Nonlinear strong absorption wave in an optically bistable semiconductor

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Fronts of switching from a state of weak absorption to a state of strong absorption and vice versa may propagate in an extended optically bistable medium. A combination of two such fronts forms a nonlinear wave. A study is reported of the motion of a traveling wave inside the medium and the behavior of the fronts at a boundary of a medium. At the forward boundary (considered relative to the exciting light) such a wave is localized and may broaden on increase in the intensity. This is a mechanism alternative to the propagation of kinks. Various types of behavior of the rear boundary are possible: wave localization, formation of self-waves, and detachment of a traveling wave.

INTRODUCTION

Optical bistability under the influence of laser pumping of an extended medium has been investigated for gases and semiconductors. A laser spark may appear in gases[1] and it may be localized near the focus of a laser beam or may travel in the opposite direction to meet the beam. In the case of semiconductors, studies have been made of the motion of a strong absorption kink[2,3] localized at the forward boundary of a sample and propagating in jumps into the interior as the laser pump intensity is increased. Such kinks appear when the transport processes are suppressed (for example, in polycrystalline samples). On the other hand, crystals exhibit localization, self-oscillations, and propagation of a strong-absorption domain opposite to the beam,[4] depending on the pump intensity.

All these processes represent the motion of a front switching from a state of weak absorption to a state of strong absorption and vice versa. We shall investigate continuous propagation of fronts in semiconductors, which is related to the processes of transport in a medium and differs from the jumps (kinks) based on the delay of switching.

A nonlinear strong-absorption wave has leading and trailing fronts and its motion occurs as follows.

The absorption of light creates electrons and holes which recombine after a time τ_R and having traveled a certain distance. Nonradiative recombination in a plasma of concentration n increases the lattice temperature T. Propagation of a plasma and of heat in space occurs because of ambipolar diffusion (represented by the coefficient D) and because of thermal diffusivity (λ). Finally, the optical absorption edge, i.e., the width of the band gap E_g and the absorption coefficient α, depends on the concentration n and on the temperature, $\alpha(n, T)$, and it is found that an increase in n and T reduces the width of the band gap, therefore during the next stage the absorption of light appears at a new place. The existence of feedback facilitates propagation of an initial perturbation (wave) to those parts of the crystal which are illuminated (motion of the leading front). The attenuation of light, recombination processes, and heat conduction cause relaxation of a perturbation in those parts of a crystal where it has been found earlier (motion of the trailing front). There are altogether three transport-nonlinearity pairs: $\lambda - \alpha(T)$, $D - \alpha(T)$, $D - \alpha(n)$, responsible for the formation of waves. We shall label these waves as follows (in the order just given): a thermal (TT) wave; a thermal-concentration (NT) wave; and a concentration (NN) wave. It must be stressed directly that the behavior of all three waves is generally different and requires an individual analysis. These three types of wave represent different limiting cases of a nonlinear strong-absorption wave and, therefore, it is interesting to study the motion of a mixed wave.

This paper is organized as follows. In Sec. 1 we shall introduce a model transport equation and consider it derivation for each of the three types of waves. In Sec. 2 we shall obtain an integral equation for the motion of two fronts of a wave in an infinite medium, from which we shall derive a system of two nonlinear differential equations suitable for analysis. We shall study the motion of a wave at a constant velocity and its stability. The motion of a wave in a bounded medium will be considered in Secs. 3–5. Section 3 is devoted to a wave localized at the leading front and its broadening on increase in the pumping rate, and to an alternative propagation of kinks.[2,3] Periodic self-waves created at a contact between bistable and strongly absorbing media are discussed in Sec. 4. A steady-state wave localized at the trailing front is investigated in Sec. 5. An increase in the pumping rate results in detachment of the wave from the rear boundary and its disappearance. On further increase in the pumping rate the wave becomes detached and changes into a traveling one.

Finally, Sec. 6 deals with waves of different types traveling in an infinite medium and with the competition between them. Expressions for the velocity of a mixed TT-NT wave, which changes to an NN wave at the highest pumping rates, are obtained for the case of strong pumping.

1. PRINCIPAL EQUATIONS OF THE PROBLEM

The plasma concentration $n(r, t)$, the temperature $T(r, t)$, and the intensity of light $J(r, t)$ are described by transport equations. We shall adopt a one-dimensional model because the effects under consideration are manifested even in this simple model:

$$\partial J/\partial r = -\alpha(n, T)J, \quad (1)$$

$$\partial n/\partial t = \alpha(n, T)J/\hbar\omega + D\partial^2 n/\partial r^2 - n/\tau_n, \quad (2)$$

$$\partial T/\partial t = n\hbar\omega/\tau_n c\rho + \lambda\partial^2 T/\partial r^2 - T/\tau_T. \quad (3)$$

Here, $\alpha(n, T)$ is the absorption coefficient governing the bistability effect; τ_n and τ_T are the relaxation times of n and

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T; c and ρ are the specific heats and the density of the medium; λ is the thermal diffusivity. The term n/τ_n models the diffusion of a plasma across the beam and recombination, whereas the term T/τ_T models the thermal diffusivity across the beam. The following expressions apply:

$$1/\tau_n=1/\tau_R+A_nD/R_n^2, \quad 1/\tau_T=A_T\lambda/R_T^2, \tag{4}$$

where R_n and R_T are the transverse dimensions of a plasma and of the temperature field; A_n and A_T are numerical coefficients of the order of unity.

