Emission from a stationary spherical "corona" produced by beams of heavy ions

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Conversion of the energy of a heavy-ion beam incident on a spherical target into radiation from the produced plasma is considered in the radiant heat-conduction approximation. It is shown that the problem has a single parameter γ having the meaning of the ratio of the depth of bulk heating by heat conduction to the range of the ions in the bulk. The conversion coefficient increases with increase of γ and amounts to ~ 1 for $\gamma \approx 10^2$. The beam and target parameters typical of present inertial thermonuclear fusion projects correspond to $\gamma \approx 10$ and to conversion coefficients exceeding 50%. Thus, during the quasistationary stage of the "corona" dispersal the target acceleration and contraction conditions are basically radiative.

1. INTRODUCTION

We consider here theoretically the conversion of the energy of heavy ions incident on a spherical target into radiation emitted by the resultant plasma. This process has a direct bearing both on the problem of inertial thermonuclear fusion (ITF) and on the need for sources of incoherent x radiation.

The use of heavy-ion beams in ITF is the subject of most studies (see, e.g., the review[1]) in which, as a rule, use is made exclusively of numerical solution of the equations that describe the nonstationary processes of hydrodynamic spreading and deceleration of the ions in the target material and of the radiation transport. One of the analytic papers in this field[2] considers the stationary spherical "corona" produced by a beam of heavy ions under conditions when the dominant energy-transport mechanism is electronic heat conduction. However, for the parameters of the ion beam and target proposed for use in planned ionic ITF designs (see Ref. 3) the decisive energy-transport process is radiative exchange. Note that the idea of converting the beam energy into radiation was first set forth in Ref. 4 as applied to electron beams.

We consider in the present paper the stationary problem in the radiant heat-conduction approximation. This makes it possible, on the one hand, to estimate the coefficient of conversion of the incident energy into radiation from the plasma, and on the other to obtain the dependence of the hydrodynamic characteristics of the "corona" (ablation pressure, density, and temperature) on the beam and target parameters.

2. PHYSICAL FORMULATION OF PROBLEM

The equations and the boundary conditions describing stationary spherical flow of a plasma irradiated by heavy ions from a source at $R = +\infty$ take the form

$$\rho v R^2=\rho_* v_* R_*^2, \quad \frac{d}{dR}(p+\rho v^2)=-\frac{2\rho v^2}{R},$$

$$\rho v R^2\left(\varepsilon+\frac{v^2}{2}+\frac{p}{\rho}\right)-R^2\varkappa_r\frac{dT}{dR}=\begin{cases} Q_0\left(1-g_0^{-1}\int_R^\infty \rho\, dr\right), & R>R_b \\ 0, & R<R_b\end{cases},$$

$$v(R_0)=T(R_0)=0, \quad \rho(R_0)=\infty, \quad T(\infty)=0,$$

$$v(\infty)=\text{const}\neq 0, \quad g_0=\int_{R_b}^{\infty}\rho\, dR, \tag{1}$$

where ρ, v, p, ε, and T are the density, velocity, pressure, specific internal energy, and electron temperature of the plasma; ρ_*, v_*, and T_* are the corresponding values at the Jouguet point ($R = R_*$); R_0 is the target radius; $\varkappa_r = \varkappa_0 T^n \rho^m$ is the coefficient of radiant heat conduction ($\varkappa_0 =$ const; n and m are numbers that depend on the substance and on the conditions); Q_0, R_b, and g_0 are the energy flux, the absorption boundary, and the penetration depth of the ions into the bulk. The right-hand side of the third equation of (1) is written in a form corresponding to an energy release uniform over the mass. The relations between the hydrodynamic quantities are

$$p=\frac{\rho z T}{M_i}, \quad \varepsilon=\frac{3}{2}\frac{p}{\rho}=\frac{3}{2}\frac{zT}{M_i}, \quad v_*^2=\alpha\frac{zT_*}{M_i}, \tag{2}$$

where α is equal to 1 or 5/3 respectively for the isothermal and adiabatic Jouguet points; z is the average ionization multiplicity of the plasma ions and is assumed to be independent of the coordinate.

