Temperature dependences of the conductivity of *n*-type InSb and *n*-type InAs in the extreme quantum limit

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The temperature dependences of the longitudinal, transverse, and Hall conductivities of n-type InSb and n-type InAs with electron densities $n = 1.2\text{–}2.7\times 10^{16}\,\mathrm{cm}^{-3}$ were determined in the extreme quantum limit in respect of the magnetic field. The experimental results were found to be in agreement with an earlier prediction [S. S. Murzin, JETP Lett. **45**, 283 (1987)] based on an analysis allowing for the influence of localization effects and scattering correlation on the conductivity.

It has recently become clear that in dealing with the transport properties of an electron gas in the extreme quantum limit (EQL) in respect of the magnetic field it is necessary to allow for the localization effects (Refs. 1 and 2)[1)] and for the correlation in the scattering[2] if electrons are scattered mainly by ionized impurities. The need to allow for the localization effects arises because during the time τ_1 in which an electron travels along a magnetic field **H** until it is scattered backward there are not only collisions with many impurities, but also a transverse displacement across the magnetic field to a distance shorter than the magnetic length $\lambda = (\hbar c/eH)^{1/2}$, which is the characteristic size of the wave functions at right-angles to **H**. If we ignore the change in the impurity potential across a magnetic field over distances $\sim\lambda$, we find in the first approximation that in view of the one-dimensional nature of its motion an electron is localized along **H** (Ref. 3). The localization radius is of the order of the distance l traveled by the investigated electron along the magnetic field. The transverse component of the inhomogeneous electric field of the impurities causes-an electron wave packet (of size $\sim l$ along the magnetic field and $\sim\lambda$ across the magnetic field) to drift across **H**. This transverse displacement as a result of drift causes delocalization of an electron along **H** in a time

$$\tau_2\sim\tau_1(k_z^2+{}^1/_4\varkappa_0^2)^{-1}\lambda^{-2}\gg\tau_1,$$

where k_z is the wave vector of the electron along **H**; $\varkappa_0 = [(4\pi e^2/\varepsilon_0)\partial n/\partial\mu]^{1/2}$ is the reciprocal of the Debye screening radius; ε_0 is the permittivity of the crystal lattice; $\partial n/\partial\mu$ is the density of states at the Fermi level. At a temperature $T\neq 0$, when the frequency of inelastic scattering processes obeys

$$\tau_1^{-1}\ll\tau_{in}^{-1}(T)\ll\tau_2^{-1},$$

we can say that the conductivity along **H** appears because of jumps between quasilocalized states and if $kT<\hbar/\tau_1$, it is given by[2]

$$\sigma_\parallel\sim\frac{ne^2\tau_1}{m}\frac{\tau_1}{\tau_{in}},\tag{1}$$

where n is the electron density and m is the effective mass of an electron. Since τ_{in}^{-1} increases on increase in T, it follows that $\sigma_\parallel$ also increases with T, whereas in the absence of the localization effects the rise of τ_{in}^{-1} has the converse effect and it reduces $\sigma_\parallel$.

In an analysis of the transverse conductivity we must allow for the correlation in the scattering process. This correlation is due to the fact that in the EQL case the value of λ is less than the screening radius d across the magnetic field and, therefore, an electron regarded as a classical particle returns in the course of its motion along **H** many times to the field of the same impurity and it shifts across **H** by less than d. It is then scattered every time (the scattering process can be regarded as drift in the field **H** at right-angles to the electric field of an impurity) in the same direction. Consequently, the transverse motion over distances less than d cannot be regarded as diffusion and it becomes diffusive only over distances greater than d. An allowance for the correlation in the case of a degenerate electron gas in an uncompensated semiconductor leads to the formula

$$\sigma_\perp=\alpha\left(\frac{e^3c\,\partial n/\partial\mu}{\varepsilon_0H}\right)^{4/3}N^{4/3}d^{4/3}\sigma_\parallel^{-1/3},\tag{2}$$

where $\alpha\sim 1$ is a numerical coefficient and N is the concentration of ionized impurities. Equation (2) is valid if $\tau_{\mathrm{in}}\ll\tau_{3,0}$ where $\tau_{3,0}$ is the time at which an electron is displaced a distance $\sim d$ across the field **H** at $T=0$. This time obeys the inequality $\tau_{3,0}\gg\tau_2$. The question how $\sigma_\parallel$ and $\sigma_\perp$ should behave at low temperatures when $\tau_{\mathrm{in}}>\tau_{3,0}$ has not yet been solved and will not be considered here.

