Steady-state spin current in ^{3}He-*B*

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Solutions of the equations of the low-frequency spin dynamics of the superfluid *B* phase of ^{3}He are analyzed which describe a steady-state flow of a spin current superposed on the spin precession in a magnetic field. The maximum spin current and the critical phase gradient at which it decays are found. A steady-state solution describing a spin vortex is also analyzed.

1. GENERAL RELATIONS

Equations describing the low-frequency dynamics of a spin superposed on its precession in a magnetic field H_0 were constructed for the superfluid *B* phase of ^{3}He in Ref. 1. This is a Hamiltonian system for the two conjugate variables ψ and $u = \cos\beta$:

$$\omega_p\frac{\partial\psi}{\partial t}=\frac{\partial V}{\partial u}-\frac{\partial}{\partial x_k}\left(\frac{\partial V}{\partial u_{,k}}\right),\qquad \omega_p\frac{\partial u}{\partial t}=\frac{\partial}{\partial x_k}\left(\frac{\partial V}{\partial \psi_{,k}}\right). \tag{1}$$

The angles ψ and β determine the orientation of the spin **S** with respect to a coordinate system which is rotating around $\mathbf{H}_0$ at the precession frequency ω_p. The angle ψ is the longitude, and β is the latitude, for which the origin of the scale is placed at $-\mathbf{H}_0$. Under conditions such that Eqs. (1) are applicable, β varies over the range $0<\beta<\arccos(-1/4)$. The Hamiltonian V is

$$V=F_\nabla+\omega_p(\omega_p-\omega_L)u, \tag{2}$$

where ω_L is the Larmor frequency corresponding to the field H_0, and F_∇ is a "gradient energy" which depends on the spatial variables $\psi_{,k}$ and $u_{,k}$. The units have been chosen here in the same way as in Ref. 1; i.e., the gyromagnetic ratio for the ^{3}He nuclei and the magnetic susceptibility per unit volume of the ^{3}He-*B* are set equal to unity. The energy density then has the units frequency squared.

System (1) can be used to study various cases of spin flow. In the present paper we analyze steady-state solutions of this system which describe a one-dimensional spin flow and a spin vortex. These solutions are of interest in connection with recent experiments by Borovik-Romanov, Bun'kov, Dmitriev, and Mukharskiĭ on spin currents and critical velocities in ^{3}He-*B* (Ref. 2). In both of the cases under consideration here, the flow occurs in the plane perpendicular to the magnetic field direction, i.e., the gradient energy contains derivatives with respect to the coordinates x and y only. According to Eq. (32) of Ref. 1 we then have

$$F_\nabla=F_{\nabla\perp}={}^1/_2(\rho_{11}\psi_{,k}\psi_{,k}+2\rho_{12}\psi_{,k}u_{,k}+\rho_{22}u_{,k},\ u_{,k}) \tag{3}$$

where

$$\rho_{11}=(1-u)\left[(1-u)c_\parallel^2+(1+u)c_\perp^2\right],$$

$$\rho_{12}=\mp c_\parallel^2\frac{1-u}{1+u}\left(\frac{3}{1+4u}\right)^{1/2},$$

$$\rho_{22}=\frac{c_\perp^2}{1-u^2}+\frac{3c_\parallel^2}{(1+u)^2(1+4u)},$$

$\psi_{,k}=\partial\psi/\partial x_k$, etc. A summation is to be understood over the index k, which takes on the values 1, 2. The double sign arises in the definition of ρ_{12} as a result of the elimination from the expression for F_∇ of the third angle Φ, which is related to u by[1]

$$u+(1+u)\cos\Phi={}^1/_2. \tag{4}$$

This relation associates two values of Φ, differing in sign, with each value of u. The states corresponding to these values of Φ are physically different, as can be seen easily if we take the customary approach of specifying as an order parameter the rotation matrix R_{ik}, using the direction of the rotation axis, **n**, and the angle $\theta(0\leqslant\theta\leqslant\pi)$ through which the rotation is made. Relation (4) expresses the fact that for the solutions under consideration here we have $\theta=\theta_0=\arccos(-1/4)\approx104°$. In this case, there is a one-to-one correspondence between **n** and R_{ik}. The Cartesian components of **n** in a coordinate system which is rotating at an angular frequency ω_p are expressed in the following way in terms of ψ, β, and Φ:

$$n_x=-\frac{\sin\beta}{\sin\theta_0}\cos\frac{\Phi}{2}\sin\left(\psi-\frac{\Phi}{2}\right),$$

$$n_y=\frac{\sin\beta}{\sin\theta_0}\cos\frac{\Phi}{2}\cos\left(\psi-\frac{\Phi}{2}\right),$$

$$n_z=\frac{\sin\Phi}{2\sin\theta_0}(1+\cos\beta). \tag{5}$$

We see that when the sign of Φ changes the vector **n** changes direction.