If a light beam has a cylindrical shape, then $J=I$, where I is the surface power density ($\text{erg}\cdot\text{cm}^{-2}\cdot\text{s}^{-1}$), whereas in the case of a conical beam we have $J=r^2I$, where r is the distance to the focus. In the latter case we have to modify Eq. (2) by replacing α with α/r^2 and, moreover, in Eqs. (2) and (3) it is assumed that the longitudinal (along the beam) part of the Laplacian $r^{-2}(r^2(...)'_r)'_r$ reduces to $(...)''_{rr}$, i.e., the thickness of the wave front is much less than r.

Equations (1)–(3) were used in Refs. 2 and 3 to describe the propagation of kinks in an optically bistable semiconductor $CdZn_xS_{1-x}$, but in the case of longitudinal transport it was assumed that $D=\lambda=0$. In this limit the observed motion of kinks remains the only theoretical mechanism of broadening of the strong-absorption region on increase in the pumping rate. In this approach the terms which are dropped are of the same order as those which are retained.

We can solve Eqs. (1)–(3) by making some assumptions about the nature of the function $\alpha(n, T)$. In an optically bistable semiconductor the dependence of α on the plasma concentration n, associated with exchange, correlation, and degeneracy of electrons and holes, appears at concentrations of $n\sim 10^{17}$–10^{18} cm^{-3}. The concentrations achieved experimentally have been lower and generally in the range $n\lesssim 10^{16}$ cm^{-3} (Refs. 2 and 4), so that optical bistability is due to the influence of temperature on the width of the band gap. We shall assume that $\alpha(n, T)=\alpha_0+(\alpha_T-\alpha_0)\theta(T-T_0)+(\alpha_n-\alpha_0)\theta(n-n_0)$, where $\theta(x)$ is the step function, whereas T_0 and n_0 are the critical temperature and concentration. Such a form of the function $\alpha(n, T)$ corresponds to the strong bistability case. Moreover, the need to allow for α_0 does not always arise and unless otherwise stated, we shall assume that $\alpha_0=0$.

Equations (1)–(3) should be supplemented by the boundary and initial conditions. For Eq. (1), we have $I(\infty, t)=I_0$. The initial conditions should be considered separately in each specific case for Eqs. (2) and (3) because the solution depends strongly on these conditions. In the case of the boundary conditions we can consider infinite and semi-infinite samples. In the former case, we have

$$n(\infty, t)=T(\infty, t)=n(-\infty, t)=T(-\infty, t)=0. \tag{5}$$

At the boundary of a sample we have

$$n_r'(t)=T_r'(t)=0. \tag{6}$$

The principal system of equations, which we shall discuss in Secs. 2–5, is as follows:

$$y_t'=y_{xx}''-\beta^2y+j\theta(y-y_0), \qquad j_x'=-\theta(y-y_0). \tag{7}$$

We can easily show that any of the three types of waves can be reduced to the system of equations (7) if suitable assumptions are made. For example, the system (7) describes a thermal (TT) wave as long as the solution of Eq. (2) is of quasisteady and local nature corresponding to $n=\alpha J\tau_n/\hbar\omega<n_0$ and the following substitution of the variables is made:

$$x=\alpha_Tr, \quad t\leftrightarrow t\lambda\alpha_T^2, \quad y=T/T_I, \quad y_0=T_0/T_I, \quad \beta=(\tau_T\lambda\alpha_T^2)^{-1/2}, \quad T_I=\frac{I_0}{\lambda\alpha_Tc\rho}\frac{\tau_n}{\tau_R}. \tag{8}$$

We can describe a thermal-concentration (NT) wave if in turn Eq. (3) has a quasistationary local solution $T=n\hbar\omega\tau_T/\tau_Rc\rho$ and if the following substitution of variables is made:

$$x=\alpha_Tr, \quad t\leftrightarrow tD\alpha_T^2, \quad y=n/n_T, \quad y_0=n_{T_0}/n_T, \quad \beta=(\tau_nD\alpha_T^2)^{-1/2}, \quad n_T=\frac{I_0}{\hbar\omega D\alpha_T}, \quad n_{T_0}=\frac{T_0c\rho}{\hbar\omega}\frac{\tau_R}{\tau_T}. \tag{9}$$

Finally, a description of a concentration (NN) wave is obtained if we drop Eq. (3) and make the substitutions

$$x=\alpha_nr, \quad t\leftrightarrow tD\alpha_n^2, \quad y=n/n_n, \quad y_0=n_0/n_n, \quad \beta=(\tau_nD\alpha_n^2)^{-1/2}, \quad n_n=\frac{I_0}{\hbar\omega D\alpha_n}. \tag{10}$$

The behavior of the waves of different types and of mixed waves is outside the range of validity of the system (7) and is therefore discussed in Sec. 6.

