Negative-energy waves in a plasma with structured magnetic fields

M. P. Ryutova

Institute of Nuclear Physics, Siberian Branch, Academy of Sciences of the USSR
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The properties of a plasma which contains structured magnetic fields (magnetic tubes) are analyzed in the case with shear flows of matter along the magnetic tubes. If the flows are sufficiently fast, they give rise to several new effects: the appearance of negative-energy waves and reversal of the sign of the radiative damping. In the nonlinear stage, they drive an explosive instability. If the velocity exceeds an upper threshold, they drive a gross (linear) hydrodynamic instability. The corresponding processes are analyzed. Calculations of the growth rates for dissipative instabilities associated with the excitation of sound waves and anomalous absorption in a resonant layer are illustrated by examples. Conditions for the occurrence of the explosive instability are identified. The results derived here may be of interest in connection with the problem of the buildup and release of energy in the solar atmosphere and also for reaching an understanding of the dynamics of various processes which occur in plasmas with structured magnetic fields in space and in the laboratory.

1. INTRODUCTION

Situations in which the magnetic flux in a plasma is concentrated in distinct and relatively thin "tubes," while the magnetic field is weak over the greater part of the plasma volume, are fairly common in various astrophysical objects and also in laboratory plasmas. In particular, observational data indicate that the entire magnetic field of the sun is concentrated in narrow tubes, mostly far apart from each other, in which the magnetic induction is ~ 2000 G. Magnetic spots constitute an ensemble of closely packed magnetic tubes (see, for example, the monograph[1] by Priest). The properties of both individual tubes and ensembles of tubes must be studied in order to reach an understanding of various magnetic-field-dominated processes which occur in the solar atmosphere, in particular, the transport of energy from the lower layers of the atmosphere to the upper layers and the buildup and release of energy. Research on the properties of plasmas with structured magnetic fields is important for explaining various processes which occur in objects in space. Furthermore, it is of general physical interest because of the wealth and diversity of the wave processes which occur in such structures. It is therefore not surprising that problems of this sort have received considerable attention in recent years.

One of the first theoretical publications in this field was a report[2] of a study of the bending oscillations of individual tubes (as part of this research, the "radiative damping" of these oscillations, associated with the emission of sound waves into the space around the tube, was found). That paper also analyzed the propagation of long acoustic oscillations through a plasma containing an ensemble of randomly positioned magnetic tubes. A specific "dissipationless" mechanism for the damping of these oscillations was revealed. That mechanism involves a transfer of the energy of these oscillations to the bending oscillations of tubes (and is somewhat analogous to the Landau damping mechanism).

Defouw[3] has called attention to the existence of some specific quasilongitudinal oscillations of a tube in which a longitudinal compression (or expansion) of the plasma within a tube is accompanied by an increase (or decrease) in the cross-sectional area of the tube, with the result that the sum of the plasma pressure (the gas-kinetics pressure) and the magnetic pressure is not perturbed. These oscillations, which constitute an analog of slow magnetosonic waves in a homogeneous plasma and which are sometimes called "slow" or "varicose," are interesting since their radiative damping is very slight.[4] Various mechanisms acting to damp the oscillations of magnetic tubes, in particular, the mechanism which results from an "Alfvén resonance,"[6] which occurs in a region in which the phase velocity of the oscillations becomes equal to the local value of the Alfvén velocity, were studied in Refs. 4 and 5. The dispersion properties of a plasma containing an ensemble of closely packed magnetic tubes were studied in Ref. 7. It was found that random variations (which are not assumed to be small: the magnetic field, the density, and the pressure of the plasma vary by amounts on the order of unity from tube to tube) lead to a dissipation of the energy of long-wave oscillations which is substantially more rapid than in the homogeneous case.

All of the studies cited above were carried out for systems in which the plasma is at rest in its unperturbed state. There are, on the other hand, situations in which the plasma outside a tube is moving along the magnetic field with respect to the plasma inside the tube. In particular, according to observational data flows of matter are observed in essentially all parts of the solar atmosphere where there are structured magnetic fields. As a rule, the velocities of these flows are different inside and outside the magnetic structures (Ref. 1, for example). In other words, there are always shear flows along the magnetic tubes in the solar atmosphere. In the present paper we study the oscillations of an individual tube in the presence of such flows.

