Manifestation of quantum-chromodynamic coherence in deep inelastic scattering

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The structure of the final hadronic states in deep inelastic scattering processes is discussed. It is shown that allowance for chromodynamic coherence leads to angular ordering of the bremsstrahlung accompanying the development of the parton system in the spacelike region of momenta.

1. INTRODUCTION

The corroboration of parton ideas, and later of quantum chromodynamics (QCD), is inseparably linked with the physics of lepton-hadron deep inelastic scattering (DIS) (Refs. 1–3). However, until recently there has been no systematic theoretical analysis of the structure of the final hadronic states in DIS,[1)] and, in particular, no understanding of how chromodynamic coherence affects this structure.

As is known from the investigation of jets in e^+e^- annihilation, the phenomenon of the coherence of soft gluon emission limits in an essential way the bremsstrahlung multiplication of partons in the cascade that develops in the spacelike region of momenta. A consequence of the coherence—the angular ordering of successive parton decays—leads, in particular, to a "humped" plateau in the energy spectrum of the particles in the jet, and the maximum of this plateau increases with frequency and is displaced into the high-energy region with increase of the hardness (annihilation energy) of the process.

In scattering processes the picture of the development of the parton system differs from that in the annihilation process. Here it is necessary to analyze the structure of the parton wavefunction of the target hadron, which is formed long before the instant of the hard interaction and corresponds to a spacelike bremsstrahlung cascade. As we shall see below, the effect of the coherence here, as in the case of a timelike cascade, leads to a picture of soft bremsstrahlung emission with angular ordering.

When discussing the structure of the final state of a scattering process with momentum transfer $-Q^2 \gg \mu^2$ and a fixed value of the Bjorken variable x one must take two phenomena into account: the decay of the produced parton fluctuation, the coherence of which is destroyed by the "removal" of a virtual quark (target fragmentation), and the evolution of the ejected quark (current fragmentation). Below we shall discuss the inclusive distributions of the final particles in the most natural frame for the DIS process—the Breit frame ($Q_0 = 0, 2x\mathbf{P} = -\mathbf{Q}$), in which the products of the fragmentation of the target and current are sharply distinguished in their direction of motion (parallel and antiparallel to the target momentum $\mathbf{P}$).

Not the least role in the justification of the Feynman hypothesis that there is a single hadronic plateau in DIS (see Fig. 1) has been assigned to the argument that it is necessary to compensate the fractional charges, since the role of the charged partons in the Feynman picture had been given unconditionally (and intuitively) to quarks. At the same time, serious doubts have been expressed about the possibility of organizing such a state dynamically if one starts from natural ideas about multiple production of particles as a result of successive decays of outgoing partons.

The problem has been formulated most clearly by V. N. Gribov in Ref. 6, in which it was shown that in a DIS process the products of the decay of the produced fluctuation leave the rapidity interval $0 < \ln \omega < \ln Q$ in the region of fragmentation of the target unoccupied (Fig. 1b). At the basis of this conclusion lay an analysis of the spatiotemporal pattern of the development of the process in the framework of a field-theoretical approach to the description of the wave function (WF) of the target hadron as a coherent system of partons. In a brief account the absence of hadrons with momenta $\omega \ll Q$ in the target fragmentation in the Gribov picture is explained by the fact that the coherence of the parton WF in this region of the spectrum is, in fact, not destroyed by hard ejection of a parton with momentum $xP \simeq Q/2$, as a consequence of which the upper part of the parton comb "collapses" as if there were no scattering at all (see Fig. 1b).

The experimental detection of an even plateau (Fig. 1a), which was interpreted as a proof of the correctness of the identification of partons with quarks, did not, in reality, eliminate the Gribov-Feynman paradox (GFP) but only sharpened it, since it required one to point to a physical mechanism responsible for the filling of the "hole" that

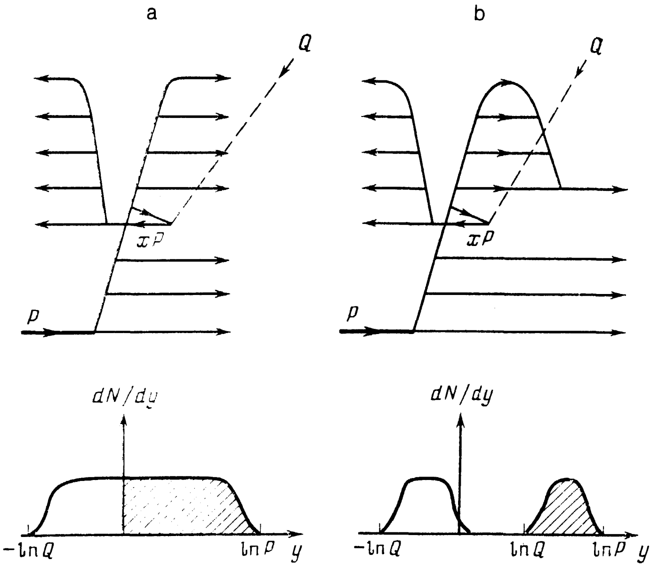

FIG. 1. Structure of the hadron plateau in a DIS process according to Feynman (a) and Gribov (b) (the region of fragmentation of the target is shaded).

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would not come into conflict with causality and quantum mechanics.

From present-day standpoints the foundations for the Gribov phenomenon have only been strengthened, since QCD indeed corroborates the interpretation of the structure of a relativistic hadron in terms of a field fluctuation composed of quasireal quarks and gluons—the partons of QCD.

We shall show that the way in which the GFP is resolved in the context of QCD is, in a certain sense, eclectic. On the one hand, here the Gribov phenomenon does indeed occur; namely, the energy spectrum of the products of the fragmentation of the partons—elements of the fluctuation ("ladder") that determines the DIS cross section—is concentrated in the region of large energies $Q \leqslant \omega \leqslant P$. On the other hand, there is a mechanism combining the fragmentation of the target and current in the spirit of the Feynman picture. This role is taken on by coherent bremsstrahlung emission of soft gluons at relatively large angles, which is determined by complete transfer of color in the scattering process and is therefore insensitive to details of the structure of the parton WF of the target hadron.

From the point of view of the color transfer, in the Breit frame the process appears as the flying apart of color 3 and $\bar{3}$ states (the ejected q and the "excited" nucleon as a whole). The similarity with e^+e^- annihilation makes it possible to expect similarities in the nature of the spectra of the final particles as well. This expectation is fully justified in respect of the current fragmentation. The bremsstrahlung processes accompanying the emission of a bare quark lead to the formation of a jet identical to the q-jet in e^+e^- annihilation at energy $W^2 = -Q^2$. In particular, a "humped" plateau arises in the energy spectrum of the products.

Much more complicated (and, at the same time, more interesting) is the organization of the region of fragmentation of the target, in which the final state is formed by a system of color currents distributed in phase space. The "valence" mechanism of DIS (Fig. 2a) generates a particle spectrum which, in the regions of fragmentation of both the target and the current, is similar to the annihilation spectum at[2)] $W^2 = -Q^2$. This statement is valid for values of x not too close to unity, when the limitation of the phase volume of the final hadronic state becomes important.

The internal structure of the parton fluctuation of the target is manifested most clearly in DIS for $x \ll 1$, when the cross section of the process is determined by many-step ladders of the type shown in Fig. 2b. Cascade processes of multiplication of partons lead in this case (when interference phenomena are taken into account) to an energy distribution of particles that differs substantially in its form from the spectrum in the region of current fragmentation. In the formation of the resulting yield of hadrons an important role is played by the fact that in QCD the source of the sea quarks that determine the DIS structure functions at small values of x consists of bremsstrahlung pairs, in an octet color state, of a q and a $\bar{q}$ with similar rapidities (with gluon exchange in the t-channel; see Fig. 2b).

