Radiofrequency surface resistance of tungsten in weak magnetic fields

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An experimental investigation was made of the surface impedance of tungsten single crystals under the anomalous skin effect conditions in the presence of a magnetic field H. In the range 0–1 kOe for $\mathbf{H}_\perp$ $\mathbf{n}_\perp$ $\mathbf{E}_\sim$ ($\mathbf{n}$ is the normal to the surface of a sample) the surface impedance R of tungsten varied nonmonotonically with the field and exhibited several extrema on the magnetic field scale. The positions of these extrema depended on the mean free path of conduction electrons l, on the surface roughness, and on the frequency of the incident electromagnetic wave: $H_{ext} \propto \delta^{-1} \propto \omega^{0.36}$. It was established that this behavior of $R(H)$ was due to a change in the nature of the scattering of conduction electrons at the metal-ambient interface when the field H was increased. The geometric dimensions of the surface irregularities (height $\sim 10^{-7}$ cm, width $\sim 10^{-6}$cm) were determined for the surfaces which scattered carriers diffusely. A comparison was made with theoretical calculations and several serious contradictions between the theory and experiment were found. A study was made of the influence of the magnetic field $H_\sim$ of an electromagnetic wave on the conductance of tungsten in the absence of a static field H.

INTRODUCTION

A theoretical description of the surface impedance $Z = R + i\chi$ of metals (R and χ are the active and reactive components) under the conditions of the extremely anomalous skin effect, when the mean free path of conduction electrons l is much greater than the depth of the skin layer δ, was given over 40 years ago by Reuter and Sondheimer[1] and has been confirmed firmly by many experimental investigations.

The results of Reuter and Sondheimer served as the basis of subsequent calculations of hf properties of metals in a magnetic field H, particularly in analyses of the positions and profiles of the oscillatory and nonresonance singularities of Z due to cyclotron and paramagnetic resonances, Shubnikov–de Haas and de Haas–van Alphen effects, size effects, and other phenomena which are of interest because they provide means for determination of the electron spectrum of carriers and Fermi surface parameters. The considerable difficulties in mathematical calculations and in the design of reliable experiments have hindered thorough theoretical and experimental investigations of the smooth part of the dependence $Z(H)$. This applies in particular to studies of Z in weak magnetic fields ($\delta/r \ll 1$) applied parallel to the surface of a sample (r is the radius of an electron orbit in a magnetic field). The knowledge of the magnetic-field dependence of the impedance is essential to prevent possible serious errors in measurements of the amplitude and profiles of cyclotron resonance lines, rf size effect, characteristic features of nonlinear absorption of waves, and other changes in Z in this range of magnetic fields. Moreover, investigations of this kind are of considerable scientific interest also for another reason: they provide information on the nature of the scattering of electrons on the surface of a metal and on propagation of electromagnetic radiation in the bulk of a metal.

Two approaches are being used currently in the calculation of Z in a parallel magnetic field, with allowance for the surface scattering of electrons. The first is based on the Fuchs approximation utilizing a phenomenological diffuseness coefficient q equal to the relative number of electrons reflected diffusely from the surface of a metal, whereas the second approach[2] is based on the assumption that in the case of a real surface the value of q should depend on the angle of incidence of electrons and a characteristic feature of such an angular dependence is a reduction in the value of q to zero for electrons with low glancing angles. Until recently the first of these approaches was much more popular and, therefore, it was used in the majority of the recent theoretical calculations of the scattering of electrons on the surface of a conductor under the anomalous skin effect conditions. A typical analysis is given in Ref. 3, where the following approximate expression is obtained for δ:

$$\delta^{-3} = \frac{4\pi\omega\sigma_0}{ic^2 l}\left\{\operatorname{cth}\frac{\pi r}{l} + \left[\frac{(\delta r)^{1/2}}{l} + \frac{q}{2-q}\right]^{-1}\right\}. \quad (1)$$

Here, ω is the frequency of the incident wave; σ_0 is the conductivity of the investigated metal in the absence of a magnetic field; $r = cp_F$ is the radius of an electron orbit in a magnetic field; e is the electron charge; p_F is the Fermi momentum; c is the velocity of light.