It turns out that the presence of shear flows along structured magnetic fields gives rise to some qualitatively new effects. First, when the velocity of the relative motion exceeds a certain threshold, negative-energy waves[1)] appear in the system. These waves may become unstable as a result of various dissipative processes (in particular, as a result of the emission of sound waves into the surrounding medium). In addition, since the system simultaneously contains positive-

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energy waves, a nonlinear "explosive" instability can occur in it. Finally, if the velocity of the relative motion exceeds a second threshold (if the first velocity threshold is exceeded, there is an instability with respect to the excitation of negative-energy waves) a coarser (linear) instability occurs. This coarse instability is related to the tangential-discontinuity instability.

Let us examine long-wave oscillations of a tube, by which we mean oscillations whose wavelength $\lambda = 1/k$ is large in comparison with the tube radius R, $kR \ll 1$. These are the oscillations which are most easily excited by large-scale motions of a plasma and which have a relatively small damping rate.

In Sec. 2 we examine the linear equations describing bending and slow oscillations, and we find the conditions for the existence of bending oscillations with a negative energy and the condition for the gross instability of the tube. In Sec. 3 we formulate the conditions for the dissipative instability of bending oscillations, and we estimate its growth rate. In Sec. 4 we discuss that instability of bending and slow oscillations which is associated with the emission of sound waves. In Sec. 5 we examine the nonlinear explosive instability of negative-energy waves. In Sec. 6 we demonstrate some properties of the gross instability of the bending oscillations which arise when the flow velocity exceeds the second threshold. In Sec. 7 we briefly discuss the results. Computational questions are set apart in appendices.

2. LINEAR THEORY OF BENDING AND SLOW OSCILLATIONS

Let us consider the model of a homogeneous tube of circular cross section in the presence of a flow which is directed along the tube axis. We adopt for the analysis a coordinate system in which the matter inside the tube is at rest, while the flow velocity outside the tube has a value u and is directed toward increasing z.

We begin with the bending oscillations. We describe the displacement of the tube with respect to its unperturbed position by the vector $\xi_\perp(z,t)$, which lies in the plane perpendicular to the axis of the tube. As in Ref. 2, we can assume that the vector $\xi_\perp(z,t)$ satisfies the equation

$$\rho_i\frac{\partial^2\xi_\perp}{\partial t^2} = -\rho_e\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial z}\right)^2\xi_\perp+\frac{B^2}{4\pi}\frac{\partial^2\xi_\perp}{\partial z^2}, \qquad (1)$$

where ρ_i and ρ_e are the densities of matter inside and outside the tube, and B is the magnetic field inside the tube (by analogy with Ref. 2, we assume that there is no magnetic field outside the tube; this assumption simplifies the calculations without having any fundamental effect on the problem).

Equation (1) has an energy integral, which can be written in the form

$$I=\int dz\frac{\pi R^2}{2}\left[(\rho_i+\rho_e)\left(\frac{\partial\xi_\perp}{\partial t}\right)^2 +\left(\frac{B^2}{4\pi}-\rho_e u^2\right)\left(\frac{\partial\xi_\perp}{\partial z}\right)^2\right]=\text{const}, \qquad (2)$$

where R is the tube radius. The integrand has the meaning of the energy of the oscillations per unit length along the tube.

For sinusoidal traveling waves of the type $\exp(-i\omega t+ikz)$ we find the dispersion relation

$$\omega^2+\frac{1}{\eta}(\omega-ku)^2-k^2a^2=0 \qquad (3)$$

from (1), where $\eta=\rho_i/\rho_e$, and $a=(B^2/4\pi\rho_i)^{1/2}$ is the Alfvén velocity inside the tube. From (3) we find

$$\frac{\omega}{k}=\frac{1}{1+\eta}[u\pm\{\eta[a^2(1+\eta)-u^2]\}^{1/2}]. \qquad (4)$$

We see that under the condition

$$u>u_c^b=a(1+\eta)^{1/2} \qquad (5)$$

(the superscript b on the u_c specifies that we mean the critical velocity for the excitation of bending oscillations) the system becomes unstable. This instability may be called "gross" in the sense that its growth rate is comparable to the frequency when the threshold is exceeded by an amount on the order of unity ($u-u_c^b\sim u_c^b$). We will analyze this instability in more detail in Sec. 6; at this point we will instead examine the effects which occur in the region $u<u_c^b$.