The article is organized as follows. In Secs. 2 and 3 we analyze coherent effects in the cross section and in the structure of the DIS final states for $x \ll 1$. It is shown that the ordering in the transverse momenta of the parton fluctuations that determine the structure functions for small values of x is a consequence of the coherence. The result of Sec. 3 is a description of the structure of the DIS final state in probability language in terms of the accompanying emission, which is organized in a certain way with respect to the angles and transverse momenta. Section 4 is devoted to the effect of quark insertions into the structure gluon ladders. In Sec. 5 we obtain approximate analytical formulas describing the inclusive energy spectra of the final particles and give the results of calculations.

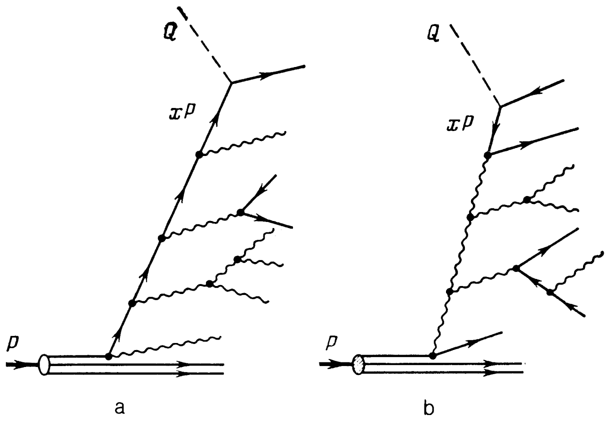

FIG. 2. Typical parton QCD fluctuations: a) valence contribution ($x \sim 1$); b) sea contribution ($x \ll 1$).

2. DIS FOR $x \ll 1$. COHERENT EFFECTS IN THE CROSS SECTION

For small values of x the DIS cross section (with power accuracy in x) is determined by the distribution of sea quarks. A common feature of the diagrams corresponding to this cross section is a gluon cut in the t-channel, ensuring (thanks to the vector nature of the gluon) that the structure functions $D_F^{F'}(x)$ increase as $1/x$ with decrease of x (see Fig. 3).

Before turning to the discussion of the final states corresponding to such fluctuations, we note that for $x \ll 1$ coherence phenomena already have a substantial effect on the magnitude of the cross section of the process. In fact, whereas for $x \sim 1$, as is well known, it is possible to interpret the structure of the parton fluctuations equally successfully in terms of ladders that are strongly ordered with respect to the transverse momenta, the angles of emission of the real partons, or the virtualities of the t-channel rungs, in our case (for $x \ll 1$) carelessness in the choice of the evolution parameter risks loss of control over significant contributions of the order of

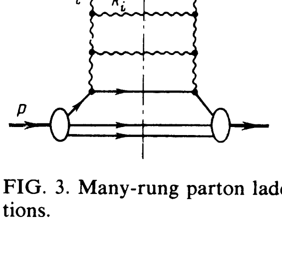

FIG. 3. Many-rung parton ladder determining the DIS structure functions.

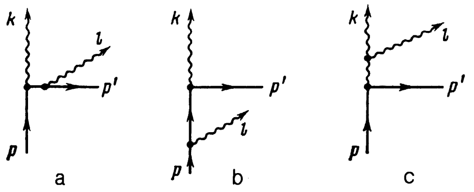

FIG. 4. Lowest-order amplitudes for the emission of a soft gluon l from a ladder cell $p \to p' + k$.

$$\alpha_s \ln^2(1/x) \gg \alpha_s. \quad (1)$$

The ordering of the $k_{i\perp}$ of the partons turns out to be correct in this situation.[7,8] To verify this we shall consider a fragment of the parton fluctuation, choosing for definiteness the lower cell of Fig. 3.

We are interested in the kinematic region (see Fig. 4)

$$\beta_{p'} \equiv \beta' \gg \beta_l \gg \beta_k, \quad (2)$$

where β is the Sudakov component of the momenta, parallel to the initial momentum p of the target (quark),

$$\beta' + \beta_l + \beta_k = \beta_p \equiv \beta_0. \quad (3)$$

A contribution to the emission by a quark of a relatively soft bremsstrahlung gluon l is given by the amplitudes corresponding to the three Feynman diagrams shown in Fig. 4.

We can integrate over two independent variables from the three $k_{i\perp}$ of the partons, which are connected by the relation

$$\mathbf{k}_\perp + \mathbf{l}_\perp + \mathbf{p}_\perp' = \mathbf{p}_\perp = 0. \quad (4)$$

To single out the region in which each of the three amplitudes has logarithmic behavior it is necessary to choose the variables in such a way that one of the independent integrations is associated with the virtuality $\varkappa^2$ of the intermediate state. This virtuality is determined by the relative angle at which the final partons p' and l fly apart in diagram 4a, and, correspondingly, by the angle of emission of l and p' (with respect to the initial direction p) in diagrams 4b and 4c:

$$\varkappa_a^2 = (p'+l)^2 \approx 2p'l = (\alpha'\beta_l + \beta'\alpha_l)s - 2(\mathbf{p}_\perp'\mathbf{l}_\perp) = \frac{p_\perp'}{\beta'}\beta_l + \frac{l_\perp^2}{\beta_l}\beta' - 2(\mathbf{l}_\perp\mathbf{p}_\perp') = \beta'\beta_l(\theta' - \theta_l)^2,$$
$$\varkappa_b^2 = (p-l)^2 \approx -\beta_0\beta_l\theta_l^2, \quad \varkappa_c^2 = (p-p')^2 \approx -\beta_0\beta'\theta'^2, \quad (5)$$

where we have introduced a two-dimensional vector describing the angle of emission of a parton[3)]:

$$\theta_i \equiv \mathbf{k}_{i\perp}/\beta_i. \quad (6)$$

A straightforward calculation leads to the following representations for the virtuality of parton k:

$$-k^2 = \frac{\beta_k}{\beta'+\beta_l}\{\beta'\beta_l(\theta'-\theta_l)^2 + \beta_0\beta_k\theta_k^2\}, \quad (7a)$$

$$-k^2 = \frac{\beta_k}{\beta'+\beta_k}\{\beta_0\beta_l\theta_l^2 + \beta'\beta_k(\theta'-\theta_k)^2\}, \quad (7b)$$

$$-k^2 = \frac{\beta_k}{\beta_l+\beta_k}\{\beta_0\beta'\theta'^2 + \beta_l\beta_k(\theta_l-\theta_k)^2\}, \quad (7c)$$

where

$$\theta_k \equiv \frac{\mathbf{k}_\perp}{\beta_k} = -\frac{1}{\beta_k}(\beta'\theta' + \beta_l\theta_l).$$

Comparison with (5) shows that the representations (7) are convenient for determining the upper limit of the logarithmic integration over $|\varkappa^2|$, since the first term in each of the formulas (7) is proportional to the quantity $\varkappa_\alpha^2$ ($\alpha = a, b, c$) that appears together with k^2 in the denominator of the corresponding Feynman amplitude of Fig. 4.

