Zel'dovich effect in atomic and nuclear physics

V. M. Karnakov,* A. E. Kudryavtsev, V. D. Mur,* and V. S. Popov

Institute of Theoretical and Experimental Physics

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Restructuring of the atomic spectrum (the Zel'dovich effect) can occur in systems coupled by a Coulomb interaction that is distorted at short distances. The features of this phenomenon are discussed for states with arbitrary angular momentum l. The analytic properties of the solution of the basic equation for this effect in the case hadronic-atom spectra are investigated in the presence of absorption. The conditions under which the spectrum restructuring is replaced by an oscillatory regime are obtained. Some manifestations of the Zel'dovich effect in the physics of hadronic atoms and mesic molecules are considered.

1. Hadronic-atom levels are shifted and broadened by strong interactions at short distances. Measurement of these quantities ($\Delta E_{nl}, \Gamma_{nl}$) can yield valuable information on strong-interaction parameters.

Experimental data indicate that atomic-level nuclear shifts are anomalously large in some cases. This is usually due to the presence of a near-zero level in a strong short-range potential $V_s(r)$. This results in a phenomenon called restructuring of the atomic spectrum. It was first observed by Zel'dovich[1] in an investigation of the spectrum the s levels of a valence electron in a doped semiconductor. The possibility of the onset of this effect in hadronic atoms was indicated in Refs. 2 and 3. In Refs. 4–6 the spectra of hadronic atoms were analyzed using the equation

$$2\left[\psi(1-\nu)-\ln\nu+\frac{1}{2\nu}\right]\prod_{j=1}^{l}(j^{-2}-\nu^{-2}) = \frac{1}{\zeta^{2l+1}a_l^{(cs)}}+\frac{r_l^{(cs)}}{2\nu^2\zeta^{2l-1}}, \tag{1}$$

which does not depend on the model of the strong potential V_s and relates the widths and shifts of atomic l-levels with low-energy scattering parameters, viz., nuclear Coulomb scatterng length $a_l^{(cs)}$ and effective radius $a_l^{(cs)}$. Here[1)]

$$\nu=\zeta(-2E)^{-1/2}, \quad E=E_{nl}^{(0)}+\Delta E_{nl}-i\Gamma_{nl}/2,$$

E is the nl-level energy ($n=l+1,\ l+2,\ldots$), $E_{nl}^{(0)} = -\zeta^2/2n^2$, $\zeta = -Z_1Z_2>0$, l) is the orbital momentum, and $\psi(z)=\Gamma'(z)/\Gamma(z)$. The parameter ν is the analog of the principal quantum number n ($\nu=n$ for unshifted Coulomb levels).

Spectrum restructuring in the case of s levels was investigated in full detail[1–7] and its following properties were elucidated:

a) the nuclear s level that perturbs the Coulomb spectrum is always pushed out of the atomic region;

b) the width of the spectrum-restructuring region is small,[2)] of order r_0/a_B, and in this region all the atomic ns levels are strongly displaced relative to the unperturbed Coulomb values $E_{ns}^{(0)}$;

c) for $\nu\gtrsim 1$ (the region of the atomic spectrum) the term in (1) with the effective radius is a small correction.

The present paper deals with the restructuring of an atomic spectrum for states with $l\neq 0$ and also for s states in the presence of a Coulomb barrier. Qualitative differences from the picture described above appear in these cases and can be manifested in experiment: simultaneous restructuring of the entire atomic spectrum gives way to successive collisions of the nuclear l level with the atomic nl levels. Let us describe briefly the content of the paper. In Sec. 2 are presented simple equations for the level shifts, which are valid everywhere including the term crossing region; we discuss also the width of the restructuring region for $l\neq 0$. Section 3 is devoted to the application of the WKB method to the Coulomb problem with short-range interaction. In Sec. 4 is considered the spectrum restructuring in the presence of absorption in a system, a situation typical of hadronic atoms. The applications of the theory are considered in Secs. 5–7. In Sec. 5 we discuss new experimental data on the pp and pd hadronic atoms, in which rather large atomic-level widths were recently observed. In Sec. 6 we calculate, without the use of a model, the nuclear shifts and widths of the levels in mesic molecules, and present a quasiclassical estimate of these quantities. In Sec. 7 we consider the sticking coefficient of a muon to an α particle and the nuclear-fusion reaction $dt\to n\alpha$.

The results of the present paper were reported in part in Refs. 8–11. Following Refs. 7–9, we shall refer to the restructuring of the atomic spectrum (for states with arbitrary l) as the Zel'dovich effect.

2. *Zel'dovich effect for states with* $l\neq 0$. Equation (1) is valid for all $l=0,\ 1,\ 2,\ldots$ under the condition $(-E)^{1/2}r_0 \ll 1$.. The effective radia $r_0^{(s)}$ and $r_0^{(cs)}$ for s levels are of the order of the radius r_0 of the forces; if, however, $l\neq 0$, we have[12,13] at the instant when a level is produced in the potential V_s

$$r_l^{(s)}<0, \quad |r_l^{(s)}|\sim r_0^{1-2l}\gg 1. \tag{2}$$

For $l\geqslant 1$, the term with the effective radius is therefore significant in the right-hand side of Eq. (1), so that this equation becomes a steep function of the parameter ν. Taking this into account, it is clear that the system contains only a nuclear $\nu=\nu_N$ and a Coulomb nl level, which are (in their own scales) weakly displaced (see Fig. 1)

$$\nu_{nl}\approx n+\alpha_{nl}^2(1/\nu_N^2-1/n^2)^{-1}, \tag{3}$$

where $\nu_N = (-\zeta^2 a_l^{(cs)} r_l^{(cs)}/2)^{1/2}$, while α_{nl} is a dimensionless parameter that decreases rapidly with increase of l:

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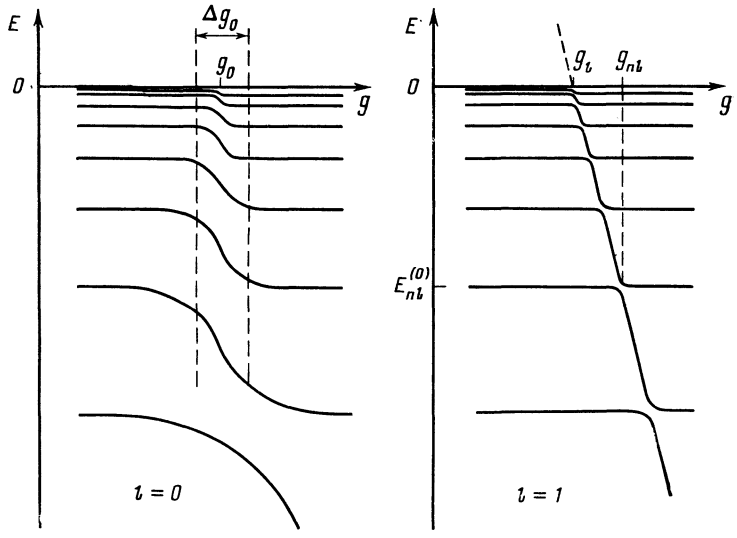