We shall limit an analysis of these model representations to the rf frequency range $\omega \ll \nu$ and to magnetic fields limited by the inequalities

$$r\delta \ll l^2, \quad (2a)$$

$$r/\delta > 1 \quad (2b)$$

(ν is the collision frequency of electrons in the bulk of a metal). The condition (2a), which sets the lower limit to the magnetic fields, is governed by the range of validity of Eq. (1). The term $\coth(\pi r/l)$ describing the oscillatory part of the impedance of a metal (cyclotron resonance) in Eq. (1) can be ignored in the subsequent estimates because the term is small everywhere with the exception of a narrow region of a resonance at high values of $\Omega\tau$ (Ω is the cyclotron frequency and τ is the electron relaxation frequency).

When these simplifications are made, we can readily show that in the Fuchs approximation for the characteristics of the surface scattering of electrons we can expect only a monotonic fall of $Z(H)$, described by a power law, in the selected range of magnetic fields. In particular, when q is close to zero and the skin current is governed by surface

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electrons, it follows from Eq. (1) that

$$\delta=(ic^2r^{1/2}/4\pi\omega\sigma_0)^{2/5}, \tag{3}$$

and the surface impedance $Z = 4\pi\omega\delta/ic^2$ is proportional to $\omega^{3/5}$, $l^{-2/5}$, and $H^{-1/5}$. However, if the scattering of surface electrons predominates over the scattering in the bulk of a metal $[q \gg (\delta r)^{1/2}/l]$, the impedance Z becomes

$$Z\approx(16\pi\omega^2lq/i^2c^4\sigma_0)^{1/3}, \tag{4}$$

which predicts that the impedance should be independent of the magnetic field. This last expression was refined in Ref. 4, where it was shown that if $q = 1$, then the impedance of a metal should be proportional to $H^{-2/3}$ and there should be a subsequent continuous transition at $\nu/\Omega = 1$ to the dependence $H^{-1/3}$.

The current status of the theory, reflecting the second point of view on the processes of surface scattering of carriers, is analyzed in the review of Okulov and Ustinov[2], so that we shall confine ourselves to two relatively recent investigations[5,6] of the topic. In spite of the many differences between the results of these investigations, which will be given below, a common feature is a strongly nonmonotonic $Z(H)$ dependence which can be manifested in a clear form[5] by replacing the variables in the square brackets of Eq. (1) with the following polynomials

$$(\delta r)^{1/2}/l+(\delta/r)^{1/2}Q, \quad \text{if} \quad k_F\beta\ll r/\delta, \tag{5a}$$

$$(\delta r)^{1/2}/l+(r/\delta)Q_1, \quad \text{if} \quad k_F\beta\gg r/\delta. \tag{5b}$$

Here, k_F is the Fermi wave vector; $Q\sim(\alpha k_F)^2(\beta k_F)^{-1/2}$; $Q_1\sim(\alpha/\beta)^2$; α and β are parameters representing the state of the surface of the investigated samples, particularly the rms height α and width β of the surface irregularities.

In the derivation of these equations it is assumed that the bulk and surface scattering make additive contributions to the impedance. The impedance is then given by the expressions

$$Z\approx\frac{4\pi\omega}{ic^2}\left(\frac{ic^2l(Q+r/l)}{4\pi\sigma\omega r^{1/2}}\right)^{2/5}, \quad \text{if} \quad \varphi\ll\frac{1}{(k_F\beta)^{1/2}}, \tag{6a}$$

$$Z\approx\frac{4\pi\omega}{ic^2}\left(\frac{ic^2lrQ_1}{4\pi\sigma\omega}\right)^{1/4}, \quad \text{if} \quad \varphi\gg\frac{1}{(k_F\beta)^{1/2}}. \tag{6b}$$

The range of validity of Eq. (6b) is limited to the equality $r\approx\delta$, and when this is satisfied, the impedance of a metal should increase monotonically on increase in H. It follows from these results that the active $R(H)$ and reactive $\chi(H)$ parts of the impedance have several extrema, the magnitudes and positions of which (on the magnetic field scale) are governed by the relative contributions of the bulk and surface carrier scattering mechanisms.