The conditions

$$\beta'\beta_l(\theta'-\theta_l)^2 \ll \beta_0\beta_k\theta_k^2, \quad (8a)$$

$$\beta_0\beta_l\theta_l^2 \ll \beta'\beta_k(\theta'-\theta_k)^2, \quad (8b)$$

$$\beta_0\beta'\theta'^2 \ll \beta_l\beta_k(\theta_l-\theta_k)^2 \quad (8c)$$

for logarithmic behavior lead, in the region of energies (2), to the inequalities

$$\mathbf{r}_\perp^2 \ll \frac{1}{\varepsilon}\mathbf{k}_\perp^2, \quad (9a)$$

$$\mathbf{l}_\perp^2 \ll \frac{1}{\varepsilon}\mathbf{k}_\perp^2, \quad (9b)$$

$$\mathbf{p}_\perp'^2 \ll \frac{1}{\varepsilon}\mathbf{k}_\perp^2, \quad (9c)$$

where

$$\mathbf{r}_\perp \equiv {}^1/_2(\mathbf{p}_\perp' - \mathbf{l}_\perp) \quad (10)$$

is the relative transverse momentum of the final partons and

$$\varepsilon \equiv \frac{\beta_0}{\beta'}\frac{\beta_k}{\beta_l} \ll 1. \quad (11)$$

It follows from this that in a rather wide kinematic region, where the transverse momenta of the partons are large and cancel each other:

$$|\mathbf{k}_\perp| \ll |\mathbf{r}_\perp| \approx |\mathbf{l}_\perp| \approx |\mathbf{p}_\perp'|, \quad (12)$$

the three amplitudes of Fig. 4 are logarithmic simultaneously. However, as can be seen by direct calculation, their sum is found to be equal to zero. The process of Fig. 4 can be interpreted as the decay of a relativistic particle p into two particles (p' and l) in an external field k. The phenomenon under discussion is then fully analogous to the vanishing of the forward ($k_\perp \to 0$) diffraction-dissociation amplitude, which was observed by V. N. Gribov and has played an important role in Regge field theory by demonstrating the possibility, in principle, of a so-called weak-coupling regime (see Ref. 10).

In space-time language the condition (12) implies that the transverse size of the fluctuation

$$p \to p' + l \to p, \quad (13)$$

in the form of which the particles pass through the interaction region, has turned out to be smaller than the wavelength of the scattering field. If here the interaction is associated with a conserved current (and this is the situation in QCD), the absorption of a quantum of a field (k) that does not "feel" the fluctuation and so cannot change the internal state of the incident particle leads only to elastic scattering.

From the above reasoning it is clear that the fact of the complete destructive interference in the region (12) does not depend on the type of colored particle (or decay mode) and is valid not only for the cell $q \to qg$ that we have considered but also in the case of an initial gluon.

Thus, we have demonstrated the coherent nature of the phenomenon that gives rise to the ordered increase of the $k_{i\perp}$ of the partons from the bottom to the top of the ladder fluctuation determining the DIS cross section for $x \ll 1$ (Refs. 7, 8).

Eliminating the region (12) from consideration, we finally obtain the following conditions:

$$\beta'\beta_l(\theta'-\theta_l)^2 \ll \beta_0^2\mathbf{k}_\perp^2/\beta'\beta_l, \quad (14a)$$

$$\mathbf{l}_\perp^2 \ll \mathbf{k}_\perp^2 \approx \mathbf{p}_\perp'^2, \quad (14b)$$

$$\mathbf{p}_\perp'^2 \ll \mathbf{k}_\perp^2 \approx \mathbf{l}_\perp^2. \quad (14c)$$

The last two inequalities imply natural ordering of the corresponding ladder diagrams with respect to the $k_{i\perp}$ of the partons. The first inequality, which limits the invariant mass $\varkappa_a^2$ of the pair, has a somewhat more complicated structure. In the case when the gluon l is hard, i.e.,

$$\beta_l \sim \beta' \sim \beta_0 \quad (\gg \beta_k),$$

the condition (14a) gives

$$\varkappa_a^2 \ll k_\perp^2 \quad (15)$$

In terms of the emission angles $\theta_{i\perp}$ the inequalities (14) take the form

$$|\theta'-\theta_l| \ll \theta' \approx \theta_l, \quad (16a)$$

$$\theta_l \ll \theta', \quad (16b)$$

$$\theta' \ll \theta_l. \quad (16c)$$

The parton configurations corresponding to the inequalities (16) do not overlap kinematically. Therefore, the amplitudes in the logarithmic approximation that correspond to them do not interfere and give independent contributions to the cross section of the scattering process.

The squares of the amplitudes corresponding to Figs. 4b and 4c generate standard ladder diagrams that are ordered with respect to the $k_{i\perp}$ of successive cells; diagram a (decay in the final state) effectively participates, together with virtual corrections, in the formation of the "running coupling constant" α_s $(k_\perp^2)$ [the dispersion relation with respect to the mass $\varkappa_a^2$, within the limits of the inequality (15); see Refs. 11 and 2]. If the gluon l is relatively soft [see (2)], interference of amplitudes a and b is possible, since, in addition to the bremsstrahlung cones about the initial quark and final quark, the apex angles of which are determined by the scattering angle θ', according to (16a) and (16b) the inequalities (14a) and (14b) are also satisfied simultaneously by soft emission through large angles in conditions when

$$\theta' \ll \theta_l \ll \theta'\beta_0/\beta_l \quad (\text{i.e., } l_\perp \ll k_\perp \approx p_\perp'). \quad (17)$$

3. COHERENCE IN THE DISTRIBUTIONS OF THE FINAL PARTICLES

In the analysis of the cross section of a process the coherence in the distributions of the final particles usually remains "out of sight", since (both at small and at large angles) the soft emission of this type cancels completely with the corresponding virtual graphs, while having no effect on the magnitude of the DIS structure functions.[11,2] However, if we are interested not in the total cross section of the process but in the distribution of the particles that are formed in it, the problem of accurate allowance for interference phenomena is resurrected.

As is not difficult to understand, coherent addition of the amplitudes a and b (Fig. 4) in the kinematic region (17) leads to a result that is fully analogous to the annihilation result: The large angle emission is determined not by the color characteristics of the initial parton p and scattered parton p' separately, but by the total t-channel color transfer (i.e., by the gluon color for the cell of Fig. 4 under consideration).

For $\theta \cong |\theta - \theta'| \gg \theta'$ the displacement of the quark in the transverse plane

$$\Delta\rho_\perp \approx t_{\text{form}}\,\theta' \quad (18)$$

in the time of formation of the emission

$$t_{\text{form}} \sim p/|(p-l)^2| \approx p'/(p'+l)^2 \approx 1/l\theta^2 \quad (19)$$

turns out to be smaller than the wavelength:

$$\Delta\rho_\perp \approx \frac{1}{l\theta}\frac{\theta'}{\theta} \ll \frac{1}{l\theta} = \frac{1}{l_\perp} \equiv \lambda. \quad (20)$$

In the case of quantum-electrodynamic (QED) scattering in this situation bremsstrahlung emission would be absent, since for large angles of observation [see (17)] the coherence of the proper quark field would not be destroyed—it would be as if the charge had not changed the direction of motion at all. In QCD, however, the color transition current appears not only upon change of direction of the momentum of the particle but also as a result of its color "charge exchange." Therefore, despite the smallness of the angle θ', wide-aperture bremsstrahlung gluon emission does arise, and its intensity is determined by the Casimir operator of the t-channel parton.