Expression (3) for the gradient energy can be simplified by replacing Φ by the new variable w in accordance with the definition

$$\frac{\partial w}{\partial x_k}=\frac{\partial\psi}{\partial x_k}+\frac{\rho_{12}}{\rho_{11}}\frac{\partial u}{\partial x_k}. \tag{6}$$

In terms of the variables w, u, the system (1) remains of Hamiltonian form:

$$\omega_p\frac{\partial w}{\partial t}=\frac{\partial V}{\partial u}-\frac{\partial}{\partial x_k}\left(\frac{\partial V}{\partial u_{,k}}\right),\qquad \omega_p\frac{\partial u}{\partial t}=\frac{\partial}{\partial x_k}\left(\frac{\partial V}{\partial w_{,k}}\right). \tag{7}$$

For the transformed Hamiltonian we have

$$V={}^1/_2(\rho w_{,k}w_{,k}+\mu u_{,k}u_{,k})+\omega_p(\omega_p-\omega_L)u,$$

where

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$$\rho=\rho_{11},\quad \mu=\frac{\rho_{11}\rho_{22}-\rho_{12}^2}{\rho_{11}}$$

$$=\frac{c_\perp^2}{(1-u)(^1/_4+u)}\frac{(1-u)c_\parallel^2+(^1/_4+u)c_\perp^2}{(1-u)c_\parallel^2+(1+u)c_\perp^2}.$$

The flux **j** of the quantity $\omega_p(1-u)$ is now given by

$$j_k=\frac{\partial V}{\partial w_{,k}}=\rho\frac{\partial w}{\partial x_k};\qquad (9)$$

i.e., the variable w has the meaning of a two-dimensional velocity potential. To proceed, it is useful to also introduce an analog of the momentum-flux tensor:

$$\Pi_{ik}=w_{,i}\frac{\partial V}{\partial w_{,k}}+u_{,i}\frac{\partial V}{\partial u_{,k}}-V\delta_{ik}=\rho w_{,i}w_{,k}+\mu u_{,i}u_{,k}-V\delta_{ik}.\quad (10)$$

In the steady-state case,this quantity satisfies the continuity equation $\partial\Pi_{ik}/\partial x_k=0$.

Steady-state solutions of system (7) are extrema of the functional $\mathscr{F}=\int Vd^2r$, so an analysis of steady-state spin currents becomes similar to an analysis of superconducting currents in the Ginzburg-Landau theory.[3] The variable w is analogous to a phase, and the quantity $1-u$ to a square of the magnitude of the order parameter in superconductors. Comparing the first and second terms in Hamiltonian (8), we can determine the length scale $\xi=c_\perp[\omega_p(\omega_p-\omega_L)]^{-1/2}$: an analog of the correlation length in the Ginsburg-Landau theory. Dissipative terms have been ignored in Eqs. (7). The applicability of this approximation can conveniently be discussed in connection with specific applications.

2. ONE-DIMENSIONAL SPIN FLOW

We consider a steady-state spin flow along a channel of constant cross section which is long (has a length $L\gg\xi$) and narrow (has a transverse dimension $a\ll\xi$). The channel is oriented perpendicular to the magnetic field H_0. Under the limitations imposed here, the flow is one-dimensional; i.e., w and u vary in only the direction along the channel (the y axis). Setting $\partial w/\partial t=0$ and $\partial u/\partial t=0$, we obtain two equations from (7) which describe a flow of this sort. These equations are Euler-Lagrange equations for the functional $\int Vdy$. To analyze the solutions, it is convenient to rewrite these equations as Hamilton's equations with respect to the coordinate y. We introduce the currents $j=\partial V/\partial w'$ and $q=\partial V/\partial u'$ which are conjugates of the variables w and u, respectively; the prime means differentiation with respect to y. The role of the Hamiltonian is played by the component $\Pi_{yy}=\Pi$ of the momentum flux tensor (10):

$$\Pi=\frac{j^2}{2\rho}+\frac{q^2}{2\mu}-\omega_p(\omega_p-\omega_L)u.\qquad (11)$$

