# Inelastic tunneling across thin amorphous films

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An investigation is made of a transition from resonant tunneling across a disordered semiconductor film to hopping charge transport. It is shown that at low temperatures the behavior of the conductance $G(T)$ is governed by inelastic tunneling via pairs of localized states and the temperature dependence is $G(T) \propto T^{4/3}$. The current-voltage characteristic due to two-impurity channels is nonlinear: $I(V) \propto V^{7/3}$. The fluctuation component of the characteristic $\delta I(V)$ for samples of small thickness is different for two different voltage scales: $\Delta V_1 \sim V$ and $\Delta V_2 \sim T/e$. The correlation functions characterizing mesoscopic fluctuations of the current-voltage characteristic are determined. An increase in temperature brings into action chains with increasing numbers of impurities. This results in a transition to the Pollak–Hauser hopping conduction mechanism [M. Pollak and J. J. Hauser, Phys. Rev. Lett. **31**, 1304 (1973); A. V. Tartakovskiĭ, M. V. Fistul', M. É. Raĭkh, and I. M. Ruzin, Sov. Phys. Solid State **21**, 370 (1987)]. The transition region considered in the present investigation may be observed in experiments on tunnel junctions with an amorphous spacer and also in small metal–insulator–semiconductor structures and in heterostructures.

## 1. INTRODUCTION

The electron structure of amorphous materials is characterized by a high density of localized states, the energies of which are distributed over a wide band. In the case of an energy spectrum of this type the conductivity $\sigma$ of a sample of macroscopic dimensions obeys the Mott law[1] in a fairly wide range of temperatures:

$$\ln \sigma(T) \propto -(T_0/T)^{1/4}, \tag{1.1}$$

where $T_0 = \beta/ga^3$, $g$ is the density of localized states, $a$ is the radius of these states, and $\beta$ is a numerical parameter. Lowering of $T$ increases the lengths of electron jumps responsible for the law (1.1) and the conductance of a sample begins to depend on the longitudinal dimension $d$. The limiting case is represented by tunneling. In this case, as in the case of hopping conduction, an important role is played by localized states since for not-too-low values of $d$ (see Ref. 2) the processes of resonant tunneling via these states become dominant. For example, in the case of tunneling across an amorphous silicon layer the resonant processes become dominant in the range $d > 60$ Å (Ref. 3). This raises the natural question: what is the temperature dependence of the conductance at low temperatures? Obviously, the processes of elastic tunneling in the case of an energy-independent density of states $g$ make the conductance $G$ temperature-dependent. Phonon-stimulated inelastic resonant tunneling along one-impurity channels can only result in a weak $G(T)$ dependence. The corresponding correction $\delta G_1(T)$ to the value of $G(0)$ is small at temperatures below the Debye value, irrespective of the magnitude of the electron–phonon interaction.[2] The relative magnitude of the correction $\delta G_1(T)/G(0)$ and its characteristic temperature scale of the changes are independent of $d$. On the other hand, the temperature above which the Mott law applies decreases as $d$ increases. Therefore, the low-temperature dependence $G(T)$ should include contributions whose relative magnitude increases with $d$.

We shall show that the processes of inelastic tunneling in channels containing pairs of localized states with a scatter of the energies of the order of $T$ play a definite role in low-temperature conduction. The relative importance of the corresponding contribution $G_2(T)/G(0) \propto \exp(d/6a)$ increases exponentially with $d$. Moreover, as temperature is lowered, the value of $G_2(T) \propto T^{4/3}$ decreases more slowly than $\delta G_1(T)$ or the contributions of the channels with a larger number of impurities. It follows from these considerations that in the case of the thicknesses $d$ used in experiments there should be a fairly wide range of temperatures when two-impurity channels determine the electrical characteristics of an amorphous layer. In addition to the unusual temperature dependence $G_2(T)$, such channels give rise to power-law nonlinearities of the current-voltage characteristic: $I(V) \propto V^{7/3}$. In contrast to nonresonant tunneling, the values of $G_2(T)$ and $I(V)$ represent the passage of electrons through separate channels separated by large distances. Therefore, $G_2$ and $I$ are self-averaging only when the cross-sectional area $S$ of a sample is sufficiently large. We shall show that mesoscopic fluctuations of $G_2(T)$ are small if $Sg^2a^3dT^2 \gg 1$. The dependence $I(V)$ is, firstly, due to new channels and, secondly, due to a change in the conductance of the existing channels because of a change in the voltage $V$ across a contact. The characteristic scale of the voltages $\Delta V$ in which a new channel is activated is equal to $T$, whereas the scale in which there is a change in the conductance of the already existing channel is $V$. Therefore, for $V \gg T$, fluctuations of the differential conductance $G_2(V)$, which appear in a sample of finite thickness, have two scales. The amplitude of small-scale fluctuations exceeds considerably the amplitude of large-scale fluctuations. Therefore, the variance of $G_2(V)$ is governed directly by small-scale fluctuations and can be found from the current-voltage characteristics of one sample.

Our two-impurity configurations are initial parts of the sequences of chains with large numbers of impurities, which

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become optimal as temperature increases. A mechanism of conduction along these chains was proposed by Pollak and Hauser[4] and investigated in detail by Tartakovskiĭ *et al.*[5] The general description of the transition from resonant tunneling to hopping conduction involving long chains is given in the section labeled Conclusions.