The currently available experimental data on the nature of the behavior of the smooth part of the surface resistance of metals in the range $\omega\ll\nu$ subjected to weak magnetic fields parallel to the surface generally confirm (see, for example, Ref. 7) the nonmonotonic behavior, but because of their qualitative nature they cannot be used in a comparison of the theory with experiment.

The present paper reports effectively the first experimental study which provided detailed information on the nature of the dependence $R(H)$ for metals subjected to a magnetic field parallel to the surface, obtained in a wide range of frequencies, temperatures, and fields.

EXPERIMENTAL METHOD

Our experiments were carried out on tungsten disks oriented in the (100) plane and cut from single-crystal ingots characterized by $\rho_{300\,K}/\rho_{4.2\,K}\approx3.5\cdot10^4$. These disks were ground with silicon carbide of the M14 grade and then were etched away to a depth of $\sim200\,\mu$m in a polishing etchant.[8] The diameter of the investigated samples was within the range 6–8 mm and the thickness was $d\approx0.5$–1 mm. We determined the change in the active part R of the surface impedance in magnetic fields 0–1 kOe at temperatures 1.5–10 K using electromagnetic radiation of frequencies 3–260 kHz.

An electromagnetic field was excited in a sample using a double-sided antisymmetric method and a rectangular induction coil. We used the apparatus shown as a block diagram in Fig. 1. An inductance coil 1 with a sample 2 were placed in a helium bath which was located inside a solenoid 3. The coil was part of an rf circuit which was tuned to a resonance by continuous variation of the frequency of a measuring oscillator. The frequency range indicated above was covered using a discrete set of capacitances C.

The maximum change in the voltage due to the absorption of rf power by the sample was observed when the total current in the circuit was kept constant by including, in series with the oscillator, an additional active resistance R_{add}, the value of which was more than one order of magnitude greater than the equivalent resistance of the circuit.

The voltage applied to this circuit was detected, in the absence of a magnetic field, by an amplitude detector, compensated by a special circuit, and then applied to the input of an X-Y automatic plotter which recorded the changes in the voltage circuit as a function of the applied magnetic field H. This system for recording the signal constituted an amplitude bridge known from the literature, which ensured practically complete exclusion of the influence of frequency changes in the circuit so that it became possible to measure the active losses alone.

The voltage across the circuit was varied by controlling, in the range from several millivolts to tens of volts, the oscil-

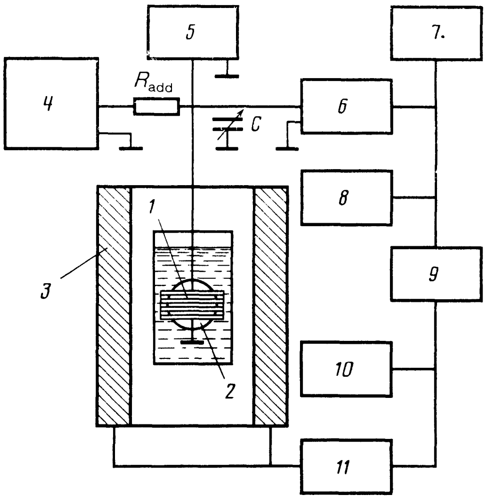

FIG. 1. Block diagram of the apparatus: 1) inductance coil; 2) sample; 3) solenoid; 4) measuring oscillator; 5) digital voltmeter; 6) amplitude detector; 7) compensation circuit; 8) digital voltmeter; 9) X-Y plotter; 10) automatic field-scanning circuit; 11) stabilized power supply.

lator voltage which created an alternating magnetic field $H_\sim$ ranging from 0.1 to 50 Oe. This field was estimated from

$$H_\sim = 0.4\pi \frac{n_c U_c}{(D^2/l_c^2 + 1)^{1/2}\omega L_c},$$

where n_c is the number of turns of the inductance coil per unit length; D is the diameter of the coil; l_c is the length of the coil; L_c is the inductance of the coil; U_c is the voltage across the circuit. The validity of this expression was confirmed by a direct measurement of the emf induced in a measuring inductance coil by a magnetic field created by an external solenoid which was calibrated using NMR data. The sensitivity of the recording devices was deduced from the magnitude of the signal which appeared as a result of transition from the superconducting to the normal state in high-purity tin samples, the geometric dimensions of which were identical with the dimensions of the tungsten.