The problem of determining the maximum coefficient of conversion of the ion-beam energy Q_0 into radiation from the plasma consists, in the considered formulation, of finding solutions $\rho(R)$, $v(R)$, and $T(R)$ such that the radiant heat-conduction flux Q_r tends as $R\to\infty$ to a nonzero constant, i.e.,

$$Q_r(\infty)=\lim_{R\to\infty}\left(R^2\varkappa_r\frac{dT}{dR}\right)\neq 0, \tag{3}$$

The ratio $Q_r(\infty)/Q_0$ is regarded here as the sought limiting conversion coefficient.

The equations and boundary conditions (1) and (3), jointly with the conditions (2), contain five specified dimensional parameters: $\varkappa_0$, Q_0, R_0, g_0, M_i/z.

3. DIMENSIONLESS PROBLEM

In dimemsionless form, the problem (1)–(3) has for $n = 6$ and $m = -2$ (Ref. 5) the form

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$$
\begin{aligned}
&\xi = x^2/\eta^{1/2},\\
&x\eta'(\alpha-\theta/\eta)+2x\theta'+4\theta=0,\\
&\eta+\frac{5}{\alpha}\theta+\beta\theta^6\theta'\frac{\eta}{x^4}=\varphi,\\
&\varphi=\begin{cases} 0, & x\geqslant x_b,\\ \dfrac{\varphi(0)}{\Phi}\displaystyle\int_x^{x_b}\eta^{-1/2}\,dx, & x<x_b \end{cases}
\end{aligned}
\tag{4}
$$

$$
\Phi=\int_0^{x_b}\eta^{-1/2}\,dx,\quad \frac{Q_r(\infty)}{Q_0}=\lim_{x\to 0}\beta\theta^6\theta'\frac{\eta}{x^4}\Big/\varphi(0)\neq 0,
$$

$$
\eta(x_0)=\theta(x_0)=0,\quad \theta(0)=0,\quad \eta(0)\neq 0,
$$

where

$$
x=\frac{R_*}{R}\left(x_0=\frac{R_*}{R_0},\ x_b=\frac{R_*}{R_{\rm rp}}\right),
$$

$$
\xi=\rho/\rho_*,\quad \theta=T/T_*,\quad \eta=v^2/v_*^2,
$$

and the dimensionless problem parameters $\varphi(0)$, Φ, and β are connected with the dimensional parameters and scales of the hydrodynamic quantities by the relations

$$
\varphi(0)=2Q_0/\rho_*v_*^3R_*^2,\quad \beta=2\varkappa_0T_*^7/\rho_*^3v_*^3R_*,
$$

$$
\Phi=g_0/\rho_*R_*,\quad v_*=\alpha zT_*/M_i. \tag{5}
$$

It follows from (5) that $\varphi(0)$ and β are indicative of the ratios of the ion flux at infinity and the radiant flux at the Jouguet point to the hydrodynamic flux at the Jouguet point, while the parameter Φ, accurate to x_0, is the ratio of the density scales g_0/R and ρ_*.

The dimensionless problem (4) reduces to finding the functions $\theta(x)$, $\eta(x)$, $\varphi(x)$, $\xi(x)$ and the dimensionless parameters x_0, x_b, $\varphi(0)$, β, Φ and $Q_r(\infty)/Q_0$. Introducing in analogy with Ref. 2 the parameter γ:

$$
\gamma=\frac{g_T}{g_0}\equiv\frac{Q_0^{11/20}\varkappa_0^{3/20}(M_i/z)^{21/20}}{g_0R_0^{1/4}}, \tag{6}
$$

where g_T has the meaning of the characteristic bulk depth of the "corona" heating by radiant heat conduction.

It will be shown below that the problem (4) is closed and single-parameter, i.e., it has a unique solution for each parameter, and that analysis of the problem requires consideration of the three cases $\gamma=0$, $\gamma\geqslant 17.6$ and $0<\gamma<17.6$, respectively.

