz. **73**, 2142 (1977) [Sov. Phys. JETP **46**, 1122 (1977)].
[5]E. A. Oks and V. A. Rantsev-Kartinov, Zh. Eksp. Teor. Fiz. **79**, 99 (1980) [Aov. Phys. JETP **52**, 50 (1980)].
[6]K. H. Finken, R. Buchwald, G. Bertschinger, and H.-J. Kunze, Phys. Ref. **A21**, 200 (1980).
[7]K. H. Finken, Fortschr. Phys. **B31**, 1 (1983).
[8]Nguen-Hoe, H. W. Drawin, and L. Herman, J. Quant. Spectr. and Radiat. Transf. **7**, 429 (1967).
[9]Yu. N. Demkov, B. S. Monozon, and V. N. Ostrovskiĭ, Zh. Eksp. Teor. Fiz. **57**, 1431 (1969) [Sov. Phys. JETP **30**, 775 (1970)].
[10]L. D. Landau and E. M. Lifshitz, *Quantum Mechanics Nonrelativistic Theory*, Pergamon, 1977.
[11]A. Cohn, P. Bakshi, and G. Kalman, Phys. Rev. Lett. **29**, 324 (1972) [Arrata: *ibid.* **31**, 620 (1973)].
[12]V. P. Gavrilenko and E. A. Oks. Methods of Atomic Calculations [in Russian] Publ. by USSR Spectroscopy Council, 1986, p. 89.
[13]V. P. Gavrilenko and E. A. Oks, Fiz. Plazmy **13**, 39 (1987) [Sov. J. Plasma Phys. **13**, 22 (1987)].
[14]V. P. Gavrilenko and E. A. Oks, Zh. Eksp. Teor. Fiz. **80**, 2150 (1981) [Sov. Phys. JETP **63**, 1122 (1981)].
[15]D. G. Yasevich and V. Yu. Yakovlev, *ibid.* **92**, 74 (1987) [**65**, 40 (1987)].
[16]M. V. Babykin, A. I. Zhuzhunashvili, E. A. Oks, *et al.*, *ibid.* **65**, 175 (1973) [**38**, 86 (1974)].
[17]N. B. Delone and V. P. Kraĭnov, Atom in a Strong Optical Field [in Russian], Atomizdat, 1978.
[18]S. Autler and C. Townes, Phys. Rev. **100**, 703 (1955).
[19]S. G. Rautian, G. I. Smirnov and A. M. Shalagin, Nonlinear Resonances in Spectra of Atoms and Molecules [in Russian], Nauka, 1979.

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