2. INFINITE SAMPLE. TRAVELING WAVE

The solution of the system (7) in the form of a wave traveling at a constant velocity was investigated by Raĭzer.[1] He sought the solution of Eq. (7) dependent on an argument $x-ut$ and having the form of a wave with leading and trailing fronts, separated by a distance (width) d. Raĭzer obtained a system of two transcendental equations for u and d, which was solved numerically (Fig. 1). The question of stability of these solutions was not considered. The problem of stability does not reduce to the usual linearization because there is a nonlinearity of an "infinite" order (θ function).

In this section we shall use the system (7) to derive integrals and then approximate differential equations of motion of two fronts of a wave in an infinite sample. The initial and boundary conditions for the system (7) are then as follows:

$$y(x, 0)=Y_0(x), \quad j(\infty)=1, \quad y(\infty, t)=y(-\infty, t)=Y_0(\infty)=Y_0(-\infty)=0. \tag{11}$$

We shall assume that the leading and trailing fronts of a wave are located at the points $x_1(t)$ and $x_2(t)$ and that the condition $x_1(t)<x_2(t)$ is satisfied. Integrating the equation for $j(x)$ and substituting it into the first equation, we find that

$$y_t'=y_{xx}''-\beta^2y+\Xi(x, t), \quad \Xi(x, t)=\exp[x-x_2(t)]\,\theta[x-x_1(t)]\,\theta[x_2(t)-x]. \tag{12}$$

We shall apply the coordinate Fourier transformation to this equation. Solving then the differential equation with respect to time and returning to the coordinate representation, we have to satisfy two conditions $y[x_1(t), t]=y[x_2(t), t]=y_0$,

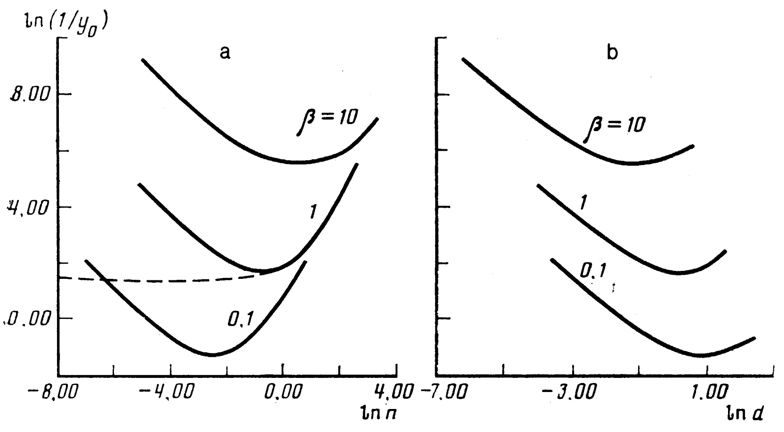

FIG. 1. Dependences of the dimensionless velocity u (a) and of the dimensionless thickness d (b) of a nonlinear strong-absorption wave on the pumping rate ($1/y_0$), plotted for different values of the parameter β. The dashed curve near the $u(1/y_0)$ graph for $\beta=1$ is the solution of Eq. (17) obtained for $\beta=1$. Equation (17) is derived ignoring the existence of a trailing front, so that there is no threshold pumping and the curve $u(1/y_0)$ does not have a minimum. A leading front (if it exists independently, see Secs. 3–5) may appear at intensities below the threshold value. After the formation of the trailing front the wave above the threshold disappears.

which are integral equations for the function $x_{1,2}(t)$. These equations are as follows:

$$2\pi y_0=\int_{-\infty}^{\infty} dk\exp[-ikx_{1,2}(t)-(k^2+\beta^2)t]\Big[\bar{Y}_0(k)+\int_0^t \bar{\Xi}(k,t')\exp[(k^2+\beta^2)t']dt'\Big], \qquad (13)$$

where $\bar{Y}_0$ and $\bar{\Xi}$ are the Fourier transforms of the functions without a bar.

The approximate method described below involves the replacement of the actual functions $x_{1,2}(t')$ with $x_{1,2}(t')=x_{1,2}(t)+u_{1,2}(t'-t)$, where the front velocities $u_{1,2}$ are assumed to be a constant calculation of the integral with respect to dt'. This method has some features in common with the Fokker-Planck approximation in kinetics, but there is one important difference. On the one hand, no use is made of the second derivatives, i.e., it is assumed that the fronts are not accelerated and, on the other hand, no linearization is made in respect of the velocities $u_{1,2}$ in the kernel of the integral equation. Therefore, the resultant differential equations, where the velocities $u_{1,2}$ are regarded now as the derivatives $dx_{1,2}/dt$, are now valid for any front velocities but fail at high accelerations when we have to assume that $(du_{1,2}/dt)/u_{1,2}\ll\beta^2$.

