Temperature dependence of conductivity in high mobility MIS structures on a base of (001) silicon

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Measurements of the temperature dependence of the conductivity of two-dimensional electrons in silicon MIS structures were carried out. It is shown that the observed dependence is well described by the equation

$$\sigma(T)=\sigma(0)\{1-Q(kT/\varepsilon_F)-P(kT/\varepsilon_F)^{3/2}+O[(kT/\varepsilon_F)^2]\}.$$

The variation of the coefficient Q with the density N_s of the two-dimensional electrons is determined, and it is shown that the observed trend of the $Q(s)$ curve is described by consideration of the temperature dependence of the dielectric function of a two-dimensional electron gas.

Following development of high mobility silicon MIS structures it was observed that at liquid-helium temperature the mobility of two-dimensional electrons increased with decreasing temperature.[1] Since the combined effects of electron-electron interactions and weak localization for electronic channels on the (100) surface of silicon decrease conductivity with decrease of temperature,[2] it could be assumed that the observed effect is connected with electron–phonon scattering. However, measurements[3,4] of the energy relaxation time in the same specimens and in the same temperature interval showed that the frequency of electron-phonon scattering is too low to explain the temperature dependence of the conductivity.

Stern[5] suggested that the increase of the conductivity on reduction of temperature may be caused by the temperature variation of the screened scattering potential in elastic scattering of electrons. His calculations led to agreement between the scales of the expected changes in the conductivity and of the experimentally observed changes. On the basis of an analytic calculation,[6,7] it was shown that allowance for the temperature dependence of the screening results indeed in a linear decline of the conductivity with increase of temperature, to lowest order in parameter kT/ε_f.

The collision broadening of the energy levels influences the screening, therefore for temperatures lower than the energy smearing of the levels the temperature dependence of the frequency of the elastic scattering processes must disappear.[6] Probably just this effect was observed in Ref.9.

The aim of the present work was the investigation of the temperature dependence of the conductivity in silicon (100) MIS structures with different mobility of the two-dimensional electrons, and a comparison with experimental results with calculations.

QUALITATIVE CONSIDERATIONS AND FUNDAMENTAL FORMULAE

In real silicon MIS structures, the layer of two dimensional electrons is not an equipotential surface. Fluctuations of the potential, in principle, may be caused by the following: differing thickness of the insulator at various points of the structure, inhomogeneity of the insulator, changes in the insulator, and inhomogeneity of the doping of the silicon substrate.

The bare potential topography will be screened to various degrees, depending upon the characteristic scale L of the inhomogeneity in the plane of the two-dimensional electron gas. If L exceeds the thickness of the insulator, then the electrons of the metallic gate may participate in the screening. For $L > (2k_F)^{-1}$ (we are considering a degenerate electron gas) the Thomas–Fermi screening is effected, and finally, for $L < (2k_F)^{-1}$ the screening deteriorates additionally, (see, e.g., Ref. 10), since the wavelength of the electron exceeds the scale of the inhomogeneity of the potential. What is essential is the correlation between the characteristic size of the fluctuations of the potential and the mean free path of the electrons: fluctuations with scale less than or of order of the mean free path may be regarded as forming the path, and the long-period fluctuations as altering the average density of the two-dimensional electrons.

So long as the maximum mobility of electrons in silicon MIS structures at liquid helium temperatures was on the order of 10^3 cm/V·s, it could be assumed that the insulator-thickness variation due to the irregularities of the SiO_2-metal interface at oxide thickness exceeding 1000 Å affect mainly the electron density, and irregularities of the SiO_2-semiconductor interface determine the contribution to the scattering. In better silicon MIS structures, possessing mobility of 20000–30000 $cm^2/V\cdot s$ at liquid-helium temperatures, the mean free path of the electrons is ~2000 Å. In this case roughnesses on both boundaries of the insulator contribute to the scattering, and the irregularities of the boundary between the SiO_2 and the metal cause fluctuations with $L \gtrsim d$.

Little is known about the degree of inhomogeneity of the insulator. It is usually assumed that the insulator is homogeneous.