The depth of penetration of the rf field into the samples did not exceed $\delta \approx 3\times 10^{-3}$ cm, which ensured that the condition $\delta \ll l$ was satisfied, and this corresponded to the anomalous skin effect ($l = 0.1$ cm). An external magnetic field H was parallel to the surface and perpendicular to the electric component $\mathbf{E}_\sim$ of the electromagnetic wave.

RESULTS OF MEASUREMENTS

The nature of changes in the surface resistance R of tungsten in a magnetic field applied parallel to the surface of a sample is demonstrated in Fig. 2. We can see that such a dependence was strongly nonmonotonic and had several extrema, the amplitudes and positions (on the magnetic field scale) depended on many factors. For example, an increase in the mean free path of electrons as a result of cooling increased the relative change in the resistance $\Delta R(H)$ and the positions of the extremal values of R on the magnetic field scale changed as follows: the position of the maximum was unaffected whereas the minima shifted in opposite directions: the former shifted toward weaker fields, and the latter toward stronger fields (Fig. 2).

The magnitude of the change in the resistance of the investigated samples in a magnetic field $\Delta R(H)$, relative to its value in the case when the impedance was measured in $H = 0$, was determined by measuring the resistance of an rf circuit containing tin of high purity in the normal and superconducting states. The results of these measurements were included in Fig. 2 (curve 3). A comparison of $\Delta R(H)$ with the value of $R(H)$ for tin indicated that their ratio did not exceed 4%.

Under the anomalous skin effect conditions the rf energy losses in the samples, in spite of the difference and conductivities of tungsten and tin in the normal state, differed only very slightly and, therefore, we could assume that R_W was close to R_{Sn}, in contrast to the case when an external magnetic field was present. Consequently, in this range of fields the resistance of tungsten samples differed only by a few percent from the value of the impedance in $H = 0$. An increase in the surface irregularities reduced strongly this quantity and the position of the maximum of $R(H)$ shifted toward lower fields. A change in the positions of the minima under these conditions was more complex and would require additional investigations which were outside the scope of the present study.

An increase in the frequency of the incident wave shifted the position of all the extrema proportionally to $\omega^{0.36}$, as demonstrated clearly in Fig. 3. The measurement results presented in this figure were obtained in a field $H_\sim \approx 0.1$ Oe, which was kept constant at all the frequencies. This approach was adopted because of the need to avoid nonlinear effects which began from $H_\sim > 0.2$ Oe and were manifested in particular by a considerable (up to 15%) increase in the resistance of a sample in zero external static magnetic field. An example of such changes in $R(H)$ at high values of $H_\sim$ is demonstrated in Fig. 4. We can see that in the range $H < 10$ Oe the dependence became steeper, compared with the curves in Fig. 2, and seemed to become oscillatory. An increase in H first increased the resistance of the sample (inset in Fig. 4) and then reduced it strongly to a value less than the resistance in the linear case. According to the results presented in Fig. 5, the nonlinear change in the resistance ΔR_{nl} was directly proportional to $H_\sim$, beginning with the lowest

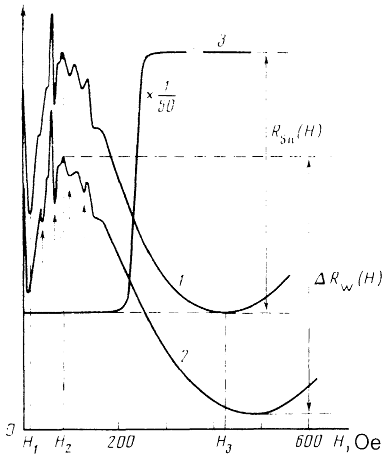

FIG. 2. Experimental curves showing the changes in the surface resistance of tungsten and tin in a magnetic field: **H**|| [100], diameter 0.0520 cm, $f = 47.5$ kHz; 1) W, $T = 4.2$ K; 2) W, $T = 1.5$ K; 3) change in the resistance of tin due to the superconducting transition at $T = 1.5$ K. The factor 1/50 alongside curve 3 shows the reduction in the gain of the recording system. The arrows identify the rf size effect lines.