Equations describing a steady-state one-dimensional spin current can then be written in the form

$$\frac{du}{dy}=\frac{q}{\mu},\quad \frac{dq}{dy}=-\frac{q^2}{2}\frac{d}{du}\left(\frac{1}{\mu}\right)-\frac{j^2}{2}\frac{d}{du}\left(\frac{1}{\rho}\right)+\omega_p(\omega_p-\omega_L),\qquad (12)$$

$$\frac{dw}{dy}=\frac{j}{\rho},\quad \frac{dj}{dy}=0.$$

The variable w is cyclic, so its conjugate current j is conserved, and there exist solutions of the type $du/dy=0$, $dq/dy=0$, $dw/dy=h=\text{const}$. These solutions are extrema of the potential $\Pi=\Pi-hj$ with respect to the explicit variables u, q, j. These are the solutions of primary interest in connection with the problem under consideration here. To construct these solutions we should substitute $du/dy=0$, $dq/dy=0$, and $dw/dy=h$ into the left sides of Eqs. (12) and solve the resulting algebraic system for u, q, and j. From the first equation we then find

$$q\left(\frac{1}{4}+u\right)(1-u)\frac{(1-u)c_\parallel^2+(1+u)c_\perp^2}{(1-u)c_\parallel^2+(^1/_4+u)c_\perp^2}=0.\qquad (13)$$

This equation is satisfied in three cases: 1) $q=0$, 2) $u=-1/4$, 3) $u=1$. In all cases we have $j=\rho(u)h$, where u is found from the second equation of system (12):

$$\omega_p(\omega_p-\omega_L)=h^2[(1-u)c_\parallel^2+uc_\perp^2]+\frac{q^2}{2}\frac{d}{du}\left(\frac{1}{\mu}\right).\qquad (14)$$

We now consider each of these three cases in succession.

In case 1), Eq. (14) leads to the following dependence of u on h:

$$u=\frac{c_\perp^2}{c_\parallel^2-c_\perp^2}\left[\frac{c_\parallel^2}{c_\perp^2}-\frac{1}{(h\xi)^2}\right].$$

The values of u lie in the allowed interval $-1/4<u<1$ if h lies in the interval $h_{c1}<h<h_{c2}$, where

$$h_{c1}^2=\frac{4c_\perp^2}{5c_\parallel^2-c_\perp^2}\frac{1}{\xi^2},\quad h_{c2}^2=\frac{1}{\xi^2}.$$

The h dependence of the current becomes

$$j=\frac{c_\perp^4}{c_\parallel^2-c_\perp^2}\left[\frac{1}{(h\xi)^4}-1\right]h.\qquad (15)$$

In case 2), Eq. (14) is rewritten as

$$4\omega_p(\omega_p-\omega_L)=(5c_\parallel^2-c_\perp^2)h^2+\frac{5c_\parallel^2+3c_\perp^2}{2c_\parallel^2c_\perp^2}q^2;$$

if $\omega_p>\omega_L$, this equation can be satisfied only by values $h\leqslant h_{c1}$. In this case the current is proportional to h:

$$j=^5/_{16}(5c_\parallel^2+3c_\perp^2)h.\qquad (16)$$

At $h=h_{cl}$, expressions (15) and (16) predict the same maximum current $j_{\max}$.

In case 3), the vanishing of ρ at $u=1$ causes Eqs. (12) to contain zeros in their denominators, and it is convenient to transform to the variable $\sigma=\beta\exp(iw)$ in order to find steady-state solutions. Retaining the leading terms in the limit $\beta\to0$ in system (7), we find the following equation for σ:

$$-i\frac{\omega_p}{c_\perp^2}\frac{\partial\sigma}{\partial t}=\frac{\partial^2\sigma}{\partial y^2}+\frac{\sigma}{\xi^2}.\qquad (17)$$

Its steady-state solutions are of the form $\sigma=\sigma_0\times\exp(\pm ih_{c2}y)$. The same result can be found from solutions 1) by taking the limit $u\to1$ in them. At the point $u=1$ itself,the angle ψ and thus the phase w are undefined. For solutions 3) we have $j=0$, and for $h\geqslant h_{c2}$ a steady-state one-dimensional spin flow turns out to be impossible.