We note that an analogous phenomenon—dependence of the accompanying emission on the color exchange in the t-channel—is responsible for the formation of the so-called true plateau—the flat universal distribution of hadrons in the rapidity interval

$$-\tfrac{1}{2}\ln(s/|t|) < \eta < \tfrac{1}{2}\ln(s/|t|)$$

in processes with large P_T (for $|t| \ll s$).[12]

Up to now we have discussed the kinematic region (2), in which the gluon l, while soft in relation to the target parton, remains harder than the momentum transfer:

$$\beta_0 \gg \beta_l \gg \beta_k. \quad (21)$$

More typical is the situation when the accompanying gluon l is found to be the softest:

$$\beta_l \ll \beta_k, \quad \beta' \lesssim \beta_0. \quad (22)$$

Returning to the initial conditions (8) for logarithmic behavior, we obtain in this case for the first two amplitudes of Figs. 4a and 4b ($|\mathbf{k}_\perp + \mathbf{p}_\perp'| = l_\perp \ll k_\perp \approx p_\perp'$)

$$(\theta_l - \theta')^2 \ll \theta'^2/\bar{\varepsilon}, \quad (23a)$$

$$\theta_l^2 \ll \theta'^2/\bar{\varepsilon} \quad (23b)$$

and for the amplitude of Fig. 4c

$$\theta_l^2 \gg \theta'^2/\bar{\varepsilon}. \tag{23c}$$

Here $\bar{\varepsilon} \equiv \beta_l \beta_k / \beta_0 \beta' \ll 1$. The quantity $\theta'^2/\bar{\varepsilon}$ in the role of a limit of integration over the gluon-emission angle θ_l^2 (an artefact of the Feynman technique) disappears when interference phenomena are taken into account. Indeed, for the diagrams a and b one can identify the already familiar proper cones of the initial quark and final quark, on which the amplitudes do not interfere:

$$|\boldsymbol{\theta}_l - \boldsymbol{\theta}'| \ll \theta' (\approx \theta_l), \quad dw = |a|^2 \propto C_F d^2\theta_l / |\boldsymbol{\theta}_l - \boldsymbol{\theta}'|^2 \tag{24a}$$

and

$$\theta_l \ll \theta' (\approx |\boldsymbol{\theta}_l - \boldsymbol{\theta}'|), \quad dw = |b|^2 \propto C_F d^2\theta_l / \theta_l^2. \tag{24b}$$

For large emission angles ($\theta_l \approx |\boldsymbol{\theta}_l - \boldsymbol{\theta}'| \gg \theta'$)

$$dw = |a+b|^2 \quad \text{for } \theta'^2 \ll \theta_l^2 \ll \theta'^2/\bar{\varepsilon}, \tag{25a}$$

$$dw = |c|^2 \quad \text{for } \theta_l^2 \gg \theta'^2/\bar{\varepsilon}. \tag{25b}$$

But the amplitudes a and b in (25a) differ only in the sequence of the quark generators in the color factor and, when summed, reproduce the structure c. As a result, in the entire range of angles $\theta_l > \theta'$,

$$dw \propto N_c d^2\theta_l / \theta_l^2, \tag{26}$$

i.e., the t-channel parton k can be regarded as the only source of soft accompanying emission.

It is clear that the physical reason ensuring that the contributions that interfere for $\theta_l > \theta'$ combine to produce independent t-channel emission is the conservation of the current. Therefore, the analysis performed is valid for any ladder cell

$$A(p) \to B(p') + T(k), \tag{27}$$

where A, B, and T are the type of the initial parton, final parton, and exchange parton ($A = B = q$ and $T = g$ in the example considered in Fig. 4).

Each parton decay (27) is accompanied by two narrow bremsstrahlung cones:

$$\theta_l < \theta' \quad \text{for } |\boldsymbol{\theta}_l - \boldsymbol{\theta}'| < \theta'$$

about the initial parton A and final parton B, with emission intensity determined by the color charges C_A and C_B, respectively, and by the soft gluon emission through large angles

$$\theta_l > \theta',$$

which is proportional to C_T. This result can be generalized without difficulty to the physically interesting case of a many-step parton fluctuation (Fig. 3). As we know, the transverse momenta $k'_{i\perp}$, and with them the parton-emission angles θ_i, increase as we move up the ladder determining the DIS cross section. The following serve as sources of the accompanying soft emission: 1) n final bremsstrahlung cones with apex angles $\Delta\theta \lesssim \theta'_i$ and emission intensity proportional to C'_i; 2) the intial cone $\theta_l < \theta'_1$ (with intensity proportional to C_0), and also 3) the t-channel virtual partons k_i, the color C_i of each of which determines the emission intensity in the range of angles

$$\theta_i' < \theta_l < \theta_{i+1}', \quad i = 1, \ldots, n-1$$

(in the top cell $i = n$, $\theta'_n < \theta_l < 1$), as in the case of e^+e^- annihilation; in the formation of emission through a relatively large angle θ_l,

$$\theta_l \gg \theta_m' \gg \theta_{m-1}' \gg \ldots \gg \theta_1', \tag{28a}$$

the beam of partons k'_i ($i = 1, \ldots, m$) with comparatively small angles of emission takes part as a single entity. The difference of the color currents of the initial parton and the S-channel beam determines the emission intensity in the range of angles

$$\theta_m' < \theta_l < \theta_{m+1}' \tag{28b}$$

through the quantity C_m [the Casimir operator is the "squared color charge" of the t-channel parton m: $C_m = C_F$ ($m = q, \bar{q}$) or $C_m = N_c$ ($m = g$)]. We recall that in the special case when the energy of a gluon l that is soft in relation to the beam:

$$\beta_l \ll \beta_i', \quad i = 1, \ldots, m, \tag{29a}$$

turns out to be greater than the energy transfer

$$\beta_l \gtrsim \beta_m, \tag{29b}$$

the emission angle $\theta > \theta'_n$ is bounded from above by the condition (17) that the transverse momentum be small:

$$l_\perp < k_{m\perp}' \approx k_{m\perp},$$

which replaces the weaker condition (28b):

$$\theta_l = \frac{l_\perp}{\beta_l} < \frac{1}{\beta_l} k_{m\perp} \ll \frac{1}{\beta_l} k'_{m+1,\perp} \approx \frac{\beta_{m+1}}{\beta_l} \theta'_{m+1} \sim \frac{\beta_m}{\beta_l} \theta'_{m+1}. \tag{30}$$

Such contributions, which arise for $x \ll 1$, will be called anomalous in the following.

Thus, we have arrived at the conclusion that, as in the case of a timelike cascade, angular ordering lies at the basis of the probabilistic picture of the emission that accompanies the development of a spacelike parton fluctuation.

4. QUARK INSERTIONS IN THE GLUON LADDER

As was shown above, the result of taking interference into account can be formulated in terms of the probabilities of accompanying emission that is organized in a certain way in the phase space of the structure ladder. This circumstance makes it possible to perform partial summation over the parton configurations and to obtain for the contributions (structure, soft, and anomalous) to the inclusive spectrum expressions containing two D-functions describing the evolution of the spacelike fluctuation before and after a particular cell; the fragmentation function $\bar{D}(\omega/l, \ln l_\perp/\Lambda)$ of the structure parton, in the framework of the logic of local parton-hadron duality,[4] can be regarded as a hadron (h), with $\bar{D}^h$ identified with the corresponding hadronic e^+e^--annihilation spectra. Here and below we use the standard notation D and $\bar{D}$ for the DIS parton distribution and the fragmentation function for $F \to h + X$ (the definitions of these functions, and also a discussion of the methods and results of the calculation of these functions in QCD perturbation theory, can be found in Refs. 2, 3, and 14).

First of all we recall the analytical expressions for the parton distributions $D(x, Q^2, Q_0^2)$ in the leading logarithmic approximation (LLA). In the representation in terms of moments,

$$D_A^B(x) = \int \frac{dj}{2\pi i} x^{-j} D_A^B(j) \tag{31}$$

the functions $D(j)$ are linear combinations of exponentials (for more detail, see Ref. 2)

$$C_A^B(\sigma, j) \exp[\nu_\sigma(j)\xi], \tag{32}$$

where $\xi = b^{-1}\ln[\alpha_s(Q_0^2)/\alpha_s(Q^2)]$ is the evolution parameter.

