Influence of boundary transparency on the critical current of "dirty" *SS'S* structures

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We have investigated the influence of boundary transparency on the stationary properties of dirty *SS'S* superconductor structures within the framework of the microscopic theory of superconductivity. We have derived boundary conditions for the Green's function of a contact between two dirty metals which are valid for arbitrary boundary transparency, and have calculated the resistivity of such a boundary. In the limits of small and large weak-link lengths d, we obtain analytic expressions for the function $I_s(\varphi)$. In the case of *SNS* transition (i.e., the critical temperature of the weak-link material $T_c = 0$) we have computed numerically the temperature dependences of the product of the critical current I_c and the normal-state transition resistivity R_N for a number of values of the weak-link length and boundary transparency.

Recently, the study of properties of Josephson structures based on semiconductor materials has attracted more and more experimental[1–5] and theoretical[6] interest. When the semiconductor is heavily doped or an inversion layer is created in some way at a surface which accumulates carriers, the bulk properties of the material differ very little from the properties of a high-resistivity normal metal. However, at a boundary between superconducting and semiconducting materials a Schottky barrier can appear whose transparency is less than unity, which automatically leads to a decrease in the weak-link critical current. A small-transparency barrier can also appear when the value of the Fermi momentum p_2 of the weak link material is considerably smaller than the Fermi momentum p_1 of the superconducting electrode material, if the boundary between the materials is abrupt on an atomic scale. A final point is that study of the processes in *SS'S* Josephson structures with barriers whose transparency differs from unity is a integral part of the problem of current flow in polycrystalline superconducting films with Josephson interactions between the small-size grains.[7]

It is therefore clear that theoretical study of the influence of finite-transparency boundaries on the critical current in dirty *SS'S* structures is a relevant topic. This problem was discussed earlier for the case of large thicknesses of weak-link material[8,9] and for a specific form of the potential barrier at the boundary of the superconductor with the weak-link material; the latter was assumed to have no metallic conductivity.[6]

The goal of this article is to investigate the stationary properties of a dirty *SS'S* structure with barriers of arbitrary transparency at the *SS'* boundaries for the case where the *S'* material has some metallic conductivity; the barriers are assumed to be abrupt on the scale of an electron mean free path. In the case of an *SNS* junction (i.e., the critical temperature of the weak-link material $T_{c2} = 0$), the calculations are carried out for arbitrary thicknesses d of the weak-link material; for the case of small d the results are applicable for arbitrary T_{c2}.

1. MODEL AND DESCRIPTION OF THE CONTACT

We will assume that the "dirty limit" conditions are fulfilled for the materials which make up the *SS'S* structure:

$$l_{1,2} \ll \xi^*_{1,2}, \tag{1}$$

where $l_{1,2}$ and $\xi^*_{1,2}$ are the electron mean free paths and coherence lengths of the *S* and *S'* materials. Since the weak-link material has a small metallic conductivity, the parameters γ and γ_J which describe the suppression of superconductivity in the electrodes due to the proximity effect[9] and the current flowing through the junction[10] are small even for *SS'* boundaries of unit transparency:

$$\gamma = \frac{\sigma_2\xi_1^*}{\sigma_1\xi_2^*} \ll 1, \qquad \gamma_J = \frac{\sigma_2\xi_1^*}{\sigma_1 d} \ll 1. \tag{2}$$

Here, $\sigma_{1,2}$ are the conductivities of the *S* and *S'* materials and d is the weak-link length along the direction of current flow. In addition to this, we assume that the *SS'* boundaries are planes which are sharp on an interatomic scale and are characterized by a transparency D, while the structure itself is quasi-one-dimensional, so that all quantities vary only along the z-axis, which is perpendicular to the boundaries.

Let us identify the coordinate origin with the center of the *S'* layer and choose the gauge with zero vector potential. Taking into account the above assumptions concerning the stationary properties, we can investigate this structure by using the Usadell equations,[11] which in the weak-link region can be written in the form

$$\Phi_2 = \frac{\pi T_{c1}}{\omega G_2}[G_2^2\Phi_2']'\xi_2^{*2} + \Delta_2, \qquad G_2 = \omega/(\omega^2 + \Phi_2\Phi_2^*)^{1/2}, \tag{3a}$$

$$\Delta_2 \ln\frac{T}{T_{c2}} + 2\pi T\sum_{\omega>0}\omega^{-1}(\Delta_2 - \Phi_2 G_2) = 0, \qquad \xi_2^* = \left(\frac{v_{F2}l_2}{6\pi T_{c1}}\right)^{1/2}, \tag{3b}$$

$$j = \frac{2\sigma_2\pi T}{e}\,\mathrm{Im}\sum_{\omega>0}\omega^{-2}G_2^2\Phi_2^*\Phi_2'. \tag{3c}$$

Here, Δ_2 is the order parameter, $\omega = (2n+1)\pi/T$ are the Matsubara frequencies, and T_{c1} and T_{c2} are the critical temperatures of the *S* and *S'* materials; the function Φ_2 is connected with the Usadell functions F_2 by the relation $F = \omega^{-1}G_2\Phi_2$ and the dash denotes differentiation with respect to z. By virtue of condition (2), the function Φ_1 in the superconducting electrodes equals its unperturbed value:

$$\Phi_1(\pm d/2) = \Delta_1\exp(\pm i\varphi/2), \tag{4}$$

where Δ_1 is the modulus of the order parameter and φ is the phase difference of the order parameters of the electrodes.