## 2. INELASTIC TUNNELING VIA A PAIR OF IMPURITIES

We shall consider the contribution of one two-impurity configuration to the tunnel current across an amorphous film located between two metal edges. The states at the edges will be described by quasimomenta **k** and **p** and we shall assume that the impurity 1 is located closer to the left-hand edge, whereas impurity 2 is close to the right-hand edge. In the absence of the electron–phonon interaction the tunnel Hamiltonian governing the **k** → **p** transitions is

$$\begin{aligned}H_0 = &\sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} a_{\mathbf{k}}{}^+ a_{\mathbf{k}} + \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} a_{\mathbf{p}}{}^+ a_{\mathbf{p}} + \varepsilon_1 a_1{}^+ a_1 + \varepsilon_2 a_2{}^+ a_2 \\ &+ \sum_{\mathbf{k}} (T_{\mathbf{k}1} a_{\mathbf{k}}{}^+ a_1 + T_{\mathbf{k}1}{}^* a_1{}^+ a_k) \\ &+ \sum_{\mathbf{p}} (T_{\mathbf{p}2} a_{\mathbf{p}}{}^+ a_2 + T_{\mathbf{p}2}{}^* a_2{}^+ a_{\mathbf{p}}) + T_{12} a_1{}^+ a_2 + T_{12}{}^* a_2{}^+ a_1. \qquad (2.1)\end{aligned}$$

Here, $\varepsilon_j$ and $a_j^+$ are the energies and operators for creation of electrons in appropriate states ($j = \mathbf{k}, \mathbf{p}, 1, 2$). The matrix elements $T_{k1}$, $T_{p2}$, and $T_{12}$ depend exponentially on the corresponding distances, so that Eq. (2.1) includes transitions from an impurity to the nearest edge. The electron–phonon interaction causes essentially all the constants in the Hamiltonian (2.1) to depend on the phonon variables. However, as pointed out in Ref. 2, inclusion of such a dependence in $\varepsilon_{\mathbf{k}}$, $\varepsilon_{\mathbf{p}}$, $T_{\mathbf{k}1}$, and $T_{\mathbf{p}1}$ gives rise to corrections which are small in the parameter $\omega_D/E_F$ ($\omega_D$ is the Debye frequency, $E_F$ is the Fermi energy of an electron at the edges, and $\hbar = 1$). We shall show later that the phonon corrections to $T_{12}$ can also be ignored. After these simplifications the term due to phonons in the Hamiltonian becomes

$$\begin{aligned}H_{e\text{-}ph} = &\sum_{\mathbf{q}} \omega_{\mathbf{q}} b_{\mathbf{q}}{}^+ b_{\mathbf{q}} + a_1{}^+ a_1 \sum_{\mathbf{q}} (\alpha_{\mathbf{q}1} b_{\mathbf{q}}{}^+ + \alpha_{\mathbf{q}1}{}^* b_{\mathbf{q}}) \\ &+ a_2{}^+ a_2 \sum_{\mathbf{q}} (\alpha_{\mathbf{q}2} b_{\mathbf{q}}{}^+ + \alpha_{\mathbf{q}2}{}^* b_{\mathbf{q}}). \qquad (2.2)\end{aligned}$$

Here, $\omega_q$ and $b_q^+$ are the spectrum and the phonon creation operator, the quantities $\alpha_{\mathbf{q}n}$ are related to the constants of the deformation potential $\Lambda_n$ of impurities by[1)]

$$\alpha_{\mathbf{q}n} = \frac{i\Lambda_n q}{(2\rho\omega_{\mathbf{q}})^{1/2}} \exp(i\mathbf{q}\mathbf{R}_n), \qquad (2.3)$$

where $\rho$ is the density of matter in the film, $\mathbf{R}_n$ is a coordinate of an impurity, and $n = 1, 2$. The Hamiltonian of Eq. (2.2) can be diagonalized by the unitary transformation

$$\hat{U} = \exp\left\{\sum_{\mathbf{q}} \frac{\alpha_{\mathbf{q}1} a_1{}^+ a_1 + \alpha_{\mathbf{q}2} a_2{}^+ a_2}{\omega_{\mathbf{q}}} b_{\mathbf{q}}{}^+ - \text{H.c.}\right\}. \qquad (2.4)$$

The exact form of the transformed total Hamiltonian $H = \hat{U}(H_0 + H_{e-ph})\hat{U}^+$ is fairly cumbersome and we shall not write it down. We shall consider only the case of a weak electron–phonon interaction, which allows us to limit the Hamiltonian to the terms linear in $\alpha_{\mathbf{q}n}$. Moreover, we shall ignore the possibility of phonon-assisted transitions between impurities and the edges, because such channels are shunted by more effective zero-phonon transitions in the case of low values of $\alpha$. Bearing all these points in mind, we find that

$$\begin{aligned}H = &\sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} a_{\mathbf{k}}{}^+ a_{\mathbf{k}} + \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} a_{\mathbf{p}}{}^+ a_{\mathbf{p}} + \varepsilon_1 a_1{}^+ a_1 + \varepsilon_2 a_2{}^+ a_2 \\ &+ \sum_{\mathbf{q}} \omega_{\mathbf{q}} b_{\mathbf{q}}{}^+ b_{\mathbf{q}} \\ &+ \sum_{\mathbf{k}} (T_{\mathbf{k}1} a_{\mathbf{k}}{}^+ a_1 + T_{\mathbf{k}1}{}^* a_1{}^+ a_{\mathbf{k}}) + \sum_{\mathbf{p}} (T_{\mathbf{p}2} a_{\mathbf{p}}{}^+ a_2 + T_{\mathbf{p}2}{}^* a_2{}^+ a_{\mathbf{p}}) \\ &+ (T_{12} a_1{}^+ a_2 - T_{12}{}^* a_2{}^+ a_1) \sum_{\mathbf{q}} (\lambda_{\mathbf{q}} b_{\mathbf{q}}{}^+ - \lambda_{\mathbf{q}}{}^* b_{\mathbf{q}}) \\ &+ (T_{12} a_1{}^+ a_2 + T_{12}{}^* a_2{}^+ a_1), \qquad (2.5)\end{aligned}$$

where $\lambda_{\mathbf{q}} = (\alpha_{\mathbf{q}1} - \alpha_{\mathbf{q}2})/\omega_{\mathbf{q}}$. A characteristic energy which occurs in $T_{12}$ is the depth of the localized state $E_0 \sim 1/ma^2$ in the region of the barrier. It follows from Eq. (2.5) and from the definition $\lambda_{\mathbf{q}}$ that the phonon renormalization constants of $T_{12}$ dropped from Eq. (2.2) are small compared with terms of the type $\lambda_{\mathbf{q}} T_{12} a_1^+ a_2 b_{\mathbf{q}}^+$, of order the parameter $\omega_{\mathbf{q}}/E_0 \sim T/E_0$, which are included in Eq. (2.5).