The behavior $D(x) \propto 1/x$ for $x \ll 1$ is determined, according to (31), by the rightmost singularity of $D(j)$ in the j-plane. This is the singularity of the leading singlet trajectory ν_+ (the anomalous dimension) at the point $j = 1$.

When analyzing the accompanying emission it is necessary to know the internal structure of the ladder—in particular, how often q and g are encountered in intermediate states in the t-channel.

The growth of the "sea" in QCD is due to a spacelike gluon cascade. As can be seen from the expansion of $\nu_+(j)$ about $j = 1$:

$$\nu_+(j) = \frac{4N_c}{j-1} - \left(\frac{11}{3}N_c + \frac{2}{3}\frac{n_f}{N_c^2}\right) + O(j-1), \tag{33}$$

the role of quark insertions in the gluon ladder is limited to a numerically small correction to the regular part of the anomalous dimension. The singular term in (33) gives rise to the characteristic growth of $xD(x)$, and the constant determines the pre-exponential factor:

$$D(x) \infty \frac{1}{x}\exp\left[16N_c\xi \ln\frac{1}{x}\right]^{1/2}\left[\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right]^{-a/b}, \tag{34}$$

where

$$a = \frac{11}{3}N_c + \frac{2}{3}\frac{n_f}{N_c^2}, \qquad b = \frac{11}{3}N_c - \frac{2}{3}n_f.$$

It is not admissible to use a wrong value of a in the calculation of the structure functions, since then the relative error of the QCD prediction would begin to grow with Q^2. However, in describing the structure of the final state it turns out to be possible to simplify the situation by replacing the real parton fluctuation by a purely gluon cascade without a substantial loss of accuracy.

The relative frequency of appearance of a quark $(q, \bar{q})$ in the t-channel at a certain stage of the evolution can be estimated as (see Ref. 2)

$$\frac{F}{G} = 2n_f\frac{C_A^F(j)C_F^B(j)}{C_A^G(j)C_G^B(j)} = 2n_f\frac{\Phi_G^F(j)\Phi_F^G(j)}{(\nu_+ - \nu_0)^2} \approx (j-1)\frac{n_f C_F}{3N_c^2}$$
$$\sim\left[\frac{4N_c\xi}{\ln(1/x)}\right]^{1/2}\frac{n_f C_F}{3N_c^2}, \tag{35}$$

where we have made use of a steepest-descent estimate for the values characteristic in (31):

$$\Delta \equiv \langle j-1\rangle \approx \left[\frac{4N_c\xi}{\ln(1/x)}\right]^{1/2} < 1. \tag{36}$$

Expansion in the parameter Δ makes sense if $\ln(1/x)$ is a large quantity. For real values of x it is difficult to regard this parameter as small (typically, $\xi \sim 0.1$–0.2); nevertheless, for an estimate of the role of the q-cells in the gluon cascade the formula (35) is entirely suitable and shows that the probability of encountering a quark instead of a gluon in the ladder is lower than 15% (for $n_f = 3$, $F/G = \Delta\cdot 4/27$). In fact, the error that arises from neglect of the quarks is smaller than F/G by a further order of magnitude, since a $q\bar{q}$ pair deep in the sea acts effectively like one gluon as a source of final particles (see Fig. 5).

In fact, according to the analysis performed in the preceding section, the accompanying emission associated with the particular $q\bar{q}$ cell in Fig. 5a is composed of two bremsstrahlung cones (q-jets) with apex angles

$$\Delta\theta = \theta_1, \quad \Delta\theta = \theta_2 \tag{37}$$

and additional t-channel emission concentrated in the range of angles

$$\theta_1 < \theta < \theta_2, \tag{38}$$

with intensity that is also proportional to C_F. Integrated over the angles this is equivalent to emission of doubled intensity, proportional to $2C_F$ in the region (38), plus a "doubled" q-jet with apex angle θ_1. The first contribution supplements the t-channel accompanying emission of the gluon current below $(\theta < \theta_1)$ and above $(\theta > \theta_2)$ the cell under consideration, while the second coincides approximately with the fragmentation of a g-jet. The relative error in identifying Fig. 5a with Fig. 5b is

$$N_c^{-1}(N_c - 2C_F) = N_c^{-2}.$$

Thus, with good accuracy (an error of $F/GN_c^2 \leqslant 2\%$) we can assume that the intensity of the accompanying t-channel emission through all angles is determined by the gluon color (N_c), despite the mixing of the g- and q-states.

As regards the fragmentation of the structure rungs, here too the contribution of a quark insertion (Fig. 5a) is effectively equivalent to that of a g-jet with apex angle equal to the angle θ_1 of emission of the lower q-line. This factorization—the universality of the final-parton fragmentation factor $\bar{D}_G^h[\omega/l, \ln(l_\perp/\Lambda)]$—simplifies our problem substantially.

5. ANALYTICAL FORMULAS FOR THE SPECTRUM OF PARTICLES IN THE FINAL STATE IN DEEP INELASTIC SCATTERING

We shall represent the final answer in the form of a sum of three contributions. Two of them are asociated with soft emission with

$$l < Q \sim xP. \tag{39}$$

The first of these is determined by the upper q-cell in Fig. 3 and combines a bremsstrahlung cone of an antiquark (with

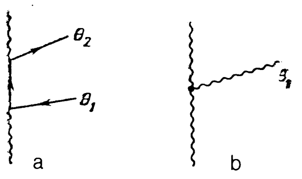

FIG. 5. For the comparative analysis of the accompanying emission by a quark cell (a) and a gluon (b).

energy $\sim Q/2$ and apex angle $\Delta\theta_l=\theta_n')$ from a broken sea $q\bar q$ pair with a t-channel accompaniment from the vertical quark line in the range of angles

$$\theta_n'<\theta_l<1.$$

As a result the spectrum acquires a contribution that coincides with the fragmentation of the ejected q (current fragmentation):

$$\frac{dN^{(I)}}{d\ln\omega}=\frac{\omega}{Q}\bar D_F^h\left(\frac{\omega}{Q},\ln\frac{Q}{\Lambda}\right). \tag{40}$$

The second contribution combines the soft emission from inside the ladder through angles not exceeding θ_n', and can be represented in the form

$$\frac{dN^{(II)}}{d\ln\omega}=\frac{1}{D_A^B}\int_{\mu^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\frac{\alpha_s(k_\perp^2)}{4\pi}\frac{\partial}{\partial\xi_k}D_A^B(x,k_\perp^2,\mu^2)\times\left[\frac{\omega}{Q}\bar D_G^h\left(\frac{\omega}{Q},\ln\frac{k_\perp}{\Lambda}\right)\right]. \tag{41}$$

It is not difficult to see that the derivative of the D-function fixes the angle of emission of the $\bar q$:

$$\theta_n'\approx k_\perp/Q;$$

integration over the energy of l and over the angle θ_l of emission of the soft g with the conditions

$$l<Q,\quad \theta_l<\theta_n'$$

leads, by virtue of the evolution equation for $\bar D$ (Ref. 14), to the appearance of the factor

$$\bar D_G^h[\omega/Q,\ \ln(k_\perp/\Lambda)].$$

The third and last contribution to the spectrum, arising from the kinematic region

$$Q<l<P, \tag{42}$$

is organized in a somewhat more complicated manner. It, in turn, consists of two parts—fragmentation of horizontal rungs, when l is one of the elements of the structure ladder, and an anomalous contribution (see Sec. 3).