Equation (3) is not valid at distances on the order of $l_{1,2}$ from the boundaries, and must be supplemented by boundary conditions. By virtue of the symmetry of the problem, the function $\Phi_2 = \mathrm{Re}\Phi_2 + i\mathrm{Im}\Phi_2$ must satisfy the requirements

$$\mathrm{Re}\Phi_2'(0)=0, \quad \mathrm{Im}\,\Phi_2(0)=0. \tag{5}$$

The second condition on the function Φ_2 is sufficient to specify it at one of the boundaries. The question of boundary conditions for Eq. (3) at $z = d/2$ in the case of arbitrary transparency D requires a separate investigation.

2. BOUNDARY CONDITIONS FOR THE USADELL EQUATIONS

In order to find the conditions we need, it is necessary to solve the system of Eilenberger equations in the immediate vicinity of the boundary, i.e., at distances on the order of $l_{1,2}$ from it.[12] By neglecting terms proportional to ω and $\Delta_{1,2}$ in these equations, we arrive at

$$2l_{1,2}\nabla\hat{g}_{1,2}{}^a=\hat{g}_{1,2}{}^c\langle\hat{g}_{1,2}{}^c\rangle-\langle\hat{g}_{1,2}{}^c\rangle\hat{g}_{1,2}{}^c, \tag{6a}$$

$$2l_{1,2}\nabla\hat{g}_{1,2}{}^c=\hat{g}_{1,2}{}^a\langle\hat{g}_{1,2}{}^c\rangle-\langle\hat{g}_{1,2}{}^c\rangle\hat{g}_{1,2}{}^a. \tag{6b}$$

Here, the angular brackets imply an integration over the total solid angle: $\langle\ldots\rangle = \oint d\Omega/4\pi$, while the subscripts c and a denote the symmetric and antisymmetric parts of the matrix $\hat{g}_{1,2}$ which is made up of Eilenberger functions:

$$\hat{g}_{1,2}^{c(a)} = {}^1/_2\,[\hat{g}_{1,2}(p_{1,2}^z)\pm\hat{g}_{1,2}(-p_{1,2}^z)], \quad \hat{g}_{1,2}=\begin{pmatrix} g_{1,2} & f_{1,2} \\ f_{1,2}^+ & -g_{1,2}\end{pmatrix} \tag{7}$$

Far from the boundaries, i.e., at distances which exceed $l_{1,2}$, solutions of the system (6) must reduce to the isotropic Usadell functions:

$$\hat{g}_{1,2}{}^c=\langle\hat{g}_{1,2}{}^c\rangle\equiv\hat{G}_{1,2}, \tag{8a}$$

$$\hat{g}_{1,2}{}^a=l_{1,2}\hat{G}_{1,2}\nabla\hat{G}_{1,2}, \tag{8b}$$

and at the boundaries themselves these functions must satisfy the conditions[13]

$$\hat{g}_1{}^a(d/2)=\hat{g}_2{}^a(d/2)\equiv\hat{g}^a(d/2), \tag{9a}$$

$$\hat{g}^a(d/2)\,[R-R(\hat{g}^a(d/2))^2+D(\hat{g}_-{}^c)^2]=D\hat{g}_-{}^c\hat{g}_+{}^c \tag{9b}$$

on a drift path, and

$$\hat{g}_1{}^a=0 \tag{10}$$

for electrons reflected from the boundaries whose component of momentum parallel to the boundary $p_\parallel$ satisfies the condition $p_2 < p_\parallel < p_1$. Here, $R = 1 - D$ is the reflection coefficient of electrons from the boundary, while

$$\hat{g}_\pm{}^c={}^1/_2[\hat{g}_1{}^c(d/2)\pm\hat{g}_2{}^c(d/2)]. \tag{11}$$

By averaging the right and left sides of Eq. (6a) over the total solid angle, we find that the quantity

$$\langle x_{1,2}\hat{g}_{1,2}{}^a\rangle=\hat{C}_{1,2}, \quad x_{1,2}=p_{1,2}{}^z/p_{1,2} \tag{12}$$

is constant in each of the metals. The constants of integration $\hat{C}_{1,2}$ can be calculated far from the boundaries by using the relation (8b):

$$\hat{C}_{1,2}={}^1/_3 l_{1,2}\hat{G}_{1,2}d\hat{G}_{1,2}/dz. \tag{13}$$

On the other hand, from (9a), (10), (12) and the condition of momentum conservation, it follows that

$$p_1{}^2\hat{C}_1=p_2{}^2\hat{C}_2. \tag{14}$$

Relations (13), (14) determine the first boundary conditions for the Usadell equations:

$$p_1{}^2l_1\hat{G}_1d\hat{G}_1/dz=p_2{}^2l_2\hat{G}_2d\hat{G}_2/dz, \tag{15}$$

which ensure that the current flowing through the boundary is continuous for any value of D.