FIG. 1. Restructuring of atomic spectrum for s states and at $l \geqslant 1$. The dimensionless coupling constant is $g \propto |V_s| r_0^2$, where V_s and r_0 are the depth and radius of the strong potential; $E_{nl}^{(0)} = -\zeta^2/2n^2$. At $g < g_l$ the nuclear level $\nu = \nu_N$ is on the unphysical sheet.

$$\alpha_{nl} = 2\zeta^{l-1/2}[p_{nl}/|r_l^{(cs)}|]^{1/2},$$
$$p_{nl} = \frac{1}{(l!)^2}\left(1-\frac{1^2}{n^2}\right)\ldots\left(1-\frac{l^2}{n^2}\right), \quad p_{n0} \equiv 1. \tag{4}$$

Note that $\alpha_{nl} \sim (r_0/a_B)^{l-1/2}$, i.e., α_{nl} is a small parameter if $l \geqslant 1$.

Equation (3) no longer holds when ν_N approaches an integer $n \geqslant l+1$. The nuclear and atomic levels interact in the narrow region $|\nu_N - n| \sim (\zeta r_0)^{l-1/2}$, and their energies are

$$E^{(1,2)} = -\frac{\zeta^2}{2n^2} + \frac{1}{2}\{\delta E_{nl} \pm [(\delta E_{nl})^2 + 2\zeta^4 \alpha_{nl}^2 n^{-3}]^{1/2}\}, \tag{5}$$

where δE_{nl} is the "detuning from resonance" frequently used in atomic physics:

$$\delta E_{nl} = {}^1/_2\zeta^2(n^{-2} - \nu_N^{-2}) = [a_l^{(cs)} r_l^{(cs)}]^{-1} - E_{nl}^{(0)}.$$

Thus, the well known "term quasicrossing" [14] takes place, see Fig. 1.

The physical cause of this phenomenon is that for states with $l \neq 0$ there are two attraction regions separated by a centrifugal barrier $l(l+1)/2r^2$. One of the wells is within the action range of the nuclear forces, and the other is at $r_- < r < r_+$, where

$$r_\pm = \nu^2[1 \pm (1-(l+{}^1/_2)^2/\nu^2)^{1/2}]a_B$$

are the quasiclassical turning points. The barrier penetrability is here $D_l \sim (\zeta r_0)^{2l+1} \ll 1$. The two levels interact when the energy of the nuclear state (localized at $r \lesssim r_0$) approaches the energy of the atomic nl level localized in the region $r_- \lesssim r \lesssim r_+$. At exact resonance ($\nu_N = n$), in both states, which are almost degenerate in energy, the particle can be located, with equal probability at $r \sim r_0$ and in the region of atomic distances $r \lesssim n^2 a_B$. The wave functions corresponding to these states, while equal in the atomic region, differ in sign in the nuclear region $r \sim r_0$ (this ensures their orthogonality). Note that even in this case the shifts of all the l levels are small: $\Delta E_{nl} \sim \zeta^2 \alpha_{nl}$.

Thus, in contrast to the s states, The Zel'dovich effect for $l \neq 0$ comprises the following: a nuclear l level, which drops as the potential $V_s(r)$ becomes deeper, collides in succession with each of the atomic nl levels. The atomic levels are then shifted upward if $n > \nu_N$ and downward if $n < \nu_N$. The nuclear can manifest itself in this case as a resonance in the scattering.

Let us estimate the width of the spectrum restructuring region. Let $V_s(r) = -(g/2r_0^2)v(r/r_0)$, where g is the dimensionless coupling constant. The energy of the weakly coupled nuclear level can be estimated from the equations

$$E_l^{(N)} \approx \begin{cases} -1/a_s^2, & l=0 \\ 1/a_l r_l, & l \geqslant 1 \end{cases} \tag{6}$$

(we neglect here the influence of the Coulomb interaction, so that the quantities a_l and r_l refer here to the potential V_s with the Coulomb interaction "turned off"). In order of magnitude we have $a_l \sim r_0^{2l+1}/(g - g_l)$, where g_l are the coupling-constant values corresponding to the instant of the appearance of the bound state. The nuclear level with angular momentum l passes through the atomic-spectrum region at $l+1 < \nu_N < \infty$, yielding in terms of the scattering length

$$|a_l| \gtrsim \begin{cases} a_B, & l=0. \\ l^2(a_B/r_0)^2 r_0^{2l+1}, & l \geqslant 1 \end{cases} \tag{6'}$$

This leads to

$$\frac{\Delta g_l}{g_l} \sim \begin{cases} r_0/a_B, & l=0 \\ l^{-4}(r_0/a_B)^2, & l \geqslant 1 \end{cases}, \tag{7}$$

where $\Delta g_l = g_{l+1,l} - g_l$ (see Fig. 1). We emphasize that this estimate is valid for an arbitrary potential $V_s(r)$. It was obtained for $l=0$ by Zel'dovich,[1] and for $l \neq 0$ by Band and Fomichev[15] with a square well as the example. The term-crossing region is of the order of

$$\delta g \sim (r_0/a_B)^{l+1/2}, \quad l \geqslant 1, \tag{8}$$

i.e., much narrower. The estimates (7) and (8) can be improved by providing them with a quantitative meaning, as detailed in Appendix A.

3. *The* **WKB** *approximation.* We have started above from Eq. (1). It is of interest to note that practically all the results can be easily derived by a semiclassical approxima-

tion. Application of the WKB method to a Coulomb field distorted over small ($r \sim r_0 \ll a_B$) distances leads to the quantization rule

$$\int_{r_-}^{r_+} p(r)\,dr = \left(n_r + \frac{1}{2}\right)\pi + \operatorname{arctg}\left\{\frac{2\pi}{(l!)^2 a_B^{2l+1}}\left[\frac{1}{a_l^{(cs)}} - r_l^{(cs)}E\right]^{-1}\right\}, \qquad (9)$$

where $p(r) = [2E + 2\zeta/r - (l+1/2)^2/r^2]^{1/2}$ is the quasiclassical momentum and $n_r = 0, 1, 2, \ldots$ is the radial quantum number (see Appendix B). In the absence of short-range action we have $a_l^{(cs)} = 0$ and this equation becomes the usual Bohr–Sommerfeld quantization rule.[14,16] In the case of a Coulomb field the radial integral in (9) can be easily calculated in the form

$$\operatorname{ctg}\pi\nu = \frac{(l!)^2}{2\pi}\left[\frac{a_B^{2l+1}}{a_l^{(cs)}} + \frac{a_B^{2l-1}}{2\nu^2} r_l^{(cs)}\right]. \qquad (10)$$

This equation determines the spectrum of the highly excited states. For s levels, in particular, recognizing that $|r_0^{(cs)}| \sim r_0 \ll a_B$, we have

$$\nu_{ns} = n + \frac{1}{\pi}\operatorname{arctg}(2\pi a_{cs}/a_B), \qquad (10')$$

where $a_{cs} \equiv a_0^{(cs)}$. We see hence that even at $|a_{cs}| \sim a_B/2\pi$ all the levels are greatly shifted, i.e., we are in the atomic-spectrum restructuring region. Owing to the large numerical factor 2π, the spectrum restructuring sets in already at $|a_{cs}| \ll a_B$.