The structure contribution can be represented (with neglect of asymptotically small corrections $\sim\alpha_s^{1/2}$) in the simple form

$$\frac{dN^{str}}{d\ln\omega}=\frac{1}{D_A^B}\int_Q^P\frac{dl}{l}\int^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\left[-\frac{\partial}{\partial\xi_k}D_G^B\left(\frac{Q}{l},\ Q_2,\ k_\perp^2\right)\right]\times D_A^G\left(\frac{l}{P},k_\perp^2,\mu^2\right)\frac{\alpha_s(k_\perp^2)}{4\pi}\left[\frac{\omega}{l}\bar D_G^h\left(\frac{\omega}{l},\ \ln\frac{k_\perp}{\Lambda}\right)\right], \tag{43}$$

where

$$d\xi_k\equiv\frac{\alpha_s(k_\perp^2)}{4\pi}d\ln k_\perp^2.$$

In the notation of Fig. 6a the structure contribution corresponds to the kinematic configuration

$$k<l<p,\quad k_\perp\approx l_\perp\gg p_\perp'. \tag{44}$$

Essentially, the energies of the partons in (44) can be assumed to be strongly ordered, although formula (43) with the derivative of the D-function takes correct account of hard decays ($k\sim l$) as well—in particular, the contribution of internal quark cells [to order $O(n_f/[\ln(1/x)]^{1/2}N_c^3)$; see the preceding section].

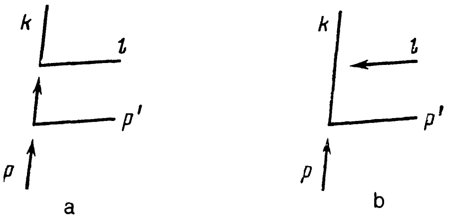

FIG. 6. Kinematics of the structure contribution (a) and anomalous contribution (b); $k\ll l$. Continuous lines link partons with transverse momenta of equal magnitudes.

The anomalous contribution is organized in practically the same way as the structural contribution, differing from the latter only in the region of integration over the momenta. We recall that here the integration over the angle of emission of an accompanying gluon l of t-channel origin is bounded by the condition (30):

$$k\ll l\ll p,\quad k_\perp\approx p_\perp'\gg l_\perp. \tag{45}$$

In the analysis of the anomalous emission the preferred cell turns out to be the first one (as one goes up the ladder of Fig. 3), with respect to which the momentum l can no longer be regarded as soft.

In the anomalous situation the momentum k always belongs to a gluon line, whereas at the basis of the cell (p,p') there can be a parton of either kind $\alpha=q,g$.

The analytical expression corresponding to Fig. 6 has the form

$$\frac{dN^{an}}{d\ln\omega}=\frac{1}{D_A^B}\int^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\frac{\alpha_s(k_\perp^2)}{4\pi}\int_Q^P\frac{dp}{p}\sum_\alpha\int_Q^p\frac{dk}{k}\Phi_\alpha^G\left(\frac{k}{p}\right)\times D_A^\alpha\left(\frac{p}{P},k_\perp^2,\mu^2\right)\int_k^p\frac{dl}{l}4N_c\int^{k_\perp^2}\frac{dl_\perp^2}{l_\perp^2}\frac{\alpha_s(l_\perp^2)}{4\pi}\times\left[\frac{\omega}{l}\bar D_G^h\left(\frac{\omega}{l},\ln\frac{l_\perp}{\Lambda}\right)\right]D_G^B\left(\frac{Q}{k},Q^2,k_\perp^2\right). \tag{46}$$

In the soft approximation

$$\Phi_\alpha^G(k/p)\approx 4C_\alpha p/k+\dots \tag{47}$$

it is possible, by changing the order of integration over the energies, to make use of the evolution equations

$$\int_Q^l\frac{dk}{k}4N_c\left[\frac{Q}{k}D_G^B\left(\frac{Q}{k},Q^2,k_\perp^2\right)\right]\approx-\frac{\partial}{\partial\xi_k}\left[\frac{Q}{l}D_G^B\left(\frac{Q}{l},Q^2,k_\perp^2\right)\right], \tag{48a}$$

$$\sum_\alpha\int_l^P\frac{dp}{p}4C_\alpha\left[\frac{p}{P}D_A^\alpha\left(\frac{p}{P},k_\perp^2,\mu^2\right)\right]\approx\frac{\partial}{\partial\xi_k}\left[\frac{l}{P}D_A^G\left(\frac{l}{P},k_\perp^2,\mu^2\right)\right], \tag{48b}$$

and to bring (46) to the following representation:

$$\frac{dN^{an}}{d\ln\omega}=\frac{1}{D_A^B}\int_{\mu^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\int_Q^P\frac{dl}{l}\left[-\frac{\partial}{\partial\xi_k}D_G^B\left(\frac{Q}{l},Q^2,k_\perp^2\right)\right]$$

$$\times\frac{\partial}{\partial\ln k_\perp^2}D_A^G\left(\frac{l}{P},k_\perp^2,\mu^2\right)$$

$$\times\int^{k_\perp^2}\frac{dl_\perp^2}{l_\perp^2}\frac{\alpha_s(l_\perp^2)}{4\pi}\bar{D}_G^h\left(\frac{\omega}{l},\ln\frac{l_\perp}{\Lambda}\right). \quad (49)$$

Adding (43) to (49) and integrating by parts, we finally obtain

$$\frac{dN^{(III)}}{d\ln\omega}=\frac{1}{D_A^B}\int_Q^P\frac{dl}{l}\int_0^{\xi_Q}d\xi_k\frac{\partial^2}{\partial\xi_k^2}D_G^B\left(\frac{Q}{l},Q^2,k_\perp^2\right)$$

$$\times D_A^G\left(\frac{l}{P},k_\perp^2,\mu^2\right)\int^{k_\perp^2}\frac{dl_\perp^2}{l_\perp^2}\frac{\alpha_s(l_\perp^2)}{4\pi}\bar{D}_G^h\left(\frac{\omega}{l},\ln\frac{l_\perp}{\Lambda}\right). \quad (50)$$

It should be noted that the formula (43) for the structure fragmentation contains an element of double counting, since it incorporates not only the products of the decay of the gluons and the internal $q\bar{q}$ pairs but also a contribution that mimics the fragmentation of the quark-parton k'_n, which has already been taken into account in $dN^{(I)}$. Therefore, from the derivative of the upper D-function:

$$-\frac{\partial}{\partial\xi_k}D_G^B\left(\frac{Q}{l},Q^2,k_\perp^2\right)$$

in (43) one ought, strictly speaking, to subtract a constant—the contribution of the Born diagram:

$$g(l)\to q(k_n)+\bar{q}(k_n').$$

This remark also pertains to the formula (49). However, the final expression (50) no longer contains anything superfluous [the integrated term arising in the integration by parts cancels the parasitic contributions to (43) and (49) that correspond to the situation when the upper D-function "collapses" ($\xi_k\to\xi_Q$, $l\sim Q$)]. The expressions (40), (41), and (50), when summed, determine the complete energy spectrum of the final particles in the region of fragmentation of the target:

$$\frac{dN}{d\ln\omega}=\frac{dN^{(I)}}{d\ln\omega}+\frac{dN^{(II)}}{d\ln\omega}+\frac{dN^{(III)}}{d\ln\omega}. \quad (51)$$

Figure 7 shows the structure of the individual terms in (51) for certain specific values of $\ln(1/x)$ and $\ln(Q/\Lambda)$. The contributions II and III, which are negligibly small for $x\sim1$, increase with decrease of x.