4. RESULTS OF NUMERICAL SOLUTION

We obtain first a numerical solution of the problem (4) for $\gamma=0$ i.e., with allowance for only the energy release due to deceleration of the ion beam; we refine by the same token the approximate analytic solution (2). The condition $\gamma=0$ means $\beta=0$ and leads to the onset of a singularity at the point in which the outflow velocity reaches the adiabatic velocity of sound ($\alpha=5/3$). Differentiating the third equation of the system (4) we get

$$
x\eta'(5/3-\theta/\eta)+2x\theta'+4\theta=0,
$$

$$
\eta'+3\theta'=-\frac{\varphi(0)}{\Phi}\frac{1}{\eta^{1/2}}. \tag{7}
$$

From the condition $\theta(1)=\eta(1)=1$ we determine the only parameter of Eqs. (7):

$$
\varphi(0)/\Phi=6, \tag{8}
$$

after which the system (7) becomes universal.

Numerical integration (after resolving the indeterminacy at the point $x=1$, $\theta'(1)=-\frac{1}{3}(5-10^{1/2})$, $\eta'(1)=-1-10^{1/2}$) yields $\varphi(0)=7.43$; $x_b=x_0=1.29$; and the isothermal Jouguet-point coordinate $x_{is}=1.15$. Figure 1 shows universal profiles of $\theta(x)$ and $\eta(x)$. The relations used convert to dimensional quantities are

$$
R_*=x_0R_0,\quad \rho_*=\frac{6g_0}{\varphi(0)x_0R_0},
$$

$$
v_*=\left(\frac{Q_0}{3g_0x_0R_0}\right)^{1/3},\quad T_*=\frac{3}{5}\frac{M_i}{z}v_*^2.
$$

If $\gamma\neq 0$, the problem (4) is easiest to solve in the case $\gamma\geqslant 17.6$, which corresponds to the condition $x_b\leqslant 1$, i.e., when the region of the ion energy release is supersonic. From an analysis of (4) for $x=1$, i.e., at the isothermal Jouguet point ($\alpha=1$), we get

$$
\theta'(1)=-2,\quad \beta=3,\quad \eta'=-8/3-4/3\sqrt{43}.
$$

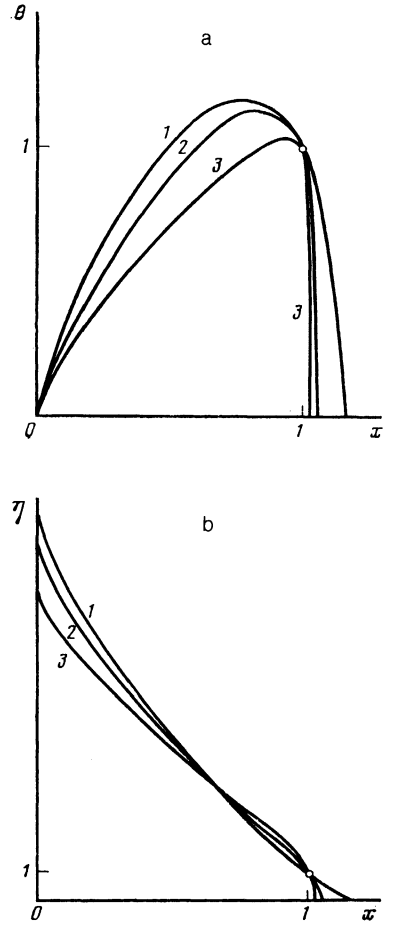

FIG. 1. Dimensionless plasma temperature θ (a) and the square of the plasma velocity η (b) vs the dimensionless reciprocal radius x for various values of the parameter γ: $\gamma=0$ (curve 1), $\gamma=62.7$ (curve 2), and $\gamma=7.9$ (curve 3).