Our aim will be to calculate the current via a two-impurity channel in the nonresonant case, i.e., when the dissipation between the levels is large compared with the overlap integral: $\Delta\varepsilon = |\varepsilon_1 - \varepsilon_2| \gg T_{12}$. This inequality makes it possible to apply the kinetic equation because the states do not become hybridized at the impurity centers and the lifetime of an electron at each of them (limited by the interaction with phonons) is given by $\tau \sim \rho s^5/\Lambda^2 \Delta\varepsilon T_{12}^2$ and is known to be longer than the time taken for the formation of a state in the case of an interimpurity transition $1/\Delta\varepsilon$. The system of kinetic equations derived using standard methods based on the Hamiltonian of Eq. (2.5) can be written in the form

$$\begin{aligned}\dot{f}_1 = &\sum_{\mathbf{k}} 2\pi |T_{\mathbf{k}1}|^2 (f_{\mathbf{k}} - f_1) \delta(\varepsilon_{\mathbf{k}} - \varepsilon_1) \\ &+ \sum_{\mathbf{q}} 2\pi |T_{\mathbf{q}}|^2 [f_2(1 - f_1) N_{\mathbf{q}} - f_1(1 - f_2)(N_{\mathbf{q}} + 1)] \delta(\varepsilon_2 + \omega_{\mathbf{q}} - \varepsilon_1), \qquad (2.6\text{a})\end{aligned}$$

$$\begin{aligned}\dot{f}_2 = &\sum_{\mathbf{q}} 2\pi |T_{\mathbf{q}}|^2 [f_1(1 - f_2)(N_{\mathbf{q}} + 1) - f_2(1 - f_1) N_{\mathbf{q}}] \delta(\varepsilon_2 + \omega_{\mathbf{q}} - \varepsilon_1) \\ &+ \sum_{\mathbf{p}} 2\pi |T_{\mathbf{p}2}|^2 (f_{\mathbf{p}} - f_2) \delta(\varepsilon_{\mathbf{p}} - \varepsilon_2), \qquad (2.6\text{b})\end{aligned}$$

$$\dot{f}_{\mathbf{k}} = 2\pi |T_{\mathbf{k}1}|^2 (f_1 - f_{\mathbf{k}}) \delta(\varepsilon_{\mathbf{k}} - \varepsilon_1), \qquad (2.6\text{c})$$

$$\dot{f}_{\mathbf{p}} = 2\pi |T_{\mathbf{p}2}|^2 (f_2 - f_{\mathbf{p}}) \delta(\varepsilon_{\mathbf{p}} - \varepsilon_2). \qquad (2.6\text{d})$$

In writing down the system of equations (2.6) we are assuming that $\varepsilon_1 > \varepsilon_2$, which does not limit the generality of the treatment[2)]; $N_{\mathbf{q}} = N(\omega_{\mathbf{q}})$ is the equilibrium distribution function of phonons; $f_{\mathbf{k}}$, $f_{\mathbf{p}}$, $f_1$, and $f_2$ are the distribution functions of electrons at the edges and at the centers; and $T_{\mathbf{q}} = \lambda_{\mathbf{q}} T_{12}$. In Eqs. (2.6c) and (2.6d) we have included only the terms associated with electron transitions from the edges to the impurities. The collision integrals resulting in relaxation of the distribution function at the edges will not be given. Nevertheless, we shall assume that the relevant relaxation times are short, so that in solving the tunnel problem we can regard the distributions $f_{\mathbf{k}}$ and $f_{\mathbf{p}}$ as given. The relationships (2.6c) and (2.6d) determine the tunnel currents from the states **k** and **p** at the edges. The accuracy of the kinetic equation is insufficient to allow for the tunnel widths $\Gamma_1$ and $\Gamma_2$ of the impurity levels.[6] Therefore, $f_{\mathbf{k}}$ and $f_{\mathbf{p}}$ in the system (2.6) should be regarded as smooth functions of energies on the scale of $\Gamma$, which in the case of equilibrium distributions $f_{\mathbf{k}} = f_l(\varepsilon_{\mathbf{k}})$ and $f_{\mathbf{p}} = f_r(\varepsilon_{\mathbf{p}})$ sets restrictions on temperature: $T \gg \Gamma_1$, $\Gamma_2$. Under steady-state conditions ($\dot{f}_1 = \dot{f}_2 = 0$) the current flowing via a two-impurity channel can be represented with the aid of Eqs. (2.6c) and (2.6d) in the form

$$J=2e\Gamma_1[f_l(\varepsilon_1)-f_1]=2e\Gamma_2[f_2-f_r(\varepsilon_2)], \qquad (2.7)$$

where

$$\Gamma_1=\pi\sum_{\mathbf{k}}|T_{\mathbf{k}1}|^2\delta(\varepsilon_{\mathbf{k}}-\varepsilon_1), \qquad \Gamma_2=\pi\sum_{\mathbf{p}}|T_{\mathbf{p}2}|^2\delta(\varepsilon_{\mathbf{p}}-\varepsilon_2).$$

It follows from Eqs. (2.6a), (2.6b), and (2.7) that

$$J=\frac{4e\chi}{R+(R^2-4\chi/\Gamma_1\Gamma_2)^{1/2}}, \qquad (2.8)$$

where

$$R=\frac{f_l+N}{\Gamma_2}+\frac{1-f_r+N}{\Gamma_1}+\frac{1}{\gamma}, \qquad (2.9)$$
$$\chi=(f_l-f_r)N+f_l(1-f_r),$$

$f_l \equiv f_l(\varepsilon_1)$, $f_r \equiv f_r(\varepsilon_2)$, $N = N(\Delta\varepsilon)$. The value of $\gamma$ determines the reciprocal of the electron lifetime in the case of the $1 \to 2$ transition for $N = 0$:

$$\gamma=\pi\sum_{\mathbf{q}}|T_{\mathbf{q}}|^2\delta(\Delta\varepsilon-\omega_{\mathbf{q}}). \qquad (2.10)$$