The transverse momenta of the partons—the sources of the jets II and III—are bounded from above by k'_n. With decrease of x the average number of cells in the structure ladder increases, and with it the characteristic angle of emission of the antiquark k'_n. Therefore, the degree of development of the secondary cascades also increases, on the average: The contribution of III increases with increase of $\ln(1/x)$. The smooth decrease of the plateau density with increase of the energy of the particles ($\omega>xP$) is explained by the fact that the more energetic partons are genetically

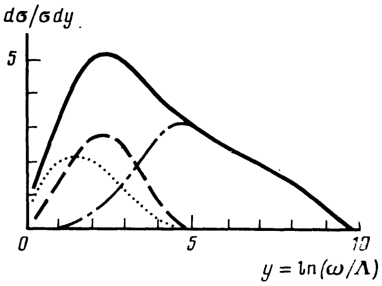

FIG. 7. Contributions to the energy spectrum of particles from the region of fragmentation of the target for $\ln(Q/\Lambda)=5$ and $\ln(1/x)=5$. The dotted line is the quark contribution I, the dashed line is the coherent t-channel emission II, the dashed-dotted line corresponds to the fragmentation of the structure partons (III), and the solid line is the total spectrum.

related to the earlier stage of development of the parton fluctuation (the lower part of the ladder of Fig. 3), where the $k_{i\perp}$, and consequently the cascading effects as well, are smaller.

It is important to stress that the decay products of the partons determining the wavefunction of the target (III) practically do not fall into the energy region $\omega<xP$. This is the chromodynamic realization of the general physical coherent phenomenon predicted by V. N. Gribov in the framework of the orthodox parton picture.[6] At the same time we see that, besides the direct products of the structure partons, which are described by the contribution III displaying the "Gribov hole" phenomenon, in the target-fragmentation region bremsstrahlung particles saturating the region $\omega<xP$ also appear. They carry information about the general structure of the color transfer in the DIS process—about the fact that the total color of the parton system corresponds to the representation 3 (the contribution I), and about the gluon nature of the sea distribution (an octet 8 distributed over the momenta—the contribution II).

As in e^+e^- annihiation, coherence in DIS leads to hardening of the energy spectra: The yield of particles with finite (in the Breit frame) energies

$$\ln(\omega/\Lambda)\sim1$$

should remain practically constant with increase of the hardness of the process. The evolution of the spectra of the final particles with increase of $\ln(Q/\Lambda)$ and $\ln(1/x)$ is displayed in Figs. 8 and 9.

The distributions of Figs. 7–9 under discussion were calculated using the formulas of the LLA for the structure functions D (Ref. 2) and a modified LLA expression for the

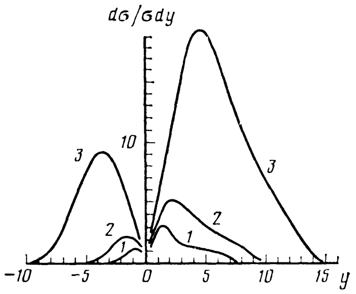

FIG. 8. Evolution of the energy distribution of the hadrons in the Breit frame with variation of Q^2 [$\ln(1/x)=5$]: 1) $\ln(Q/\Lambda)=3$; 2) $\ln(Q/\Lambda)=5$; 3) $\ln(Q/\Lambda)=10$.

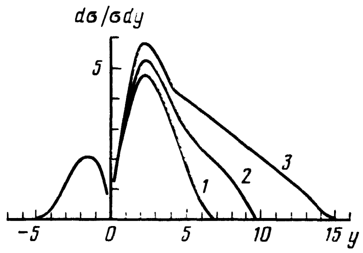

FIG. 9. Evolution of the spectrum with variation of x [$\ln(Q/\Lambda) = 5$]; 1) $\ln(1/x) = 2$; 2) $\ln(1/x) = 5$; 3) $\ln(1/x) = 10$.

fragmentation function $\bar{D}$ (Refs. 14, 10). Neglecting terms of order $O(j-1)$ in (32), it is not difficult to obtain formulas expressing the parton distributions $D_A^B(x, Q^2, Q_0^2)$ for small x in terms of modified Bessel functions. In particular,

$$D_G{}^G(z,\xi)=\left[\frac{4N_c\xi}{\ln(1/z)}\right]^{1/2} I_1\left[\left(16N_c\xi\ln\left(\frac{1}{z}\right)\right)^{1/2}\right] e^{-a\xi},$$
$$\frac{\partial}{\partial\xi}D_G{}^F(z,\xi)\approx\frac{2}{3}D_G{}^G(z,\xi),\qquad D_A{}^G\approx\frac{C_A}{N_c}D_G{}^G. \quad (52)$$

The corrections to (52) are of order $O[(\ln(1/z))^{-1/2}]$. This rough approximation is not obligatory, since the standard LLA makes it possible to monitor the x-dependence of the functions $D_A^B(x, \xi)$ exactly. The approximation made, however, is fully adequate for a qualitative analysis of the character of the spectra and makes it possible to get by without numerical integration of the evolution equations, using the analytical formulas (52).

The applicability of the LLA analysis is limited, strictly speaking, by the condition

$$\frac{\alpha_s}{\pi}\ln\frac{1}{x}\ll 1. \quad (53)$$

The point is that, in our above discussion of the structure of a parton fluctuation, we assumed that the $k_{i\perp}$ of the ladder cells are strongly ordered. It is this basic LLA region, in which each power of α_s is cancelled by a logarithm in $k_\perp$, that determines the terms $\sim\alpha_s$ in the anomalous dimension

$$\gamma_j(\alpha_s)=\frac{\alpha_s}{4\pi}\frac{4N_c}{j-1}-\frac{\alpha_s}{4\pi}a+\left[\left(\frac{\alpha_s}{4\pi}\right)\frac{c}{j-1}\right]^2+\ldots \quad (54)$$

The term in the square brackets corresponds to the situation when the $k_{i\perp}$ of two cells are of the same order of magnitude. The resulting correction to $D(x)$ can be estimated as

$$\int^{Q^2}\frac{dk_\perp{}^2}{k_\perp{}^2}\left(\frac{\alpha_s(k_\perp{}^2)}{4\pi}\right)^2\left(\frac{1}{\langle j-1\rangle}\right)^2\sim\frac{\alpha_s}{\pi}\ln\frac{1}{x}, \quad (55)$$

where the characteristic moment $\langle j-1\rangle$ is determined by (36).

Upon parametric decrease of x the regime of successively increasing values of $k_{i\perp}$ along the ladder should be replaced by a diffusion regime, when the ordering disappears and the characteristic values of $\langle k_{i\perp}^2\rangle$ inside the ladder can significantly exceed Q^2. Physically, this implies a transition from a DIS process to Regge scattering.[15] For a real problem this asymptotic behavior is inaccessible. Nevertheless, in the realistic transition region

$$\frac{\alpha_s(Q^2)}{\pi}\ln\frac{1}{x}\lesssim 1 \quad (56)$$

corrections of the type (55) from integration over values of $k_{i\perp}$ of the same order in a group of cells can substantially modify the x-dependence of the parton distributions. The DIS structure functions $D_A^B(x, Q^2, \mu^2)$ are in the transition region (56) are in need of further theoretical analysis. However, it seems to us that the character of the distributions of the final particles are not substantially affected by this modification of the D-functions, since "clusterization" of the cells with respect to $k_{i\perp}$ with preservation of the general ordering of the fluctuations (monotonic increase of $k_{i\perp}$ with decreasing β_i as we go up the ladder in Fig. 3) does not destroy the strict ordering of the angles of emission of the structure partons—a property that lies at the basis of our analysis of the accompanying emission in Sec. 3. The reason for such an extensive discussion of the applicability of our results in the region (56) is that it is this parameter which determines the relative magnitude of the contributions I–III to the spectrum. This can be seen intuitively from a calculation of the total multiplicity of the particles. Integrating over the energy ω of the detected hadron, we obtain