On the right-hand side of Eq. (2) the only term that depends on temperature is $\sigma_\parallel$ and, therefore, if the localization effects reduce $\sigma_\parallel$ as a result of lowering of T, then they should increase $\sigma_\perp$, where $\sigma_\perp(T)\propto\sigma_\parallel^{-1/3}(T)$. Therefore, the dependences of $\sigma_\perp$ and $\sigma_\parallel$ on T are opposite to those deduced without allowance for the localization effects and for the scattering correlation. Thus, in checking the validity of the results of Ref. 2 it is best to study the temperature dependences of $\sigma_\parallel$ and $\sigma_\perp$. Although there have been many investigations of the magnetoresistance of semiconductors in the EQL case, results from which we can obtain simultaneously the dependences $\sigma_\parallel(T)$ and $\sigma_\perp(T)$ under the required conditions have been found only by us for just one sample of $Hg_{0.79}Cd_{0.21}Te$ (See Ref. 2). In the present paper a fuller check of the theoretical predictions will be made against measurements carried out on n-type InSb and n-type InAs.

We selected the electron densities n to be moderately high so that in magnetic fields available to us (up to 120 kOe) it would be possible to exceed considerably the field corresponding to the EQL. On the other hand, n should be sufficiently high to ensure a major difference between the

TABLE I

Sample	n, 10^{16} cm^{-3}	μ, 10^4 cm$^2\cdot$V$^{-1}\cdot$s^{-1}	H, kOe	E_F^0, K	τ_1, 10^{-13} s	$\hbar/\tau_1$, K	E_F, K
InSb-1	1.2	13.0	63	20	5.6	14	27
InSb-1	1.2	13.0	79	12	4.9	16	17
InSb-2	2.3	9.3	93	32	5.1	15	40
InAs-1	1.8	3.0	70	21	3.5	22	30
InAs-2	2.7	4.0	121	16	3	25	22

field necessary for transition to the EQL ($H_{\rm EQL} \propto n^{2/3}$) and a field H_b in which magnetic freezeout of electrons takes place ($H_b \ln H_b \propto n$). Apart from the logarithm, we have $H_b/H_{\rm EQL} \propto a_B n^{-1/3}$, where $a_B = \hbar^2\varepsilon_0/me^2$ is the Bohr radius.

SAMPLES AND MEASUREMENT METHOD

Our measurements were carried out on two samples of n-type InSb and two samples of n-type InAs. The densities of electrons and their mobilities are given in Table I. Samples about 1 cm long with transverse dimensions of about 0.1 cm were cut by spark machining and then etched in CP-4A. The measurements were carried out using the four-point method and an alternating current of 30 Hz frequency and about 1 mA amplitude. A magnetic field up to 121 kOe was created in a superconducting solenoid. The main measurements at temperatures from 1.5 to 30 K were carried out in a can with double walls placed inside liquid helium. A small amount of gaseous helium was injected into the space between the walls. Inside the can the temperature was increased by a heater. Moreover, measurements down to 0.35 K were made on one of the samples using a low-temperature attachment. This temperature was reached by pumping out He^3 vapor.

EXPERIMENTAL RESULTS

All the experimental results obtained for the four samples were basically similar. In the EQL case we found the field in which oscillations of the transverse magnetoresistance $\rho_\perp$ were finally suppressed. In the case of sample No. 1, this field was, for example, 30 kOe (Fig. 1). This feature was not manifested very strongly in the magnetoresistance $\rho_\parallel$ (Fig. 2). In the EQL case the value of $\rho_\parallel$ increased as a result of cooling. The value of $\rho_\perp$ in fields somewhat higher than the field of the transition to the EQL (35 kOe in Fig. 1) fell as a result of cooling. In stronger magnetic fields the dependence of $\rho_\perp$ on T was reversed, but it still remained weaker than the dependence of $\rho_\parallel$ on T. The Hall resistivity depended even less on temperature than did $\rho_\perp$ (Fig. 1).

