Three-pulse hydrodynamic echo in an inhomogeneous plasma–vacuum transition layer

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We investigate temporal echoes of hydrodynamic type, produced in a plasma–vacuum transition layer by applying three successive pulsed perturbations. We show that it is possible to deduce the effective electron-collision frequency from the dependence of the amplitude of a three-pulse echo signal on the third-pulse delay time.

1. The damping of a surface wave in plasma with a diffuse boundary has a resonant collisionless character due to the wave coupling with the local Langmuir oscillations in the inhomogeneous transition layer.[1,2] It is shown in Ref. 3 that the surface-wave energy is transferred in the course of time to Langmuir oscillations that are localized near the plasma-resonance point in a region much narrower than the transition-layer width. These oscillations are not damped in a cold collisionless plasma, so that an echo of hydrodynamic nature can be excited in a strongly inhomogeneous transition layer. This echo, produced by two external-perturbation pulses and corresponding to allowance for second-order nonlinearities in the equation of motion of the electron fluid, was investigated in Refs. 5 and 6. At the same time, higher-order effects are possible in this system, such as three-pulse echoes produced in a homogeneous plasma[6,7] in response to three successive pulse perturbations. Plasma wave echoes of higher order, excited by two external perturbations, were the subject of a number of studies, which are reviewed in Ref. 8.

We consider here a three-pulse hydrodynamic echo in a strongly inhomogeneous plasma–vacuum transition layer. The perturbations chosen are pulses of surface waves traveling along the boundary. An interesting situation arises when the delay time T of the third pulse is much longer than the time interval τ between the first two. The echo signal is produced at a time τ after the third pulse, and its amplitude depends substantially on the electron-collision frequency ν if $\nu T \sim 1$. This circumstance can be used to determine ν from the dependence of the three-pulse echo amplitude on the time delay T.

2. We consider a plasma occupying the space $x>0$, uniform along y and z, and with vacuum ($x<0$) as its boundary. Let the equilibrium plasma density $n_0(x)$ increase monotonically in a transition region $0<x<a$ from zero to a value $n_0(a)$ and remain constant in the region $z>a$. We confine ourselves to high-frequency electron oscillations for which the plasma ions can be regarded as an immobile neutralizing background, and assume that the plasma is cold. The electron fluid is then described by the hydrodynamic continuity and momentum equations

$$\frac{\partial n}{\partial t}+\operatorname{div}(n\mathbf{v})=0,\qquad \frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}=-\frac{e}{m}\mathbf{E}-\nu\mathbf{v} \tag{1}$$

together with the Maxwell equations for the electromagnetic field

$$\operatorname{rot}\mathbf{B}=\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}-\frac{4\pi e}{c}n\mathbf{v},\qquad \operatorname{rot}\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},$$
$$\operatorname{div}\mathbf{B}=0,\qquad \operatorname{div}\mathbf{E}=-4\pi e[n-n_0(x)]. \tag{2}$$

Here n and $\mathbf{v}$ are the density and velocity of the electron fluid, e and m the electron charge and mass, c the speed of light, and ν the effective electron-momentum transfer frequency; in other words, the term $-\nu\mathbf{v}$ in the right-hand side of the equation of motion describes the friction force.

We solve the set of Eqs. (1) and (2) perturbatively, representing the variables in the form

$$A=A_0+A^{(1)}+A^{(2)}\ldots,$$

where A_0 is the equilibrium value of A, and $A^{(i)}$ is the ith-order response to the external perturbations. For a two-dimensional problem in which all the system perturbations are independent, say, of the coordinate z, Eqs. (1) and (2) break up into two independent systems for TE modes (with electromagnetic-field components E_z, B_x, B_y) and TM modes (with components E_x, E_y, B_z). We confine ourselves below to TM modes, which admit of a solution in the form of surface waves.