$$N(Q^2,x)=\frac{C_F}{N_C}N_G\left(\ln\frac{Q}{\Lambda}\right)+\int_{\mu^2}^{Q^2}\frac{dk_\perp{}^2}{k_\perp{}^2}\frac{\alpha_s(k_\perp{}^2)}{4\pi}N_G\left(\ln\frac{k_\perp}{\Lambda}\right)$$
$$\times\frac{[D_A{}^B(x,k_\perp{}^2,\mu^2)]'}{D_A{}^B(x,Q^2,\mu^2)}+\int_{\mu^2}^{Q^2}\frac{dk_\perp{}^2}{k_\perp{}^2}\frac{\alpha_s(k_\perp{}^2)}{4\pi}\int^{k_\perp{}^2}\frac{dl_\perp{}^2}{l_\perp{}^2}\frac{\alpha_s(l_\perp{}^2)}{4\pi}$$
$$\times N_G\left(\ln\frac{l_\perp}{\Lambda}\right)\frac{[D_A{}^B(x,Q^2,\mu^2)]''}{D_A{}^B(x,Q^2,\mu^2)}. \quad (57)$$

Here $N_G[\ln(Q/\Lambda)]$ is the multiplicity of the particles in a gluon jet with hardness parameter Q, and the prime denotes the derivative with respect to ξ. Obviously, two of the three terms correspond to (40) and (41). We shall clarify the origin of the third. After integration of (50) over $\ln\omega$ (contribution III) the dependence on l of the factor involving $\bar{D}$ disappears, since the total multiplicity of the particles in the jet is determined entirely by the quantity $l_\perp$. Therefore, the integral over l can be taken using the completeness relation. First we must replace $\partial^2/\partial\xi_k^2$ by $\partial^2/\partial\xi_Q^2$, which is possible owing to the fact that $D_G^B(x, Q^2, k_\perp^2)$ depends, in the LLA, on the difference $\xi_Q-\xi_k$.

Now we can estimate the magnitudes of the contributions to (57). The multiplicity in a cascade increases fairly rapidly with increase of the transverse momentum. Therefore, the integrals in (57) are determined by the interval $\Delta\ln k_\perp^2\sim\Delta\ln l_\perp^2\lesssim[\ln(Q/\Lambda)]^{1/2}$ near the upper limit. Taking into account the approximate asymptotic relations

$$\int^{Q^2}\frac{dk_\perp{}^2}{k_\perp{}^2}\frac{\alpha_s(k_\perp{}^2)}{4\pi}N_G\left(\ln\frac{k_\perp}{\Lambda}\right)$$
$$\approx\frac{1}{(4N_c)^{1/2}}\left[\frac{\alpha_s(Q^2)}{2\pi}\right]^{1/2}N_G\left(\ln\frac{Q}{\Lambda}\right), \quad (58)$$
$$[D_A{}^B]'=\frac{\partial}{\partial\xi}D_A{}^B(x,\xi)\approx\left[4N_c\frac{\ln(1/x)}{\xi}\right]^{1/2}D_A{}^B(x,\xi),$$

we obtain the following estimate for the multiplicity of the particles in the region of fragmentation of the target:

$$N_{\rm M}^{\rm (DIS)} \approx N_q^{(e^+e^-)}(Q^2)\left\{1+\frac{N_c}{C_F}\left[\frac{\alpha_s(Q^2)}{2\pi}\,\frac{\ln(1/x)}{\xi}\right]^{1/2} + \frac{N_c}{C_F}\,\frac{\alpha_s(Q^2)}{2\pi}\,\frac{\ln(1/x)}{\xi}\right\}, \qquad (59)$$

where $N_q^{(e^+e^-)}$ is the multiplicity of the particles in one q-jet. The total multiplicity of the hadrons in the DIS process is obtained simply by replacing 1 by 2 in the curly brackets in (59).

Thus, we see that the general scale of the multiplicity is determined by the hardness of the process. At the same time, the internal structure of the parton wavefunction determines the natural logarithmic growth of $N^{\rm DIS}$ with decrease of x (the contribution III from the fragmentation of the structure partons).

6. CONCLUSION

The question of the structure of the final states in deep inelastic scattering acquires particular interest in connection with the SSC and LHC collider projects involving the use of proton beams ($Q^2_{\rm max} \cong 10^6\ {\rm GeV}^2$) and with the new experimental possibilities that will be realized on the HERA facility (the ep collider, with $Q^2_{\rm max} \cong 10^5\ {\rm GeV}^2$). Experimental investigations on this problem have already begun (the EMC group).

In discussing the QCD picture of the formation of the final hadronic states in DIS processes we have highlighted the role of coherent phenomena, which are manifested most sharply for small values of x. The fact that, under the condition of sea dominance, the scattering amplitude corresponds to the transfer of gluon color in the t-channel makes the accompanying bremsstrahlung emission in the region of fragmentation of the target twice as intense as that in the current fragmentation.[16]

However, cascade effects substantially modify this simple picture. As in e^+e^- annihilation, instead of a flat inclusive distribution $\omega dN/d\omega$ there appears a spectrum of the humped-plateau type, the density of which decreases monotonically with increase of the energy ω of the particle from $\omega \sim Q \sim xP$ to $\omega \lesssim P$. The shape of the spectrum evolves in the predicted manner with change of Q^2 and x. In particular, the ratio of the heights of the two maxima (fragmentation/current) should increase slightly with decrease of x.

It is not out of place to note that when the QCD predictions are compared with experiment for moderately small values of x it is necessary to take into account also the contribution of the valence mechanism (the nonsinglet component of the structure functions), allowance for which decreases the pronounced difference between the target fragmentation and current fragmentation.

Standard allowance for the contribution of weak interactions, which has a substantial effect on the cross sections for large values of Q^2 but should not modify the character of the spectra of the particles produced, also does not present any special difficulty.

The observation of the asymmetric two-humped energy distribution of the hadrons in the Breit system and of the evolution of the maxima with Q^2 (and, in the region of fragmentation of the target, with $\ln(1/x)$ as well), and the verification of the most important consequence of QCD-coherence and the hypothesis of local parton-hadron duality—the prediction of finiteness (or, more accurately, constancy with Q^2 and x) of the number of particles with energies bounded by $\ln(\omega/m) < 1$ (the region of the "dip")—these are the principal problems facing present-day and future experimenters in the physics of deep inelastic scattering.

The analysis performed in this paper is also completely applicable to the structure of the regions of fragmentation of the colliding hadrons in processes with large $P_\perp$, and to any hard interactions in which the problem of describing the development of QCD cascades in the spacelike region of momenta arises.

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[1)] In recent years this question has been addressed in Refs. 4 and 5.

[2)] We shall ignore the finite (not affecting the character of the asymptotic picture of the hadron distributions) contribution of the spectators and discuss henceforth deep inelastic scattering by a colored particle with characteristic virtuality $\sim\mu^2 = R_{\rm had}\,[x \equiv Q^2/2(QP_{\rm valence\ parton})]$.

[3)] The numerators of the Feynman amplitudes can also be represented in terms of the variables θ_i and β_i; the corresponding technique is developed in Ref. 9.

[4)] Details on the essence and usefulness of the hypothesis of local parton-hadron duality can be found in the bibliography to Ref. 13.

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