4. *Equation* (1) *in the presence of absorption.* Equation (1) remains valid also for complex values of the scattering length, indicating that the problem involves also open channels (e.g., the processes $\bar{p}p \to 2\pi$, 3π, etc., for the $\bar{p}p$ atom). The atomic levels are not only shifted but also acquire a width on account of the strong interaction at short distances.

Let us examine in greater detail the properties of Eq. (1) in the complex $\nu = \zeta(-2E)^{-1/2}$ plane. It is more convenient to investigate the level lines of the function $[a_l^{(cs)}(\nu)]^{-1}$ defined by Eq. (1). In fact, the onset of a level in the system produces a smooth change of the quantity $\operatorname{Im}[1/a_l^{(cs)}]$

$$1/a_l^{(cs)} = \alpha_1 + i\alpha_2, \quad \alpha_1 \propto g - g_l, \quad \alpha_2 \approx \text{const},$$

where $g \propto \operatorname{Re}(-V_s) r_0^2$.

The results of the calculations for the case $l = 0$ (for $\zeta = 1$) are shown in Fig. 2. One can see distinctly the saddle points S_1, S_2, ..., of the function $1/a_0^{(cs)}(\nu)$, as well as the periodicity of the entire pattern in the atomic region ($\operatorname{Re}\nu \gtrsim 1$). It is easy to track with the aid of this figure the motion of the S-matrix poles corresponding to bound states of the system with change of the coupling constant g. For real a_{cs}, the poles move along the real axis. In the case of low absorption the pole trajectories shift to the lower complex ν half-plane, but the character of the trajectories remains qualitatively unchanged (see the trajectory with value $\operatorname{Im} 1/a_{cs} - 2\pi \leqslant -0.1$ on Fig. 2).

Further increase of the absorption, i.e., growth of $\operatorname{Im} a_{(cs)}^{-1}$, leads to a qualitative change of the pole-motion pattern. It is convenient to describe the absorption in a hadronic atom by the parameter

$$\xi = \frac{1}{2\pi}\operatorname{Im}(a_B/a_{cs}), \quad l = 0. \qquad (11)$$

For [3] $\xi = \xi_1$ the trajectories of the poles located for small g near $\nu = 1$ and $\nu = 2$ intersect at the saddle point S_1. After the intersection one of these poles returns to the point $\nu = 1$, and the second moves to $\nu = 0$, i.e., it leaves the Coulomb region of the spectrum and becomes a nuclear level. At $\xi > \xi_1$ the trajecotries of the poles from $\nu = 1$ are closed, i.e., the restructuring regime is replaced by a regime of oscillations about the position of the unperturbed pole $\nu = 1$.

The poles whose trajectories intersect at the saddle points S_n behave similarly. For $\xi_n < \xi < \xi_{n+1}$, the levels $1s$, $2s$, ..., ns move in the oscillation regime, and the levels $(n+1)s, (n+2)s, \ldots$ in the restructuring regime. This illustrates the important role of the saddle points: level lines pass-

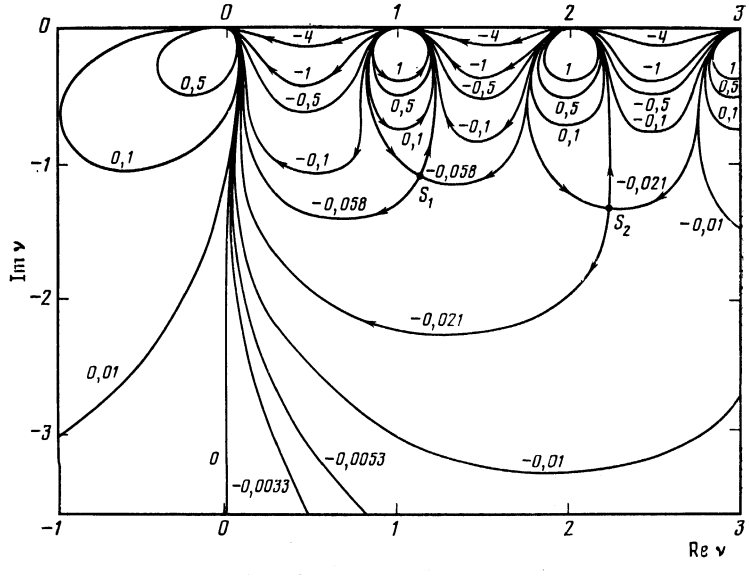

FIG. 2. $\operatorname{Im} a_{cs}^{-1}(\nu)$ level lines in the complex ν plane in the case $l = 0$. The curves are marked by the values of the parameter $2\pi(\xi - 1)$. The motion of the poles, indicated by the arrows, corresponds to an increase of the coupling constant g.

ing through them separate these two regimes. The saddle points S_n correspond to values $\xi = \xi_n$ close to unity:

$$\xi_1 = 0.991, \quad \xi_2 = 0.997, \quad \xi_3 = 0.999;$$

$$\xi_n = 1 - \frac{\ln n}{4\pi^2 n^3} + \ldots, \quad n \gg 1.$$

We conclude hence that the intermediate region of the transition from the restructuring regime to oscillations is very narrow in the case $l = 0$. As to the nuclear level, for $\xi < 1$ it stems from Coulomb levels and for $\xi < 1$ it comes to the region $\nu \approx 0$ from the second sheet of the complex energy variable plane; see Ref. 10 for details.

The results indicated were obtained from the fundamental equation (1) for $l = 0$ at $r_{cs} = 0$ (note that if $l = 0$ all the curves of Fig. 2 are practically independent of r_{cs}).

