Parametric x-ray emission by crystals irradiated by an ultrasound wave

V. G. Baryshevskiĭ and I. V. Polikarpov

Research Institute for Nuclear Problems, Belorussian State University
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The theory of formation of parametric x-ray emission by crystals acted upon by an ultrasound wave is considered. Analytic expressions are obtained for the angular and integral characteristics of the radiation in the cases of weakly and strongly absorbing crystals. The existence of ultrasound-excited extinction beats is predicted. It is shown that for certain ultrasound wave vectors the spectral-angular density of the parametric x radiation is resonantly enhanced (acoustoparametric resonance). The influence of ultrasound on the dependence of the parametric x radiation on the energy of the particles incident on the crystal is investigated.

1. INTRODUCTION

The theoretical and experimental study of parametric x radiation (PXR) has become by now the subject of many publications (see, e.g., Refs. 1–11). According to analysis,[1,2,6–9] the PXR is produced when, under conditions of diffraction of the produced radiation, the effective refractive index of the crystal becomes larger than unity even in the x-ray band. This makes possible the Cherenkov mechanism of photon generation. All the studies made to date deal with particle motion in the crystal in the absence of external action. One of the authors of Ref. 12 has shown, however, that in the presence of an external alternating field the crystal has a certain effective frequency-dependent refractive index.

It is shown in the present paper that the dependence of the effective refractive index on the frequency and amplitude of the external field alters substantially the PXR distribution in spectrum and in angle. Explicit equations are derived for the angular distribution and for the integral number of photons in the PXR reflection of a crystal acted upon by an ultrasound wave (USW) in Laue geometry.

It is demonstrated that the USW action causes the PXR to increase, in the case of small oscillation amplitudes, in proportion to the square of the USW amplitude. It is shown that for USW vectors exceeding a value $\varkappa_{\rm thr}$ [see Eq. (21)] the USW increases radically the differential number of PXR photons emitted within a small angle along the reflection direction. The angular divergence of the photons produced in this case is determined by the characteristics and is independent of the energy of the particles incident on the crystal. It follows from analysis also that the action of the USW alters the dependence of the rate of PXR formation on the energy of the particles incident on the crystal and on the average multiple-scattering angle, and the PXR production threshold energy can be substantially decreased.

2. DISTRIBUTION, IN SPECTRUM AND ANGLE, OF PARAMETRIC X RADIATION OF A CRYSTAL ACTED UPON BY AN ULTRASOUND WAVE

Let a fast charged particle pass through a crystal acted upon by an alternating external field. Analysis[9,12] shows that at large distances from the crystal, when the electromagnetic wave generated in the crystal becomes spherical, the number of photons of frequency ω and polarization $s=\pi, \sigma$ in the direction $\mathbf{n}=\mathbf{k}/k$, where $\mathbf{k}$ is the wave vector of the photon in vacuum, is given by

$$N^s_{\mathbf{n},\omega}=\frac{\omega}{4\pi^2\hbar c}\left|\int \mathbf{E}_{\mathbf{k}}^{s(-)*}(\mathbf{r},\omega)\,\mathbf{j}(\mathbf{r},\omega)\,d^3\mathbf{r}\right|^2, \qquad (1)$$

where the Fourier transform of the current is

$$\mathbf{j}(\mathbf{r},\omega)=\int e^{i\omega t}e\mathbf{v}\delta(\mathbf{r}-\mathbf{v}t)\,dt, \qquad (2)$$

$\mathbf{v}$ is the charged-particle velocity, e is its charge, and $\mathbf{E}_{\mathbf{k}}^{s(-)}$ is the radiation field and is an exact solution of the homogeneous Maxwell equations with Hermitian-adjoint dielectric constant. This solution describes the scattering, by the crystal, of a plane wave of unit amplitude $\mathbf{e}_s\exp(i\mathbf{k}\cdot\mathbf{r})$ (e_s is the photon polarization vector), and has an asymptote in the form of a decreasing plane wave plus a converging spherical wave. Given the solution $\mathbf{E}_{\mathbf{k}}^{s(+)}$ of the homogeneous Maxwell equations that describe the photon scattering by the crystal, with the usual asymptote in the form of a diverging spherical wave, the field $\mathbf{E}_{\mathbf{k}}^{s(-)}$ can be obtained by using the relation[9,12]

$$\mathbf{E}_{\mathbf{k}}^{s(-)}=(\mathbf{E}_{-\mathbf{k}}^{s(+)})^*. \qquad (3)$$