Before we use these solutions to describe a spin current, we need to ensure that they are stable. In case 1), we can use the standard procedure for analyzing stability. In other words, we should substitute perturbed steady-state solutions $w = hy + \varphi(y,t)$ and $u = u_0(h) + \eta(y,t)$ into Eqs. (1) and linearize them in terms of the small perturbations φ and η. As a result we find the system of equations

$$\omega_p \frac{\partial\varphi}{\partial t} - \frac{d\rho}{du} h \frac{\partial\varphi}{\partial y} + \mu \frac{\partial^2\eta}{\partial y^2} - \frac{1}{2} h^2 \frac{d^2\rho}{du^2}\eta = 0,$$
$$\omega_p \frac{\partial\eta}{\partial t} - \frac{d\rho}{du} h \frac{\partial\eta}{\partial y} - \rho \frac{\partial^2\varphi}{\partial y^2} = 0. \tag{18}$$

Substituting $\varphi, \eta \sim \exp[i(ky - \omega t)]$ into these equations, we find the dispersion law for small oscillations of the spin and the order parameter, i.e., for spin waves superposed on a spin current:

$$\omega = -\frac{h}{\omega_p}\frac{d\rho}{du}k \pm \frac{k}{\omega_p}\left(\frac{h^2}{2}\rho\frac{d^2\rho}{du^2} + \rho\mu k^2\right)^{1/2}. \tag{19}$$

For $-1/4 < u < 1$ and $c_\parallel^2 > c_\perp^2$, the expression in the radical is positive; i.e., solutions 1) are stable with respect to small perturbations. By studying the definiteness of the energy density (2), Sonin[4] recently concluded that the solutions 1) are unstable as a result of a violation of the Landau criterion. The conclusion that the Landau criterion is violated for these solutions is correct. That conclusion also follows from the dispersion law (19), which is analogous to the dispersion law for sound propagating in a liquid moving at a velocity v. In this case the combination—$(h/\omega_p)(d\rho/du)$ plays the role of the velocity of the liquid and the spin-wave velocity $s = (h/\omega_p)(1/2\rho d^2\rho/du^2)^{1/2}$ plays the role of the sound velocity. It follows from the inequality found by Sonin that for the solutions 1) we have $v > s$, i.e., spin waves may be emitted at the expense of the energy of the spin current. Note, however, that the spin current in this case is superposed on a nonequilibrium and time-dependent state. This state relaxes to equilibrium even in the absence of a current. When there is a current, there is an additional component of the relaxation rate because of spin diffusion; i.e., the spin current is not dissipationless, even if the Landau criterion does hold. A violation of this criterion means that yet another relaxation mechanism has come into play, and the question is the importance of this mechanism. Simple estimates show that, under those assumptions regarding the nature of the perturbations introduced in the flow which are the most favorable assumptions for an analysis of spin waves, the ratio of the wave component of the dissipation to the spin-diffusion component is a bounded quantity of order $(c^2/D\omega_L)\times[(\omega_p - \omega_L)/\omega_L]$ (D is the spin diffusion coefficient). Under the experimental conditions of Ref. 2 this ratio was $\sim 10^{-2}$–10^{-3}. The wave components may prove important only at temperatures well below T_c, where the roles played by other dissipation mechanisms fade, and the only competing mechanism which is left is the emission of spin waves at the walls by the precessing spin (an emission which is unrelated to the current).[5] A violation of the Landau criterion thus does not lead to a disruption of the current state. A more important circumstance is that for $v > s$ the spin flow becomes analogous to supersonic flow in gasdynamics,[6] and this circumstance influences its nature even if dissipation is ignored.