In the calculation of the current-voltage characteristics for voltages $eV \gg T$ across a contact we need the expression for the current in the limit when $N = 0$, $f_l = 1$, and $f_r = 0$. We then have

$$\chi=1, \quad R=1/\Gamma_1+1/\Gamma_2+1/\gamma. \qquad (2.11)$$

The value of the current given by Eq. (2.8) differs by no more than a factor of $2e/R$. Therefore, the current is in fact governed by the sum of the resistances of three links joining the opposite edges [see Eq. (2.11)]. Such an estimate is valid for any ratio of $eV$ to $T$, provided both $f_l$ and $f_r$ are not exponentially small in the parameter $d/a$.

If $eV \ll T$, then in the approximation linear in respect of $eV/T$, it follows from Eq. (2.8) that

$$J=V\mathscr{G}(\mathbf{r}_1,\mathbf{r}_2,\varepsilon_1,\varepsilon_2), \qquad (2.12)$$

$$\mathscr{G}(\mathbf{r}_1,\mathbf{r}_2,\varepsilon_1,\varepsilon_2) =\frac{e^2}{2T}\left[\Gamma_1^{-1}\operatorname{ch}^2\frac{\varepsilon_1}{2T}+\Gamma_2^{-1}\operatorname{ch}^2\frac{\varepsilon_2}{2T} +2\gamma^{-1}\operatorname{sh}\frac{|\varepsilon_1-\varepsilon_2|}{2T}\operatorname{ch}\frac{\varepsilon_1}{2T}\operatorname{ch}\frac{\varepsilon_2}{2T}\right]^{-1}. \qquad (2.13)$$

Equation (2.13) is valid for any sign of the difference $\varepsilon_1 - \varepsilon_2$; the energies $\varepsilon_1$ and $\varepsilon_2$ are measured from the Fermi level.

## 3. CONDUCTANCE AND CURRENT-VOLTAGE CHARACTERISTIC OF A LARGE-AREA JUNCTION

We shall calculate the conductance of a junction in the case which is linear in the voltage when the inequality $eV \ll T$ is obeyed. This can be done by averaging the conductance for one channel over the coordinates and energies of a pair of impurities in this channel. Integration with respect to the two coordinates of the first impurity $(x, y)$ gives the contact area $S$ and

$$\langle G_2\rangle=Sg^2\int d\varepsilon_1\,d\varepsilon_2\,dz_1\,dz_2\,d^2\rho\,\mathscr{G}(z_1,z_2,\rho,\varepsilon_1,\varepsilon_2), \qquad (3.1)$$

where $z_1$ and $z_2$ are the distances from the impurities to the corresponding edges, and $\boldsymbol{\rho}$ is the projection of the vector $\mathbf{r}_1 - \mathbf{r}_2$ on the $xy$ plane.

Since the denominator of Eq. (2.12) is in the form of a sum of three quantities exponentially dependent on $z_1$, $z_2$, and $|\mathbf{r}_1 - \mathbf{r}_2|$, its extremum is attained at the values of $\mathbf{r}_1^{(0)}$ and $\mathbf{r}_2^{(0)}$ for which these three quantities are equal and we have $\rho = 0$. Replacing $z_1$ and $z_2$ with new coordinates $\zeta_{1(2)} = z_{1(2)} - z_{1(2)}^{(0)}$ and introducing the expansion

$$|\mathbf{r}_1-\mathbf{r}_2|=|\mathbf{r}_1^{(0)}-\mathbf{r}_2^{(0)}|-(\zeta_1+\zeta_2)+\rho^2/2|\mathbf{r}_1^{(0)}-\mathbf{r}_2^{(0)}|,$$

we find that

$$\langle G_2\rangle=\frac{\pi}{2^{4/3}}\frac{e^2}{T}g^2S\int d\varepsilon_1\,d\varepsilon_2\,\Gamma_0(|\varepsilon_1-\varepsilon_2|) \cdot\left[\operatorname{sh}^{1/3}\frac{|\varepsilon_1-\varepsilon_2|}{2T}\operatorname{ch}\frac{\varepsilon_1}{2T}\operatorname{ch}\frac{\varepsilon_2}{2T}\right]^{-1} \cdot\int d\zeta_1\,d\zeta_2\,d^2\rho\left\{\exp\frac{2\zeta_1}{a}+\exp\left(\frac{2\zeta_2}{a}\right) +\exp\left[-\frac{2(\zeta_1+\zeta_2)}{a}+\frac{\rho^2}{a|\mathbf{r}_1^{(0)}-\mathbf{r}_2^{(0)}|}\right]\right\}^{-1}, \qquad (3.2)$$

$$\Gamma_0=(\Gamma_1\Gamma_2\gamma)^{1/3}|_{\rho=0}.$$

The integral with respect to the coordinates can be calculated by introducing dimensionless variables. The integral with respect to the energies can be found by identifying the dependence of $\gamma$ on $\Delta\varepsilon$. For one pair of impurities it follows from Eq. (2.10)

$$\gamma=\Delta\varepsilon\,\frac{\Lambda^2T_{12}^{\;2}}{2\pi\rho s^5}, \qquad 2\Lambda^2=\begin{cases}\Lambda_1^{\,2}+\Lambda_2^{\,2}, & \Delta\varepsilon d/s\gg 1,\\ (\Lambda_1-\Lambda_2)^2, & \dfrac{\Delta\varepsilon d}{s}\ll\dfrac{|\Lambda_1-\Lambda_2|}{(\Lambda_1\Lambda_2)^{1/2}},\end{cases} \qquad (3.3)$$