In order to obtain a second boundary condition on the function $\hat{G}_{1,2}$, we will assume that for any value of boundary transparency the inequality

$$D\hat{g}_-{}^c\ll\hat{g}_+{}^c \tag{16}$$

holds (from here on, for brevity we will write conditions at the boundary in the form of matrix inequalities using a notation which implies that the inequalities hold for all the nonzero components of the matrices).

From the boundary condition (9b) it follows within this approximation that

$$\hat{g}^a(d/2)\ll\hat{g}_+{}^c \tag{17}$$

in which terms proportional to $R(\hat{g}^a)^2$ in the left side of (9b) are neglected. Because condition (8b) implies that $\hat{g}_{1,2}{}^a$ is also small compared to $\hat{g}_{1,2}{}^c$ far from the boundaries, we have for all z that

$$\hat{g}_{1,2}{}^a\ll\hat{g}_{1,2}{}^c. \tag{18}$$

Using (18), we obtain a solution for the boundary problem (6), (8) in the form

$$\hat{g}_{1,2}{}^c=\langle\hat{g}_{1,2}{}^c\rangle=\hat{G}_{1,2}, \quad \hat{g}_{1,2}{}^a=l_{1,2}x_{1,2}\hat{G}_{1,2}d\hat{G}_{1,2}/dz. \tag{19}$$

Substituting (19) into the boundary conditions (9) and assuming that the inequality

$$R\cdot\hat{1}\gg D(\hat{g}_-{}^c)^2, \tag{20}$$

holds, we obtain the relation

$$\hat{g}^a\,(d/2)=(D/4B)\,(\hat{G}_2\hat{G}_1-\hat{G}_1\hat{G}_2), \tag{21}$$

from which, taking into account (12), (13), there follows the desired boundary condition to the Usadell equations

$$l_2\hat{G}_2\frac{d}{dz}\hat{G}_2=\frac{3}{4}\left\langle\frac{x_2D}{R}\right\rangle(\hat{G}_2\hat{G}_1-\hat{G}_1\hat{G}_2). \tag{22}$$

It follows from (22) that the quantity $D\hat{g}^c_-$ is proportional to $(Rl_2/\xi_2^*)\hat{g}^c_+$, therefore, conditions (16), (20) which define the range of applicability of condition (22) are found to be fulfilled for any values of the transmission coefficient of the boundary as long as the conditions of the "dirty limit" (1) are valid.

Relations (15), (22) reduce to two conditions on the functions $\Phi_{1,2}$; these functions must satisfy these conditions at the interface between the S and S' materials (for $z = d/2$):

$$p_1{}^2l_1G_1{}^2\Phi_1'=p_2{}^2l_2G_2{}^2\Phi_2', \tag{23a}$$

$$\xi_2{}^*\gamma_BG_2\Phi_2'=G_1(\Phi_1-\Phi_2), \qquad \gamma_B=\frac{2}{3}\frac{l_2}{\xi_2{}^*}\left\langle\frac{x_2D}{R}\right\rangle^{-1}. \tag{23b}$$

Equations (23) automatically ensure continuity of the supercurrent flowing through the boundary. As was pointed out in Ref. 13, for small values of γ_B conditions (23) reduce to the previously established requirements[14] that the functions Φ and the quantities $p^2 l\Phi' \propto \sigma\Phi'$ be continuous; for $\gamma_B \gg 1$ they reduce to conditions at the boundary of an insulator.[15]

The boundary conditions for the Usadell equations were studied previously in Refs. 16 and 17, where only the first condition, corresponding to current continuity through the boundary, was obtained correctly. In Ref. 16 boundary conditions for the Usadell functions were obtained by averaging boundary conditions obtained by the authors for the Eilenberger equations over the Fermi surface for the case of identical materials separated by a δ-function potential barrier. The boundary conditions (23b) differ from the ones obtained in Ref. 16, where different boundary conditions for the original Eilenberger equation were used and a different dependence on the boundary transmission coefficient was given, although the test evaluation of these conditions given by the authors of Ref. 16 for the cases $D=1$ and $D=0$ gave the correct results of Refs. 14 and 15, respectively. In Ref. 17 a derivation of the boundary conditions was presented for temperatures close to the critical temperature, based on the solution of the linear integral equations. However, the expression used in this paper for the kernel of the integral equation for Δ took into account only the portion of the waves reflected from the boundary, which in the general case leads to incorrect results.

It is qualitatively clear that the relations (23) which we have derived are correct for boundaries which are smooth on an atomic scale but abrupt on the scale of the electronic mean free path, if we understand R and D to be the reflection and transmission coefficients of electrons through these boundaries.