Let the crystal be acted upon by a transverse USW of type

$$\mathbf{u}(\mathbf{r},t)=\mathbf{a}\sin(\varkappa\mathbf{r}-\Omega t+\varphi'),$$

where $\mathbf{a}$ is the amplitude of the USW, $\varkappa$ its wave vector directed along the normal to the crystal surface, and $\Omega(\varphi')$ the frequency (phase) of the sound. Since the time of flight of the particle and of its radiation through a crystal acted upon by the USW is much shorter than the period of the USW oscillations, a semiclassical approximation can be used. The expression calculated in this case for $N_{\mathbf{n}}\omega^s$ must next be averaged over the USW phase φ'. We consider in the present paper US oscillations with amplitude much smaller than the distance between the crystal planes, when $\xi=\frac{1}{2}\boldsymbol{\tau}\cdot\mathbf{a}\ll 1$. In this situation, in the two-wave approximation, the number of electromagnetic waves interacting with the crystal is restricted to six. The Maxwell equation for the fields $E_{\mathbf{k}}^{s(-)}$ takes the form

$$\operatorname{rot}\operatorname{rot}|\mathbf{E}^{s(-)}(\mathbf{r},\omega)\rangle-\frac{\omega^2}{c^2}|\mathbf{E}^{s(-)}(\mathbf{r},\omega)\rangle-\frac{\omega^2}{c^2}|E^{s(-)}(\mathbf{r},\omega)\rangle\hat{\chi}^+=0. \qquad (4)$$

the row matrix $|\mathbf{E}^{s(-)}(\mathbf{r},\omega)\rangle$ is equal to $|\mathbf{E}^{s(-)}(\mathbf{r},\omega)\rangle=(\mathbf{E}^{s(-)}_{k+\varkappa}(\mathbf{r},\omega);\ \mathbf{E}^{s(-)}_{k_1+\varkappa}(\mathbf{r},\omega)\ \ \mathbf{E}^{s(-)}_{k}(\mathbf{r},\omega);\ \mathbf{E}^{s(-)}_{k_1}(\mathbf{r},\omega);$

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$\mathbf{E}^{s(-)}_{k-\varkappa}(\mathbf{r},\omega)$; $\mathbf{E}^{s(-)}_{k_1-\varkappa}(r,\omega)$), $\mathbf{E}^{s(-)}_{k}$,$\mathbf{E}^{s(-)}_{k\pm\varkappa}$($\mathbf{E}^{s(-)}_{k_1}$,$\mathbf{E}^{s(-)}_{k_1\pm\varkappa}$) are waves with wave vectors $\mathbf{k}$, $\mathbf{k}\pm\varkappa(\mathbf{k}+\boldsymbol{\tau}$, $\mathbf{k}+\boldsymbol{\tau}\pm\varkappa)$, respectively ($\boldsymbol{\tau}$ is the reciprocal-lattice vector).

The crystal polarizability tensor $\hat{\chi}^+$ is given by

$$\hat{\chi}^+=\hat{\chi}_0{}^++\hat{V}^+,$$

with a diagonal block matrix

$$\hat{\chi}_0{}^+=\hat{X}^+_{\rm St}\begin{pmatrix}\hat{I}&0&0\\0&\hat{I}&0\\0&0&\hat{I}\end{pmatrix};\quad \hat{I}=\begin{pmatrix}1&0\\0&1\end{pmatrix};\quad \hat{X}^+_{\rm St}=\begin{pmatrix}g_0^*&g_{\bar\tau}^*\\g_\tau^*&g_0^*\end{pmatrix},$$

$$\hat{V}^+=\xi\hat{G}^+\begin{pmatrix}0&e^{-i\varphi'}\hat{I}&0\\-e^{i\varphi'}\hat{I}&0&e^{-i\varphi'}\hat{I}\\0&-e^{i\varphi}\hat{I}&0\end{pmatrix},\quad \hat{G}^+=\begin{pmatrix}0&-g_{\bar\tau}^*\\g_\tau^*&0\end{pmatrix}.$$