We proceed now the case of $l \neq 0$. The level trajectories and the saddle-point trajectories depend here substantially on the effective radius $r_l^{(cs)}$. It is therefore convenient to identify the number n of the saddle point $S_n^{(l)}$ with the principal quantum number n. We introduce in analogy with (11) the quantity

$$\xi^{(l)} = \frac{(l!)^2}{2\pi} \operatorname{Im}(a_B^{2l+1}/a_l^{(cs)}), \tag{12}$$

which is indicative of the absorption in the system. We denote by $\xi_n^{(l)}$ the value of the variable $\xi^{(l)}$ corresponding to the saddle point $S_n^{(l)}$. It follows from Eq. (1) that $\lim_{n\to\infty} \xi_n^{(l)} = \xi_{cr}^{(l)} = 1$ (just as for $l = 0$), but this asymptote is reached only at large $n \gg (a_B/r_0)^{(2l-1)/3} \gg 1$. A numerical calculation using Eq. (1) shows[11] that at $n \sim l + 1$ the values of $\xi_n^{(l)}$ differ substantially from $\xi_{cr}^{(l)} = 1$. Here $\xi_n^{(l)}$ exceeds unity and, in contrast to the s-wave case, the sequence $\{\xi_n^{(l)}\}$ decreases with increase of n.

The pattern of the pole collision in the case $l \geqslant 1$ differs from that described above for $l = 0$. Now an atomic level coming from the point $\nu = n$ collides at the saddle point with a nuclear level coming from the second sheet of the energy plane. After the collision, one of the poles moves to the point $\nu = n - 1$, and the other returns to the point $\nu = n$ (this pertains to a specified absorption value $\xi^{(l)} = \xi_n^{(l)}$). All the levels that emerge from the points $\nu = l + 1, l + 2, \ldots, n - 1$ are then in the restructuring regime, and those from the points $\nu = n + 1, n + 2, \ldots$, are in the oscillation regime. Just as for $l = 0$, the level lines corresponding to $\xi^{(l)} = \xi_n^{(l)}$ are separatrices in the vicinity of the saddle points $S_n^{(l)}$, i.e., they separate the spectrum-restructuring region from the oscillation region. To obtain the complete pattern of the level motion one must investigate the level lines $\operatorname{Im}[1/a_l^{(cs)}(\nu)]$ for arbitrary absorption [(see Fig. 3, which pertains to the case $l = 1, \zeta = 1$ and $|r_1^{(cs)}| = 16\pi$, and corresponding for a model of $V_s(r)$ in the form of a δ-function potential, to the ratio $r_c/a_B = 3/20\pi \approx 0.05$ typical of hadronic atoms]. Note that whereas the pattern shown in Fig. 2 is practically independent of the effective radius, this is not the case for Fig. 3.

Note that the possibility of an oscillating motion of the levels in strong absorption was observed in Refs. 17 and 18. Kok[19] investigated the model "Coulomb field plus separable potential" problem ($l = 0$) and pointed out the important role of the saddle points on going from restructuring of the atomic spectrum to the oscillation regime. The motion of the poles and the positions of the saddle points in the presence of absorption were investigated in Ref. 20 using the "Coulomb field plus δ potential" model and in Refs. 10 and 11 on the basis of the model-independent equation (1).

5. *Application to hadronic atoms.* Equation (1) was applied[4-6] for certain hadronic systems, such as $\bar{p}p$, $\Sigma^- p$, K^-^{4}He. Let us discuss the new experimental data.

a) the $\bar{p}p$ atom ($a_B = 57.6$ Fm, $E_c = 25.0$ keV) was considered[4,10] in connection with the published[21] indication of a large shift of the ground level ($\Delta E_{1s} \approx 3$ keV compared with $E_{2s}^{(0)} E_{1s}^{(0)} = 9.4$ keV). A contemporary LEAR experiment gives

$$\Delta E_{1s} = (0.5 \pm 0.3) \text{ keV}, \ \Gamma_{1s} \lesssim 1 \text{keV (Ref. 22)};$$
$$\Delta E_{1s} = (0.73 \pm 0.15) \text{ keV}, \ \Gamma_{1s} = (0.85 \pm 0.39) \text{ keV (Ref. 23)};$$
$$\Delta E_{1s} = (0.657 \pm 0.126) \text{ keV}, \ \Gamma_{1s} = (1.125 \pm 0.228) \text{kev (Ref. 24)};$$

For these values of ΔE_{1s} and Γ_{1s} the level shifts are determined by perturbation theory in the scattering length,[25] and in the s state the $\bar{p}p$ system is far from reconstruction.

A recently refined width of the $2p$ level of the $\bar{p}p$ atom [26] is $\Gamma_{2p} = (38.8 \pm 10.7) \cdot 10^{-3}$ eV (no $2f$-level shift is given in Ref. 26). Assuming $\Delta E_{2p} \sim \Gamma_{2p}$, we have

$$\delta_{2p} = |\Delta E_{2p}| / (E_{3p}^{(0)} - E_{2p}^{(0)}) \sim 10^{-5}.$$

Varying the strong-interaction radius in the range $r_0 = 1.2$–1.4 Fm we obtain from Eq. (11) of Ref. 27 $\delta_{2p}^{(cr)} = 1/2(r_0/a_B)^3 \approx (4\text{–}7) \cdot 10^{-6}$. Thus, $\delta_{2p} \sim \delta_{2p}^{(cr)}$, mean-

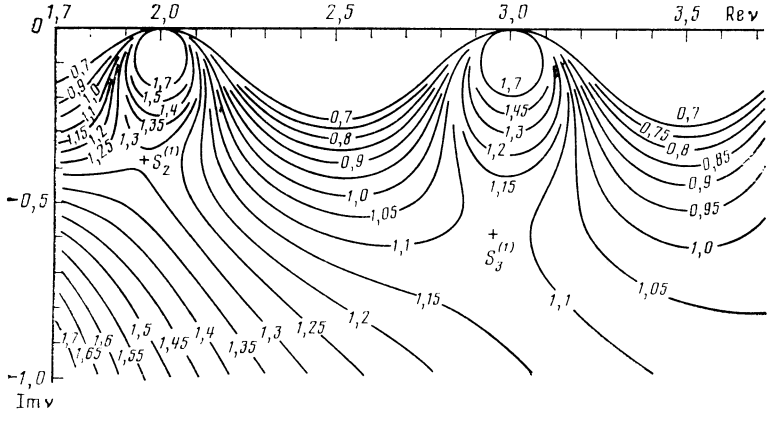

FIG. 3. Level lines $\operatorname{Im}[1/a_l^{(cs)}(\nu)]$ for $l = 1$. The curves are marked by the values of $\operatorname{IM}[1/a_l^{(cs)}]$ and the crosses indicate the positions of the saddle ponts $S_n^{(1)}$.