The equations in the regions $0 < x \leqslant x_b$ and $x_b < x \leqslant x_0$ take respectively the forms

$$\theta' = -\frac{(\eta+5\theta-\varphi)x^4}{3\theta^6\eta}, \quad \eta' = \frac{4\theta+2x^5(\eta+5\theta-\varphi)/3\theta^6\eta}{x(1-\theta/\eta)},$$

$$\varphi' = -\frac{\varphi(0)}{\Phi}\frac{1}{\eta^{1/2}}, \tag{9}$$

$$\theta' = -\frac{(\eta+5\theta)x^4}{3\theta^6\eta}, \quad \eta' = \frac{2x\theta'-4\theta}{x(1-\theta/\eta)}. \tag{10}$$

The system (10) is universal, and numerical integration, in particular, yields $x_0 = 1.045$.

Equations (9) contain a single unknown parameter $\varphi(0)/\Phi$. The boundary conditions for them are

$$\theta(x_b) = \theta_{un}(x_b), \ \eta(x_b) = \eta_{un}(x_b), \ \varphi(x_b) = 0,$$
$$\theta(0) = 0, \quad \eta(0) = \text{const} \neq 0, \quad \varphi(0) = \text{const} \neq 0,$$
$$\lim_{x\to 0} \theta^6\theta' \frac{\eta}{x^4} \Big/ \varphi(0) = \text{const} \neq 0, \tag{11}$$

where $\theta_{un}(x)$ and $\eta_{un}(x)$ are the solutions of the universal system (10). By specifying $x_b \leqslant 1$ and $\theta_{un}(x)$ and $\eta_{un}(x)$, we can integrate (9) numerically for each value of $\varphi(0)/\Phi$. A numerical analysis shows that for each $x_b \leqslant 1$ the problem (9)–(11) is closed, i.e., one obtains $\theta(x)$, $\eta(x)$, and $\varphi(x)$ profiles that depend parametrically on x_b and have a universal part, as well as the constants $\varphi(0)$, Φ, $\eta(0)$, and $Q_r(\infty)/Q_0$. The connection between γ and these parameters obtained from (2) and from the relation $R_* = x_0 R_0$ (5):

$$\gamma = \frac{3^{3/20}}{2^{7/10}} \frac{x_0^{1/4}\varphi(0)^{11/20}}{\Phi}. \tag{12}$$

The results of the numerical solution of (9)–(11) are gathered in Table I. The minimal γ corresponding to the formulation (9)–(11) is 17.6, which corresponds in turn to $x_b = 1$. Figure 1 shows the $\theta(x)$ and $\eta(x)$ profiles for $\gamma = 17.6$ ($x_b = 1$) and $\gamma = 62.7$ ($x_b = 0.8$). Conversion to numerial scales is possible by successively calculating

$$R_* = x_0 R_0, \quad \rho_* = g_0/\Phi R_*,$$
$$v_* = (2Q_0/\varphi(0)\rho_* R_*^2)^{1/3}, \quad T_* = M_i v_*^2/z. \tag{13}$$

In the range $0 < \gamma < 17.6$, which corresponds to $1 < x_b \leqslant x_0$, the equations at the isothermal Jouguet point yield