For the explicit form of the crystal polarizability-tensor components g_0 and g_τ see, e.g., Ref. 13. A solution of the matrix equation (4) can be found in analogy with Refs. 14–16. Since we shall be interested hereafter in the properties of radiation propagating at a large angle to the particle-velocity $\mathbf{v}$, we write in explicit form only the expression for the diffracted waves

$$\mathbf{E}^{s(-)}_{k_1}(\mathbf{r},\omega)=\sum_{p=-1}^{1}\mathbf{E}^{s(-)}_{k_1+p\varkappa}(r,\omega)$$
$$=\frac{\mathbf{e}_s\beta g_{\tau}{}^s\exp(i\mathbf{k}_1\mathbf{r}-(i/4\gamma_0)k(z-L)x^s)}{2((x+2g_0)^2+4\beta r_s)^{1/2}}$$
$$\times\sum_{a=1,2}\sum_{b=\pm1}\Big[\,b\Big(1+(-1)^a\frac{\delta_-{}^s}{\delta_0{}^s}\Big)$$
$$\times\exp\Big\{i\frac{k(z-L)}{4\gamma_0}b\Big(2\frac{\varkappa}{k}\gamma_0-(-1)^a\delta_0{}^s\Big)\Big\}$$
$$-\frac{\xi}{2}\zeta_b{}^s\frac{(-1)^a}{\delta_0{}^s}\exp\Big\{i\varphi'-\frac{ik}{4\gamma_0}(z-L)\Big((-1)^a\delta_0{}^s+b\frac{2\varkappa}{k}\gamma_0\Big)\Big\}$$
$$-\frac{\exp(-ib\varphi')}{\delta_+{}^s}\Big(\exp\Big\{ib\frac{k(z-L)}{4\gamma_0}\Big(\delta_+{}^s+2\frac{\varkappa}{k}\gamma_0\Big)\Big\}$$
$$-\exp\Big\{ib\frac{k(z-L)}{4\gamma_0}\Big(\frac{\delta_+{}^s+\delta_-{}^s}{2}\Big)\Big\}\Big]. \qquad (5)$$

We have introduced here the symbois

$$\delta_0=(\delta_-{}^2+4\beta r_s\xi^2)^{1/2},\quad \zeta_{\pm1}=(x+2g_0)\pm((x+2g_0)^2+4\beta r_s)^{1/2},$$
$$\delta_\pm=((x+2g_0)^2+4\beta r_s)^{1/2}\pm2\frac{\varkappa}{k}\gamma_0,\quad x=\alpha\beta-g_0(\beta+1),$$
$$r_s=g_\tau{}^sg_{\bar\tau}{}^s,\quad \beta=\frac{\gamma_0}{\gamma_1},\quad \alpha=\frac{2\mathbf{k}\boldsymbol{\tau}+\tau^2}{\omega^2}c^2,$$

γ_0 and γ_1 are the cosines of the angles between $\mathbf{k}$ and $\mathbf{k}_1$ and the inward normal $\mathbf{N}\|\boldsymbol{\varkappa}\|\mathbf{z}$ to the crystal. In the case of a standing USW it is necessary to replace $\pm e^{\pm i\varphi'}$ in (4) by $i\sin\varphi'$. It follows from (5) that the phase of the waves $E^{s(-)}_{k_1}$ acquires additional terms that depend on the USW parameters. The refractive index of a crystal acted upon by a USW is thus modified by the appearance of an effective refractive index that depends on the frequency and amplitude of the sound wave ($b,c,d=\pm1$):

$$n_{b,c}=1+\frac{1}{4}\Big(-x+b\Big(2\frac{\varkappa}{k}\gamma_0+c\delta_0\Big)\Big);$$
$$n_d=1+\frac{1}{4}\Big(-x+d\Big(4\frac{\varkappa}{k}\gamma_0+((x+2g_0)^2+4\beta r_s)^{1/2}\Big)\Big).$$

Substitution of (5) in (1) yields for the PXR distributions in spectrum and angle of the crystal, in the presence of USW, the expression