A detailed determination of the dependences of $\rho_\perp$, $\rho_\parallel$, and ρ_H on T was made in magnetic fields which were at least twice as high as the field for the transition to the EQL. Then, these dependences were corrected to allow for the dependences of the components of the conductivity tensor on T:

$$\sigma_\parallel=\rho_\parallel^{-1},\quad \sigma_\perp=\rho_\perp/(\rho_\perp^2+\rho_H^2),\quad \sigma_H=\rho_H/(\rho_\perp^2+\rho_H^2).$$

Figure 3 shows the dependences of $\sigma_\parallel$ on T for three samples. In the range $T>5$ K the value of $\sigma_\parallel$ increased superlinearly with T ($\partial^2\sigma_\parallel/\partial T^2 \geqslant 0$). The low-temperature part of the curve 1 appeared quite different and was characterized by $\partial^2\sigma_\parallel/\partial T^2<0$. The Hall conductivity σ_H was practically independent of temperature and equal to its classical value $\sigma_H = nec/H$, where n was determined from the Hall coefficient in weak magnetic fields and from the period of the Shubnikov–de Haas oscillations.

The transverse conductivity $\sigma_\perp$ fell on increase in T in the range $T>3$ K (Fig. 4). We plotted also in Fig. 4 the temperature dependences $\beta\sigma_\parallel^{-1/3}$. The constants β were selected so that at $T=5$ K we obtained $\beta\sigma_\parallel^{-1/3}=\sigma_\perp$. Clearly, there were temperature intervals where $\sigma_\perp(T)\propto\sigma_\parallel^{-1/3}(T)$.

DISCUSSION

1. Our measurements were made in magnetic fields less than the freezeout field, which was indicated by the fact that $\sigma_H = nec/H$ and $\sigma_\perp<\sigma_H<\sigma_\parallel$. The results were in qualitative agreement with the predictions of Ref. 2, namely that $\sigma_\parallel$ fell and $\sigma_\perp$ increased as a result of lowering of T. There was a temperature interval where $\sigma_\perp \propto \sigma_\parallel^{-1/3}$.

2. Deviations from this relationship at high temperatures were clearly due to violation of the condition $kT \ll E_F$. In the EQL case the Fermi energy E_F decreased on increase

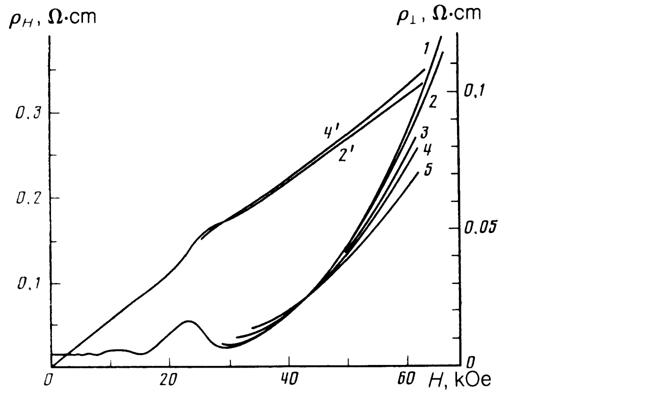

FIG. 1. Dependences of the resistivity $\rho_\perp$ of sample InSb-1 across the magnetic field (curves 1–5) and of the Hall resistivity σ_H (curves 2′ and 4′) on the magnetic field H applied at different temperatures T (K): 1) 1.5; 2), 2′) 4.4; 3) 7.5; 4), 4′) 10; 5) 15.

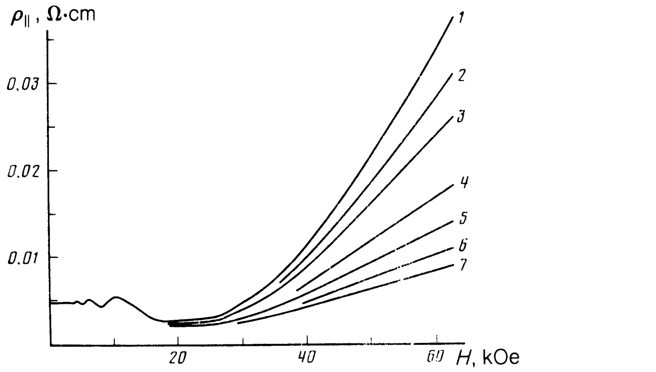

FIG. 2. Dependences of the resistivity $\rho_\parallel$ of sample InSb-1 along a magnetic field on the field H applied at different temperatures T (K): 1) 1.5; 2) 3; 3) 4.4; 4) 7.5; 5) 10; 6) 12.5; 7) 15.