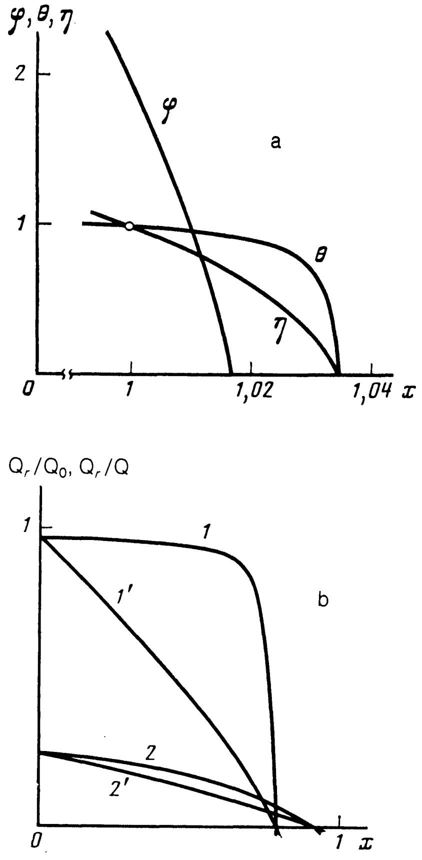

FIG. 2. a) Variation of the dimensionless values of the ion flux φ, the plasma temperature θ, and the squared plasma velocity η near the target at $\gamma = 12.6$; b) Ratios of the current radiant flux to the total and current ion fluxes, Q_L/Q_0 (curves 1' and 2') and Q_L/Q (curves 1 and 2) vs the dimensionless reciprocal radius x for different values of the parameter γ, viz., $\gamma = 62.7$ (curves 1 and 1') and $\gamma = 2.7$) (curves 2 and 2').

$$\beta = 3 - \varphi(1)/2, \tag{14}$$

and the connection between γ and the parameters of the dimensionless problem take the form

$$\gamma = \frac{\beta^{3/20}}{2^{7/10}} \frac{x_0^{1/4}\varphi(0)^{11/20}}{\Phi}. \tag{15}$$

Numerical analysis shows that x_b does not coincide with x_0 for $10.2 < \gamma < 17.6$. On the other hand, $x_b = x_0$ in the range $0 \leqslant \gamma \leqslant 10.2$. Relations (13), naturally, remain in force. The results of the numerical calculation for $\gamma = 7.9$ and $\gamma = 12.6$ are shown in Figs. 1 and 2.

TABLE I.

γ	$Q_r(\infty)/Q_0$	x_0	x_0/x_b	$\eta(0)$	$\varphi(0)$	$1/\Phi$	β
0	0	1.152	1	14.2	14.2	1.25	0
2.7	0.22	1.089	1	11.9	15.1	1.35	0.1
3.4	0.32	1.070	1	11.8	16.9	1.44	0.2
3.9	0.42	1.057	1	11.8	20.6	1.61	0.4
7.9	0.64	1.032	1	11.3	31.8	1.89	1
10.3	0.71	1.031	1.001	11.2	41.2	2.00	1.5
12.6	0.77	1.032	1.015	11.4	51.4	2.08	2
17.6	0.85	1.045	1.045	11.4	76.2	2.21	3
62.7	0.97	1.045	1.306	13.2	392	3.18	3

TABLE II.

Parameters	First example	Second example
z	30	25
γ	15	5
β	2,5	0,5
$1/\Phi$	2,15	1,7
$\varphi(0)$	64	24
$Q_r(\infty)/Q_0$	82%	50%
ρ_*	3,1 g/cm^3	2,43 g/cm^3
v_*	$1{,}05\cdot10^7$ cm/s	$0{,}73\cdot10^7$ cm/s
T_*	0,79 keV	0,46 keV
l_{R*}	$1{,}3\cdot10^{-4}$ cm	$4{,}1\cdot10^{-5}$ cm
$T\Big/\frac{dT}{dR}\, l_R\Big\vert_{x_T=0.3}$	$63\gg1$	$200\gg1$
$\frac{v_*\tau_{\rm pul}}{R_0}$	$5>1$	$4>1$
$\frac{Q_r(R_*)}{Q_e(R_*)}$	$\frac{3{,}7\cdot10^{20}\ \text{erg/s}}{0{,}65\cdot10^{18}\ \text{erg/s}}\gg1$	$\frac{1{,}3\cdot10^{19}\ \text{erg/s}}{1{,}2\cdot10^{17}\ \text{erg/s}}\gg1$

Note: The parameter z was chosen to be consistent with the values of ρ_* and T_*.