Using the dispersion relation (4), we easily find from (2) that for traveling waves the energy density per unit length of the tube, W, is

$$W=\frac{\pi R^2\rho_e}{2}\xi_\perp{}^2k^2\left[(1+\eta)\frac{\omega^2}{k^2}+a^2\eta-u^2\right]$$

or

$$W=\frac{1}{1+\eta}\pi R^2k^2\rho_e\xi_\perp{}^2(x^2\pm ux), \qquad (6)$$

where we have introduced $x=\{\eta[a^2(1+\eta)-u^2]\}^{1/2}$. Here it is to be understood that the expression in the radical is positive, i.e., that the gross instability does not occur. Since we are assuming $u>0$, the only wave which can have a negative energy is one which corresponds to the minus sign in dispersion relation (4), i.e., the wave which would propagate in the negative z direction in the absence of a flow. For this wave we find from (6)

$$W=\pi R^2k^2\rho_e\xi_\perp{}^2(\eta a^2-u^2)x/(x+u).$$

We see from this expression that the energy of the wave goes negative at

$$u>u_c^n=a\eta^{1/2} \qquad (7)$$

(the superscript n specifies that we mean the threshold for the appearance of negative-energy waves). Comparing (5) and (7), we see that the relation $u_c^n<u_c^b$ holds, i.e., that negative-energy waves do in fact appear in a plasma which is still stable with respect to the gross hydrodynamic instability. At the lower boundary of the interval in which negative-energy waves exist (at $u=u_c^n$) the phase velocity of the wave corresponding to the minus sign in dispersion relation (4) is zero. At $u>u_c^n$, the negative-energy waves propagate along the direction of the flow.

We turn now to slow oscillations.[3,4] A characteristic feature of the slow oscillations is that there is almost no perturbation of the sum of the magnetic pressure p_M and the plasma pressure (gas-kinetics pressure) p inside the tube in the case of these oscillations, while each of these components separately is substantially perturbed:

$$\frac{\delta p}{p}\sim\frac{\delta\rho}{\rho},\quad \frac{\delta p_M}{p_M}\sim\frac{\delta\rho}{\rho},\quad \frac{\delta(p_M+p)}{p_M+p}\sim(kR)^2\frac{\delta\rho}{\rho}\ll\frac{\delta\rho}{\rho}.$$

Because of this distinctive feature of the slow oscillations, the plasma parameter values outside the tubes have only a slight effect on the dispersion relation for these oscillations (they introduce corrections of order k^2R^2). In particular, external flows have only a slight effect on these oscillations. We thus reach the conclusion that the dispersion relation for slow oscillations, even in the case of a flow, can be written as follows,[3,4] within small corrections:

$$c_T=\left(\frac{\omega}{k}\right)_T=\frac{as_i}{(a^2+s_i^2)^{1/2}}, \qquad (8)$$

where $s_i=(\gamma p_i/\rho_i)^{1/2}$ is the sound velocity inside a tube, and we will use the subscript T, for "tube," to specify slow oscillations.

The complete equations for the bending and slow oscillations are derived in Appendix 1.

3. DISSIPATIVE INSTABILITIES OF NEGATIVE-ENERGY BENDING OSCILLATIONS

In the interval

$$u_c^n<u<u_c^b \qquad (9)$$

there may be an instability of negative-energy waves as a result of dissipative processes in the plasma. In other words, incorporating dissipative effects results in a transfer of energy away from the negative-energy waves and thus an increase in their amplitude. A remarkable property which magnetic tubes exhibit, because of their particular nature, is that even in the absence of dissipative processes of any type this ("fine") instability may be caused by the mechanism of a collisionless dissipation of bending oscillations which was studied in Ref. 5. It was shown in that paper that when a radial variation of the magnetic tube is taken into account an anomolous and strong absorption of oscillations at a resonant point is manifested. The "resonant point" here is the point at which the phase velocity of the oscillations becomes equal to the local value of the Alfvén velocity. A corresponding effect occurs in the case of a flow of matter along magnetic tubes. Here we will follow the approach taken in Ref. 5.