$$dN_{\mathbf{n}\omega}{}^s=\frac{(\mathbf{e}_s\mathbf{v})^2}{4\pi^2c\hbar\omega}\Big|\frac{\beta g_{\bar\tau}}{2((x+2g_0)^2+4\beta r_s)^{1/2}}\sum_{a,d=1,2}\sum_{b,c=\pm1}\Big[\Big(b\Big(1$$
$$+(-1)^a\frac{\delta_-}{\delta_0}\Big)+\frac{\xi}{2}\zeta_b\frac{(-1)^a}{\delta_0}\exp(-i\varphi')\Big)$$
$$\times\Big(\frac{1}{q_b{}^a}-\frac{1}{q_0}\Big)\Big(1-\exp\Big(-ikq_b{}^a\frac{L}{\gamma_0}\Big)\Big)$$
$$-(-1)^d\xi\exp(ic\varphi')\frac{\zeta_c}{\delta_+}\Big(\frac{1}{q_d{}^c}-\frac{1}{q_0}\Big)$$
$$\times\Big(1-\exp\Big(-ikq_d{}^c\frac{L}{\gamma_0}\Big)\Big)\Big]\Big|^2, \qquad (6)$$

where

$$q_b{}^a=q_0+\frac{1}{4}\Big(x+b\Big(\frac{2\varkappa}{k}\gamma_0-(-1)^a(\delta_-{}^2+4\beta r_s\xi^2)^{1/2}\Big)\Big)$$
$$q_d{}^c=q_0+\frac{1}{4}\Big(x+c\Big(\frac{2\varkappa}{k}\gamma_0(1-(-1)^d)+((x+2g_0)^2+4\beta r_s)^{1/2}\Big)\Big),$$
$$q_0=\frac{1}{2}(\vartheta^2+\gamma^{-2}).$$

ϑ is the angle between the PXR wave vector and the vector $\omega\mathbf{v}/cv+\boldsymbol{\tau}$, $\gamma=E/mc^2$ is the particle gamma vector, E is the particle energy, m the particle mass, c the speed of light, and L the crystal thickness. Analysis of (6) shows that the emission cross section is a maximum if the real part of the longitudinal momentum transferred to the medium vanishes, i.e.,

$$\mathrm{Re}(q_j{}^i)=0. \qquad (7)$$

In this case the corresponding coherence length $l^i_j=(kq^i_j)^{-1}$ reaches its maximum. Condition (7) differs from the Vavilov-Cherenkov condition for an amorphous medium in that the corresponding refractive index is replaced by the refractive index of the crystal under the influence of the USW. Equations (6) were obtained neglecting multiple scattering of the charged particles. Under real experimental conditions[3–5] this scattering can be substantial. It becomes therefore necessary to take into account the influence of multiple scattering on the PXR formation. It follows from an analysis of this question[2,17,18] that the influence of the multiple scattering can be phenomenologically taken into account by replacing the real part of the momentum kq_j^i transferred to the medium by $(1/2)k\,\bar{\vartheta}_s^2$. That is to say, in this case $q_0=\frac{1}{2}(\vartheta^2+\gamma^{-2}+\bar{\vartheta}_s^2)$, where $\bar{\vartheta}_s^2$ is the average multiple-scattering angle[17]

$$\overline{\vartheta_s^2}=(E_s/E)^2L/L_R,$$

L_R is the crystal radiation length, and E_s is equal to 21 MeV for electrons.

3. PXR ANGULAR CHARACTERISTICS AND NUMBER OF PHOTONS

We consider first a crystal of thickness smaller than the PXR absorption length at the given frequency, but much larger than the extinction length, i.e., we investigate the case of a weakly absorbing crystal. It is known[14,15,19,20] that USW exerts a maximum influence in dynamic diffraction of x rays (thermal neutrons, and others) if the x-ray-acoustic resonance condition

$$\delta_-=0 \tag{8}$$

is met, i.e., when the USW wavelength is equal to the extinction length. Conditions (7) and (8) can be met simultaneously for waves with refractive indices

$$n=1+{}^1/_4(-x+2\varkappa\gamma_0/k\pm\delta_0),$$
$$n=1+{}^1/_4(-x+((x+2g_0)^2+4\beta r_s)^{1/2}\pm4\varkappa\gamma_0/k),$$

if the USW vector satisfies the equality

$$\varkappa=\frac{k}{2\gamma_0}\frac{(\beta r_s+(\vartheta_{sc}^2+\vartheta_b^2)^2)}{(\vartheta_{sc}^2+\vartheta_b^2)},\qquad \vartheta_b^2=\gamma^{-2}-g_0+\overline{\vartheta_s^2}. \tag{9}$$

In this situation the radiation field contains waves whose refractive indices differ by a small amount proportional to the USW amplitude.