ing that the nuclear level (if it exists) is quite far from the region of the atomic spectrum. Its location cannot be predicted on the basis of the model-independent equation (1); a rough estimate in accordance with (6) yields $|E_N| \gtrsim 15$ MeV. The situation calls for a more detailed study of the shifts and widths of the $2p$ levels of the proton-antiproton atom (at low energies, the contribution of the p wave is significant in the $\bar{p}p$ system [28]).

b) $\bar{p}d$ atom ($a_B = 43.2$ Fm, $E_c = 33.35$ keV). Observation of strong absorption from the $3d$ state, $\Gamma_{3d} \approx 5 \cdot 10^{-5}$ eV, was reported at the Villar conference.[26] This exceeds by an order of magnitude the theoretical $\Gamma_{3d} = 6 \cdot 10^{-6}$ eV, obtained by solving the three-body problem using standard potential models for the NN interaction.[29] In analogy with the foregoing, we have $\delta_{3d}^{(cr)} = (4/729)(r_0/a_B)^5 \approx (1\text{–}3) \cdot 10^{-9}$ for $r_0 = 2\text{–}2.5$ Fm (the mean squared deuteron radius is $\langle r^2 \rangle^{1/2} = 1.966$ Fm, Ref. 30). The experimental value $\delta_{3d} \gtrsim 3 \cdot 10^{-8}$ is at the same time considerably higher than $\delta_{3d}^{(cr)}$. This seems to attest to the existence of a weakly bound nuclear level in the $\bar{p}d$ system. A final conclusion calls for higher experimental accuracy with measurement, in particular, of not only the width but also the shift of the $3d$ level.

c) Equation (1) was used also to analyze other hadronic atoms, $\Sigma^- p$ (Refs. 5 and 10) and K^- ^{4}He (Refs. 6 and 27). The most definite is the situation with the K^- ^{4}He system, for which the experimental values[31] $\Delta E_{2p} = (43 \pm 8)$ eV and $\Gamma_{2p} = (55 \pm 34)$ eV predicted the existence of a weakly bound nuclear leve with binding energy ε and width γ of the order of 1 MeV.

6. *Nuclear shifts of mesic-molecular levels.* Let us examine the application of the approach described above to the question of μ-catalysis of nuclear fusion reactions.[32,33] Ponomarev and coworkers[34,35] obtained by laborious computations the molecular-level energies in the $dd\mu$ and $dt\mu$ systems. The most important results of these calculations can be taken to be the predicted existence of weakly bound mesic-molecular levels (thus, in the $dt\mu$ molecule, $\varepsilon_{01} = 34.9$ eV and $\varepsilon_{11} = 0.64$ eV., where ε_{Jv} is the level binding energy and J and v the rotational and vibrational quantum numbers.

To obtain the velocities λ_f^{Jv} of the fusion reaction $d + t \to {}^4\text{He} + n$ from various states of a mesic molecule, and the nuclear shifts of mesic-molecular levels, calculations were preformed [36,37] in which the potential $V_s(r)$ were chosen to be model optical potentials whose parameters were fitted to the experimental data on elastic dt scattering and of the reaction $d + t \to {}^4\text{He} + n$ near $E \lesssim 200$ keV ($a_B = 24.0$ Fm, $E_c = 59.9$ keV, and $\zeta = -1$ for the dt system). It has been found[36] for the states with $J = 0$ that the reaction rate is $\lambda_f^{00} = 1.0 \cdot 10^{12}$ s^{-1}, the nuclear shift is $\Delta E_{00} = -0.70 \cdot 10^{-3}$ eV (the binding energy of the unperturbed level is $\varepsilon_{00} = 319.2$ eV), and $\lambda_f^{01} = 0.80 \cdot 10^{12}$ s^{-1}, $\Delta E_{01} = -0.60{,}10^{-3}$ eV ($\varepsilon_{01} = 34.9$ eV). The essential result of these calculation is the smallness of the molecular-level shift due to the nuclear dt interaction (this agrees with calculations[37] using another strong-potential model).

On the other hand, it is stated in Refs. 38 that the presence of the $3/2^+$ quasistationary level (s-wave resonance with $E_r = 64$ keV and $\Gamma_r/2 = 70$ keV) in the dt system exerts a substantial influence on the locations of the intermolecular levels, owing to the spectrum-restructuring effect.

This question can be considered without a model on the basis of Eq. (B.6) of Appendix B. Putting in this equation $l = 0$ and $\zeta = -1$ we have

$$E - E_{Jv}^{(0)} = \frac{1}{2} Q_{Jv}^2 \left\{ \frac{1}{a_{cs}} + \left(\frac{1}{3} - r_{cs} \right) E \right\}^{-1}, \tag{13}$$

where $E_{Jv}^{(0)}$ and Q_{Jv} are the level energy and the normalized wave function at zero [see Eq. (B.9)], calculated without allowance for the strong interaction. The dimensionless factor Q_{Jv} is proportional to the penetrability of the Coulomb barrier at the energy

$$E = \frac{Z_1^2 m_1 e^4}{2\hbar^2} - \frac{Z_2^2 m_2 e^4}{2\hbar^2} - \varepsilon_{Jv}, \tag{14}$$

where $Z_1 = 2$, $Z_2 = 1$, $m_1 = m_{dt} m_\mu / (m_{dt} + m_\mu)$, $m_2 = m_t m_\mu / (m_t + m_\mu)$, so that $E \approx 8.3$ keV for states with $J = 0$. Note that (14) is the energy of the relative motion of the dt system over short distances, where the effective potential in the $dt\mu$ mesic molecule is of the form $V = V_s + 1/r + o(r)$. The penetrability of the Coulomb barrier is here $\exp(-2\pi/ka_B) \approx 5 \cdot 10^{-6}$. Substituting in (13) the values of the low-energy scattering parameters

$$a_{cs} = -(3.88 + 1.27i) a_B, \quad r_{cs} = (0.238 + 0.016i) a_B, \tag{15}$$

extracted from the experimenal data on the cross section of the reaction $d(t\alpha)n$ in the triton energy interval $E_t = 12.5\text{–}117$ keV (Ref. 39), and the values $Q_{00} = 1.02 \cdot 10^{-4}$, $Q_{01} = 0.927 \cdot 10^{-4}$ obtained from calculations of the wave functions of the mesic molecule, [35–37] we get[40]

$$\begin{gathered} \lambda_f^{00} = \Gamma_{00}/\hbar = 1.36 \cdot 10^{12} \text{ s}^{-1}, \quad \Delta E_{00} = -1.25 \cdot 10^{-3} \text{ eV}, \\ \Gamma_{00} = 0.90 \cdot 10^{-3} \text{ eV}, \\ \lambda_f^{01} = 1.13 \cdot 10^{12} \text{ s}^{-1}, \quad \Delta E_{01} = -1.04 \cdot 10^{-3} \text{ eV}, \\ \Gamma_{01} = 0.74 \cdot 10^{-3} \text{ eV}. \end{gathered} \tag{16}$$

These values agree with the numerical calculations of Refs. 36 and 37. This is not surprising, since the mesic-molecular level shifts and widths due to the nuclear dt interaction are determined, as seen from (13), by the low-energy parameters a_{cs} and r_{cs}, and are independent of the specific form of the strong potential $V_s(r)$.