Charges found in the insulator and within the semiconductor may be considered in the same manner. It is essential that two-dimensional electrons are effectively scattered only by those charges which lie in a layer of thickness k_f^{-1} surrounding the two-dimensional electron gas[10]. Therefore it is frequently assumed that all Coulomb scattering centers are

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located on the boundary between the insulator and semiconductor.

Thus, in the simplest model for determining the conductivity of a two-dimensional electron gas, it suffices to consider two types of scattering: first, charges on SiO_2–Si interface, and second, the roughnesses of this interface. Three parameters are shown to be necessary for a description of the potential fluctuations: number N_i of the Coulomb scattering centers, N_i, average height Δ of the steps on the boundary, and the average distance Λ between them.

The square of the matrix element of the potential for transition with a momentum transfer $\hbar q$ has the form[11,12]

$$\langle |U_q|^2\rangle = N_i (2\pi e^2/\varepsilon_L q)^2 F_i(q)$$

$$+\pi\Delta^2\Lambda^2 q_s^2 \varepsilon_F^2 \left(1+\frac{2N_D}{N_S}\right)^2 \exp(-q^2\Lambda^2/4). \qquad (1)$$

Here ε_L is the average dielectric function of the insulator and of the silicon, $F_i = (1+q/b)^{-3}$ is the form factor due to the finite extent of the electron wave function in the z direction,

$$b=[48\pi m_\perp e^2 \varepsilon_{sc}(N_D+11N_S/32)]^{1/3},$$

q_s is the screening parameter in the Thomas-Fermi approximation; N_D is the total charge in the depletion layer per square centimeter.

In the Born approximation the relaxation time, which determines the dc conductivity, is given for electrons with energy ε by

$$\frac{1}{\tau(\varepsilon,T)} = \frac{1}{2\pi\hbar\varepsilon}\int_0^{2k} \frac{q^2\,dq}{(4k^2-q^2)^{1/2}} \frac{\langle |U_q|^2\rangle}{\varepsilon^2(q,T)} \qquad (2)$$

where k is the electron wave vector, and $\varepsilon(q,T)$ is the dielectric constant of the two dimensional electron gas. The conductivity at finite temperature is written as

$$\sigma(\varepsilon_F,T) = \frac{1}{4kT}\int_0^\infty d\varepsilon \frac{\sigma(\varepsilon,T)}{\mathrm{ch}^2[(\varepsilon-\varepsilon_F)/2kT]},$$

$$\sigma(\varepsilon,T) = \frac{N_S e^2 \tau(\varepsilon,T)}{m}. \qquad (3)$$

Expressions (1)–(3) allow one to determine the conductivity for arbitrary temperature and density of the two dimensional electrons. The terms of lowest order in the expansion of the conductivity in a series in the parameter kT/ε_f stem from the characteristic form of the dielectric constant $\varepsilon(q,T)$.[13,14]

The qualitative form of the $\varepsilon(q)$ dependence is shown in Fig. 1. It is important that the $\varepsilon(q)$ plot has a singularity for $q = 2k_F$. Since the scattering processes with a momentum change $2k_F$ enter in the collision frequency with large weight (because of the square-root singularity in Eq. (2)), the smearing of the singularity at $q = 2k_f$ owing to the finite temperature (see Fig. 1) leads to a temperature dependence of the conductivity. Another mechanism leading to a temperature dependence of the conductivity is thermal excitation of the electrons into states of energy exceeding ε_F. As a result, momentum transfer larger than $2k_F$ becomes possible. As shown in Refs. 6 and 7, both mechanisms lead at low temperatures to the linear relation

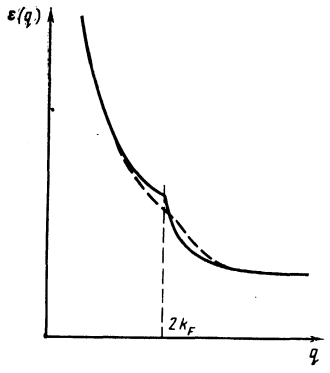

FIG. 1. Dependence of the dielectric function ε of a two-dimensional electron gas on the wave vector q (continuous curve—$T=0$, dashed curve—$T\neq 0$).