We shall be interested in time intervals $t\gg\beta^{-2}$, so that the initial distribution $Y_0(x)$ is already relaxed and in the interval along dt' we can replace the required limit with $-\infty$. This yields the following equations

$$y_0=y_1=\frac{\exp[-d(1-a_2+\varphi_2)]}{2\varphi_2(a_2-\varphi_2)}-\frac{e^{-d}}{2\varphi_1(a_1-\varphi_1)}, \qquad (14)$$

$$y_0=y_2=\frac{1}{2\varphi_2(a_2+\varphi_2)}-\frac{\exp[-d(a_1+\varphi_1)]}{2\varphi_1(a_1+\varphi_1)}, \qquad (15)$$

where $y_1=y(x_1(t),t)$, $y_2=y(x_2(t),t)$, $a_{1,2}=1+u_{1,2}/2$, $\varphi_1^2=\beta^2+u_1-u_2+u_1^2/4$, $\varphi_2^2=\beta^2+u_2^2/4$, $d=x_2(t)-x_1(t)$. Assuming that $u_1=u_2$, we obtain a system of equations from the review of Raĭzer.[1] We shall now consider the solution of this system. Its distinguishing feature is the existence of the threshold pumping rate $(y_0^{-1})_{\min}$ (we recall that $y_0\propto I_0^{-1}$), which depends on the parameter β. At pumping rates below the threshold the trailing front catches up with the leading front and the wave disappears. Above the threshold there are two branches of the solution, i.e., one pumping rate corresponds to two pairs (u,d) or two realizations of a wave traveling at a constant velocity: slow (left-hand branch) and fast (right-hand branch, see Fig. 1).

This solution depends strongly on the parameter β. We can see from Eqs. (8)–(10) that this parameter governs the attenuation length on $y(x)$: $\Delta x\propto\beta^{-1}$. The absorption length of light is equal to unity. At high values of $\beta\gg 1$ the fronts interact weakly and vice versa (we are speaking here of the influence of term y''_{xx}).

We shall give a series of asymptotic expressions for a wave traveling at a constant velocity u, which we shall require later. The threshold values $u_t(\beta)$, $d_t(\beta)$, $(y_0^{-1})_{\min}$ are as follows:

$$u_t\sim\begin{cases}1, & \beta\gg1\\ \beta, & \beta\ll1\end{cases},\quad d_t\sim\begin{cases}\beta^{-1}, & \beta\gg1\\ 1, & \beta\gg1\end{cases}. \qquad (16)$$

$$(y_0^{-1})_{\min}>y_\beta^{-1},\quad y_\beta^{-1}=\begin{cases}2\beta^2, & \beta\gg1\\ 2\beta, & \beta\ll1\end{cases},\quad \frac{(y_0^{-1})_{\min}-y_\beta^{-1}}{y_\beta^{-1}}\sim1.$$

For the right-hand branch, far from the threshold ($u\gg u_t$, $d\gg d_t$) the wave velocity is governed by the motion of the leading front and satisfies the equation

$$2(\beta^2+u^2/4)^{1/2}[1+u/2+(\beta^2+u^2/4)^{1/2}]=1/y_0. \qquad (17)$$

The width of the wave far from the threshold is governed by the fact that the trailing front shifts to such a distance at which the intensity of the transmitted light makes possible the motion of this front at the same velocity u as the leading front. A simple formula for the wave width is obtained when the fronts do not interact and $d\gg\beta^{-1}$: $d=-\ln(\beta y_0)$.

We can see from Eq. (17) that at high light intensities we obtain $u^2=2/y_0$ which agrees with the Zel'dovich expression for the propagation of a flame.

We shall now consider the stability of a traveling wave using Eqs. (14) and (15). We shall assume that the positions of the fronts differ little from equilibrium:

$$x_2-x_1=d+\delta,\quad u_{1,2}=u+w_{1,2},$$

where d and u satisfy the system $y_1(d,u)=y_2(d,u)=y_0$. We shall introduce

$$y_{ij}=\partial y_i/\partial u_j,\quad y_{id}=\partial y_i/\partial d;\quad i,j=1,2.$$

Expanding in terms of small deviations, we obtain

$$w_1y_{i1}+w_2y_{i2}+\delta y_{id}=0.$$

Hence, we find that

$$\frac{w_1}{w_2}=\frac{y_{1d}y_{22}-y_{2d}y_{12}}{y_{2d}y_{11}-y_{1d}y_{21}}=\text{const.}$$

Consequently, the relaxation solution is of the form $(x_2 - ut) = \text{const}(x_1 - ut)$. Using this solution, we obtain the stability criterion:

$$\frac{w_1}{x_1-ut}=\frac{w_2}{x_2-ut}=\frac{w_2-w_1}{\delta}=\frac{y_{1d}(y_{21}+y_{22})-y_{2d}(y_{11}+y_{12})}{y_{11}y_{22}-y_{12}y_{21}}<0. \qquad (18)$$

The left- and right-hand branches of the solutions for a traveling wave were investigated also for stability using the criterion of Eq. (18). In those cases when the solution could be obtained analytically, the investigation was also analytic. Near the minima in Fig. 1 the investigation was numerical. The left-hand branch is unstable and the right-hand one is stable. The physical meaning of the stability of the left-hand branch is that a fluctuation-induced increase in the wave width creates a pumping rate higher than that needed to compensate for the losses (as demonstrated in Fig. 1b). In those cases when the pumping rate y_0^{-1} is a decreasing function of the width of the strong-absorption region, the corresponding state is always unstable.