Solution 2) corresponds to one end of the range of allowed values of u. The equations of motion (7) have a singularity at $u = -1/4$, and a stability analysis of solution 2) cannot be carried out by the standard method. We treat solution 2) as the limit of solution 1) in the case $h \to h_{cl}$ and, respectively, $u \to -1/4$ from the side $u > -1/4$. The second term in the radical in (19) has a singularity at $u = -1/4$. Retaining only the leading terms in the limit $u \to -1/4$ in this equation, we find

$$\omega = \frac{5c_\parallel^2 - c_\perp^2}{\omega_p} h_{c1} k \pm \frac{k}{\omega_p}\left[\frac{5}{16}(c_\parallel^2 - c_\perp^2)(5c_\parallel^2 + 3c_\perp^2) h_{c1}^2 + \frac{5c_\parallel^2 c_\perp^2}{1+4u} k^2\right]^{1/2}. \tag{20}$$

We see that no instability arises when this limit is taken, and the region of wave vectors for which the Landau criterion is violated disappears. We could also regard solution 2) as the limit of the solutions with $h = \text{const}$ for $u < -1/4$. Solutions of this form and oscillations superposed on them were analyzed in Ref. 1. The limit $u \to -1/4$ in the equations derived there is legitimate only if $h \ll 1/\xi$. In this case the oscillation frequencies remain real, and no instability arises. A more complete analysis of the vicinity of the point $u = -1/4$ will require going beyond the leading low-frequency approximation, but even the analysis presented here leads to the conclusion that solutions 2) are stable.

The dependence of the spin current density j on the phase gradient h (Fig. 1) thus has two regions: a linear increase in the interval $0 < h \geqslant h_{c1}$, in accordance with (16), and a decrease in accordance with (15) for $h_{cl} < h < h_{c2}$. The nature of the flow changes when h_{c1} is crossed. At $h < h_{c1}$, the flow is analogous to subsonic flow, while at $h > h_{c1}$ it is analogous to supersonic flow, in gasdynamics. The role of the sound velocity is being played here by the local velocity of the spin waves. By analogy with gasdynamics, one can assert that in order to achieve supersonic flow we would need a channel with a constriction and a subsequent expansion: a Laval nozzle. A transition from subsonic to supersonic flow in a channel of this sort occurs as the phase difference Δw between the ends of the channel increases, when the current density at the narrowest part of the channel reaches its maximum value j_{max}. With a further increase in the phase difference, the current density in the narrowest part remains constant, while further downstream, i.e., along the h direction, a regime corresponding to $h > h_{c1}$ is established. If the flow in

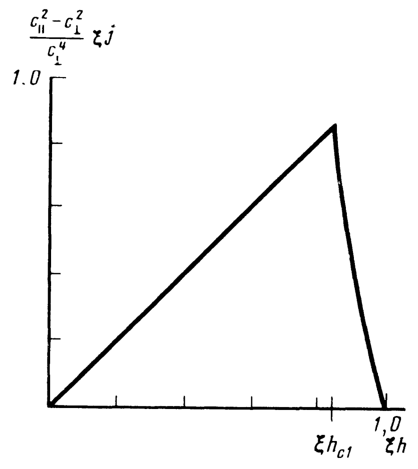

FIG. 1.

a channel with a varying cross section is to be regarded as one-dimensional, it is necessary that any substantial changes in the cross section occur over distances much longer than the length scale ξ. When the upper critical gradient h_{c2} is reached, the phase decays: There is an abrupt decrease in h. Such decays were seen directly in the experiments of Ref. 2. The behavior of the critical gradient h_c as a function of $\omega_p - \omega_L$ found in those experiments agrees satisfactorily with a square-root law, and the value of h_c does not differ greatly from $1/\xi$. A more detailed comparison of theory and experiment will require consideration of the effect of dissipation and the effect of the ends of the channel on the spin flow. When dissipative terms are taken into account in the equations of motion, the conservation of the longitudinal component of the spin is disrupted, and the current density in a channel of constant cross section decreases with distance from the inflow region. It is legitimate to ignore dissipative terms if the relative change in the current density over the length of the channel is small. If only diffusive dissipation is taken into account, the result is the following limitation on the length of the channel: $L \ll \xi c^2/D\omega_L$. Under the experimental conditions of Ref. 2, this strong inequality did not hold. Dissipation can be taken into account, but in this case it is necessary to solve the problem for a channel of finite length. We will not take up that problem in this paper.