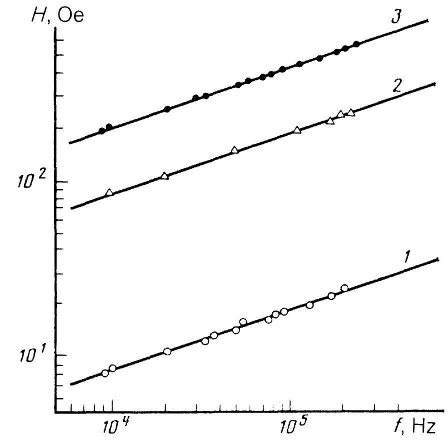

FIG. 3. Frequency dependences of the positions of the extremal values of $R(H)$ on the magnetic field scale obtained for a sample of tungsten of diameter 0.064 cm. The three dependences correspond to the extrema H_1, H_2, and H_3 (Fig. 2).

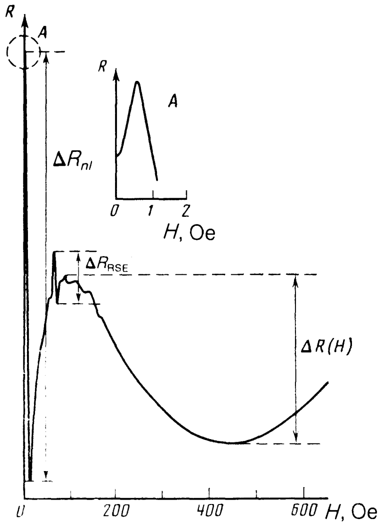

FIG. 4. Example of a record of a change in the resistance of tungsten in a magnetic field observed in the presence of high values of $H_\sim$: diameter 0.052 cm, $f = 47.5$ kHz, $H_\sim = 25$ Oe, $T = 4.2$ K.

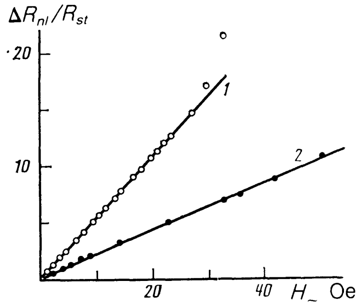

FIG. 5. Dependence of the relative changes in the resistance of tungsten on $H_\sim$: 1) $R_{st} = \Delta R_{\rm RSE}$; 2) $R_{st} = \Delta R(H)$; diameter 0.064 cm, $f = 22$ kHz, $T = 4.2$ K, $\mathbf{H}_\sim \| \mathbf{H} \|$ [100].

values of the hf field. In order to ensure that the dependence was single-valued, it was plotted in relative units of $\Delta R_{nl}/\Delta R_{\rm RSE}$ and $\Delta R_{nl}/\Delta R(H)$, where RSE refers to the rf size effect; this feature eliminated the error due to the sensitivity of the recording system.

Our investigation showed that $\Delta R(H)$ was independent of $H_\sim$ and the amplitude of the RSE lines provided a sufficiently good standard right up to those values of $H_\sim$ which corresponded to a static magnetic field in which the appearance of the size singularities of $R(H)$ was observed. Under these conditions the electrons were clearly prevented from reaching the opposite side of the sample because of considerable changes in their trajectories in the skin layer, which was the reason for the strong reduction in $\Delta R_{\rm RSE}$ and for the deviation of the experimental points from the straight line in Fig. 5.

It should also be stressed that a similar linear dependence on $H_\sim$ was exhibited by the resistance of tungsten also in the absence of a static magnetic field. This was established by investigating the field dependences $R(H)$ involving measurements of the difference between the values of the resistance of the samples in $H = 0$ and at the minimum of the RSE obtained for different values of $H_\sim$.