3. LOCALIZATION AT THE FORWARD BOUNDARY

If the intensity of light is constant, then a wave traveling along the beam reaches the forward boundary and stops. The resultant steady state of strong absorption extends over a thickness which is related to the pumping rate as follows:

$$y_0=\frac{e^{-d}}{2\beta(\beta-1)}-\frac{e^{-\beta d}}{\beta(\beta^2-1)}-\frac{\exp[-d(1+2\beta)]}{2\beta(\beta+1)}, \qquad (19)$$

and this state is stable. It is clear from Eq. (19) that there is a threshold pumping rate $(y_0^{-1})_{\min}$, dependent on β, which can maintain such a stable state. The width of the strong-absorption region d is then minimal. The asymptotic expressions describing this case are

$$(y_0^{-1})_{min}=\begin{cases}2\beta^2, & \beta\gg1\\ \beta, & \beta\ll1\end{cases}, \quad d_{min}=\begin{cases}\beta^{-1}\ln(2\beta), & \beta\gg1\\ -\ln\beta, & \beta\ll1\end{cases}.$$

It follows from the condition (6) that the boundary of a sample can stop (localize) a wave and lower the threshold of its existence [cf. Eq. (16)].

We shall now consider broadening of a localized wave on increase in the pumping rate, in connection with the results given in Refs. 2 and 3. We shall be interested in the case when $\beta \gg 1$, so that we can use Eq. (14) to describe the motion of the trailing front. Far from the threshold we obtain the equation of motion

$$2\varphi_1(\varphi_1-a_1)=e^{-d}j(t)y_0^{-1}, \qquad (20)$$

$$\varphi_1^2=\beta^2-d_t'+(d_t')^2/4, \quad a_1=1-d_t'/2,$$

where $j(t)$ is the time dependence of the pumping rate normalized to its initial value (we recall that in the adopted approximation we have $\alpha_0 = 0$, so that the presence of an initial wave and pumping is set by the initial condition). It is preferable to adopt an unknown function $s(t) = \exp(\text{-d})jy_0^{-1}$, which is proportional to the intensity of the transmitted light. We then obtain

$$s_t'=\frac{sj}{y_0}\left[\frac{j_t'}{j}-\Phi_\beta(s)\right], \qquad (21)$$

where the function $\Phi_\beta(s)$ has the asymptotic forms:

$$\Phi_\beta(s)=\begin{cases}(s-2\beta^2)(s-\beta^2)^{-1/2}, & d'\gg1\\ (s-2\beta^2)\beta^{-1}, & d_t'\ll1\end{cases}.$$

We can see that Eq. (21) is of the relaxation type. The quasisteady behavior corresponds to vanishing of the expression in the square brackets in Eq. (21). If the initial value $s(0)$ is greater (smaller) than the quasisteady-state value, then $s(t)$ tends to a quasisteady-state value from above (below). The approach itself, based on the application of a differential equation, is valid in the case of pumping rates which do not increase too rapidly with time. A sufficient condition is the approach of j_t'/j to zero with time. Then, the far asymptote of the quasisteady-state solution is of the form $s = 2\beta^2$ or

$$d=\ln[j(t)/2\beta^2y_0]. \qquad (22)$$

We shall compare now the broadening of the wave due to the motion of the trailing front as a result of the transport processes involving kink propagation.[2,3] We shall do this by dropping the term y''_{xx} responsible for the motion of the front, from the system (7) and allow for the weakly absorbing state: $\alpha_0 \neq 0$, $\zeta = \alpha_0/\alpha \ll 1$, where we have $\alpha = (\alpha_T, \alpha_n)$, depending on the type of wave and kinks. Then, for the states of strong and weak absorption, we obtain the following expressions instead of Eq. (7):

$$y_t'=-\beta^2y+j_0e^{-x}, \quad y_t'=-\beta^2y+\zeta j_0e^{-\zeta x}, \qquad (23)$$

where $j_0(t)$ is the arbitrarily normalized intensity of light incident on the front face of a sample. It is assumed in Refs. 2 and 3 that the specific dependence $j_0 \propto t$ applies. We shall normalize the intensity so that $\beta^2 y_0 = \zeta$, $j_0 = t/t_0$, and we shall assume that $t_0 \gg \beta^{-2}$, so that we can then compare the results with Eq. (22).

The motion of a kink occurs as follows. Initially, a sample is in a state of weak absorption. At the moment $t = t_0$ the critical intensity is reached and the forward boundary might be switched but the critical value y_0 has not been reached: the term y_t' is responsible for a delay with a characteristic time β^{-2}. It means that up to switching at a moment $(t - t_0) \propto \beta^{-2}$ the intensity exceeds the critical value at a depth

$$\beta^2y_0=\zeta j_0e^{-\zeta\bar{x}}, \quad \bar{x}=\frac{1}{\zeta}\ln\frac{t_0+\beta^{-2}}{t_0}\sim\frac{1}{\zeta\beta^2t_0}.$$

However, the whole $0 < x < \bar{x}$ layer may be switched to the upper state since $\beta y_0 < j_0 e^{-\bar{x}}$. Switching extends only to a depth $\bar{x}_1$:

$$\beta^2y_0=j_0e^{-\bar{x}_1}, \quad \bar{x}_1=\ln\frac{t_0+\beta^{-2}}{t_0\zeta}\sim\ln\frac{1}{\zeta}<\bar{x}.$$

This is followed by a pause which lasts until the critical value y_0 is reached at a depth $\bar{x}_1$: $\beta^2 y_0 = \zeta j_0 e^{-\bar{x}_1}$. Hence, this pause ends at

$$t_1=t_0/\zeta.$$

Proceding by induction, we then obtain

$$\frac{j_0(t_n)}{j_0(t_{n-1})}=\frac{1}{\zeta}, \quad j_0(t_n)e^{-\bar{x}_n}=1, \quad x_n=n\ln\frac{1}{\zeta} \qquad (24)$$

or, for the adopted law,

$$t_n = t_0 \zeta^{-n}. \tag{25}$$

Here, $\bar{x}_n$ is the depth where a kink appears at a moment t_n.