The remainder of the solution procedure for the first and second approximation is described in detail in Ref. 5. We use Laplace transforms in time and Fourier transforms in the coordinate y:

$$A(p,k)=\int_0^\infty dt\,e^{-pt}\int_{-\infty}^{\infty}dy\,e^{-iky}A(t,y).$$

The dispersion function $D(p,k)$, which determines the surface oscillations on the plasma–vacuum boundary with a strongly inhomogeneous transition layer ($|\partial n_0/\partial_x|\gg|kn_0|$), we have

$$D(p,k)=\varepsilon(a,p)+\frac{\varkappa}{\varkappa_0}+\varkappa\int_0^a dx\,\varepsilon(x,p)+\frac{p^2}{c^2}\frac{\varepsilon(a,p)}{\varkappa_0}\int_0^a dx\frac{N+\varepsilon(x,p)}{\varepsilon(x,p)}, \tag{3}$$

where

$$N=(kc/p)^2,\quad \varepsilon(x,p)=1+\omega_{Le}^2(x)/p(p+\nu),$$

$$\omega_{Le}^2(x)=4\pi e^2 n_0(x)/m,$$

$$\varkappa_0^2=k^2+p^2/c^2,\quad \varkappa^2=k^2+p^2\varepsilon(a,p)/c^2,$$

$$\operatorname{Re}\varkappa,\ \operatorname{Re}\varkappa_0>0.$$

Putting $p=-i\omega_0-\gamma$ in (3) and equating its real and imaginary parts to zero, we obtain the following expressions for the frequency ω_0 and for the damping rate γ of the surface wave:

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$$\omega_0^2=k^2c^2+{}^1/_2\omega_{Le}^2(a)-[k^4c^4+{}^1/_4\omega_{Le}^4(a)]^{1/2},$$

$$\gamma=p_k+{}^1/_2\nu\beta. \tag{4}$$

Here p_k is the collisionless damping rate of surface oscillations with wave number k:

$$p_k=\pi hk^2c\alpha^2(\alpha-1)^{1/2}/(2\alpha-1)(2\alpha^2-2\alpha+1),$$

$$\alpha=\left(\frac{kc}{\omega_0}\right)^2,\quad \beta=\frac{(2\alpha-1)(\alpha-1)}{2\alpha^2-2\alpha+1},$$

$$h^{-1}=\frac{d}{dx}\frac{\omega_{Le}^2(x)}{\omega_0^2}\bigg|_{\omega_{Le}(x)=\omega_0}\sim a^{-1}. \tag{5}$$

As shown in Ref. 3, the oscillations of the linear velocity component $v_x^{(1)}$ and the associated electric-field component $E_x^{(1)}$ are not damped. It is convenient to represent the above velocity component in the form

$$v_{xkp}^{(1)}(x)=-i\frac{e}{m}\frac{E_x^{\text{ext}}(k,p)}{(p+\nu)\varepsilon(x,p)D(p,k)}, \tag{6}$$

where E_x^{ext} is the electric field of the external perturbations. We choose them in the form of three successive pulses applied to the plasma–vacuum boundary at times $t=0$, $t=\tau$, and $t=T$:

$$E_x^{\text{ext}}(y,t)=E_1\cos(k_1,y)\delta(\Omega t)+E_2\cos(k_2y)\delta[\Omega(t-\tau)]$$
$$+E_3\cos(k_3y)\delta[\Omega(t-T)]. \tag{7}$$

Here Ω is a constant with the dimension of frequency. The most favorable situation for observation of the two-pulse echo that results from the action of the first two pulses (7) is

$$p_k^{-1}\ll\tau\ll\nu^{-1}. \tag{8}$$

This condition means that the macroscopic signal of the first pulse vanishes by collisionless damping even before the second pulse is applied, but its memory remains in the form of micro-oscillations, having the local plasma frequency, of the components $E_x^{(1)}$ and $v_x^{(0)}$. On the other hand, the friction force that causes loss of the phase memory of the external perturbation, hardly comes into play over times of order τ.