Note that even if the nuclear level were to land in the region of the molecular levels, so that the expression in the curly brackets of (13) were close to zero at $E = E_{Jv}^{(0)}$, the shifts would remain small even in this case. This is easily verified by solving Eq. (13) for the shift $\Delta Q_{Jv} = E - E_{Jv}^{(0)}$:

$$\Delta E_{Jv}^{(1,2)} = \frac{1}{2} \{ \delta E \pm [(\delta E)^2 + 6 Q_{Jv}^2 / (a_B - 3r_{cs}) a_B^3]^{1/2} \}. \tag{17}$$

Here $\delta E = E_0^{(N)} - E_{Jv}^{(0)}$ is the detuning from resonance, with $E_0^{(N)} \approx -3a_{cs}(a_B - 3r_{cs})$ the position of the nuclear level. This equation is similar to expression (5) that describes the Zel'dovich effect for states with angular momentum $l \neq 0$. The smallness of the shift is ensured here by the small penetrability of the Coulomb barrier

$$Q_{Jv}^2 \propto \exp(-2\pi/ka_B),$$

that separates the regions of the molecular and nuclear attractions. Moreover, an estimate using the semiclassical equation (B.7), recognizing that $\hbar\omega = \varepsilon_{01} - \varepsilon_{00} \approx 0.3$ keV

yields $Q \approx 3.5 \cdot 10^{-4}$, which agrees[4)] with the cited result of the numerical calculation.[35-37]

We arrive at the conclusion that in our problem allowance for the nuclear interaction at small distances cannot cause a significant change in the molecular-level energies calculated with the strong interaction neglected. The situation for arbitrary values of a_{cs} is here far from the situation with spectrum restructuring, differing qualitatively from the problem considered in Refs. 1–4, of the influence of a shallow s level on the atomic spectrum. We emphasize once more that the cause of this difference is the existence of Coulomb potential of low penetrability. Our conclusion are thus in full agreement with the results of the model calculations.[36,37]

7. Finally, we apply the model-free approach developed above to the question of the influence of nuclear interaction on the sticking probability of a muon to an α particle in the $dt\mu \to nt + \alpha\mu(\mu^{-4}\mathrm{He})$ reaction that proceeds from the Jv state of the $dt\mu$ mesic molecule.

The sticking coefficient w_s^0 was calculated in Ref. 41 with neglect of the influence of the nuclear interaction on the wave function Ψ^{Jv} of the mesic molecule. The equation used for the amplitude of the pickup to the nl state of the mesic atom was

$$A_{nl}^{Jv}(\mathbf{r}) = \int d\boldsymbol{\rho}\, \Psi_{nl}^{*}(\boldsymbol{\rho})\, e^{-i\mathbf{q}\boldsymbol{\rho}} \Psi^{Jv}(\boldsymbol{\rho}, \mathbf{r}), \tag{18}$$

where $r = |\mathbf{r}_d - \mathbf{r}_t| \to 0$, $\boldsymbol{\rho}$ is the muon radius vector in the c.m.s., $\Psi_{nl}(\boldsymbol{\rho})$ is the normalized wave function of the ^{4}He μ-mesic atom,

$$q = m_\mu V = \frac{m_\mu}{m_{\mu\alpha}} [2m_{n(\mu\alpha)}(\Delta + \varepsilon_{nl} + \varepsilon_{Jv})]^{1/2}, \tag{19}$$

$\mathbf{V}$ is the velocity of the mesic atom in the c.m.s. $m_{\mu\alpha}$ and $m_{n(\mu\alpha)}$ are the reduced masses of the mesic atom and of the $n(\mu^{-4}\mathrm{He})$ system, $\Delta = 17.6$ MeV is the energy released in the fusion reaction, ε_{nl} and ε_{Jv} are the binding energies of the mesic atom and the mesic molecule, and finally $\Psi^{Jv}(\boldsymbol{\rho},\mathbf{r})$ is the wave function of the mesic molecule with the nuclear interaction between the deuteron and the triton turned off. The Schrödinger-equation variables are separable in the region $r \ll a_\mu = \hbar^2/m_\mu e^2$, and Ψ^{Jv} takes the form (we assume for the sake of argument $M_J = 0$)

$$\Psi^{Jv}(\boldsymbol{\rho}, \mathbf{r}) \approx \sum_K \Phi_K(\boldsymbol{\rho}) \sum_{l=|J-L|}^{J+L} \chi_{Kl}^{Jv}(r)\, Y_{l,-m}(\mathbf{r}/r). \tag{20}$$

The summation (integration over the continuum) is carried out here over the complete system of orthonormalized states $K \equiv (NLm)$ of the ^{3}He mesic atom $E_N = -2\tilde{m}_\mu/N^2$, $\tilde{m}_\mu = m_\mu(m_d + m_t)/(m_d + m_t + m_\mu) \approx m_\mu$ for the discrete spectrum), and the functions

$$\chi_{Kl}^{Jv}(r) = Q_{Nl}^{Jv} C_l^{-1}(k_N^{Jv})^{-1} F_l(k_N^{Jv} r, \eta)/r \approx Q_{Nl}^{Jv} r^l, \quad r \to 0, \tag{21}$$

where $k_N^{Jv} \equiv [2(E_{Jv} - E_N)]^{1/2}$, E_{Jv} is the mesic-molecule energy.

From (18)–(21) it follows that the probability of muon pickup into the nl state of the mesic atom[41] is

$$w_{nl} = \left| \sum_N b_N F_N(nl) \right|^2, \tag{22}$$

where

$$b_N = Q_{N0}^{Jv} \Big/ \left(\sum_N |Q_{N0}^{Jv}|^2 \right)^{1/2},$$

$$F_N(nl) = \int d\boldsymbol{\rho}\, \Psi_{nl}^{*}(\boldsymbol{\rho})\, e^{-i\mathbf{q}\boldsymbol{\rho}} \Phi_N(\boldsymbol{\rho}). \tag{23}$$

The first term ($N = 1$) in (22) corresponds to the Born-Oppenheimer approximation, and the remaining terms constitute a correction for the non-adiabaticity, which amounts to $\approx 30\%$. Ultimately,[41] $w_s^0 = 8.48 \cdot 10^{-3}$. In addition, it is necessary to introduce many corrections for the calculation of w_s^0 (for the finite ratio m_μ/m_{He}, for the nuclear sizes, for the polarization of the vacuum, etc.), but these change w_s by less than 1% (Ref. 41).