These semiquantitative conclusions applicable to a strong bistability are similar to the estimates obtained in Ref. 3. Substituting t_n from Eq. (25) into Eq. (22), and bearing in mind that $\beta^2 y_0 = \zeta$, we obtain

$$d(t_n) = \ln \frac{t_n}{2\beta^2 y_0 t_0} = (n+1) \ln \frac{1}{\zeta}.$$

We can see that the rates of both processes are of the same order of magnitude, as pointed out in Sec. 1. Kinks regarded as the broadening mechanism cease to act at low values in the range $\beta < 1$, when the whole depth of the switched region becomes comparable with the attenuation length β^{-1}, i.e., when

$$\beta \ln \frac{1}{\zeta} \leqslant 1. \tag{26}$$

These conclusions are independent of the law describing the rate of rise of the pumping $j(t)$, provided $j'_t/j \to 0$.

4. BOUNDARY BETWEEN BISTABLE AND STRONGLY ABSORBED REGIONS. SELF-WAVES

The existence of a threshold pumping rate for a traveling wave (Sec. 2) and its reduction near the forward boundary of a sample (Sec. 3) suggest that the conditions for the existence of a wave are easier to satisfy in those regions where relaxation is difficult. Therefore, a traveling wave formed near such a region may disappear after moving deeper into a bistable region.

As a first example (see also Sec. 5) we shall consider the case when behind the rear boundary of a bistable region there is a strongly absorbing region and the other parameters of the regions are the same (a variable-gap semiconductor).

We shall assume that at an initial moment in time the absorbing region ($x < 0$) is heated near the boundary ($x = 0$) so that the temperature or concentration at the boundary is slightly higher than the critical value. Then, the leading front of a wave penetrates deeply into the bistable region. The trailing front does not exist initially, but it forms when the temperature or concentration at the boundary drops below the typical value and it catches up with the leading front, as it happens in the case of a subthreshold wave in the bistable region (Sec. 2). The wave disappears, the bistable region becomes bleached, and the process is repeated.

A calculation of periodic self-waves is made in the case when $\beta \gg 1$, when we can use the differential approximation. Then, in the course of formation of the leading front of width β^{-1}, we find from Eq. (15) that

$$2\varphi_2(a_2+\varphi_2) y_0 = 1, \quad \beta d \gg 1.$$

We shall select a pumping rate known to be below the threshold, $y_0 = (1-\varepsilon)[2\beta(1+\beta)]^{-1}$, $\beta\varepsilon \ll 1$. Then, the velocity of the leading front is constant and equal to $u_2 = 2\varepsilon(\beta+1)$. The critical value of y is reached at the boundary at a moment when $\exp[-x_2(t)] = \beta^2 y_0$, i.e., when $x_2 \approx \ln 2$ and $\beta x_1(t) \gg 1$. This is followed for the formation of the trailing front $x_1(t)$ and the motion of this front will be described for the case when $\beta x_1(t) \gg 1$. The velocity of contraction of the strongly absorbing region $d'_t = x'_2(t) - x'_1(t)$, follows from Eqs. (14) and (15):

$$d_t' = -2\beta(1-e^{-d})/(2e^{-d}-1)^{1/2}, \quad d(0) = \ln 2. \tag{27}$$

This equation can be integrated in a region defined by $\ln 2 > d \gtrsim \beta^{-1}$ and the contraction time is $t_2 = (\ln \beta)/2\beta$. Near $d \propto \beta^{-1}$ the function $t(d)$ behaves logarithmically so that the duration of motion t_2 is accurate to within a constant under the logarithm. The period of self-waves is described by

$$t_0 = t_1 + t_2 = \frac{\ln 2}{2\varepsilon(\beta+1)} + \frac{\ln \beta}{2\beta}; \quad t_1 \gg t_2. \tag{28}$$

The stage of formation of the leading front and the stage of contraction are separated by short time intervals proportional to β^{-2}. During one of these intervals the trailing front is formed and during the other both fronts disappear and a new leading front is formed.

We shall now write down the expressions for the time dependence of the intensity of light transmitted across the boundary between the regions:

$$s(t) \propto \exp[-d(t)]$$

$$= \begin{cases} \exp[-2\varepsilon t(\beta+1)], & 0 < t < t_1 \\ (1/2)[1 + (6\beta\Delta t_1)^{2/3}], & t > t_1,\ \Delta t_1 = t - t_1 \ll t_2 \\ 1 - \exp(-2\beta\Delta t_2), & t < t_1 + t_2,\ \Delta t_2 = t_2 - t \ll t_2. \end{cases} \tag{29}$$

The dependence $d(t)$ following from Eq. (27) is implicit, so that in Eq. (29) the second stage consists of two asymptotes valid at the beginning and end of this state.