Interference of waves with different refractive indices leads to an oscillatory dependence of the PXR intensity on the USW amplitude. Similar effects have been observed[19,20] in diffraction of x rays and neutrons by crystals acted upon by USW. An essential feature of this situation is that the oscillatory changes of intensity takes place in a narrow angle interval near ϑ_s. For $\vartheta_p\Delta\vartheta\ll\xi(\beta r_s)^{1/2}$ and in the angle range $\vartheta_p\pm\Delta\vartheta$ we can express the PXR spectral-angular distribution of an absorbing crystal in the form

$$dN^s_{n,\omega}=C_s^2N^s\frac{(\overline{\vartheta_s^2}+\vartheta^2)}{(1+M^2)^2}\Bigg[\frac{(1+M^2)}{(q_+q_-)^2}\times\Big((q_+^2+q_-^2)\Big(1-\cos\Big(\frac{\omega L}{\gamma_0}q_0^0\Big)\times\sin\Big(\frac{\omega L}{2\gamma_0}(\xi^2\beta r_s)^{1/2}\Big)\Big)+(q_+^2-q_-^2)\sin\Big(\frac{\omega L}{\gamma_0}q_0^0\Big)\times\sin\Big(\frac{\omega L}{2\gamma_0}(\xi^2\beta r_s)^{1/2}\Big)\Big)+2\frac{(1-M^2)}{q_+q_-}\Big(\cos\Big(\frac{\omega L}{2\gamma_0}(\xi^2\beta r_s)^{1/2}\Big)-\cos\Big(\frac{\omega L}{\gamma_0}q_0^0\Big)\Big)\cos\Big(\frac{\omega L}{2\gamma_0}(\xi^2\beta r_s)^{1/2}\Big)\Bigg], \tag{10}$$

$$q_\mp=q_0^0\pm\frac{1}{2}(\xi^2\beta r_s)^{1/2};$$

$$q_0^0=\frac{1}{4}\Big(2(\vartheta^2+\vartheta_b^2)+\mathrm{Re}(x+2g_0)-2\frac{\varkappa}{k}\gamma_0\Big),$$

$$N^s=\frac{\sin^2\theta_B}{\pi\beta r_s\omega}N_0; \tag{11}$$

$$M=\frac{(x+2g_0)+((x+2g_0)^2+4\beta r_s)^{1/2}}{(4\beta r_s)^{1/2}},\qquad N_0=\frac{e^2r_s}{4\pi\sin^2\theta_B\hbar c},$$
$$C_0=\cos\varphi\cos2\theta_B,\qquad C_1=\sin\varphi,$$

where φ is the azimuthal angle. Analysis shows that in view of the small angle interval $\Delta\vartheta$ its influence on the integral intensity of the PXR reflection can be neglected.

When

$$\varkappa<\frac{k}{2\gamma_0}\Big(\frac{\beta r_s+\vartheta_b^4}{\vartheta_b^2}\Big), \tag{12}$$

the PXR is a superposition of waves $E^{s(-)}_{k_1+p\varkappa}$ with refractive indices

$$n_p=1+{}^1/_4(-x+((x+2g_0)^2+4\beta r_s)^{1/2}-4p\varkappa\gamma_0/k), \tag{13}$$

which can in this situation be regarded as independent of the USW amplitude. The angular distribution (10) of PXR of a weakly absorbing crystal can be written in the form

$$\frac{dN}{d\Omega}=\sum_{s=0,1}C_s^2N_0\frac{\omega L}{c\gamma_0}\Bigg[\frac{(\vartheta^2+\overline{\vartheta_s^2})}{(\vartheta^2+\vartheta_b^2)^2+\beta r_s}+\xi^2\sum_{\substack{p=\pm1\\m=0,1}}\frac{(\vartheta^2+\overline{\vartheta_s^2})}{(\vartheta^2+\vartheta_b^2+2pm\varkappa\gamma_0/k)^2+\beta r_s}\,\frac{(\vartheta^2+\vartheta_b^2)^{4(1-m)}(\beta r_s)^{2m}}{((\vartheta^2+\vartheta_b^2+p\varkappa\gamma_0/k)^2+\beta r_s-(\varkappa\gamma_0/k)^2)^2}\Bigg]. \tag{14}$$

For a low-frequency USW ($\varkappa\ll k\vartheta_b^2/2\gamma_0$) and in the approximation $\beta r_s\ll\vartheta_b^2$ we can represent Eq. (14) in the form

$$\frac{dN}{d\Omega}\approx\sum_{s=0,1}C_s^2N_0\frac{\omega L}{c\gamma_0}\frac{(\vartheta^2+\overline{\vartheta_s^2})}{(\vartheta_b^2+\vartheta^2)^2}(1+2\xi^2)=\Big(\frac{dN}{d\Omega}\Big)_0(1+2\xi^2), \tag{15}$$

where $(dN/d\Omega)_0$ is the angular distribution of the crystal PXR without external actions. The influence of low-frequency USW on PXR generation in a thin crystal reduces thus to an angular-distribution amplitude increase proportional the squared amplitude of the oscillations, with practically no change of its for.