In second-order perturbation theory it is easy to find the velocity component $v_x^{(2)}$ for the expression for the three-pulse echo:

$$v_{xkp}^{(2)}=\frac{k^2\varepsilon(a,p)}{2(p+\nu)D(p,k)}\int_0^a\frac{dx}{\varepsilon^2(x,p)}\frac{\partial\varepsilon(x,p)}{\partial x}\int\frac{dk'}{2\pi}$$
$$\cdot\int\frac{dp'}{2\pi i}v_{xk-k',p-p'}^{(1)}(x)v_{xk'p'}^{(1)}(x), \tag{9}$$

and the nonlinear surface charge of the two-pulse echo signal:

$$\sigma_\pm^{(2)}(y,t)\approx\frac{eE_1E_2(t-\tau)}{24m\Omega^2}$$
$$\cdot\frac{(k_1\pm k_2)^2F(k_1)F(k_2)F(k_1\pm k_2)[\omega_{Le}^2(a)-\omega_j^2]}{\omega_j^5[4\omega_0^2(k_2)-\omega_j^2](\omega_1+\omega_3)(\omega_1-\omega_3+ip_1+ip_3)}$$
$$\cdot\left[a\omega_j^2-\int_0^a\omega_{Le}^2(x)dx\right]\exp[i(k_1\pm k_2)y$$
$$-(i\omega_j+p_j)(t-2\tau)-\nu\tau]+\text{c.c.} \tag{10}$$

Here

$$\omega_j=\omega_1\equiv\omega_0(k_1),\quad p_j=-p_1\equiv-p(k_1)\quad\text{if}\quad t<2\tau,$$
$$\omega_j=\omega_3\equiv\omega_0(k_1\pm k_2),\quad p_j=p_3\equiv p(k_1\pm k_3)\quad\text{if}\quad t>2\tau,$$
$$F(k_i)=\alpha(\alpha-1)(2\alpha-1)^{-1}(2\alpha^2-2\alpha+1)^{-1}\omega_0^2(k)|_{k=k_i}.$$

The surface charge (10) is a superposition of two signals with sum k_1+k_2 and difference $|k_1-k_2|$ wave numbers.

3. We consider now the response of the plasma to the third external-perturbation pulse applied to the plasma at the instant $t=T$, assuming that $T\gg\tau$. In third-order perturbation theory, after taking the Fourier–Laplace transform, the system of Eqs. (1) and (2) becomes

$$\frac{\partial}{\partial x}E_{xkp}^{(3)}+ikE_{ykp}^{(3)}=-4\pi en_{kp}^{(3)},$$

$$\frac{\partial}{\partial x}E_{ykp}^{(3)}-ikE_{xkp}^{(3)}=-\frac{p}{c}B_{zkp}^{(3)},$$

$$ikB_{zkp}^{(3)}=\frac{p}{c}E_{xkp}^{(3)}-\frac{4\pi en_0}{c}v_{xkp}^{(3)}$$
$$-\frac{4\pi e}{c}\int\frac{dk'}{2\pi}\int\frac{dp'}{2\pi i}(n_{k-k',p-p'}^{(1)}v_{xk'p'}^{(2)}+n_{k-k',p-p'}^{(2)}v_{xk'p'}^{(1)}), \tag{11}$$

where

$$v_{xkp}^{(3)}\approx-\frac{e}{m(p+\nu)}E_{xkp}^{(3)}-\frac{1}{2(p+\nu)}\frac{\partial}{\partial x}I_{kp}(x),$$
$$4\pi en_{kp}^{(3)}\approx\frac{\partial}{\partial x}\left[\frac{\omega_{Le}^2}{p(p+\nu)}E_{xkp}^{(3)}\right]+ik\frac{\omega_{Le}^2}{p(p+\nu)}E_{ykp}^{(3)}$$
$$+\frac{2\pi e}{p(p+\nu)}\frac{\partial}{\partial x}\left[n_0\frac{\partial}{\partial x}I_{kp}(x)\right],$$
$$I_{kp}(x)=\int\frac{dk'}{2\pi}\int\frac{dp'}{2\pi i}[v_{xk-k',p-p'}^{(1)}(x)v_{xk'p'}^{(2)}(x)$$
$$+v_{xk-k',p-p'}^{(2)}(x)v_{xk'p'}^{(1)}(x)]. \tag{12}$$

We have retained in (12), from among the terms that describe the nonlinear interaction, only those containing products of the type $v_x^{(1)}v_x^{(2)}$, for it is just these terms which describe the hydrodynamic echo.