Let us examine the influence of the resonant dt interaction on the probability w_s^0 (it is stated in Ref. 42) that this effect can change w_s by 2–3 times). Taking into account the nuclear interaction we have

$$\chi_{N0}^{Jv}(r) = Q_{N0}^{Jv}\{1 - a(E_N^{Jv})/r\}, \quad r_0 \lesssim r \ll a_B, \tag{24}$$

where

$$a(E_N^{Jv}) = a_{cs}[1 - (E_{Jv} - E_N)/E_r]^{-1}$$

and $E_r = -[a_{cs}(a_B/3 - r_{cs})]^{-1}$ is the position of the resonance [note that Eq. (24) is obtained from (21) by replacing the Coulomb function $F_0(l=0)$ that is regular at zero by $F_0(r) + \tan\delta_{cs} G_0(r)$]. It can be shown that the normalizing factor in (24) remains unchanged to within terms of order $(r_0/a_B)^3$.

To estimate the influence of the resonant dt interaction on w_s^0 we retain in (24) only the singular term and assume that it determines the behavior of $\chi_{N0}^{Jv}(r)$ also for $r \lesssim r_s$. We obtain for the sticking coefficient

$$w_{nl} \approx \left| \sum_N Q_N \left\{ 1 + \frac{E_N - E_{Jv}}{E_r} \right\}^{-1} F_N(nl) \right|^2 \Bigg/ \sum_N \left| Q_N \left(1 + \frac{E_N - E_{Jv}}{E_r} \right)^{-1} \right|^2. \tag{25}$$

This shows that in the adiabatic approximation ($Q_N = Q_1 \delta_{N,1}$) the nuclear interaction does not influence w_s^0 at all, it accounts only for the corrections for nonadiabaticity, and has an additional smallness $\sim |E_1/E_r| = \delta$. For the dt system we have $\delta \sim 8$ keV/64 keV $= 1/8$ and we obtain an estimate $\approx 4\%$ for the correction to the sticking coefficient. This agrees with the $\sim 3\%$ obtained for this correction in Ref. 41.

A statement made in Ref. 42, that the resonant dt interaction which distorts strongly the wave function Ψ^{Jv} can alter w_s^0 substantially, is in error, in view of an implicit assumption that the $dt \to n\alpha$ reaction is possible also at distances $r \sim a_\mu \gg r_0$. The model-independent estimate (25) confirms the conclusion of Ref. 41 that the corrections to the sticking coefficient $w_s^0 = 8.48 \cdot 10^{-2}$ are of the order of several percent.

8. A few concluding remarks. The atomic-spectrum restructuring, or the Zel'dovich effect, is of general quantum-mechanical character and can manifest itself in any system for which the potential interaction breaks up into two parts with greatly differing radii. We have investigated above the

characteristic features of this effect in the case when two attraction region in the potential $V(r)$ are divided by low-penetrability barrier (centrifugal if $l = 0$ or Coulomb for arbitrary l). In this case the nuclear level shifts are always small, even in the term quasicrossing region. This is precisely why in the examples considered in Secs. 6 and 7 the resonant nuclear interaction, which distorts the wave functions at short distances, does not lead to a noticeable level shift (and also leaves in fact unchanged the wave function at large distances).

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APPENDIX A

Let g_l be that value of the coupling constant g for which there appears in the potential

$$V_s = {}^1/_2 g r_0^{-2} v(r/r_0)$$

a bound state with angular momentum l, and let $\chi_l(r)$ be the corresponding wave function. This function is normalizable at $l \geqslant 1$ and its asymptotic value is $\chi_1 = A_l x^{-l} + \ldots$ as $x = r/r_0 \to \infty$. Using perturbation theory in Δg, we arrive at the equation

$$\Delta g_{nl}/g_l = c_l (r_0/na_B)^2, \quad l \geqslant 1, \tag{A.1}$$

where

$$c_l = \int_0^\infty \chi_l^2\, dx \Big/ \int_0^\infty (\chi_l'^2 + l(l+1)x^{-2}\chi_l^2)\, dx, \quad g_l = \lim g_{nl}$$

as $n \to \infty$ (the definitions of g_{nl} and g_l are clear from Fig. 1). In a number of cases the values of g_l and c_l can be calculated analytically. Thus, for a δ-like potential $[v(x) = \delta(x-1)]$ we have

$$g_l = 2l+1, \quad c_l = 2/(2l-1)(2l+3), \tag{A.2}$$

and for a rectangular well $[v(x) = \theta(1-x)]$

$$g_l = \xi^2_{p,l+1/2}, \quad c_l = (2l+1)/(2l-1)\xi^2_{p,l+1/2}, \tag{A.3}$$

where $\xi_{p,\nu}$ is the pth positive zero of the Bessel function $J_\nu(\xi)$ and $p = 0, 1, 2, \ldots$. From (A.2) we have for $l \gg 1$

$$g_l = l^2 + O(l^{4/3}), \quad c_l = l^{-2}(1 + O(l^{-2/3})).$$

Using the $1/n$ expansion [43] we can readily show that for an arbitrary smooth potential we have $g_l \sim l^2$ and $c_l \sim l^{-2}$ as $l \to \infty$. Hence

$$\Delta g_{nl}/g_l \sim l^{-2}(r_0/na_B)^2,$$

which yields for $n = l + 1$ the estimate (7).

Using Eq. (5) and the connection between the effective radius r_l and the asymptotic coefficient A_l (Ref. 12), we get

$$\frac{\delta g_{nl}}{g_l} = \frac{2^{l+2}}{(2l)!}\left[\frac{(n+l)!}{(n-l-1)!}\right]^{1/2}\left(\frac{r_0}{na_B}\right)^{l+1/2} b_l, \tag{A.4}$$

where

$$b_l = A_l \Big/ \int_0^\infty (\chi_l'^2 + l(l+1)x^{-2}\chi_l^2)\, dx.$$

This yields directly the estimate (8). The derivation of the equations above, and also the generalization of Eq. (5) to include potentials with power-law "tails" at infinity, will be published elsewhere.

APPENDIX B

Semiclassical approximation in a Coulomb potential with short-range action

The semiclassical approximation, or the WKB method, is one of the most powerful approximation methods of quantum mechanics, and for physically reasonable potentials the region of its validity extends way down to low quantum numbers (see, e.g., Refs. 16, 44, and 45). It is therefore natural to use it in the Coulomb problem with short-range action.