3. SPIN VORTEX

We now consider steady-state solutions of Eqs. (7) which describe spin vortices which are analogous to the vortices in superconductors and in superfluid ^{4}He. These solutions must contain a line on which the value $u = 1$ holds, while the phase w is not defined. When this line is circumvented along a closed contour, the phase changes by $2\pi N$, where N is an integer (the number of circulation quanta).[1)] The specific form of the vortex solution depends on the orientation of the singular line with respect to the magnetic field. The configuration simplest to analyze is that in which the singular line is a straight line running parallel to the field. In this case, the Hamiltonian (2) is axisymmetric, and axisymmetric vortex solutions can be sought. Here we will consider only a configuration of that type. The second of Eqs. (7), written in the polar coordinates r and φ, takes the following form in the steady state:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho\frac{\partial w}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial\varphi}\left(\frac{\rho}{r}\frac{\partial w}{\partial\varphi}\right)=0. \qquad (21)$$

The third axis, z, runs antiparallel to H_0, and all quantities are assumed to be independent of z. For an axisymmetric vortex, the radial component of the current, $j_r = \pi\partial w/\partial r$, must vanish; i.e., we must have $w = w(\varphi)$. Equation (21), along with the requirement that the order parameter be single-valued, leads to the customary dependence of the phase w on the polar angle for vortices: $w(\varphi) = N\varphi + w_0$. Now substituting $w(\varphi)$ into the first equation of system (7), and setting $\partial w/\partial t = 0$, we find an equation for $u(r)$:

$$\frac{1}{R}\frac{d}{dR}\left(R\tilde{\mu}\frac{du}{dR}\right)-\frac{1}{2}\frac{d\tilde{\mu}}{du}\left(\frac{du}{dR}\right)^2-\frac{N^2}{2R^2}\frac{d\tilde{\rho}}{du}-1=0. \qquad (22)$$

Here we have introduced the dimensionless coordinate $R = r/\xi$ and also $\tilde{\rho} = \rho/c_\perp^2$ and $\tilde{\mu} = \mu/c_\perp^2$. The change in u in

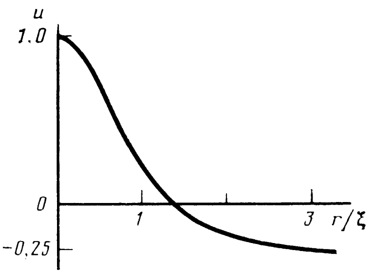

FIG. 2.

a vortex is described by the solution of Eq. (22) which is determined by the boundary conditions $u \to 1$ as $R \to 0$ and $u \to -1/4$ as $R \to \infty$. From (22) we can easily find the manner in which these asymptotes are approached. In this limit $R \to 0$ we have $u \approx 1 - AR^{2N}$, while for $R \to \infty$ we have $u \approx -1/4 + B\exp(-aR)$, where $a^2 = (5c_\parallel^2 + 3c_\perp^2)/2c_\parallel^2$. The constants A and B are found as a result of the solution of the equation. A numerical solution for a vortex with $N = 1$ leads to the functional dependence $u(R)$ shown in Fig. 2. For $c_\parallel^2/c_\perp^2$ we adopted the value 4/3, which is the value to which this ratio tends in the limit $T \to T_c$. The region near the vortex axis with a size $\sim\xi$, where the basic variation of u occurs, should be regarded as the core of the vortex; outside this core we have $u \approx -1/4$. Actually, as $r \to \infty$ the value of u tends toward a limit which is slightly smaller than $-1/4$, so that the required frequency shift $\omega_p - \omega_L$ is provided. A description of this effect goes beyond the leading approximation in l_D/ξ, which is valid as long as the condition $u + 1/4 \gg (l_D/\xi)^2$ holds, i.e., as long as the condition $r/\xi \ll \ln(\xi/l_D)^2$ holds, where l_D is the dipole length.

To determine the orientation of the spin and that of the order parameter in a vortex, we should return from the variable w to the variable ψ, which depends on not only φ but also r (Fig. 3). When we go from w to ψ, we run into double-valuedness because of the two ways in which the sign can be chosen in the definition of ρ_{12}. It can be seen from (6) that in the case $\partial w/\partial r = 0$ a change in the sign of ρ_{12} leads to a change in the sign of $\partial\psi/\partial r$. This result means that there are two types of vortices: right-handed and left-handed, which differ in the direction in which ψ changes as we go radially toward the axis of the vortex. In terms of the vector $\mathbf{n}$ [see (5)], the distinction between the two types of vortices is that at $r = 0$ we have $\mathbf{n} \| \mathbf{H}_0$ for right-handed vortices and $\mathbf{n} \| -\mathbf{H}_0$ for left-handed vortices. In the limit $r \to \infty$, $\mathbf{n}$ tends toward a direction perpendicular to $\mathbf{H}_0$ for vortices of both types.