DISCUSSION OF EXPERIMENTAL RESULTS

1. Linear regime

A comparison of the experimental results reported above with theoretical calculations of the impedance of metals made on the assumption of a constant diffuseness coefficient of the samples [Eqs. (3) and (4)] or allowing for the dependence of this coefficient on the glancing angle of electrons [Eq. (6)] provided clear evidence in support of the dependence of this approach. Following the representation based on Eq. (6), conduction electrons in a magnetic field parallel to the surface of a metal can be divided into two groups. One of them consists of bulk electrons that do not interact with the surface and the other represents surface electrons with orbit centers which lie outside the metal. The electrons which follow these orbits collide with the surface of the metal in each turn and apparently glide along the interface between the metal and the ambient. These two groups of electrons add to the hf current contributions the magnitude of which is governed by the competition between the bulk and surface scattering processes. In spite of the relative simplicity of such a model, an allowance for collisions of conduction electrons with the metal surface is a complex multifactor problem. Therefore, the published theoretical calculations[5,6] are in many respects inequivalent, which is readily demonstrated by analyzing the principal results presented in Table I and in the form of schematic dependences in Fig. 6.

It follows from these data that the $R(H)$ dependence has several extrema the positions of which are governed by the parameters of the metal surface and by the trajectories of conduction electrons. In particular, at H_1 the frequencies of the surface ν^s and bulk ν relaxations of the carriers are equal, whereas H_2 corresponds to the maximum value of ν^s and H_3 corresponds to the situation when the diameter $2r$ of the electron orbits is equal to the depth of the skin layer δ.

On the basis of these considerations, we shall estimate H_1 and H_2 using the data of Ref. 6 and employing the expressions

TABLE I

Source of data	Positions of extremal values on the magnetic field scale		
	H_1	H_2	H_3
Ref. 5	$\propto \frac{cp_F}{el}\frac{\beta^{1/2}}{k_F^{3/2}\alpha^2}$	$\propto \frac{c\hbar}{e\delta}\frac{1}{\beta}$	$\propto \frac{cp_F}{e}\frac{1}{\delta}$
Ref. 6	$\propto \frac{cp_F}{e\delta}\left(\frac{\beta^{1/2}}{k_p^{3/2}\alpha^2}\right)^2$	$\propto \frac{c\hbar}{e\delta}\left(\frac{1}{k_F\alpha^2\beta^2}\right)^{1/3}$	—

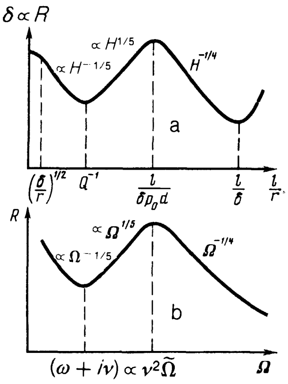

FIG. 6. Schematic representations of the dependences of the real part of the impedance on the magnetic field H (theory): a) Ref. 5; b) Ref. 6.

$$\nu^s \propto \frac{k_F^{3/2}\alpha^2}{\beta^{1/2}}\Omega, \tag{7}$$

$$\Omega \propto \frac{\hbar}{\delta m}\frac{1}{k_F\alpha^2\beta^2} \tag{8}$$

Here, Ω is the cyclotron frequency of the electron motion at which ν^s has its maximum value. Hence, replacing Ω with its value eH/mc and assuming that $\nu^s = v$ and also that the relaxation time of bulk electrons is limited by the time they spend in the skin layer, i.e.,

$$\nu = v_F/L \approx v_F/(\delta r)^{1/2}, \tag{9}$$

we can readily find H_1 and H_2. The results of such a calculation are given in Table I.

In analyzing the results in Table I we must bear in mind particularly the absence of a second minimum of $R(H)$ in Ref. 6, which is not predicted by these calculations right up to fields 10^5 Oe, and also the disagreement between the values of H_1 and H_2. It should be pointed out that the difference between the values of H_1 given in Refs. 5 and 6 is readily avoided if we assume that in Eq. (9) the quantity L is equal not to $(r\delta)^{1/2}$ but to l. However, in our opinion this is not justified because in the case of bulk electrons the application of a magnetic field should shorten the duration of interaction of these electrons with an electromagnetic wave.