For distances $r \lesssim (l + 1/2)^2 a_B$, the semiclassical wave function is

$$\varphi_l(r) = \text{const}\,[p(r)]^{-1/2} \sin\left(\int_{r_-}^{r} p\, dr + \frac{\pi}{4} + \theta_l\right), \tag{B.1}$$

where

$$p(r) = [k^2 + 2\zeta/r - (l+1/2)^2/r^2]^{1/2}$$

is the semiclassical momentum, and the phase θ_l takes into account the contribution of the potential $V_s(r)$ at short distances. Comparing (B.1) with the asymptote of the Coulomb wave function as $r \to \infty$, we get $\theta_l = \delta_l^{(cs)}(k)$, where $\delta_l^{(cs)}$ is the nucleus-Coulomb scattering phase.[46] Analytic continuation into the discrete-spectrum region, $k = i\lambda$, yields the quantization condition

$$\int_{r_-}^{r_+} p\, dr = \pi(n_r + 1/2) - \delta_l^{(cs)}(i\lambda) \tag{B.2}$$

where the energy is $E = k^2/2 = -\lambda^2/2$. To continue analytically the phases $\delta_l^{(cs)}(k)$ it is convenient to use the effective-radius expansion [46,47]:

$$K_l^{(cs)}(k^2) \equiv k^{2l+1} \prod_{j=1}^{l} (1 + \eta^2/j^2)\,[C_0^2(\eta)\operatorname{ctg}\delta_l^{(cs)} + 2\eta h(\eta)]$$

$$= -1/a_l^{(cs)} + r_l^{(cs)} k^2/2 + \ldots.$$

Here $\eta = -\zeta/k$ is the Sommerfeld parameter, $C_0^2 = 2\pi\eta(e^{2\pi\eta} - 1)^{-1}$,

$$h(\eta) = \frac{1}{2}[\psi(1+i\eta) + \psi(1-i\eta) - \ln\eta^2] = \begin{cases} \dfrac{1}{12\eta^2} + O(\eta^{-4}), & \eta \to \infty \\ -\ln|\eta| - C + O(\eta^2), & \eta \to 0 \end{cases},$$

and $C = 0.5772...$. This yields in the case of Coulomb attraction ($ka_B \ll 1$)

$$\operatorname{ctg}\delta_l^{(cs)}(k) = \frac{(l!)^2}{2\pi} a_B^{2l+1}\left[-\frac{1}{a_l^{(cs)}} + r_l^{(cs)}\frac{k^2}{2}\right] + \frac{(ka_B)^2}{12\pi} + \ldots, \quad \text{(B.3)}$$

and in the case of Coulomb repulsion

$$\operatorname{ctg}\delta_l^{(cs)}(k) = \frac{1}{2\pi} e^{2\pi/ka_B}\left\{(l!)^2 a_B^{2l+1}\left[-\frac{1}{a_l^{(cs)}} + r_l^{(cs)}\frac{k^2}{2}\right] - \frac{(ka_B)^2}{6}\right\} + \ldots. \quad \text{(B.4)}$$

Equation (9) follows directly from (B.2) and (B.3).

In a number of physical problems the potential reduces to Coulomb repulsion at short distances: $V(r) = -\zeta/r + V_0 + O(r)$, $\zeta < 0$, while an attraction region exists for large values of r (for example, in the case of mesic molecules such as $dd\mu$, $dt\mu$, and others). In such a potential there can exist bound states whose spectrum is determined from the quantization condition

$$\int_{r_-}^{r_+} p\,dr = \pi\left(n_r + \frac{1}{2}\right) + \operatorname{arctg}\left\{\left[2\pi e^{-2\pi/ka_B}\left[(l!)^2 a_B^{2l+1}\left(\frac{1}{a_l^{(cs)}} - r_l^{(cs)}\frac{k^2}{2}\right) + \frac{(ka_B)^2}{6}\right]^{-1}\right\}, \quad \text{(B.5)}$$

where now

$$p(r) = [2(E - V(r)) - (l+1/2)^2/r^2]^{1/2}$$

and $k^2 = 2(E - V_0) > 0$ (which is usually realized in mesic molecules). The exponential factor corresponds to the penetrability of the Coulomb barrier. The nuclear level shifts for states with arbitrary angular momentum, including $l = 0$, are unusually small if $ka_B \ll 1$. With this taken into account, Eq. (B.5) takes the simpler form

$$E - E_{nl} = \frac{1}{2}[(2l+1)!!]^2 \times \frac{Q_{nl}^2}{a_B^{2l+3}}\left\{\frac{1}{a_l^{(cs)}} + \frac{k^2}{2}\left[\frac{1}{3(l!)^2 a_B^{2l-1}} - r_l^{(cs)}\right]\right\}^{-1} \quad \text{(B.6)}$$

with

$$Q_{nl} = 2^{l+1}[(2l+1)!]^{-1} a_B \omega_{nl}^{1/2}\, e^{-\pi/ka_B}, \quad \text{(B.7)}$$

where

$$\omega_{nl} = \pi\left[\int_{r_-}^{r_+} p_{nl}^{-1}\,dr\right]^{-1}$$

is the frequency of the classical radial motion in a long-range well. It can be shown[48] that relation (B.6) is valid not only in the semiclassical approximation. It is a generalization of a perturbation-theory equation in terms of the scattering length[25]

$$E - E_{nl} = \frac{1}{2} Q_{nl}^2[(2l+1)!!]^2 a_l^{(cs)}/a_B^{2l+3}. \quad \text{(B.8)}$$

Here Q_{nl} is a dimensionless coefficient that determines the behavior of the normalized wave function φ_{nl} (unperturbed by the short-range potential V_s) a zero:

$$\varphi_{nl}(r) \underset{(r\to 0)}{\approx} \frac{Q_{nl}}{a_B^{l+1/2}} r^{l+1} + \ldots, \quad \int_0^\infty \varphi_{nl}^2(r)\,dr = 1. \quad \text{(B.9)}$$

Thus, in a pure Coulomb attraction potential ($V = -\zeta/r$) we have[14]

$$Q_{nl} = 2[(2l+1)!!]^{-1}(p_{nl}/n^3)^{1/2}.$$

*Moscow Engineering-Physics Institute.

[1)] We use the atomic units $e = \hbar = m = 1$, where m is the reduced mass of the system; the energy unit is $E_c = me^4/\hbar^2$.

[2)] Here r_0 is the effective radius of the nuclear forces and $a_B = |\zeta|^{-1}$ is the Bohr radius. It is assumed henceforth that $r_0 \ll a_B$.

[3)] The value $\xi_1 = 1 - 0.058/2\pi = 0.9908$ corresponds to the sadle point S_1, see Fig. 2.

[4)] The cause of difference between the semiclassical estimate of the wave function at zero and the exact value is that the mesic-molecular potential near the turning point is not a Coulomb potential.

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