If $\beta \ll 1$, the period of self-waves is $t_0 \propto \beta^{-2}$.

5. BEHAVIOR NEAR THE REAR FACE

In a study of the steady state and its stability near the rear face it is necessary to allow for the boundary condition $y'_x(0, t) = 0$ in the derivation of the differential equations (14) and (15). This is done by allowing for the propagation of a source $\Xi(x,t)$ in Eq. (12) over the whole axis in accordance with the expression $\Xi(x) = \Xi(|x|)$. Instead of the system (14)–(15), we now have a more general system

$$y_0 = y_1 = \frac{\exp[-d(1-a_2+\varphi_2)]}{2\varphi_2(a_2-\varphi_2)} \{1+\exp[-2x_1(1-a_2+\varphi_2)]\} - \frac{e^{-d}}{2\varphi_1(a_1-\varphi_1)} \{1+\exp[-2x_1(1-a_1+\varphi_1)]\}, \tag{30}$$

$$y_0 = y_2 = \frac{1}{\varphi_2^2 - a_2^2} + \frac{1+\exp[-2x_2(1-a_2+\varphi_2)]}{2\varphi_2(a_2-\varphi_2)} - \frac{\exp[-d(a_1+\varphi_1)]}{2\varphi_1(a_1+\varphi_1)} - \frac{\exp[-d-(x_1+x_2)(1-a_1+\varphi_1)]}{2\varphi_1(a_1-\varphi_1)}, \tag{31}$$

where $d = x_2 - x_1$. We shall consider only the limiting cases $u_{1,2} = 0$ and $x_1 = 0$, and we shall begin with consideration of the first of these two cases.

Under steady-state conditions we shall assume that $\beta \gg 1$ and $\beta \ll 1$ and we shall be interested in the values $y(d)$ and $y(0)$. If $d \ll 1$, we then obtain

$$y(0) = \frac{1-z}{\beta(\beta-1)}, \quad y(d) = \frac{\beta - 1 + 2z - (\beta+1)z^2}{2\beta(\beta^2-1)}, \quad z = e^{-d\beta}.$$

The condition $y(d) < y(0)$ is now satisfied and this implies the absence of the trailing front under the steady-state conditions when $d \ll 1$. The function $y(d)$ increases in the region $0<d<d_0$, where $d_0 = \beta^{-1}\ln(\beta - 1)$ has a maximum at $d = d_0$, $y(d)_{\max} = [2\beta(\beta+1)]^{-1}\beta(\beta^2-1)^{-1}$, and falls then to $y(d) = [2\beta(\beta+1)]^{-1}$. Such a value of $y(d)$ is retained for $d \gtrsim 1$ and represents the threshold for the motion of the leading front (Sec. 4). Therefore, the steady state appears if $(y_0^{-1}) > [y(d)_{\max}]^{-1}$, but a study of its stability shows that this occurs only if $d > d_0$, when an increase in the pumping rate is accompanied by an increase in d. This continues as long as $d \leqslant \ln 2$ [compare with Eq. (27)], i.e., as long as $(y_0^{-1}) \leqslant 2\beta(\beta-1)$. The pumping range $2\beta(\beta-1) < y_0^{-1} < (y_0^{-1})_{\min}$, where $(y_0^{-1})_{\min}$ is the threshold pumping rate in the interior [described by the asymptotic expression in Eq. (16)], corresponds to the motion of one wave of the same kind as a self-wave in Sec. 4. When this wave is lost, the sample becomes bleached because in this model we have $\alpha_0 = 0$ and only one wave can be created subject to the initial condition. We can assume that allowance for $\alpha_0 \neq 0$ does not alter the situation if the beam is not focused.

At pumping rates in the range $(y_0^{-1}) > (y_0^{-1})_{\min}$ a traveling wave becomes detached from the boundary.

We shall now consider the case when $\beta \ll 1$, which is more difficult to study and for which we shall simply give the results. As before, there is a distance $d_0 = \ln\beta$, where $y(d)$ is maximal. This corresponds to the threshold pumping rate $(y_0^{-1}) = \beta(1 - \beta\ln\beta + \beta)$. At higher pumping rates there are two steady states, of which the stable one corresponds to an increase in $d(y_0)$. When the pumping rate exceeds the value $(y_0^{-1}) = \beta[1 + (2\beta)^{1/2}]$, the width reaches its maximum value $d = (2/\beta)^{1/2}$ if the condition $y(d) = y(0)$ is satisfied, and a trailing front appears at $x_1 > 0$. The profile of the steady state $y(x)$ is in the form of a hanging drop. An increase in the pumping rate reduces the thickness of this drop, $d \to 0$, and its distance from the boundary increases; however, we can show that the steady state is unstable. If $(y_0^{-1}) > (y_0^{-1})_{\min}$, a traveling wave becomes detached from the boundary.

It follows that, irrespective of the value of β, an increase in the pumping gives rise to the following four consecutive stages: 1) absence of a wave; 2) a localized wave; 3) detachment of a wave from a boundary and its disappearance; 4) detachment of a traveling wave.