where $s$ is the velocity of sound. We shall assume that for $\Delta\varepsilon \sim T$, we have one of the limiting cases and $\Gamma_0 \propto \Delta\varepsilon^{1/3}$. In the first case we can ignore the scatter $\delta\Lambda$ of the values of the deformation potential constant and regard $\Lambda$ in Eq. (3.3) as its characteristic value. In the second limiting case Eq. (3.2) makes it possible to carry out additional averaging over the values of $\Lambda$. This averaging in all the subsequent expressions, replaces $\Lambda$ with the characteristic quantity $\delta\Lambda$. Introduction of dimensionless variables into the energy integral in Eq. (3.2) gives the average conductance in the leading order in $a/d$:

$$\langle G_2\rangle=\nu(\Lambda^2E_0/\rho s^5)^{1/3}e^2g^2a^3dST^{4/3}\Gamma, \qquad (3.4)$$

where the numerical coefficient is $\nu \approx 50$. The main thickness dependence in Eq. (3.4) is contained in the quantity $\Gamma$, which determines the characteristic width of the impurity levels in the channels under discussion:

$$\Gamma=E_0 \exp(-2d/3a). \qquad (3.5)$$

The limitations of the adopted method have the effect that Eq. (3.4) is valid at temperatures $T > \Gamma$. At low temperatures $T \sim \Gamma$ the processes of inelastic scattering of electrons are weakened and give rise only to exponentially small corrections to the conductance $\langle G_1\rangle \propto e^2 gaSE_0 \exp(-d/a)$, which is due to the resonant tunneling processes.[6]

The contribution made to the conductance by inelastic two-impurity channels exceeds the contribution of $\langle G_1\rangle$ at temperatures

$$T \gtrsim (ga^2dE_0)^{-3/4}(\Lambda^2E_0^2/\rho s^5)^{-1/4}E_0 \exp(-d/4a). \qquad (3.6)$$

We can naturally expect a further increase in temperature to result in activation of the processes of conduction in chains consisting of three or more impurities. Estimating the contribution of three-impurity channels to the average conductance by the method described above, we obtain

$$\langle G_3\rangle \sim (\Lambda^2E_0/\rho s^5)^{1/2}e^2g^3a^5d^2ST^{5/2}E_0 \exp(-d/2a).$$

The faster rise of $\langle G_3\rangle$ with temperature has the effect that Eq. (3.4) determines the conductance of a film when

$$T \lesssim (ga^2dE_0)^{-6/7}(\Lambda^2E_0^2/\rho s^5)^{-1/7}E_0 \exp(-d/7a). \qquad (3.7)$$

Therefore, at low temperatures defined by Eq. (3.7) the main contribution to the temperature dependence of the conductance of an amorphous film comes from two impurity channels. The only exception is the case of exponentially small values of the density of states

$$g \lesssim \frac{1}{a^2\,dE_0}\left(\frac{\Lambda^2E_0^2}{\rho s^5}\right)^{4/11} \exp\left(-\frac{3d}{11a}\right), \qquad (3.8)$$

when the temperature correction $\delta G_1$ mentioned in Sec. 1 is due to inelastic tunneling across single localized states which may exceed the contribution of many-impurity configurations.

We can find the average current $\langle I(V)\rangle$ flowing through two-impurity channels provided we average Eq. (2.8) for the current via one pair of impurities over the coordinates and energies of these impurities:

$$\langle I(V)\rangle = g^2S\int dz_1\,dz_2\,d^2\boldsymbol{\rho}\,d\varepsilon_1\,d\varepsilon_2\,J(z_1, z_2, \boldsymbol{\rho}, \varepsilon_1, \varepsilon_2). \qquad (3.9)$$

Equation (3.9) can be simplified in the nonlinear limit we are interested in, $eV \gg T$. We can use the relationships (2.11) for $\chi$ and $R$ and integration with respect to energy in Eq. (3.9) can be limited to the range

$$-eV/2 < \varepsilon_2 < \varepsilon_1 < eV/2. \qquad (3.10)$$

The error introduced by these simplifications into $\langle I(V)\rangle$ is of the order of $T/eV \ll 1$.

Integration with respect to coordinates can be carried out in the same way as in the calculation of $\langle G_2\rangle$. The result is that, to leading order in $a/d$ and $T/eV$, the average current is

$$\langle I(V)\rangle \sim (\Lambda^2E_0/\rho s^5)^{1/3}eg^2a^3dS(eV)^{7/3}\Gamma. \qquad (3.11)$$

The validity of Eq. (3.11) is governed by the efficiency of two-impurity channels. The corresponding limitations on $V$ can be found in the same way as the limitations on $T$ derived in an analysis of Eq. (3.4). They reduce to the replacement of $T$ with $eV$ in the inequalities used above. In particular, the $I \propto V^{7/3}$ law applies in spite of restrictions on $V$ deduced from Eqs. (3.6) and (3.7).

## 4. MESOSCOPIC FLUCTUATIONS OF THE CONDUCTANCE AND THE CURRENT-VOLTAGE CHARACTERISTIC

In the case of a sample of finite thickness $S$ the conductance $G_2(T)$ and the current-voltage characteristic $I(V)$ are random quantities. The condition for these quantities to be close to their average values, calculated in Sec. 3, is the large number of effective two-impurity channels. By an effective channel for the quantity $G_2$ we understand a pair of impurities for which $\mathcal{G}(\mathbf{r}_1,\mathbf{r}_2,\varepsilon_1,\varepsilon_2)$ is close to the maximum value. An analysis of Eq. (2.12) shows that the width of the maximum in terms of the coordinates $\delta z_1$ and $\delta z_2$ is $\sim a$; $\delta\rho \propto (ad)^{1/2}$ and the width of the maximum in respect of the energies $\delta\varepsilon_1$ and $\delta\varepsilon_2$ is $\sim T$. Therefore, in a sample of cross-sectional area $S$ the effective number of the conduction channels is

$$N_G \sim Sg^2a^3dT^2. \qquad (4.1)$$

In estimating fluctuations of the quantity $G_2$ we shall first find the variance of the conductance:

$$\langle(\delta G_2)^2\rangle = \langle G_2^2(T)\rangle - \langle G_2(T)\rangle^2, \qquad (4.2)$$

where the angular brackets denote averaging for a batch of samples. Calculation of the variance (4.2) reduces to averaging of the square of the conductance in one channel[7]:

$$\langle(\delta G_2)^2\rangle = Sg^2\int dz_1\,dz_2\,d^2\boldsymbol{\rho}\,d\varepsilon_1\,d\varepsilon_2\,\mathcal{G}^2(z_1, z_2, \boldsymbol{\rho}, \varepsilon_1, \varepsilon_2), \qquad (4.3)$$

yielding the result

$$\langle(\delta G_2)^2\rangle \sim e^4g^2a^3dST^{2/3}\Gamma^2(\Lambda^2E_0/\rho s^5)^{2/3}. \qquad (4.4)$$