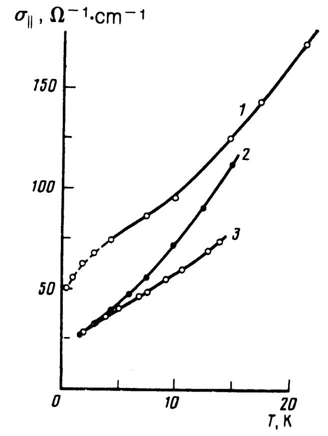

FIG. 3. Dependences of the longitudinal conductivity $\sigma_{\parallel}$ on the temperature T of a sample: 1) InSb-2, $H = 93$ kOe; 2) InSb-1, $H = 63$ kOe; 3) InAs-2, $H = 121$ kOe.

in the magnetic field. If $E_F \gg \hbar/\tau_1$, then

$$E_F = 2\pi^4 n^2 \lambda^4 \hbar^2 / m \infty H^{-2}.$$

The values of the Fermi energy E_F^0 calculated from the above expression are listed in Table I. This table includes also the values of τ_1 and $\hbar/\tau_1$ found from the following expression taken from Ref. 4:

$$\frac{\hbar}{\tau_1} = \pi \frac{\partial n}{\partial \mu} \frac{\hbar\omega}{4E_F + E_s} \frac{\lambda}{4}, \tag{3}$$

where ω is a cyclotron frequency and $E_s = \hbar^2 \varkappa_0^2 / 2m$. We can see from Table I that $\hbar/\tau_1$ is comparable with E_F^0 and, therefore, in the determination of E_F we must allow for the fact that the energy dependence of the density of states changes as a result of collisional broadening of the energy levels.[5] A rough calculation on the assumption that the broadening was Lorentzian and that $\Gamma \approx \hbar/\tau_1(E)|_{E=\Gamma}$ (Ref. 5) gave values of E_F which were larger than E_F^0 (Table I).

The ratio kT/E_F reached approximately 0.5 in our measurements. Therefore, the corrections to the conductivity $\sim (kT/E_F)^2$, associated with the thermal broadening of the Fermi level could result in deviations from the dependence $\sigma_\perp \propto \sigma^{-1/2}$. However, these corrections could not account by themselves for the dependences of $\sigma_\parallel$ and $\sigma_\perp$ on T, because, firstly, the dependence of $\sigma_\parallel$ on T was too strong and, secondly, the dependences of both $\sigma_\parallel$ and $\sigma_\perp$ on T were not described by an expression of the $\sigma = A + bT^2$ type.

3. In an analysis of the temperature dependences of the longitudinal conductivity $\sigma_\parallel$ [see Eq. (1)] we shall bear in mind that in the EQL case not only τ_{in} but also the elastic relaxation time τ_1 for the scattering by impurities could depend on temperature. This was because Eq. (3) was derived on the assumption that the permittivity of the electron gas is

$$\varepsilon_e(q) = 1 + \varkappa_0^2/q^2.$$

In reality, in the EQL case for the wave numbers $q \ll \lambda^{-1}$ (Ref. 6), we find that

$$\varepsilon_e(q) = 1 + \frac{\varkappa_0^2 k_F}{q^2 q_z} \ln \left| \frac{q_z + 2k_F}{q_z - 2k_F} \right| \tag{4}$$

diverges in the limit $q_z \to 2k_F$ ($k_F\hbar$ is the Fermi momentum along the magnetic field). This divergence is removed if we allow for the collisional broadening or finite temperatures. If $kT > \hbar/\tau_1$, then $\varepsilon_e(2k_F)$ depends on temperature[7]

$$\varepsilon_e(2k_F) = 1 + \frac{\varkappa_0^2}{8k_F^2} \ln \frac{2eE_F}{kT}. \tag{5}$$

Here, e is the base of natural logarithms. The dependence of ε_e on T leads to a dependence of the matrix element of the elastic scattering on T and, consequently, on τ_1. In contrast to the localization effects, the dependence of τ_1 on T should increase $\sigma_\parallel$ and, consequently, reduce $\sigma_\perp$ as a result of cooling. It follows from the experimental data that, as expected, the dependence of τ_1 on T has less effect on the temperature dependences of the conductivity than do the localization effects. Since $\hbar/\tau_1$ is comparable with E_F, the logarithm in the expression for $\varepsilon_e(q)$ at $q_z = 2k_F$ is of the order of 1 and it does not affect greatly the value of τ_1.