All these stages were clearly observed in the report given in Ref. 4: the third stage was called "oscillations of a localized domain" and the fourth stage was called a "traveling domain." The appearance of new waves during the third stage is associated with $\alpha_0 \neq 0$ and the presence of a focused beam. The loss of a "domain" traveling along an expanding beam is due to the fact that $\beta \propto r^{-1}$ and $y_0 \propto r^2$ and this occurs not in the case when $\beta \gg 1$ characterized by $(y_0^{-1})/(y_0^{-1})_{\min} \sim (y_0^{-1})/2\beta^2 = \text{const}$, but in the case when $\beta \ll 1$, characterized by $(y_0^{-1})/(y_0^{-1})_{\min} \sim (y_0^{-1})/2\beta \propto r^{-1}$.

6. BEYOND THE LIMITS OF THE MODEL. FAST AND MIXED WAVES

We shall first consider the competition between TT and NT waves which appears at moderately high pumping rates when the effects of the concentration renormalization of E_g are not yet important.

We shall seek the solution of the system (1)–(3) in the form of a traveling wave and then Eqs. (1)–(3) reduce to a system of two equations

$$\begin{cases} D\dfrac{d^2n}{dr^2} - U\dfrac{dn}{dr} - \dfrac{n}{\tau_n} + \dfrac{\alpha I_0}{\hbar\omega} j(r) = 0, \\ \lambda\dfrac{d^2T}{dr^2} - U\dfrac{dT}{dr} - \dfrac{T}{\tau_T} + \dfrac{n\hbar\omega}{\tau_R c\rho} = 0, \end{cases} \tag{32}$$

$$j(r) = e^{\alpha_T r}\theta(-r)\theta(D+r),$$

which corresponds to the following eigenvalues:

$$\Lambda_{1,2} = \frac{U}{2D} \pm \left(\frac{U^2}{4D^2} + \frac{1}{D\tau_n}\right)^{1/2}, \quad L_{1,2} = \frac{U}{2\lambda} \pm \left(\frac{U^2}{4\lambda^2} + \frac{1}{\lambda\tau_T}\right)^{1/2}.$$

Each of the equations of the system (32) should be considered in three regions $r > 0$, $0 > r > -D$, $-D > r$, and we then have twelve integration constants which have to satisfy eight matching conditions at a point $r = 0$, $r = -D$, and four conditions of the type given by Eq. (5) at infinity. We can then assume that $T(0) = T(-D) = T_0$ and this yields two equations for U and D. Unfortunately, these equations are too cumbersome to quote here so that we shall consider only some special cases. We shall introduce two parameters:

$$\eta_1 = D/\lambda, \quad \eta_2 = \tau_n/\tau_T,$$

when at low velocities U, we have

$$L_{1,2}/\Lambda_{1,2} = \eta_1\eta_2,$$

and at high velocities we obtain

$$L_1/\Lambda_1 = \eta_1, \quad L_2/\Lambda_2 = \eta_2.$$

Therefore, a thermal wave appears if $\eta_1, \eta_2 \ll 1$ and in the case when $\eta_1, \eta_2 \gg 1$ the wave is of the thermal-concentration type. However, if one of the parameters is large and the other small compared with unity, the nature of the wave changes on increase in the velocity (pumping rate) and it may become mixed. Pure thermal and pure thermal-concentration waves can be dealt with using the model discussed above, but we must introduce additional conditions. For a thermal wave the condition is $U^2 \ll \lambda/\tau_n$, whereas for a thermal-concentration wave the condition is $U^2 \ll D/\tau_T$. At high velocities we cannot use the quasisteady behavior assumed in the model. However, at such velocities the leading edge moves independently and determines the wave velocity (Sec. 2). We shall now give the resultant equation for the velocity of a TT-NT wave

$$(L_1 - L_2)(\Lambda_1 - \Lambda_2)\frac{T_0}{\alpha_T{}^2 T_I} = \frac{\Lambda_1 - \Lambda_2}{L_1} - \frac{\Lambda_2}{\Lambda_1 - L_2} - \frac{\Lambda_1}{L_1 - \Lambda_2},$$

which reduces to Eq. (17) for pure TT and pure NT waves when the substitutions given by Eqs. (8) and (9) are made. At high intensities the velocity rises so much that $U^2 \gg \max(\lambda/\tau_n, \lambda/\tau_T, D/\tau_T, D/\tau_n)$, we obtain a simple expression

$$U^4 = \frac{\alpha_T I_0}{T_0 \tau_R c\rho}(D^2 + D\lambda + \lambda^2). \tag{33}$$

We can now see that the asymptote $U \propto I_0^{1/2}$, which appears in the model at high pumping rates of Eq. (17), is intermediate and changes in the case of a TT-NT wave to $U \propto I_0^{1/4}$ [Eq. (33)].

We shall now consider briefly a generation or NN wave. In this case the above model is not subject to limitations on the velocity because this wave is completely independent of temperature. Consequently, the final asymptote in the presence of an *NN* wave is $U \propto I_0^{1/2}$.

We shall conclude by noting that the absorption of light creates only one particle (wave) in a nonlinear system of the type considered here. Light transmitted by the wave is insufficient to maintain a second wave. At high pumping rates in the case of *NN* waves when the re-emission and amplification of light becomes important, it is possible to create systems with several waves.

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