We shall test the reliability of the theoretical models underlying the results reported above by comparing these data with the main characteristics of the behavior of the experimentally determined positions of the extrema of $R(H)$. Bearing in mind that the range of validity of the theories of Refs. 5 and 6 is limited by the inequality $L \ll l$ on the low-field side, we shall find this limit of H in the specific case of tungsten. In general, in view of the narrowness of the effective "belt" of values, we can assume that the trajectory of an electron in the skin layer subjected to low magnetic fields is a part of a circle. Therefore, assuming that $L = (8\delta r)^{1/2} = l$, we can readily show that

$$H^{L=l} = 8cp_F\delta/el^2. \tag{10}$$

We shall estimate this value of H for the minimum frequency of an electromagnetic field in an experiment and the maximum value of the Fermi momentum of conduction electrons. According to Ref. 8, this momentum is exhibited by carriers belonging to an "electron jack" of the Fermi surface and the value of this momentum is $p_F^{\max} = 1.1 \times 10^{-19}$ g·cm·s^{-1}. Therefore, in the case of the samples used in the present investigation ($l \approx 0.1$ cm) we find that at the frequency of 3 kHz we have $H^{L=l} \approx 5$ Oe. Although this value is of the same order of magnitude as H_1 (Fig. 3), there is no doubt that we can use the expressions in Table I in the interpretation of the experimental results because a reduction in the electron momentum and an increase in the frequency will shift this limit toward lower fields.

In an investigation of the anomalous skin effect in metals reported in Ref. 9 the value $H^{L=l}$ was identified with the magnetic field corresponding to the maximum of the function $R(H)$. However, this approach to the definition of $H^{L=l}$ cannot be accepted not only because of the considerable value of the magnetic field H_2 in which this maximum is observed, but also because of the constancy of its position when temperature is lowered and because the frequency dependence (Fig. 3) is opposite to that given by Eq. (10).

We shall now consider the characteristic features of the behavior of each of the extrema of $R(H)$ in greater detail. As pointed out already, the experimental results indicate that the magnetic field H_1 corresponding to the first of these extrema (which is a minimum) decreases on increase in the mean free path of electrons and increases on increase in the frequency of a wave incident on a sample proportionally to $\omega^{0.36}$. The good agreement between the numerical exponent of the frequency dependence of the position of this extremum in a magnetic field with the value 1/3 shows that H_1 is inversely proportional to the depth of the skin layer δ because we know that $\delta \propto \omega^{-1/3}$. Comparing these results with the estimates of H_1 given in Table I, we can easily see that the theoretical results do not describe fully these dependences. Thus, according to Ref. 5, the value of H_1 is independent of ω, whereas according to Ref. 6 the electron mean free path l does not affect this field. The disagreement between the experiments and theory was found also for the value of H_3 representing the position of the second minimum of $R(H)$. The field H_3 increases on increase in the electron mean free path and the difference between the ratios H_3/H_2 is considerable. According to the experimental results, this ratio is ≈ 1.5–3, whereas the expressions given in Table I suggest that it should be $\approx \beta k_F$, the numerical value of which is shown below to be $\approx 10^2$.

We shall now consider the main aspects of the behavior of the maximum of $R(H)$. According to the experimental and theoretical data, its position H_2 in a magnetic field shifts, on increase in the frequency of the incident electromagnetic wave, proportionally to $\omega^{0.36} \propto \delta^{-1}$ and is independent of the mean free path of conduction electrons. The good agreement between the theoretical and experimental results and the relatively high sensitivity of the amplitude and position of this extremum to the quality of the surface treatment of the samples suggests that the experimental results can be used to estimate the parameters of surface defects of metals. In particular, it follows from the results of the calculations in Ref. 5 that the average horizontal size β of the surface irregularities of the investigated samples can be estimated from

$$\beta \approx c\hbar/e\delta H_2. \tag{11}$$

We shall select the frequency f of the incident wave to be 50 kHz. It then follows from the graph in Fig. 3 that $H_2 \approx 100$ Oe and the depth of penetration δ of the electromagnetic field found from Eq. (1) is $\approx 6 \times 10^{-4}$ cm. Using these data in Eq. (11), we can readily show that $\beta \approx 10^{-6}$ cm for samples of tungsten subjected to grinding with a powder consisting of small grains and removal of the disturbed layer in a polishing etchant. An estimate of the rms height of irregularities of such a surface can be obtained by substituting this value into the formulas for H_1 or into the expression which governs H_2 of Ref. 6. A simple calculation gives $\alpha \approx 10^{-7}$ cm.