In the long-wave approximation, the assumption that the fluid is incompressible (i.e., the assumption $\operatorname{div}\mathbf{v}=0$) is quite accurate. We can then replace the velocity by a stream function ψ:

$$v_r=-\frac{1}{r}\frac{\partial\psi}{\partial\varphi},\quad v_\varphi=\frac{\partial\psi}{\partial r}.$$

In this case, the general system of equations, (1.6) (Appendix 1), reduces to a single equation for ψ:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\left(\rho\Omega-\frac{k^2B^2}{4\pi\Omega}\right)r\frac{\partial\psi}{\partial r}\right]-\left(\rho\Omega-\frac{k^2B^2}{4\pi\Omega}\right)\frac{\psi}{r^2}=0. \qquad (10)$$

We are assuming that ω now contains a small imaginary increment $i\nu$, introduced in order to circumvent the singular point at $\rho\Omega=k^2B^2/4\pi\Omega$ correctly. Physically, the appearance of this correction can be explained in terms of, for example, infrequent collisions between ions and neutral particles of the plasma. For clarity in the calculations we assume, as in Ref. 5, a model tube which is homogeneous nearly throughout space, except in a narrow transition layer (the diffuse boundary of the tube), where the plasma density and the square of the magnetic field are linear functions of the radius:

$$\rho\Omega^2=\rho_i\omega^2\frac{R-r+l}{l}+\Omega^2\rho_e\frac{r-R}{l},$$

$$B^2(r)=B^2\frac{R-r+l}{l},$$

where $l\ll R$.

Solutions of Eq. (10) at constant values of the density, the magnetic field, and the flow velocity are Bessel functions in the interior and Hankel functions in the exterior. To first order in $kR\ll 1$, these solutions are, respectively,

$$\psi=\begin{cases}Ar, & r<R,\\ B/r, & r>R+l.\end{cases} \qquad (1)$$

To find solutions in the transition region $R<r<R+l$ we introduce the variable $z=(r-R)/l$ $(0\leqslant z\leqslant 1)$. We can make use of the small parameter l/R to rewrite Eq. (10) as

$$\frac{d}{dz}(z-z_0-i\varepsilon)\frac{d\psi}{dz}-\frac{l^2}{R^2}(z-z_0-i\varepsilon)\psi=0, \qquad (11)$$

where

$$z_0=(k^2a^2-\omega^2)\Big/\left(k^2a^2+\frac{\rho_e}{\rho_i}\Omega^2-\omega^2\right),$$

and the small increment $i\varepsilon$ has arisen because of $i\nu$ (the specific value of ε is unimportant, since it does not appear in the final result).

Equation (11) has a single-valued solution in the form of Bessel functions in the complex z plane with a cut along the line $\operatorname{Im} z=i\varepsilon$, $-\infty<\operatorname{Re} z<z_0$. Expanding in a series in the parameter l/R, we can write this solution in the form

$$\psi=C+D\ln(z-z_0-i\varepsilon).$$

Now using the conditions that ψ and $d\psi/dz$ are continuous at the points $r=R$ and $r=R+l$ (i.e., joining the solutions in the corresponding way), and choosing the correct branch of the logarithm, we find the dispersion relation

$$\ln\frac{z_0}{1-z_0}+\frac{R}{l}\left(\frac{1}{z_0}-\frac{1}{1-z_0}\right)+i\pi=0. \qquad (12)$$

The real part of (12) yields

$$1-z_0=z_0.$$

It is easy to verify that this expression is precisely the same as the dispersion relation (4). For the imaginary part of the frequency we find the following expression from (12):

$$\frac{\gamma}{\omega}=-\frac{\pi}{4}\frac{l}{R}\frac{\eta}{(1+\eta)^2}\frac{(\eta u\mp x)^2}{\pm x}.$$

It can now be seen that for waves with a positive energy (the upper sign) the quantity γ corresponds to a damping rate, while for waves with a negative energy (the lower sign) it corresponds to a growth rate.

The growth rate of the instability for negative-energy waves which is caused by the resonant absorption of oscillation energy is

$$\frac{\gamma_{res}}{\omega}=\frac{\pi}{4}\frac{l}{R}\frac{\eta}{(1+\eta)^2}\frac{[\eta u+\{\eta[a^2(1+\eta)-u^2]\}^{1/2}]^2}{\{\eta[a^2(1+\eta)-u^2]\}^{1/2}}. \qquad (13)$$

It must be kept in mind that this value for the growth rate is valid in a region which is not too close to the threshold, where the denominator in (13) vanishes.