$$\sigma(T) = \sigma(0)[1 - QkT/\varepsilon_F - P(kT/\varepsilon_F)^{3/2} + \ldots]. \qquad (4)$$

Between coefficients P and Q exists the analytic relationship

$$P = 0.58QC(N_S), \qquad (5)$$

where $C(N_S)$ is a function, known from calculations, of the density of the two-dimensional electrons (see Ref. 7). Equation (5) is exact in the presence of one scattering process: either by charge centers or by surface roughnesses. It becomes approximate if both mechanisms are simultaneously present. The $Q(N)_S$ dependence will be of interest to us hereafter. The use of the approximate equation (5) to get the $Q(N_S)$ dependence from experimental curves introduces errors not larger than 5% in the value of Q, certainly less than the errors due to other causes.

An analytic expression for the coefficient Q was obtained in Ref. 7:

$$Q = \frac{\Delta(\tau^{-1})}{\tau^{-1}(\varepsilon_F,0)}(kT)^{-1}, \qquad (6)$$

where $\tau^{-1}(\varepsilon_F,0)$ is determined by Eq. (2) at $T=0$, and $\Delta(\tau^{-1})$ equals

$$\Delta(\tau^{-1}) = (2\hbar^{-1}kT\ln 2)k_F^2\varepsilon_F^{-2}\langle |U(2k_F)|^2\rangle V(2k_F)[1 - G(2k_F)]\{1+V(2k_F)\rho_F[1-G(2k_F)]\}^{-3}. \qquad (7)$$

Here $V(q)$ is the Fourier transform of the potential of the interaction of the electrons in the two dimensional case (see Ref. 10, for example),

$$G(q) = q/4(q^2+k_F^2)^{1/2},\ \rho_F = 2m/\pi\hbar^2.$$

EXPERIMENTAL RESULTS AND DISCUSSION

The measurements were performed on five Hall-effect transistors with the using of four-point bridge circuit.[15] The precision of the resistance measurement was within 0.05% in the temperature interval from 0.6 to 4.5 K. The interval in which the measurements were conducted was chosen from the following considerations.

1) On the high-temperature side the measurement region is limited by two circumstances. Firstly, for reduction according to Eq. (4) and determination of the coefficient $Q(N_S)$ it is necessary that the next, not accounted-for term of the expansion be small in comparison with that linear in temperature. For a 10% accuracy limit in the determination

of Q, it is necessary to have $kT/\varepsilon_F \leqslant 0.1$. This gives a temperature 4 K for a density 6×10^{11} cm^{-2}, but with increase of the electron density the temperatures corresponding to $0.1\varepsilon_F$ rises. Secondly, the temperature interval is bounded from above by processes of electron-phonon scattering. The electron-phonon scattering frequency, which enters in the conductivity, is not known exactly, but in the considered temperature region one can expect an order-of-magnitude agreement between this frequency and the reciprocal electron-relaxation time τ_ε^{-1}. The experimental values of the electron energy relaxation time (see, for example, Ref. 4), are such that for 10% precision in the determination of Q the temperature of the measurements must have an upper bound 5–7 K for all investigated electron densities.

2) On the low-temperatures side, the region for measurement is restricted by the effects of weak localization and electron-electron interaction, which result in a conductivity decrease logarithmic in temperature with decreasing temperatures.[16,17] In addition, as shown by Das Sarma,[8] for temperatures less than the collision broadening of the energy levels

$$kT<\Gamma=\hbar/2\tau\approx 3\cdot 10^4\mu_0^{-1}[\mathrm{K}],$$

where $\mu_0=\sigma(0)/N_S e$, the singularity at $q=2k_F$ in the dielectric function is washed-out mainly by collision broadening. Under these conditions one should expect weakening of the temperature dependence of the conductivity compared with Eq. (4), and its vanishing in the limit $kT\ll\Gamma$. Therefore the noticeable decrease of the temperature below 1 K for our samples having a mobility $\mu_0\cong 20000$–30000 cm^2/sec had no meaning.