In view of this whole series of contradictions between theoretical calculations and experiments and because of the approximate nature of the expressions in Table I, the values of α and β were estimated only to the nearest order of magnitude. Refinement of these values would naturally require a better theoretical model and suitable further experiments. Nevertheless, we must stress the good agreement between these results and the geometric dimensions and irregularities on the surface of tungsten samples oriented in the (100) plane, which were determined with a tunnel microscope and reported in Ref. 10.

2. Nonlinear effects

Turning back to the experimental results reported above (Fig. 2), we must stress once again that under linear conditions in a homogeneous magnetic field H the surface resistance of tungsten first decreases smoothly, reaches a maximum at $H \approx 10$ Oe, and only then begins to rise. The value of the relative changes $\Delta R(H)/R(0)$ in fields up to 1 kOe does not exceed a few percent.

In view of the quasistatic nature of the phase of the rf electromagnetic wave during a period equal to the relaxation time of conduction electrons ($\tau \approx 10^{-9}$ s), it was natural to expect similar changes in R_W also on increase in $H_\sim$, but the results presented in Fig. 4 demonstrated just the opposite effect. In fact, when the homogeneous magnetic field H was fully compensated, the resistance of the samples did not decrease when $H_\sim$ was increased, but in fact it increased reaching values considerably greater than $\Delta R(H)$ (curve 2 in Fig. 5). The rise of ΔR_{nl} ($H_\sim$) obeyed a linear law beginning from very low values of the magnetic field.

The currently available experimental results taken as a whole demonstrate that manifestation of the nonlinear effects and the hf resistance of tungsten in the presence of a strong rf wave (see also Ref. 11) is not a consequence of the characteristics of the electronic structure of this metal, but applies to all the metals characterized by a long mean free path of electrons. Such changes in the impedance had been observed in particular for bismuth,[12] gallium,[13,14] and tin (as reported in the review of Dolgopolov[15]).

The same investigations established at an increase in the amplitude of an rf wave altered the nature of the dependence $Z(H)$ in a homogeneous magnetic field and gave rise to additional extrema, which were very sensitive to the degree of roughness of the surface of a sample and to the structural quality of the surface layer. This indicated that the nonlinear correction to the impedance was most probably due to conduction electrons colliding with the boundary of the metal.

Using these ideas, it was suggested in Ref. 9 that a localized group of electrons is present in the skin layer and it is concentrated in a narrow surface layer of the metal of thickness $\Delta \ll \delta$. In accordance with this model, the magnetic-field-induced redistribution of the contributions made to the total current by this group and by the bulk electrons should first increase the resistance quadratically on increase in $H_\sim$, and then, after passing through a maximum at $H_\sim \approx 8cp_F\delta/el^2$ governed by the equality $L = l$, it should decrease.

A comparison of these predictions with the experimental results demonstrates clearly the disagreement both in respect of the exponent of the power dependence of $R(H_\sim)$ and in respect of the position of $R(H_\sim)$ on the field scale. In the latter case, as shown above, the equality $L = l$ is readily satisfied for all the groups of conduction electrons of tungsten already in fields of ≈ 5 Oe, which is an order of magnitude less than the corresponding experimental values of $H_\sim$ in the case of an extremum of $R(H_\sim)$ (Ref. 9). However, this model makes it difficult to explain also the considerable quantitative changes in $R(H_\sim)$ found experimentally. Therefore, the principle of detailed differentiation of conduction electrons in the skin layer proposed in Ref. 9 is not very useful in the interpretation of the nonlinear changes in $Z(H_\sim)$. It seems more promising to analyze theoretically the nonlinear absorption of an rf wave by conduction electrons allowing for the possibility of the appearance, under the action of a ponderomotive force, of acoustic vibrations in metals placed in a spatially inhomogeneous magnetic field and also for estimating the role of the surface scattering of carriers characterized by $l \approx L$ under these conditions.

This analysis clearly demonstrates that in investigations of the surface impedance of metals in weak magnetic fields there has been considerable progress in the attempts to explain the results reliably, but there are still many unresolved problems. This applies to the linear and nonlinear ranges of variation of $Z(H)$. It would be undoubtedly of interest to reformulate these problems and to solve them because it would provide new information on the processes of carrier scattering at the boundary of a metal and on the nature of propagation of an electromagnetic wave in a metallic semiconductor subjected to a magnetic field.

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