The integral number of photons of the PXR reflection without allowance for second-order terms is

$$N=\sum_{s=0,1}\overline{C_s^2}\pi N_0\frac{\omega L}{c\gamma_0}$$
$$\times\left\{\sum_{p=0,\pm1}\xi^{|2p|}\ln\left(\frac{(\vartheta_D^2+\vartheta_b^2+p\varkappa\gamma_0/k)^2+R_s}{(\vartheta_b^2+p\varkappa\gamma_0/k)^2+R_s}\right)\right\},$$
$$\overline{C}_s^2={}^1/_2(\cos 2\theta_B)^{2s},\ R_s=\beta r_s-(p\varkappa\gamma_0/k)^2. \tag{16}$$

The action of low-frequency sound on the formation of PXR in a weakly absorbing crystal is similar in many respects to its action on x-ray diffraction. The integral number of photons changes by 2–20% when $\xi=\frac{1}{2}\tau a$ is changed from 0.1 to 0.3.

The substantial changes of the USW influence on PXR generation in a thick crystal [Eq. (12)] are due to the fact that the coherence length is in this case substantially shorter than the crystal thickness. In fact, from the very definition of the coherence length it follows, if the Vavilov-Cherenkov condition (7) is met, that

$$l_j^i=(kq_j^{i''})^{-1}, \tag{17}$$

where $g_j^{i''}$ is the imaginary part of g_j^i. In the case of crystal without USW (Ref. 8) we have

$$q_j^{i''}=\frac{g_0''\beta[(\vartheta_b^2+\vartheta^2+r_s''/g_0'')^2+r_s'-(r_s''/g_0'')^2]}{2(\beta r_s+(\vartheta_b^2+\vartheta^2)^2)}, \tag{18}$$

where $r_s'(r_s'')$ is the real (imaginary) part of $g_\tau^s g_{\bar\tau}^s$.

Thus, without external action, the coherence length in a crystal is inversely proportional to the imaginary part g_0 of the dielectric constant. For $\vartheta_b^2\gg(r_s')^{1/2},r_s''/g_0''$ and $\beta\sim 1$ we have

$$q_j^i={}^1/_2g_0''\beta. \tag{19}$$

It follows from (7), when account is taken of (13), that the coherence lengths take the form

$$l_p=(kq_p'')^{-1}$$
$$=\left(k\frac{g_0''\beta((\vartheta^2+\vartheta_b^2+r_s''/g_0''+2p\varkappa\gamma_0/k)^2+r_s'-(r_s''/g_0'')^2)}{2(\beta r_s+(\vartheta^2+\vartheta_b^2+2p\varkappa\gamma_0/k)^2)}\right)^{-1}, \tag{20}$$

where $q_p=q_0{}^{+1/4}(x-((x+2g_0)^2+4\beta r_s)^{1/2}+4p\varkappa\gamma_0/k)$; $p=0,\pm1$. Obviously, if

$$\varkappa_{\text{thr}}=k(\vartheta_p^2+\vartheta_b^2+r_s''/g_0'')/2\gamma_0 \tag{21}$$

the coherence length L_{-1} is substantially changed. In fact,

$$l_{+1}\approx({}^1/_2kg_0''\beta)^{-1}=l_0,$$
$$l_{-1}\approx({}^1/_4kg_0''(1-(g_\tau^{s''}/g_0'')^2))^{-1}. \tag{22}$$