Multiplying (23b) by $G_2\Phi_2^*$ and using (3c), we immediately obtain the well-known sinusoidal current-phase relation for a Josephson $S'IS$ junction derived from tunnelling theory, with critical current I_c equal to

$$I_c=\frac{2\sigma_2\pi T}{e\xi_2^*\gamma_B}\sum_{\omega>0}\frac{\Delta_1\Delta_2}{(\omega^2+\Delta_1^2)^{1/2}(\omega^2+\Delta_2^2)^{1/2}}. \tag{24}$$

It is necessary to point out that Eq. (24) is correct only for small values of the transparency for which we can neglect the proximity effect of the S and S' materials.

The normal resistivity of this structure equals the sum of the resistivities of the boundaries and of the weak-link material. In determining the resistivity of a plane boundary between two normal metals, we make use of the boundary conditions (22):

$$\frac{1}{3}l_2\frac{d}{dz}f_2^c=\left\langle\frac{x_2D}{R}\right\rangle(f_1^c-f_2^c)=-\left\langle\frac{x_2D}{R}\right\rangle\frac{\partial f_0}{\partial\varepsilon}eV_b. \tag{25a}$$

Here $f_{1,2}{}^c$ is the symmetric part of the electron distribution function averaged over the Fermi surface, f_0 is the Fermi distribution function and V_b is the potential jump at the boundary. From the definition of the current, together with condition (25a), we obtain a linear relation between current and the potential jump at the boundary:

$$I=R_b^{-1}V_b,\qquad R_b=\frac{2\pi^2}{e^2p_2^2S^*}\left\langle\frac{x_2D}{R}\right\rangle^{-1}=\frac{\xi_2^*\gamma_B}{\sigma_2S^*}, \tag{25b}$$

where S^* is the area of the section surface.

Thus, the resistivity of the $SS'S$ structure in the normal state equals

$$R_N=\frac{d}{\sigma_2S^*}(1+2\Gamma_B),\qquad \Gamma_B=\frac{\xi_2^*\gamma_B}{d}. \tag{26}$$

An analytic expression for the critical current of an $SS'S$ junction can be obtained only in certain special cases.

3. THE SS' BOUNDARY IN THE SMALL-TRANSPARENCY APPROXIMATION

The properties of $SS'S$ structures depend significantly on the parameter Γ_B, which in fact is equal to the ratio of the resistances of the boundary and weak-link material, along with the intrinsic superconducting properties of the layer material.

Actually, if $T_{c2}\neq 0$, then the maximum value of the supercurrent flowing through the boundaries $I\propto\sigma_2\Delta_2\Delta_1/\xi_2^*\gamma_B$ for small values of their transparency can turn out to be significantly smaller than the depairing current in the weak-link material $I_p\propto\sigma_2\Delta_2^2/\xi_2^*$. In this case,

$$\gamma_B\gg\Delta_1/\Delta_2, \tag{27}$$

and the $SS'S$ structure should possess the properties of a system consisting of two tunnelling junctions with the same critical currents. When condition (27) holds, the gradient terms in (3a) are small, and to lowest order in γ_B^{-1} it follows from (3a) and (5) that

$$\operatorname{Re}\Phi_2=\Delta_2,\quad \operatorname{Im}\Phi_2=0. \tag{28}$$

It is convenient to calculate the magnitude of the supercurrent flowing through the structure at the boundary point $z=d/2$, where using (4) and (23b) we have to first order in γ_B^{-1}

$$\xi_2^*\gamma_BG_2^2\operatorname{Im}\Phi_2'=G_2G_1\Delta_1\sin(\varphi/2),\quad G_{1,2}=\omega/(\omega^2+\Delta_{1,2}^2)^{1/2}. \tag{29}$$

From (28), (29) and relation (3c) for the supercurrent we obtain the sought-after function $I_s(\varphi)$:

$$I_s=I_c\sin(\varphi/2) \tag{30}$$

with a critical current I_c determined by Eq. (24). If the materials which make up the superconducting electrodes are different, the results obtained are easily generalized and lead to a function $I_s(\varphi)$ of the form

$$I_s=I_1I_2\sin\varphi[I_1^2+I_2^2+2I_1I_2\cos\varphi]^{-1/2},\qquad I_c=\min\{I_1,I_2\}, \tag{31}$$

where $I_{1,2}$ are the critical currents of each of the two consecutive Josephson junctions. Relations (30) and (31) are valid when condition (27) holds and when the thickness of S' material is not too great:

$$d/\xi_2^*\ll\gamma_B. \tag{32}$$

For $d>\xi_2^*\gamma_B$ it is necessary to include the linear advance in the phase of the order parameter in the S' material caused by the flowing supercurrent; this leads to a change in the function $I_s(\varphi)$, but not of the critical current, which will be primarily determined by Eq. (24).