According to Eqs. (4.4) and (4.1), the relative rms fluctuation of the conductance is governed by the number of channels:

$$\delta G_2/\langle G_2\rangle \sim 1/N_G^{1/2}. \qquad (4.5)$$

The fluctuations are small for $N_G \gg 1$, i.e., when

$$S \gg S_G \equiv (g^2a^3dT^2)^{-1}. \qquad (4.6)$$

The quantity $S_G$ introduced in this way represents the contact area containing on the average one effective two-impurity conduction channel.

Fluctuations of the conductance about the average value are small and it is sufficient to satisfy Eq. (4.6). However, the true value of the variance of the conductance of a sample is not given by the contribution of (4.4) of two-impurity channels. Moreover, there are rare but fairly transparent configurations of impurities, which determine the variance. In the range of areas[3)]

$$S_G \ll S \ll S_G/ga^2dT \qquad (4.7a)$$

the distribution function of the conductance $F(G)$ of samples is in the form of a Gaussian peak, the position and width of which are governed by two-impurity channels. If $G - \langle G_2\rangle \gg \delta G_2$, this peak is shifted to the lower but wide wing. It determines the variance of $G$, but in the case of typical samples the scatter of the conductance is described by Eq. (4.5). Raĭkh and Ruzin[7] pointed out the possibility that the magnitude of the conductance fluctuations in typical samples can be compared with the value calculated from the variance. However, for the class of models investigated in Ref. 7 the average value of the conductance and its typical fluctuations are governed by channels of a different kind so that the distribution function differs greatly from the Gaussian profile for all regions of variation of $G$.

In a study of mesoscopic fluctuations of the current-voltage characteristic of a contact we shall also restrict our attention to the areas in which only two-impurity channels are important:

$$S_I \ll S \ll S_I / g a^2 d e V. \tag{4.7b}$$

Here, $S_I = [g^2 a^3 d (eV)^2]^{-1}$ is the area of a contact with one effective current channel on the average. Fluctuations of the random quantity $I(V)$ can be described conveniently using the correlation function

$$K_I(V, U) = \langle I(V) I(U) \rangle - \langle I(V) \rangle \langle I(U) \rangle. \tag{4.8}$$

To calculate this function it is necessary to average the product $J(V)J(U)$ of the currents of Eq. (2.8) via one channel with two impurities over the coordinates and energies of these impurities. It is also necessary to allow for the changes in the energies of the impurities caused by the voltage applied to the contact:

$$\varepsilon_1 = \varepsilon_1^{(0)} + \frac{d - 2z_1}{d} \frac{eV}{2}, \quad \varepsilon_2 = \varepsilon_2^{(0)} - \frac{d - 2z_2}{d} \frac{eV}{2}. \tag{4.9}$$

In the calculation of the correlation function of Eq. (4.8) at high voltages to leading order in the parameter $T/eV \ll 1$ we can repeat the simplifications adopted in Sec. 3: we can use the relationships in Eq. (2.11) and limit the range of integration to that given by Eq. (3.10). The integration can then be carried out in terms of elementary functions. We shall give here only the principal terms of the asymptote in the case of similar voltages, $0 < U - V \ll V$:

$$K_I^{(0)}(V, U) \propto e^2 g^2 S a^3 d \Gamma^2 \left( \frac{\Lambda^2 E_0}{\rho s^5} \right)^{2/3} (eV)^{8/3} \left[ 1 + \frac{4}{9} \frac{U - V}{V} - 3^{-4/3} \left( \frac{U - V}{V} \right)^{5/3} \right]. \tag{4.10}$$

For $U = V$, Eq. (4.10) gives the variance of the current

$$\langle (\delta I)^2 \rangle = (\Lambda^2 E_0 / \rho s^5)^{2/3} e^2 g^2 S a^3 d \Gamma^2 (eV)^{8/3}, \quad eV \gg T \tag{4.11}$$

A comparison of Eq. (4.11) with the expression (3.11) for $\langle I(V) \rangle$ gives the relative magnitude of fluctuations of the current:

$$\delta I / \langle I \rangle \sim (S_I / S)^{1/2}. \tag{4.12}$$

An increase in the voltage $V$ increases the conductance due to two-impurity channels. The associated characteristics of such scale in the dependence $K_I^{(0)}(V,U)$ is $\Delta V_1 \sim V$. An attempt to calculate the variance of the differential conductance by means of Eq. (4.10) leads however to a physically unjustified divergence due to the nonanalytic dependence of $K_I^{(0)}$ on $U - V$. The nonanalyticity is due to incorrect inclusion in Eq. (4.10) of the contribution of new channels activated on increase in the voltage. These channels give rise to corrections to Eq. (4.10) which are small in absolute amplitude, but which determine an additional "small" scale, different from $V$, of fluctuations $\Delta V_2 \sim T/e \ll V$. Thus, the deviation of the dependence $I(B)$ for a given sample from the average characteristic of Eq. (3.11) has two scales: limited small-scale fluctuations are superimposed on large but smooth fluctuations of the scale $\Delta V_1$ (Fig. 1). Obviously, the fluctuations with the scale $\Delta V_2$ are more conveniently investigated by considering the voltage dependence of the differential conductance $G(V)$. The correlation function