Examples of the $\sigma(T)$ dependences for different densities N_s for one of the investigated samples is shown in Fig. 2. As seen in the figure, the temperature dependence of the conductivity can be approximated with good accuracy by a straight line in the interval 1.3–4.4 K. With further decline of temperature, a weaker temperature dependence is observed. The continuous lines in Fig. 2 were obtained by fitting the experimental points to Eq. (4) using Eq. (5) and two fit coefficients, $\sigma(0)$ and Q. Allowance for the term proportional to $T^{3/2}$ does not result in large deviation from linearity in the investigated temperature interval (see Fig. 2), but does lower the value of the coefficient Q by 15–20% for the lowest densities of the two-dimensional electrons. A deviation of experimental points from the calculated plot for temperatures 0.6 K is observed for all investigated densities and agrees in scale with that expected from the weak localization and electron-electron interaction, $\sigma_m=e^2/4\pi^2\hbar$. Application of a small ($\simeq 600$ H) magnetic field perpendicular to the surface of the samples increased the value of the coefficient Q little (10–20%), but did not alter by any noticeable amount the deviation of the experimental points from the calculated dependence for the lowest temperatures.

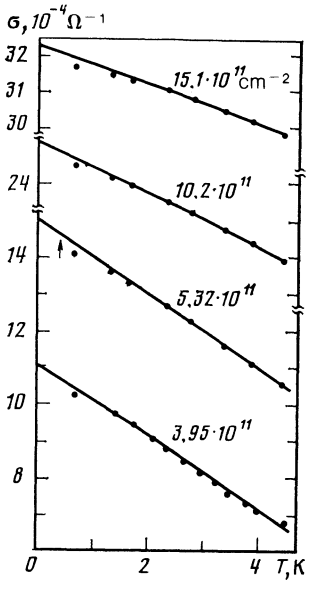

FIG. 2. Dependence of conductivity on temperature for a sample with minimal peak mobility (N4) for different values of N_S (numbers above the lines). The scale of the deviation from a straight line because of the effective localization and electron-electron interaction is marked by the arrow.

A similar behavior was typical of those four samples for which detailed data is given below. One more specimen, whose peak was of sufficient size (29000 cm^2/V·s at $T=1.55$ K and density 4.3×10^{11} cm^{-2}), was investigated by us, but the temperature dependence of the conductivity even in the interval 1.5–4.2 K noticeably deviated from a straight line. Hall-emf measurements of this sample in a weak magnetic field revealed a variation of the electron density with temperature, capable of explaining the observed deviation. Because of the complexity and low precision of the reduction of the data obtained for this specimen, it is not discussed below.

In Figs. 3 and 4 are shown results of the measurements of μ_0 and $Q(N)$ for four samples with different values of maximum mobility. Attention is called to the fact that at relatively large densities ($N_S\sim 2\times 10^{12}$ cm^{-2}) the values of mobility obtained for different samples are close. The values of the coefficients Q at these densities were also found to be close. Since at large, undoubtedly, scattering on the roughnesses of the surface prevails, it follows from the equality of the coefficients Q that the characteristic roughness scales Λ do not differ greatly for different specimens. (In agreement with Ref. 7, in the limit $k_F\Lambda\gg 1$ the value of Q is determined by one parameter Λ.) The equality of the values of μ_0 in this density region means that the characteristic heights of the roughnesses of different samples are close.

In Figs. 3 and 4 are shown the calculated $\mu_0(N_S)$ and

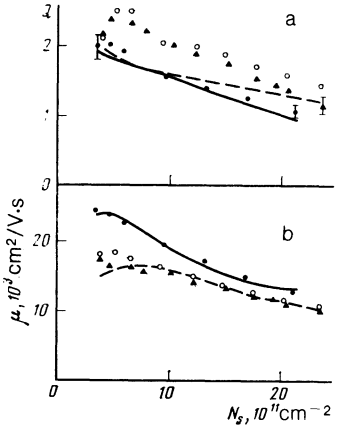

FIG. 3. Plots of $Q(N_S)$ and μ_0: ●) sample 2, ▲) sample 4. External magnetic field $H=0$; ○) sample 4, $H\approx 500$ Oe. The continuous and dashed lines correspond to calculations with the parameters indicated in Table I.