If $T_{c2} = 0$, i.e., the weak-link material is not superconducting, then for sufficiently small transparencies of the SS' boundaries the function Φ_2 can be small compared to πT; Eq. (3a) and the boundary conditions (5) and (23) can then be linearized, and it is not difficult to obtain for Re Φ_2 and Im Φ_2 the values:

$$\mathrm{Re}\,\Phi_2(z)=\frac{G_1\Delta_1\cos(\varphi/2)}{2\Gamma_B\beta\,\mathrm{sh}\,\beta+\mathrm{ch}\,\beta}\mathrm{ch}\frac{2\beta z}{d},\qquad \beta=\frac{d}{2\xi_2^*}\left(\frac{\omega}{\pi T_{c1}}\right)^{1/2}, \tag{33a}$$

$$\mathrm{Im}\,\Phi_2(z)=\frac{G_1\Delta_1\sin(\varphi/2)}{2\Gamma_B\beta\,\mathrm{ch}\,\beta+\mathrm{sh}\,\beta}\mathrm{sh}\frac{2\beta z}{d}. \tag{33b}$$

Substituting these solutions (33) into the expression for the supercurrent (3c), and taking into account (26), we obtain a sinusoidal function $I_2(\Phi)$ with a critical current I_c equal to

$$I_c=\frac{2\pi T}{eR_N}\sum_{\omega>0}\frac{2\Delta_1^2\beta(1+2\Gamma_B)}{(\omega^2+\Delta_1^2)\left[(1+4\Gamma_B^2\beta^2)\,\mathrm{sh}\,2\beta+4\Gamma_B\beta\,\mathrm{ch}\,2\beta\right]}. \tag{34}$$

Equation (34) is valid when the condition $|\Phi_2(d/2)|\ll\pi T$, is fulfilled, i.e., in the case

$$\Delta_1/\pi T\ll 1+2\beta\Gamma_B\,\mathrm{th}\,\beta. \tag{35}$$

For $\beta\gg1$, Eq. (34) implies an experimental dependence for $I_c(d)$, which coincides with the expression derived earlier for SNS junctions with low-transparency barriers.[8] In the low-temperature region $T\ll V_{F2}l_2/6\pi d^2$ the dependence of the critical current on the length of the weak link ceases to be exponential, and the result given in Ref. 8 coincides to logarithmic accuracy with the function $I_c(d)$ which follows from (34).

It follows from (35) that for $T\approx T_{c1}$ the value of the characteristic voltage of the Josephson structure $V_c = I_cR_N$ is determined by Eq. (34) for any parameter values Γ_B and β. In this temperature range, for small values of Γ_B condition (34) reduces to a result established previously for SNS bridges of variable thickness,[19] while for large values of Γ_B the product I_cR_N decreases as the value of this parameter increases:

$$V_c=\frac{\pi}{4}\frac{\Delta_1^2}{eT_{c1}}\left[8\sum_{\omega>0}\frac{T_{c1}^2}{\omega^2(\mathrm{ch}\,2\beta+\beta\Gamma_B\,\mathrm{sh}\,2\beta)}\right],\qquad \Gamma_B\gg1. \tag{36}$$

In particular, for $d\ll\xi_2^*$, it follows from (36) that the function $V_c(T)$,

$$V_c=\frac{\pi}{4}\frac{\Delta_1^2}{eT_{c1}}\left\{1-\frac{4\alpha}{\pi^2}\left[\psi\left(\frac{1}{2}+\frac{1}{2\alpha}\right)-\psi\left(\frac{1}{2}\right)\right]\right\},$$
$$\alpha=2\Gamma_B\left(\frac{d}{3\xi_2^*}\right)^2, \tag{37}$$

where $\Psi(x)$ is the digamma function, differs little from the result of Aslamazov and Larkin[20]

$$V_c=\frac{\pi}{4}\frac{\Delta_1^2}{eT_{c1}}\left[1-\frac{4\alpha}{\pi^2}\ln\frac{\pi^2}{4\alpha}\right],\qquad \alpha\ll1 \tag{38}$$

for $\gamma_B\ll\xi_2^*/d$. For large values of γ_B

$$V_c=\frac{\pi}{4}\frac{\Delta_1^2}{eT_{c1}}\frac{1}{\alpha}\frac{7\zeta(3)}{\pi^2},\qquad \gamma_B\gg\xi_2^*/d, \tag{39}$$

i.e., the parameter V_c decreases in inverse proportion to γ_B.

Thus, the stationary properties of weak links of $SS'S$ type in the presence of low-transparency barriers at the SS' boundaries differ significantly from the properties of analogous structures with transparent boundaries[21], independent of whether the weak-link material is superconducting or normal metal. As the boundary transparency increases, i.e., as γ_B decreases, the difference between these two types of junction decreases; the character of the behavior of $V_c(\gamma_B)$ depends significantly on the relation between d and ξ_2^*.