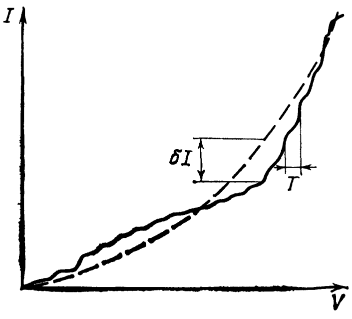

FIG. 1. Schematic representation of the current-voltage characteristic of a sample of finite area. The dashed curve shows the dependence $\langle I(V) \rangle$.

$$K_G(V, U) = \langle G(V) G(U) \rangle - \langle G(V) \rangle \langle G(U) \rangle \tag{4.13}$$

can be calculated with the aid of Eqs. (2.8) and (2.9). To leading order in $T/eV$, we find that

$$K_G(V, U) = K_G^{(0)}(V, U) + \delta K_G(V, U). \tag{4.14}$$

Two terms in Eq. (4.14) correspond to two types of fluctuations (Fig. 2). The large-scale fluctuations correspond to the correlation function $K_G^{(0)}(V,U)$. For $U - V > T$, this correlation function determines the behavior of $K_G(V,U)$ and can be found from Eq. (4.10):

$$K_G^{(0)}(V, U) = \partial^2 K_I^{(0)}(V, U) / \partial V \partial U.$$

The small-scale fluctuations of $G(V)$ are characterized by a correlation function $\delta K_G(V,U)$, which falls exponentially in the range $U - V > T$:

$$\delta K_G(V, U) \sim \delta K_G(V, V) \frac{U - V}{T} \exp\left( - \frac{U - V}{3T} \right), \tag{4.15}$$

but dominates in the range $U - V < T$ and governs the vari-

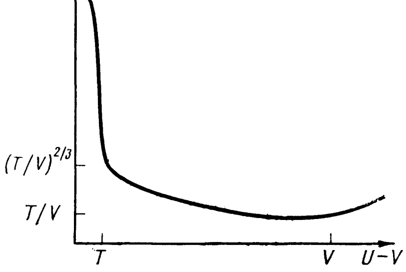

FIG. 2. Correlation function $K_G(U - V)$. The dependence shown exhibits small-scale fluctuations in the region $U - V \lesssim T$.

ance of the differential conductance

$$\langle(\delta G(V))^2\rangle = K_G(V,V) \approx \delta K_G(V,V)$$
$$\sim \left(\frac{\Lambda^2 E_0}{\rho s^5}\right)^{2/3} g^2 a^3\, dS\, \Gamma^2 \frac{(eV)^{5/3}}{T}. \qquad (4.16)$$

This makes it possible to determine the variance from measurements on one sample by the following procedure. We introduce a random function

$$\xi(V) = G(V) - \overline{G(V)}, \qquad (4.17)$$

where the bar denotes averaging in the voltage range $\Delta$:

$$\overline{G(V)} = \frac{1}{\Delta} \int_{V-\Delta/2}^{V+\Delta/2} G(V')\,dV', \quad \frac{T}{e} \ll \Delta \ll V, \qquad (4.18)$$

and we then determine the quantity $\eta(V) = \overline{\xi^2(V)}$. It is shown in the Appendix that the quantity $\eta(V)$ is self-averaging and it is identical (apart from quantities small in terms of the parameters $T/e\Delta$ and $\Delta/V$) with the variance of Eq. (4.16).

## 5. CONCLUSIONS

It is shown in Sec. 3 that the tunnel mechanism of conduction in an amorphous layer is effective only at exponentially low temperatures and when the temperature described by Eq. (3.6) is reached, it changes to inelastic tunneling via impurity pairs. Further increase in temperature activates inelastic channels characterized by large numbers of impurities. The average conductance due to $n$ impurity channels can be estimated as in the case when $n = 2$:

$$\langle G_n\rangle \sim e^2 \frac{S}{ad}\left(\frac{\Lambda^2 E_0{}^2}{\rho s^5}\right)^{(n-1)/(n+1)}$$
$$\cdot (ga^2\, dTn^2)^n \left[\frac{E_0}{T}\exp\left(-\frac{d}{a}\right)\right]^{2/(n+1)}. \qquad (5.1)$$

It is clear from Eq. (5.1) that for $T > T_n$, where

$$T_n \sim (ga^2 dn^2)^{-1}[(\rho s^5/\Lambda^2 E_0) ga^2 d \exp(-d/a)]^{2/(n^2+n+2)}, \qquad (5.2)$$

then $n$-impurity chains make a greater contribution to the average conductance than $(n-1)$-impurity chains. The average conductance of a film is given by

$$\langle G\rangle = \sum_{n=1} \langle G_n\rangle. \qquad (5.3)$$

In the temperature interval

$$T_n < T < T_{n+1} \qquad (5.4)$$

the largest value in the sum of Eq. (5.3) contains the term $\langle G_n\rangle$. If $n$ in Eq. (5.4) satisfies the condition $n \lesssim n^* = (d/a)^{1/3}$, the other terms contribute only small corrections to the value $\langle G\rangle = \langle G_n\rangle$. ["Switching" of the main term in Eq. (5.3) occurs at the limits of the intervals defined by Eq. (5.4).] Therefore, in the range $T \lesssim T_{n^*}$, we have

$$\ln(T_{n^*} gad^2) \sim -(d/a)^{1/3}, \qquad (5.5)$$

so that at each fixed temperature only the chains with equal numbers of impurities can be included in the expression for the conductance. At high temperatures ($T \gg T_{n^*}$) the number of terms in Eq. (5.3) governing $\langle G\rangle$ is