The appearance of spin vortices and their motion in the

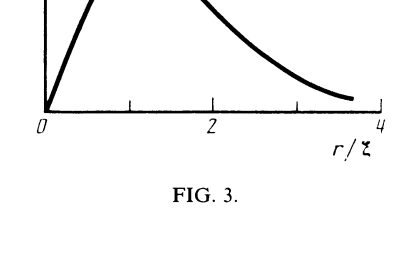

FIG. 3.

direction across the spin flow constitute a mechanism for phase slippage in those cases in which spin flow cannot be regarded as one-dimensional, i.e., in which the transverse dimension of the channel is not small in comparison with ξ in at least one direction. We should expect that vortices of this type would determine the magnitude of the critical phase gradient of the spin current in the case of a spin flow across a slit of thickness $a \ll \xi$ and width $d \gg \xi$, whose wide side runs perpendicular to the magnetic field. Since spin vortices are seen to be analogous to quantized vortices in superfluid ^{4}He, we can thus immediately transfer the estimate of the critical phase gradient of the order parameter, at which the formation of vortices becomes favored, from that other arena:

$$\nabla w \sim \frac{1}{d} \ln\left(\frac{d}{\xi}\right).$$

The critical gradient falls off basically as ξ/d, in contrast with a one-dimensional flow.

As a criterion telling us whether it is legitimate to ignore dissipative terms in the equations of motion in describing the vortex structure we can adopt the condition that the radial component of the current which arises when dissipation is taken into account must be small in comparison with the tangential component. This condition reduces to the inequality

$$\frac{D\omega_L}{c^2} \ln \frac{r}{\xi} \ll 1,$$

where r is the maximum distance out to which the approximate description is valid. In the case of spin flow along a channel of width d, the condition for the applicability of a description of this sort is the inequality $\ln(d/\xi) \ll c^2/D\omega_L$.

4. DISCUSSION

In the examples considered here, the system of spins in ^{3}He-B behaves in the same way as superfluid ^{4}He or electrons in superconductors would behave in an analogous situation. The analogy is based on the circumstance that one of the variables which determine the orientation of the order parameter in ^{3}He-B (here, this variable is w) is analogous to the phase of the order parameter in a superconductor. Since a phase is defined modulo 2π, the changes in the nature of the flow occur abruptly, as a result of the onset of phase-slippage centers. The same circumstance makes it possible to observe in spin currents effects similar to Josephson effects.[7,8] Another important point is that the initial Hamiltonian is degenerate in the phase of w; the variable which is the conjugate of w (in this case, the longitudinal component of the spin) is conserved, and we can say that it "overflows."

The method used to observe spin currents in the experiments of Ref. 2 is based on the circumstance that the motion of the order parameter under the conditions of those experiments is rigidly coupled to the motion of the spin, and the phase w determines not only the orientation of the order parameter but also the orientation of the spin, rendering this phase a directly observable quantity. However, this circumstance makes it necessary to work with time-varying states and thus the appearance of a dissipation. When dissipative effects are taken into account in the equations of motion, the analogy with currents in superconductors becomes incomplete; in particular, the conservation of the longitudinal component of the spin is violated. In a superconductor, this violation would correspond to charge nonconservation. If, however, the conditions stated above are observed, the dissipative loss will be small. Furthermore, experiments are usually carried out in such a way that the vanishing of the spin is offset by an external pump; in such a case, all of the assertions based on a continuity of the phase continue to hold. Note also that spin currents in ^{3}He-B, like superfluid currents in ^{4}He or in superconductors, arise as a response to a disruption of the spatial homogeneity of a condensate of Cooper pairs or Bose particles; i.e., the spin currents in ^{3}He-B not only are analogous to these other currents but also are of the same physical nature.

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[1)]The possible existence of vortices of this type was also mentioned by Sonin,[4] but certain assertions regarding the specific properties of the vortices which were made in that other study on the basis of general considerations are not supported by the solution being analyzed here.

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