It is easy to obtain from (11) and (12) equations for the electromagnetic-field component:

$$\frac{\partial}{\partial x}\left(\frac{\varepsilon}{N+\varepsilon}\frac{\partial}{\partial x}E_{ykp}^{(3)}\right)-\frac{p^2}{c^2}\varepsilon E_{ykp}^{(3)}$$
$$=-ik\frac{2\pi e}{p(p+\nu)}\frac{\partial}{\partial x}\left[\frac{n_0}{N+\varepsilon}\frac{\partial}{\partial x}I_{kp}(x)\right],$$
$$E_{xkp}^{(3)}=-\frac{i}{k}\frac{N}{N+\varepsilon}\frac{\partial}{\partial x}E_{ykp}^{(3)}-\frac{2\pi en_0}{p(p+\nu)}\frac{1}{N+\varepsilon}\frac{\partial}{\partial x}I_{kp}(x),$$
$$B_{zkp}^{(3)}=-\frac{c}{p}\frac{\partial}{\partial x}E_{ykp}^{(3)}+i\frac{kc}{p}E_{xkp}^{(3)}. \tag{13}$$

We find the solutions of (13) in the regions $x<0$, $0<x<a$, and $x>a$, and then match them in the planes $x=0$ and $x=a$. Substituting the solutions obtained into the equation for the jump of the normal component of the electric induction across the plasma–vacuum transition layer, we obtain the following expression for the third-order perturbation of the surface-charge density:

$$\sigma_{kp}^{(3)}=-\frac{ek^2\varepsilon(a,p)}{2p^2(p+\nu)^2D(p,k)}\left[\int_0^a\omega_{Le}^2(x)\,dx+ap(p+\nu)\right]\cdot\int_0^a dx\frac{dn_0}{dx}\frac{1}{\varepsilon^2(x,p)}I_{kp}(x). \qquad (14)$$

Using in (14) expression (9) for $v_x^{(2)}$ and taking the inverse Fourier–Laplace transform, we retain only those terms which describe the successive action on the plasma by the three external perturbations, i.e., terms containing the product of the three linear perturbations of the velocity $v_x^{(1)}$:

$$\begin{aligned}\sigma^{(3)}(y,t)\approx{}&\frac{e}{4}\int_0^a dx\frac{dn_0}{dx}\int\frac{dk}{2\pi}k^2e^{iky}\cdot\int\frac{dp}{2\pi i}e^{pt}\left[\int_0^a\omega_{Le}^2(z)\,dz+ap(p+\nu)\right]\\&\cdot\frac{\varepsilon(a,p)}{p^2(p+\nu)^2\varepsilon^2(x,p)}\frac{1}{D(p,k)}\int\frac{dk'}{2\pi}\int\frac{dp'}{2\pi i}\\&\cdot\left\{\frac{v^{(1)}_{xk-k',p-p'}}{(p'+\nu)\varepsilon(x,p')}\frac{\partial}{\partial x}\int\frac{dk''}{2\pi}\int\frac{dp''}{2\pi i}\cdot v^{(1)}_{xk'-k'',p'-p''}\cdot v^{(1)}_{xk''p''}+\frac{v^{(1)}_{xk'p'}}{(p-p'+\nu)\varepsilon(x,p-p')}\frac{\partial}{\partial x}\int\frac{dk''}{2\pi}\right.\\&\left.\cdot\int\frac{dp''}{2\pi i}v^{(1)}_{xk-k'-k'',p-p'-p''}\cdot v^{(1)}_{xk''p''}\right\}. \qquad (15)\end{aligned}$$