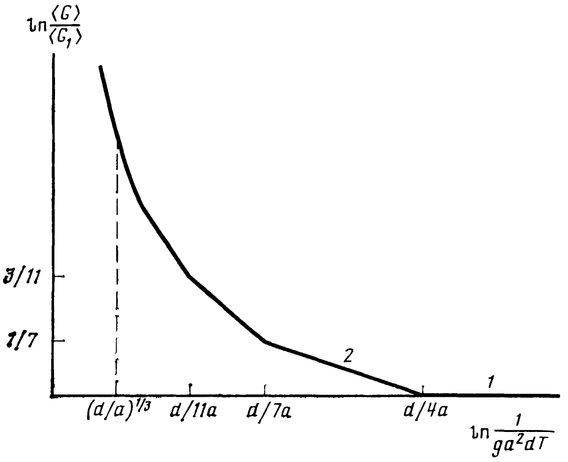

FIG. 3. Temperature dependence of the conductance $\langle G\rangle$ in the region of the transition from resonant tunneling to conduction by hopping in long chains.

$$\Delta n \sim \left[\frac{d}{a}\ln^{-3}\frac{1}{ga^2\, dT}\right]^{1/4}. \qquad (5.6)$$

We can then regard $n$ as a continuous variable and replace the sum with an integral with respect to $n$, as was done in Ref. 5. The temperature dependence of $\langle G\rangle$ can be readily plotted in terms of the coordinates $\{\ln(\langle G\rangle/\langle G_1\rangle), \lambda(T) = \ln(1/ga^2dT)\}$ (Fig. 3). Resonant tunneling at temperatures $T < T_2$ corresponds to the horizontal region 1 in Fig. 3. The temperature ranges $T_2 < T \lesssim T_{n^*}$ correspond to the broken line in which each section represents one term in Eq. (5.3). The slope of each step is $d\ln\langle G\rangle/d\lambda = -[n - 2/(n+1)]$, where $n$ varies from 2 to $n^*$. At temperature $T > T_{n^*}$ (to the left of the dashed vertical line in Fig. 3) the broken curve merges to form a smooth variation $\ln\langle G\rangle \sim -2[(2d/a)(\lambda(T) + \ln(a/d))]^{1/2}$ (see Ref. 5). We investigated the behavior of the conductance at temperatures corresponding to the initial region 2 of the broken curve.

The conductance of sufficiently thin amorphous films at $T = 0$ is governed by elastic tunneling of electrons along chains containing two or more resonant impurities,[6] and not along single states. Nevertheless, the main dependence of the average conductance on the thickness remains the same: $\langle G\rangle \propto \exp(-d/a)$, because the improvement in the width of the resonance for configurations of this kind is compensated by the loss in the probability of formation. Consequently, the condition governing the transition from elastic to inelastic tunneling is identical (apart from the preexponential factor) with Eq. (3.6). We can show that the channels in which an electron travels partly by resonance tunneling and partly due to phonon-activated jumps do not make a significant contribution to the conductance at any temperature. Therefore, the above description of the transition from the resonant to the hopping conduction mechanism remains valid also in the case of thick films.

The conductance in the transition regime can be investigated, for example, at contacts with an amorphous silicon spacer. If $d = 250$ Å, the above characteristic temperatures are $T_2 = 2$ K and $T_3 = 15$ K. Since the number of two-impurity inelastic channels per unit area is considerably less than the number of resonant one-impurity configurations, mesoscopic effects occurring in the range $T > T_2$ appear in samples of relatively large area. In our example when $T = T_3 - T_2$ the characteristic area is $S_G \approx 5\times 10^{-6}$–$2.5\times 10^{-6}$ cm$^2$ (we used $g = 10^{20}$ eV$^{-1}$ cm$^{-3}$ and $a = 8$ Å—see Ref. 3). Another object in which this effect can be observed is a metal–insulator–semiconductor transistor with a sufficiently wide and short inversion layer. The fabrication of such structures has become possible recently.[8,9]

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## APPENDIX

In view of the Gaussian statistics of fluctuations of two-impurity channels, when the condition (4.7) is satisfied, the variance of the quantity of interest to us $\eta(V)$ can be estimated from

$$\langle(\delta\eta(V))^2\rangle = \langle\eta^2(V)\rangle - \langle\eta(V)\rangle^2$$

$$= \frac{2}{\Delta^2} \iint\limits_{V-\Delta/2}^{V+\Delta/2} dV'\, dV'' \langle\xi(V')\xi(V'')\rangle^2. \qquad \text{(A1)}$$

If $T/e \ll \Delta \ll V$, the main contribution to the integral in Eq. (A1) comes from a band $|V' - V''| \lesssim T$, because outside it the integrand is small. In fact, apart from quantities of the order of $T/e\Delta$, the average value is

$$\langle\xi(V)\xi(U)\rangle = K_G(V,U) - K_G(\bar{V},U) - K_G(V,\bar{U}) + K_G(\bar{V},\bar{U}). \qquad \text{(A2)}$$

Here a bar above the argument denotes averaging in accordance with Eq. (4.18) and with respect to this argument. This averaging procedure reduces $\delta K_G$ in the $U - V \lesssim T$ case to $(T/\Delta)\delta K_G(V,V)$. Therefore, in the range of $UV$ of interest to us, we find from Eqs. (4.14) and (A2) that

$$\langle\xi(V)\xi(U)\rangle \approx \delta K_G(V,U), \qquad \text{(A3)}$$

and hence for $V = U$ we have $\langle\eta(V)\rangle \approx \langle(\delta G)^2\rangle$. Using Eqs. (A1), (A3), and the definition $\eta(V) = \overline{\xi^2(V)}$, we find that

$$\delta\eta/\langle\eta(V)\rangle \sim (T/\Delta)^{1/2} \ll 1.$$

The last inequality ensures self-averaging of $\eta(V)$.

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1) Generally speaking the deformation potential constant shows a scatter for an amorphous material.

2) In Eqs. (2.6a) and (2.6b) the correlation function $\langle a_1^+ a_1 a_2^+ a_2\rangle$ is decoupled. We can easily obtain a more rigorous system of kinetic equations which do not require this approximation and show that such decoupling does not affect the results given in Secs. 3–5. The problem of the feasibility of such decoupling was also considered in Ref. 10.

3) This integral is exponentially large in terms of the parameter $d/a$ under the conditions described by Eqs. (3.6) and (3.7).

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