4. THE APPROXIMATION OF SMALL WEAK-LINK LENGTHS

It follows from the structure of Eq. (3a), the boundary conditions (23), and the solution (33), that for $d\ll\xi_2^*$ the real part of the function Φ_2 increases with decreasing γ_B as γ_B^{-1}; however, its imaginary part increases much more slowly: $\mathrm{Im}\Phi_2\propto\Gamma_B^{-1}=(\xi_2^*\gamma_B/d)^{-1}$. This implies a range of parameters γ_B and d/ξ_2^*:

$$\min\{\Delta_1/\Delta_2,\ 1\}\gg\gamma_B\gg d/\xi_2^*, \tag{40}$$

for which the supercurrent flowing through the $SS'S$ structure exceeds the depairing current of the weak-link material for $T_{c2}\neq0$. It turns out, however, that this current is still considerably smaller than the weak-link critical current of a fully transparent barrier $I_{KO}\propto\Delta_1^2\sigma_2/d$ (Ref. 22).

For those values of γ_B which satisfy the inequality (40), we can neglect the nongradient terms in (3a) in determining Re $\Phi_2(z)$, and in the solution of the equations so obtained with the boundary conditions (5) and (23b), and find that

$$\mathrm{Re}\,\Phi_2=\Delta_1\cos(\varphi/2). \tag{41}$$

Equation (29), which determines the value of Im $\Phi_2'(d/2)$, remains valid if we insert the quantity $G_2=\omega[\omega^2+\Delta_1^2\cos^2(\varphi/2)]^{-1/2}$, for G_2; the sought-after function $I_s(\varphi)$ follows immediately from (3c) and (29):

$$I_s=\frac{2\pi T\Delta_1^2}{eR_N}\sum_{\omega>0}\frac{\sin\varphi}{(\omega^2+\Delta_1^2)^{1/2}[\omega^2+\Delta_1^2\cos^2(\varphi/2)]^{1/2}}. \tag{42}$$

Relation (42) shows that the supercurrent in the $SS'S$ structure is determined primarily by electron tunnelling through the barriers at the SS' boundaries from the superconducting electrodes to the weak-link region; in this region, the order parameter is determined by the function Re Φ_2 given by (41), and does not depend on the properties of the weak-link material.

From (42) it follows that for arbitrary temperatures the function $I_s(\varphi)$ is nonsinusoidal. In particular, for $T\ll T_{c1}$, by changing the summation to integration over ω we obtain

$$I_s(\varphi)=\frac{\Delta_1(0)}{eR_N}K\left(\sin\frac{\varphi}{2}\right)\sin\varphi, \tag{43}$$

where $K(x)$ is the complete elliptic integral of the first kind. The function $I_s(\varphi)$ defined by Eq. (43) has a maximum at $\varphi\approx1.86$; the value of the characteristic voltage V_c exceeds the corresponding value for tunnelling transitions[18] by 22% and is 8% smaller than the value of V_c for dirty weak links (based on the theory of Kulik and Omel'yanuk, e.g., KO-1 or Ref. 22). Numerical calculations show that as the temperature increases the function $I_s(\varphi)$ given by (43) gradually becomes sinusoidal, and the function $V_c(T)$ approximates the results of the KO-1 theory[22], the difference being no more than 3% for $T \gtrsim T_{c1}/2$.

When large-transparency barriers are present at the SS' boundaries, i.e.,

$$\gamma_B \ll d/\xi_2^*, \tag{44}$$

they have only a weak influence on the function $I_s(\varphi)$. In this case Re Φ_2 is primarily determined by Eq. (41); in determining Im Φ_2 we can neglect the gradient terms in (3a) and obtain the expression

$$\operatorname{Im} \Phi_2(z) = A_\omega \operatorname{tg} \frac{CzA_\omega}{\omega^2}, \quad A_\omega = [\omega^2 + \Delta_1^2 \cos^2(\varphi/2)]^{1/2}, \tag{45}$$

for which the constant of integration C is found from the conditions (23b):

$$\gamma_B \xi_2^* C = \frac{\omega^2}{A_\omega(\omega^2+\Delta_1^2)^{1/2}} \cos\delta\left(\Delta_1 \sin\frac{\varphi}{2} - A_\omega \operatorname{tg}\delta\right),$$

$$\delta = \frac{C\,dA_\omega}{2\omega^2}, \tag{46}$$

Using (3c) we can determine the value of the supercurrent flowing through the $SS'S$ structure:

$$I_s = \frac{2\pi Td}{eR_N}(1+2\Gamma_B) \sum_{\omega>0} \frac{\Delta_1 \cos(\varphi/2)}{\omega^2} C. \tag{47}$$

Condition (46) is conveniently rewritten in the form

$$2\Gamma_B\delta = \sin(\varphi_0 - \delta), \quad \varphi_0 = \operatorname{arctg}[\Delta_1 \sin(\varphi/2)/A_\omega]. \tag{48}$$

For small barrier transparencies ($\Gamma_B \gg 1$) we can obtain a solution to Eq. (48) by the method of successive approximations in the parameter Γ_B, from which, according to (47), there follows an expression for the current

$$I_s = \frac{2\pi T\Delta_1^2}{eR_N} \sum_{\omega>0} \frac{\sin\varphi}{(\omega^2+\Delta_1^2)^{1/2} A_\omega}\left(1 + \frac{1}{2\Gamma_B}\right), \tag{49}$$

which asymptotically goes to (42). In the case of large boundary transparencies, the function $I_s(\varphi)$ is determined by the expression

$$I_s = \frac{2\pi T}{eR_N} \sum_{\omega>0} \frac{2\Delta_1 \cos(\varphi/2)}{A_\omega} \varphi_0 \left(1 - \frac{4}{3}\Gamma_B^3 \varphi_0^2\right), \tag{50}$$

which for $\Gamma_B \ll 1$ reduces to the result of the KO-1 theory.