Let us estimate the change of the coherence length. Let 8-KeV PXR be generated by the (220) planes of a silicon single crystal 2.5 mm thick by passage of 500-MeV electrons. Under these conditions the coherence length l_{-1} increases by an order of magnitude compared with coherence length l_0 ($\beta=1$). The USW frequency is calculated from Eq. (21) and equals 830 MHz under these conditions. The angle interval ϑ_{ac} in which the influence of the USW leads to a substantial change of the coherence length is determined by the condition

$$\vartheta_{\text{ac}}=(r_s'-(r_s''/g_0'')^2)^{1/4}. \tag{23}$$

Obviously, ϑ_{ac} is always less than ϑ_b, which determines the PXR angle divergence of a stationary crystal. For the situation considered above $\vartheta_b=7\cdot7\times10^{-3}$ rad and $\vartheta_{\text{ac}}=1.6\cdot10^{-3}$ rad. The divergence of the wave $E^s_{k_1+x}$ is thus practically equal to the divergence of $E^s_{k_1}$, while the divergence of the wave $E^s_{k_1-\varkappa}$ is substantially smaller.

The physical cause of the increase of the coherence length under acoustoparametric-resonance conditions is the following.

The refractive index of a crystal under dynamic-diffraction conditions is given by (see, e.g., Refs. 9 and 13)

$$n_{1,2}=1+{}^1/_4(-\alpha\beta+g_0(1+\beta)\pm((\alpha\beta+g_0(1-\beta))^2+4\beta r_s)^{1/2}).$$

The condition $\text{Re}(n)=1+\frac{1}{2}(\vartheta^2+\gamma^{-2}+\bar\theta_s^2)$ for the onset of PXR is met only for n_1 at a sufficiently large deviation from the Wulf-Bragg condition $\alpha\sim-2(|g_0'|+\vartheta^2+\gamma^{-2}+\bar\theta_s^2$ ($\beta=1$). In this situation the crystal absorption coefficient for the n_1 mode tends to the absorption coefficient $M_0=\frac{1}{2}kg_0''$ of an amorphous medium. When the Wulf-Bragg condition $\alpha=0$ is met, the linear absorption coefficient for the mode n_1 is substantially smaller than the absorption coefficient of an amorphous medium. It is this which causes the anomalous passage of x radiation.[13]

The action of USW alters the refracting properties of the crystal. The altered refractive index for the $E^s_{k_1-\varkappa}$ wave can be written in the form (13)

$$n_{1,2}^-=1+{}^1/_4(-\alpha\beta+g_0(1+\beta)+4\varkappa\gamma_0/k\pm((\alpha\beta+g_0(1-\beta))^2+4\beta r_s)^{1/2}).$$

For $\varkappa=(k/2\gamma_0)(\gamma^{-2}+|g_0'|+\vartheta^2+\bar\theta_s^2)$, the condition for the onset of PXR is met if $\alpha=0$. As a result, PXR are produced in this case under conditions of anomalous passage of the x rays. Since, according to (17), the coherence length is inversely proportional to the x-ray absorption coefficient, it increases strongly. Note that $g_\tau^{\sigma''}/g_0''=\varepsilon_0^\sigma e^{-M}$, where M is the Debye-Waller factor and $\varepsilon_0^\sigma\approx1$ (Ref. 13), so that it follows from (22) that $l_{-1}\approx(1-(\varepsilon_0^\sigma e^{-M})^2)^{-1}$. The increase of the coherence length depends thus substantially on the temperature, and when the latter is lowered the coherence length increases. Thus, under the conditions of the above example, on going from room temperature to 4 K, the coherence length l_{-1} is doubled. The condition $\alpha=0$ corresponds to the maximum amplitude of the diffracted waves. The abrupt decrease of the absorption and the increase of the wave amplitude $E^\sigma_{k_1-\varkappa}$ under conditions of PXR generation leads to an increase of the angular density of the radiation. Acoustoparametric resonance sets in. A change of the wave amplitudes (8)–(10) in x-ray-acoustic resonance also increases the angular density of the PXR.