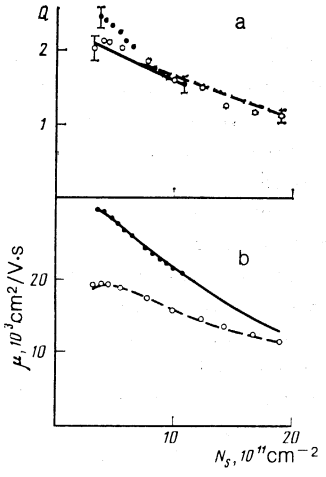

FIG. 4. Plots of $Q(N_S)$ and μ_0: ●) for sample 1, ○) sample 3. The continuous and dashed lines correspond to calculations with the parameters indicated in Table I.

$Q(N_S)$ dependences. For calculations based on the right end of the $Q(N_S)$ curve we determined beforehand the parameter Λ with the aid of relations (2), (6), and (7); next, varying all three quantities N_i, Δ and Λ, we fitted the $\mu_0(N_S)$ curves. This procedure was carried out numerically, in so much as the approximate analytic expressions of Ref. 7 do not provide the necessary accuracy. The correction to the beforehand-determined value of Λ were small (in the worse case it did not exceed 10%). The fit parameters obtained in this manner are listed in Table I. As seen from Table I, the number of Coulomb-scattering centers increases systematically with decrease of mobility; the parameter Δ varies weakly and is of the order of the interatomic distance; the characteristic roughness size Λ decreases with decreasing mobility.

From Figs. 3(a) and 4(a) it is seen that the expected values at low densities N_S really do not depend upon the fit parameters and coincide for all samples. Experimental points for two samples of intermediate mobility agree at the left ends of the curves to within the experimental error with those expected from the calculations. For two other samples the experimental values for small densities N_S exceed the calculated values by 20%.

In our opinion the agreement, within such accuracy, of the experimental value of Q with calculations in this region, where there is no strong dependence of Q on the fit parameters, shows convincingly the correctness of the calculation. Such an agreement is more readily surprising than regular. The point is that the random-phase approximation, in which the dielectric function utilized in Eq. (2) is computed, consists of approximate linearization of the equations of motion and is correct if the electron interaction can be considered weak.

TABLE I.

Sample	μ_{0max}, $cm^2/V \cdot s$	N_i, cm^{-2}	Δ, Å	Λ, Å
1	29 400	$1.7 \cdot 10^{10}$	2.98	77
2	24 000	$2.58 \cdot 10^{10}$	3.14	70
3	19 300	$2.8 \cdot 10^{10}$	2.91	66
4	17 000	$4.1 \cdot 10^{10}$	2.4	50

For the two dimensional electron gas in an MIS structure, however, the ratio of the average potential energy of an electron $U = e^2/\varepsilon_L N_S^{-1/2}$ to the kinetic energy $\varepsilon_F = \pi N_S \hbar^2/2m$ is of the order $U/\varepsilon_F \cong 2.5 \times 10^{-7} x (N_S[cm^{-2}])^{-1/2}$. In the investigated range of densities we have $U/\varepsilon_F \sim 10$ and the electron gas cannot be regarded as weakly interacting. Another condition for the validity of the random-phase approximation is the condition $q \ll k_F$. It also is not met in the given case, as the linear temperature dependence of the conductivity is connected with the singularity of the dielectric function at $q = 2k_F$.

The obtained values of fit parameters are quite reasonable. The values of Λ were found to be unusually large. It was assumed earlier (see Ref. 10), that in MIS structures on silicon the size of Λ was ~ 15 Å. In a recent paper[18] a value $\Lambda = 27$ was deduced from analysis of the $\mu(N_S)$ dependence for a maximum of electron mobility in the specimen $\cong 14000$ $cm^2/V \cdot s$. It must, however, be borne in mind that the simultaneous determination of at least three parameters from one experimental curve, utilized in these studies, does not make it possible to determine Λ with any degree of reliability. The method utilized in the present work allows one to find the size of Λ independently of other parameters that characterize the scattering. That fact, that Λ in high-mobility structures was found to be ~ 70 Å, may turn out to be essential for the interpretation of experiments on the quantum Hall effect, since such values of Λ agree with the magnetic length in the usually utilized fields from 5 to 10 T.

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