We substitute in (15) the linear velocity perturbations in form (6) with account taken of the form (7) chosen for the external perturbations. We can then integrate with respect to k'', k', and k with the aid of δ functions. The integration with respect to p'', p', and p can be carried out by closing the corresponding integration contours in the left-hand half-plane; the main contributions are made in this case by the poles of the functions $\varepsilon(x,p)$. The echo after applying the third pulse and resulting from the consecutive action of all the external perturbation pulses on the inhomogeneous transition layer takes the form of a superposition of signals with combined wave numbers $k_e=rk_1+lk_2+fk_3$ (r, l, and f take on values ± 1). These signals are described by the expression

$$\begin{aligned}\sigma_e^{(3)}(y,t)\approx{}&\frac{e^4E_1E_2E_3}{(8m\Omega)^3}k_e^2(t-T)\tau\exp\left[ik_ey-\frac{\nu}{2}(t-T-\tau)-\nu T\right]\int_0^a dx\frac{dn_0}{dx}\frac{d\omega_{Le}}{dx}\\&\cdot[\omega_{Le}^2(a)-\omega_{Le}^2]\left[\int_0^a\omega_{Le}^2(z)\,dz-a\omega_{Le}^2\right]\exp[-i\omega_{Le}(t-T-\tau)]\\&\cdot\left[\omega_{Le}^6D\left(-i\omega_{Le}-\frac{\nu}{2},k_e\right)D\left(-i\omega_{Le}+\frac{\nu}{2},k_3\right)\right.\\&\cdot D\left(-i\omega_{Le}-\frac{\nu}{2},k_2\right)\\&\left.\cdot D\left(i\omega_{Le}-\frac{\nu}{2},k_1\right)\right]^{-1}+\text{c.c.}, \qquad (16)\end{aligned}$$

where $\omega_{Le}\equiv\omega_{Le}(x)$. Since the integrand in (16) contains the rapidly oscillating function $\exp[-i\omega_{Le}(t-T-\tau)]$, the integral vanishes for all instants of time except near $t=T+\tau$. At this instant, a macroscopic surface charge with a wave number k_e is produced in the plasma–vacuum transition layer.

To integrate with respect to x in (16), an actual form of the $n_0(x)$ dependence must be specified. We approximate it by the expression

$$\omega_{Le}(x)=\omega_{Le}(a)x/a.$$

Transforming next to the complex plane $z=\omega_{Le}(a)x/a$, we displace the integration contour to $\pm i\infty$, depending on the sign of $t-T-\tau$. It can be verified that the contributions to the integral from the corresponding straight-line segments $\operatorname{Re}z=0$ and $\operatorname{Re}z=\omega_{Le}(a)$ are small compared to the contribution from the poles of the dispersion function (3). The expression for $\sigma^{(3)}$ is rather unwieldy, and we present here only its asymptotic value in the limits of short- and long-wave oscillations.

The transition to the short-wave limit $(k_ic)^2\gg\omega_{Le}^2(a)$ $(i=1,2,3,e)$ can be effected by taking formally the limit $c\to\infty$. The dispersion equation $D(-i\omega,k)=0$ for the surface waves describes then quasistatic oscillations, and Eq. (16) takes the form

$$\sigma_e^{(3)}(y,t)\approx\frac{8e^2E_1E_2E_3}{3m^2\Omega^3}\frac{k_e^2(t-T)\tau}{\pi^3a^3}\frac{\omega_{Le}(a)}{2^{1/2}}\cos\left[k_ey-\frac{\omega_{Le}(a)}{2^{1/2}}(t-T-\tau)\right]e^{-\nu T}$$
$$\times\begin{cases}\dfrac{\exp[p_1(t-T-\tau)]}{(k_1+k_e)(k_1+k_2)(k_1+k_3)}, & t<T+\tau,\\[2ex] \dfrac{\exp[-p_e(t-T-\tau)]}{(k_e-k_3)(k_e-k_2)(k_1+k_e)}+\dfrac{\exp[-p_3(t-T-\tau)]}{(k_e-k_3)(k_2-k_3)(k_1+k_3)} \\ \qquad+\dfrac{\exp[-p_2(t-T-\tau)]}{(k_e-k_2)(k_2-k_3)(k_1+k_2)}, & t>T+\tau.\end{cases} \qquad (17)$$