The angular distribution of the PXR of a thick crystal is of the form

$$\frac{dN}{d\Omega}=\sum_{s=0,1}\frac{C_s^2N_0}{\beta g_0''}\left[\frac{(\vartheta^2+\overline{\vartheta_s^2})}{(\vartheta^2+\vartheta_b^2+r_s''/g_0'')^2+r_s'-(r_s''/g_0'')^2}\right.$$

$$+\sum_{p=\pm1;\, m=0,1}\frac{\xi^2(\vartheta^2+\overline{\vartheta_s^2})(\vartheta^2+\vartheta_b^2)^{4(1-m)}(\beta r_s)^{2m}}{((\vartheta^2+\vartheta_b^2+p\varkappa\gamma_0/k)^2+R_s)^2((\vartheta^2+\vartheta_b^2+r_s''/g_0''+2pm\varkappa\gamma_0/k)^2+r_s'-(r_s''/g_0'')^2)}\Bigg]. \tag{24}$$

The integral number of photons is

$$N=\sum_{s=0,1}\frac{\pi C_s^2 N_0}{\beta g_0''}\times\sum_{p=0,\pm1}\xi^{2|p|}\Bigg[\frac{1}{2}\ln\left(\frac{(\vartheta_D^2+\vartheta_b^2+r_s''/g_0'')^2+r_s'-(r_s''/g_0'')^2}{(\vartheta_b^2+r_s''/g_0'')^2+r_s'-(r_s''/g_0'')^2}\right)$$
$$-\frac{\vartheta_b^2+r_s''/g_0''}{(r_s'-(r_s''/g_0'')^2)^{1/2}}\times\left(\operatorname{arctg}\frac{\vartheta_D^2+\vartheta_b^2+r_s''/g_0''}{(r_s'-(r_s''/g_0'')^2)^{1/2}}-\operatorname{arctg}\frac{\vartheta_b^2+r_s''/g_0''}{(r_s'-(r_s''/g_0'')^2)^{1/2}}\right)$$
$$-\frac{4p\varkappa\gamma_0/k}{R_s^{1/2}}\left(\operatorname{arctg}\frac{(\vartheta_D^2+\vartheta_b^2+p\varkappa\gamma_0/k)}{R_s^{1/2}}-\operatorname{arctg}\frac{(\vartheta_b^2+p\varkappa\gamma_0/k)}{R_s^{1/2}}\right)\Bigg],\quad R_s>0. \tag{25}$$

Analysis of expression (25) shows that a low-frequency USW acts on the integral number of PXR photons of a thick crystal just as in the case of weakly absorbing crystals. Under acoustoparameteric resonance conditions, the influence of the USW on the integral intensity is somewhat higher. The number of photons increases particularly strongly near the angle ϑ_{sc} defined by the condition (21). For example, under the conditions of the example above, for a detector-window angle $\sim\vartheta_{ac}$ near the angle ϑ_{sc} (21), the number of PXR photons entering the window is increased by the action of the USW by 1.4–5.3 times. Note that an increase of the USW amplitude, from $\xi\sim0.3$ to $\xi\sim1$, strengthens the action of the coherent sound oscillations on the x-ray diffraction and, as a result, increases the integral intensity of the diffracted radiation.[21] This tendency is preserved also in the influence of USW on the PXR process.

By virtue of (16), the conditions for resonant enhancement of the π and σ polarization do not coincide, i.e., if Eq. (16) is satisfied the PXR polarization increases simultaneously with the change of the integral intensity. Another important fact is that an USW alters the dependence of the PXR integral yield on the charged-particle energy. In fact, the PXR threshold energy of a crystal not acted upon by an USW is given by[2]

$$E_{\rm thr}=mc^2/|g_0'|^{1/2} \tag{26}$$

or

$$\gamma_{\rm thr}^{-2}=|g_0'|. \tag{27}$$

In the case of a crystal acted upon by USW, the threshold energies (the gamma factors) are different for all three waves

$$\gamma_{\rm eff}^{-2}=|g_0'|, \tag{28}$$

where

$$\gamma_{\rm eff}^{-2}=\gamma^{-2}+2p\varkappa\gamma_0/k. \tag{29}$$

Obviously, $\gamma_{\rm eff}^{-2}$ is substantially smaller than γ^{-2} if $\gamma^{-2}\approx2\varkappa\gamma_0/k$ for $p=-1$ or, which is the same, the charged-particle effective energy is substantially higher than the threshold energy. When multiple scattering of charged particies becomes noticeable ($\bar\vartheta_s^2\sim\gamma^{-2}$), Eq. (29) takes the form

$$\gamma_{\rm eff}^{-2}=\gamma^{-2}+\overline{\vartheta_s^2}+2p\varkappa\gamma_0/k. \tag{30}$$

The influence of multiple scattering on the change of $\gamma_{\rm eff}^{-2}$ can also be compensated for by the action of USW. The wave vector of the USW is then increased.

We have shown thus that an USW influences substantially the angular and integral characteristics of PXR.

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