The results we have obtained indicate that for $d \ll \xi_2^*$ there should occur a decrease in the critical current and a growth in the normal-state resistivity of the $SS'S$ structure as the transparency of the barriers at its boundaries decreases; the characteristic voltage V_c remains practically constant and does not depend on the properties of the weak-link material.

5. LARGE THICKNESSES OF WEAK-LINK MATERIAL

If the thickness of the weak-link material is large compared to ξ_2^* and its critical temperature $T_{c2} = 0$, then the function $I_s(\varphi)$ is sinusoidal, while the critical current of the structure equals[9]

$$I_c = \frac{64\pi T}{eR_N}(1+2\Gamma_B)\frac{d}{\xi_2} C_0^2 \exp\left(-\frac{d}{\xi_2}\right), \quad d \gg \xi_2 = \xi_2^* \left(\frac{T_{c1}}{T}\right)^{1/2}. \tag{51}$$

The constant C_0 is determined by the boundary conditions (23), which are related for $\omega = \pi T$ to the real parts of the solutions of the Usadell equations in the superconductor (4) and in the layer material:

$$\operatorname{Re} \Phi_2(z) = \pi T \operatorname{tg}\{4 \operatorname{arctg}[C_0 \exp((z-d/2)/\xi_2)]\}, \tag{52}$$

This constant statisfies the equation

$$\sin\left(\operatorname{arctg}\frac{\Delta_1}{\pi T} - 4 \operatorname{arctg} C_0\right) - 2\gamma_B \left(\frac{T}{T_{c1}}\right)^{1/2} \sin(2 \operatorname{arctg} C_0) = 0. \tag{53}$$

In the case of an SS' boundary with a small transparency, the constant C_0 is small:

$$C_0 = \frac{1}{4\gamma_B} \frac{\Delta_1 (T_{c1}/T)^{1/4}}{[(\pi T)^2 + \Delta_1^2]^{1/2}}, \quad \gamma_B \gg \left(\frac{T_{c1}}{T}\right)^{1/2}, \tag{54}$$

and the critical current I_c is determined by the expression

$$I_c = \frac{4\pi T_{c1}}{eR_N} \frac{1+2\Gamma_B}{\gamma_B} \frac{\Delta_1^2}{(\pi T)^2+\Delta_1^2} \frac{d}{\xi_2} \exp\left(-\frac{d}{\xi_2}\right), \tag{55}$$

which is not difficult to obtain from (34) using the appropriate limits.

In the region of small values of γ_B the critical current to first order γ_B decreases linearly as γ_B increases:

$$I_c = \frac{64\pi T}{eR_N}(C_0^2 + 2C_0C_1)\frac{d}{\xi_2} \exp\left(-\frac{d}{\xi_2}\right), \quad \gamma_B \ll \left(\frac{T_{c1}}{T}\right)^{1/2},$$

$$C_0 = \frac{b}{1+a+[2a(1+a)]^{1/2}},$$

$$C_1 = -\gamma_B \left(\frac{T}{T_{c1}}\right)^{1/2} \frac{b}{a^{1/2}\{1+a[2(1+a)]^{1/2}\}}, \tag{56}$$

where $a = (1+b^2)^{1/2}$ and $b = \Delta_1/\pi T$.

From Eqs. (55) and (56), it follows that in contrast to an $SS'S$ structure with $d \ll \xi_2^*$, for which the function $V_c(\gamma_B)$ is smooth (for $T \ll T_{c1}$, the maximum difference of the values of V_c for the cases $\Gamma_B \ll 1$ and $\Gamma_B \gg 1$ amounts to only 8%), for $d \gg \xi_2^*$ the characteristic voltage V_c of the junction decreases monotonically as γ_B increases.

If the weak-link material is a superconductor with a critical temperature $T_{c2} < T_{c1}$, then it is clear qualitatively that within the range of temperatures $T_{c2} < T < T_{c1}$ the parameter V_c also will decrease with increasing γ_B. For lower temperatures, i.e., $T < T_{c2}$, the weak-link material is found in the superconducting state, while for large boundary transparency ($\gamma_B \ll \Delta_1/\Delta_2$) the value of the critical current equals the depairing current of the weak-link material and the parameter V_c increases linearly as d increases.[21] When the condition (27) is fulfilled, the $SS'S$ junction may be treated as a system with two consecutive tunnelling junctions in series.

6. ARBITRARY VALUES OF d AND γ_B

For arbitrary values of the weak-link length and transparency of the barriers, it is necessary in investigating the stationary properties of the $SS'S$ junction to solve Eq. (3) with boundary conditions (5), (23) by numerical methods.