5. PHYSICAL RESULTS

We have thus solved numerically the one-parameter problem (4), (5) for arbitrary values of the dimensionless parameter γ that relates this problem with the dimensional problem (1)–(3). We have found the dependences of $Q_r(\infty)/Q_0$, R_*/R_0, R_b/R_0, $v^2(\infty)/v_*^2$, $\rho_*/g_0R_0^{-1}$, $\varphi(0)$, and β on γ and obtained the spatial profiles of the hydrodynamic quantities for arbitrary γ. The principal results are gathered in Table I (with renormalization to the isothermal speed of sound for $\gamma=0$) and are shown in the figures. It follows from the table that within the scope of the considered model the coefficient of conversion of the ion energy into radiation increases monotonically with increase of γ and turns out to be close to unity at $\gamma\sim100$. This result has physical meaning if the radiant flux $Q_r(x)$ becomes comparable with $Q_r(\infty)$ in a coordinate region in which there is still no violation of the radiant heat conduction approximation. The latter is valid if

$$T\Big/\frac{dT}{dR}\gg l_R, \tag{16}$$

where l_R is the Rosseland length. Putting $l_R=aT^3/\rho^2$, we rewrite (16) in the form

$$T\Big/\frac{dT}{dR}\,l_r=x_0\frac{x^2}{(-\theta')\theta^2\eta}\frac{R_0}{l_{R*}}\gg1,$$

$$l_{R*}=a\left(\frac{M_i}{z}\right)^3\frac{Q_0^2}{g_0^4}\frac{\Phi^4}{\varphi(0)^2};$$

l_{R*} is the Rosseland range at the Jouguet point. The final form of the criterion of validity of the radiant heat conduction approximation is in this case

$$\frac{x^2}{(-\theta')\theta^2\eta}\frac{x_0\varphi(0)^2}{\Phi}\chi\gg1,\quad \chi=R_0g_0^4\Big/a\left(\frac{M_i}{z}\right)^3Q_0^2. \tag{17}$$

If now $x_T(\gamma)$ is the coordinate at which the radiant heat-conduction flux is saturated (e.g., $Q_r(x_T)=0.7Q_r(0)$), the inequality (17) must be satisfied at the point x_T for the conclusions of the considered model to be valid. Analysis shows that in the range $100>\gamma>0$ the values for $x_T(\gamma)$ fluctuate about $x=0.3$, i.e., radiant heat conduction takes place all the way to the coordinate $R=(3–4)R_0$. It can be seen from (17) and from Table I that at a fixed parameter χ the criterion is met better, and in a larger coordinate range, the larger γ. Besides the criterion (17), it is necessary also to meet the stationarity condition $v_*\tau_{\rm pul}\gg R_0$, where $\tau_{\rm pul}$ is the duration of the ion pulse, as well as the condition that the electron heat-conduction flux be small compared with the radiant flux. The last condition will be illustrated in the estimates that follow. We note one more circumstance connected with the ideal equation of state used here. It is physically clear that a dense plasma can be nonideal. Allowance for this effect, however, for example by introducing into the problem a phenomenological equation of the type $p/\rho\sim T^{3/2}$ (for Pb, Au),[5] obtained from a numerical experiment, encounters in principle no difficulty. The use of such an equation, obviously, changes neither the structure of the considered problem nor the qualitative deductions.

Attention is called also to the result that the departure of the ion–beam absorption boundary from the target takes place at the minimum distance between the Jouguet point and the target, i.e., on the steepest temperature front (see Table I) and at a finite value of γ. The physical meaning of the equality $x_b=x_0$ at finite heat conduction is due to the large thermal resistance of the zone near x_0 which is heated by the ion beam. In other words, when stationary flow sets in the nonlinear thermal perturbations in the region of the ion-energy release attenuate in this same region.

We conclude with two numerical examples with different ion fluxes, based on the parameters of a numerical experiment[3]: $g_0=0.3$ g/cm^2, $R_0=0.2$ cm, $Q_{01}=5\cdot10^{21}$ erg/s, $Q_{02}=5\cdot10^{20}$ erg/s, $\tau_{\rm pul}=100$ ns, lead target ($A=207$). The coefficient a is set equal to $0.24\cdot10^{25}$ CGS.[5] The results are given in Table II. It follows from these data, in particular, that decreasing the ion flux by an order of magnitude does not change noticeably the temperature scale, since the decrease of the parameter γ reverses the roles of the energy channels, viz., in the first case the "corona" radiates more and is less heated than in the second.

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