4. Knowing τ_1 (Table I) and the experimental value of $\sigma_\parallel$, we can use Eq. (1) to estimate τ_{in}. It is found that at $T = 10$ K the value of τ_{in} for all the samples was only several times less than τ_1 ($\tau_1 \sim 10^{-12}$–10^{-13} s). The frequency of electron-phonon collisions calculated using expressions from Ref. 4, allowing for the interaction of via the deformation potential and because of the piezoelectric mechanism, was $\sim 10^9$–10^{10} s^{-1}, i.e., it was two orders of magnitude less than the experimental estimate of τ_{in}. Clearly, the hopping conduction process should occur because of the electron-electron interaction.[8] In the EQL case the electron-electron pair collisions could not give rise to transitions between quasilocalized states, since they did not redistribute the energy between the colliding electrons. However, as demonstrated in Ref. 9, in the case of a Boltzmann gas a fairly rapid redistribution of energy between electrons can occur because of triple collisions between electrons or between two electrons and an impurity. The characteristic energy mixing time is then

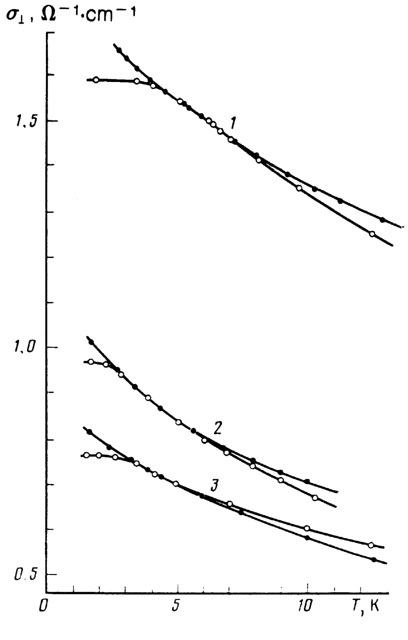

FIG. 4. Dependences of the transverse conductivity $\sigma_\perp$ (O) and $\beta\sigma_\parallel^{-1/3}$ (●) on the temperature of a sample. The coefficients β are different for different curves. Curve 1 represents sample InAs-2 in a field $H = 121$ kOe; curve 2 represents InSb-1 in $H = 79$ kOe; curve 3 represents InSb-1 in $H = 63$ kOe.

$$\tilde{\tau} \propto \tau_1 (\varepsilon_0 kT/n^{1/3}e^2)^{3/2}.$$

Bearing in mind that under our conditions we have $\varepsilon_0 E_F/n^{1/3}e^2 \sim 1$ we can expect that at $kT \sim E_F$ we can assume that $\tau_{\rm in} \sim \tilde{\tau} \sim \tau_1$ and cooling ensures that $\tau_1 < \tau_{\rm in} < \tau_{3,0}$, as predicted.

5. Deviations from the relationship

$$\sigma_\perp(T) \propto \sigma_\parallel^{-1/4}(T)$$

and the change of sign of $\partial^2\sigma_\parallel/\partial T^2$ at low temperatures is due to violation of the inequality $\tau_{\rm in} \ll \tau_{3,0}$. Under our conditions, when $k_F \sim \varkappa_0$, we find that

$$\tau_{3,0} \sim \tau_1 (k_F\lambda)^{-2} \sim 10\tau_1 \sim \tau_{in}(T)$$

at a temperature $T = 2$ K, at which the conductivity $\sigma_\perp$ ceases to rise as a result of cooling.

6. We can use Eq. (2) to estimate the transverse size d of impurities. We found that in all our samples the value of d was of the same order as the Debye screening radius: $\varkappa_0^{-1} \sim 10^{-6}$–$10^{-5}$ cm. Under our conditions we found that $k_F \sim \varkappa_0$ and, therefore, we would expect to ensure that $d \propto \varkappa_0^{-1}$.

It follows from the above that the experimental results agree basically with the predictions of Ref. 2. A more detailed comparison is difficult because the inequality $(k_F^2 + \frac{1}{4}\varkappa_0^2)\lambda^2 \ll 1$ proposed in Ref. 2 is not satisfied sufficiently well and because we do not know how $\tau_{\rm in}$ should depend on the magnetic field and temperature.

In conclusion we note once again that in the EQL case we do not know how $\sigma_\parallel$, $\sigma_\perp$, and σ_H behave in the limit $T \to 0$ when we have $\tau_{\rm in}(T) > \tau_{3,0}$ (see Refs. 1 and 10–12). In particular, we cannot exclude the possibility that $\sigma_\parallel, \sigma_\perp \to 0$ and $\sigma_H \to$ const (Ref. 12).

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[1)]It is not clear whether the localization effects are allowed for fully in Ref. 1, but nevertheless the results given there suggest that these effects have a strong influence on the conductivity.

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