The programs we have developed[12] allow us to carry out calculations for arbitrary critical temperatures of the weak-link material. However, in this paper we limit ourselves to the case which is most important from a practical point of view, i.e., $T_{c2} = 0$. The results of numerical calculations are shown in Fig. 1 in the form of a family of dependences of the

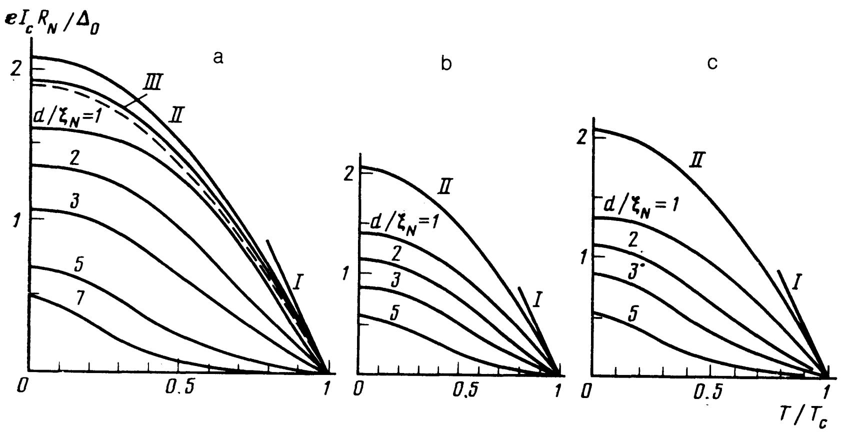

FIG. 1. Temperature dependence of the product $V_c = I_c R_N$ for a dirty *SNS* junction ($T_{c2} = 0$) for several values of the weak-link thickness d and for various values of the parameter γ_B (23b): a—0.1, b—0.3, c—1. Curve I is the asymptotic dependence of the theory of Aslamazov and Larkin, $V_c = 2\pi^3(T_c - T)/7\zeta(3)e$ (Ref. 20); curve II is the function $V_c(T)$ from the KO-1 theory (Ref. 22) for $d \ll \xi_N$, $\gamma_B = 0$; curve III is the function $V_c(T)$ following from Eq. (42) ($d \ll \xi_N, d/\xi_N \ll \gamma_B \ll 1$); the dashed curve is the function $V_c(T)$ for an *SNS* junction of width $d = \xi_N$ (Ref. 19); Δ_0 is the value of the order parameter of the electrodes for $t = 0$, e is the electron charge, $\xi_N = \xi_2^*$.

characteristic voltage V_c on temperature T for several values of the weak-link thickness and parameter γ_B. It is clear that for fixed values of the weak-link thickness d and T/T_c the parameter V_c decreases with increasing γ_B. This decrease is more marked than the larger d is. For values $\gamma_B \gtrsim 1$ the results of numerical calculations match the asymptotic dependence (34); as this parameter decreases, a smooth transition takes place for $d/\xi_2^* > \gamma_B$ to the function $V_c(T)$ calculated in Ref. 19 for weak *SNS* links with transparent boundaries (the dashed curve in Fig. 1a). As we pointed out in Sec. 4, in the region of small weak-link lengths $d \ll \xi_2^*$ the function $V_c(T)$ passes smoothly from the KO-1 theoretical result (curve II) for $\gamma_B \ll d/\xi_2^*$ to the dependence (42) for $\gamma_B \gg d/\xi_2^*$ (curve III).

CONCLUSION

From the calculations we have carried out it follows that the presence of potential barriers at the boundaries of an *SS'S* Josephson structure allows us to realize in practice a new type of Josephson junction which combines the advantages both of a tunnelling structure (the large value of R_N which for $\gamma_B > d/\xi_2^*$, is independent on the layer material thickness, and the small critical current) and a weak link (the small junction capacitance). The magnitude of the characteristic voltage V_c which determines the high-frequency properties of the junction is only a few percent smaller than the value of this parameter for weak links. From the results of numerical calculations it follows that for any thickness of the weak link and $\gamma_B \approx 1$ the value of V_c is smaller than the corresponding values for weak links with transparent boundaries by no more than 20% for $T \ll T_{c1}$; as the temperature T approaches T_{c1} this difference decreases. This new type of Josephson junction can be realized if we fabricate a weak link using a conductor with a Fermi level $p_2 < p_1(l_2/\xi_2^*)$ whose boundaries with the superconducting electrodes are sharp on an interatomic scale.

The boundary conditions derived in this paper for the Green's function in the dirty limit (23) are valid both for the Matsubara and temporal representations, and can be used in solving both stationary and nonstationary problems for the contacting dirty metals (if the thickness of the region in which the change in metallic properties takes place—the size of the boundary—is much smaller than $V_{F1,2}/\varepsilon$ and $l_{1,2}$, where $\varepsilon = \max\{T, \Delta, V, \omega\}$; V is the voltage across the junction, ω is the frequency of an external perturbation).

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