In the long-wave limit $(k_ic)^2\ll\omega_{Le}^2(a)$ $(i=1,2,3,e)$ the dispersion equation of the surface oscillations describes quasi-transverse waves; for the third-order echo signal we obtain the expression

$$\sigma_e^{(3)}(y,t) \approx \frac{e^2 E_1 E_2 E_3}{96 m^2 \Omega^3} \frac{(t-T)\tau}{\omega_{Le}^{11}(a)} k_1^6 k_2^6 k_3^6 k_e^8 c^{12} e^{-\nu T} \begin{cases} \dfrac{\sin[k_e y - k_1 c(t-T-\tau)] \exp[p_1(t-T-\tau)]}{k_1^6 (k_e^2 - k_1^2)(k_3^2 - k_1^2)(k_2^2 - k_1^2)}, & t < T + \tau, \\ \sum\limits_{i=2,3,e} \dfrac{\sin[k_e y - k_i c(t-T-\tau)] \exp[-p_i(t-T-\tau)]}{k_i^6 (k_i^2 - k_1^2) \prod\limits_{j=2,3,e}' (k_i^2 - k_j^2)}, & t > T + \tau. \end{cases} \tag{18}$$

The prime on the product symbol means that the factor with $j = i$ must be omitted, i.e., the product consists of two factors.

We have presented above expressions for two- and three-pulse echo signals in surface-charge-density form. The corresponding expressions for the current produced by the echo response can be easily obtained with the aid of the continuity equation for the surface oscillations.

4. The hydrodynamic echo in a strongly inhomogeneous plasma–vacuum transition layer has a number of features in common with cyclotron echo in a magnetized plasma.[9,11] In fact, a plasma receiving three successive pulses of frequency ω_{ce} at times $t = 0$, τ, and T emits pulses at the same frequency at the instants $(n+1)\tau$ and $T + n\tau$, where $T > 2\tau$ and $n = 1, 2, \ldots$. These pulses were named two-and three-pulse cyclotron echo, respectively. The hydrodynamic echo also takes the form of a sequence of equally spaced pulses, if account is taken of the possibility of generating responses of higher orders, excited by the two- and three-pulse echo signals considered in the present article. For example, the second external-perturbation pulse ($t = \tau$) and the two-pulse echo signal ($t = 2\tau$) generate a third-order echo at the instant $t = 3\tau$.

This analogy between the cyclotron- and hydrodynamic-echo patterns is due to the qualitative similarity of their mechanism. Cyclotron echo can be produced if the equations describing the cyclotron motion of the electrons contain nonlinearities. Such a nonlinearity may be due to the dependence of the collision frequency of the particles on their velocities, on the non-uniformity of the magnetic field, etc.[12] In our hydrodynamic-echo case, Eqs. (1) and (2) describing the plasma are intrinsically nonlinear.

The most interesting feature of the three-pulse echo is its dependence on the electron-fluid friction force, in other words, on the effective frequency ν of the electron-momentum transfer. If the delay time T of the third pulse is such that $\nu T \sim 1$, the three-pulse echo is substantially weakened by electron friction. It can be seen from Eqs. (17) and (18) that by specifying the ratio of the amplitudes $\sigma_1^{(3)}$ and $\sigma_2^{(3)}$ for two delay time T_1 and T_2, respectively, one can calculate the effective electron momentum-transfer frequency:

$$\nu = (T_2 - T_1)^{-1} \ln(\sigma_1^{(3)}/\sigma_2^{(3)}). \tag{19}$$

From the experimental standpoint, the most convenient situation is one in which the macroscopic external peturbation is attenuated, at the plasma-resonance point with decrement p_k [see Eq. (5)], within a time shorter than the interval between the pulses, i.e., $p_k \tau \gg 1$. On the other hand, bulk plasmons that preserve in the transition layer information on the extraneous perturbation, should not manage to attenuate within the delay time of the third pulse. Such an attenuation can be due either to electron friction or to thermal outflow of plasmons from the plasma-resonance region.[2] The condition for the validity of Eq. (19) is therefore

$$T_{1,2} \lesssim \frac{1}{\nu} \ll \frac{1}{\omega_{Le}} \left(\frac{a}{r_{De}} \right)^{2/3} \Big|_{x=x_0},$$

where r_{De} is the electron Debye radius and x_0 is the coordinate of the plasma-resonance point at which the surface-wave frequency is equal to the local Langmuir frequency.

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