The anomalous-absorption effect which is responsible here for the instability of negative-energy waves may also occur in the case of a homogeneous magnetic tube when there are variations in the shear flow.

4. INSTABILITY OF BENDING AND SLOW OSCILLATIONS CAUSED BY THE EMISSION OF SOUND WAVES INTO THE EXTERNAL SPACE

The dispersion relation (4) for bending oscillations has been derived by ignoring the compressibility of the medium. Incorporating compressibility corresponds at a formal level to the retention of the terms of next higher order in the parameter $kR \ll 1$ in the exact dispersion relation (cf. Ref. 2). The primary effect of incorporating the compressibility is the emission of secondary sound waves by the oscillating tube.[2] If there is no plasma flow, this effect leads to a "radiative" damping of the bending oscillations. If the plasma outside the tube instead has a nonzero velocity u, the emission of sound waves may lead to growth of the bending oscillations. This situation is possible in two cases: if the bending oscillation has a negative energy, and the emitted sound wave has a positive energy; or if the bending oscillation has a positive energy, and the sound wave has a negative energy.

The dispersion relation for plane sound waves (far from the tube, the waves can be assumed to be plane waves) is

$$(\omega/k)_s = u \pm s_e(1+k_\perp^2/k^2)^{1/2}, \qquad (14)$$

where $k_\perp$ is the component of the wave vector which is perpendicular to the z axis, and k, as before, is the component of the wave vector along the z axis. The subscript s specifies sound waves. It is easy to verify that the sound wave which can have a negative energy is that which, in the absence of a flow, would propagate in the negative z direction [the wave corresponding to the lower sign in dispersion relation (14)]. The sign of the energy of this wave is negative if

$$u \geqslant s_e(1+k_\perp^2/k^2)^{1/2}. \qquad (15)$$

The transverse component of the wave vector of the found wave is found from the condition

$$(\omega/k)_b = (\omega/k)_s.$$

We will first find the conditions under which the bending oscillations with a positive energy radiate sound waves with a negative energy, i.e., the conditions under which the following relations hold:

$$\frac{1}{1+\eta}(u+x) = u - s_e\left(1+\frac{k_\perp^2}{k^2}\right)^{1/2} > 0. \qquad (16)$$

We recall that we have $x = \{\eta[a^2(1+\eta) - u^2]\}^{1/2}$.

Simple calculations show that these relations can hold under the conditions

$$a > s_e/\eta^{1/2}, \quad u > s_e + (a^2 - s_e^2/\eta)^{1/2}. \qquad (17)$$

It follows from the condition for equilibrium of the unperturbed tube,

$$p_i + B^2/8\pi = p_e,$$

that we have

$$s_e > a(\gamma\eta/2)^{1/2},$$

where γ is the adiabatic index. Correspondingly, conditions (17) can hold only if $\gamma < 2$.

Can a bending oscillation with a negative energy emit a wave with a positive energy? In other words, can the condition

$$\frac{1}{1+\eta}(u-x) = u + s_e\left(1+\frac{k_\perp^2}{k^2}\right)^{1/2}$$

be satisfied? It is obvious that this condition cannot be satisfied, since it reduces to the equation

$$-x = \eta u + (1+\eta)s_e\left(1+\frac{k_\perp^2}{k^2}\right)^{1/2},$$

whose left side is negative, and whose right side is positive.[2)]

We thus reach the conclusion that in our model the conditions may be such that a bending wave with a positive energy (which is traveling "downstream") will go unstable as a result of the emission of secondary sound waves with a negative energy. The growth rate for this instability is calculated in Appendix 2; the result is

$$\frac{\gamma_{rad}}{\omega} = \frac{\pi}{2}\frac{(v_\varphi-u)^2[(v_\varphi-u)^2-s_e^2]}{s_e^2 v_\varphi[(1+\eta)v_\varphi-u]}k^2R^2.$$

We must of course recall that this instability occurs if a threshold in the flow velocity has been reached [see (17)].

A corresponding instability mechanism operates for slow oscillations. As we mentioned in Sec. 2, a flow has essentially no effect on these oscillations; in particular, their energy remains positive even when there is a flow. Correspondingly, an instability may be caused in this case by the emission of negative-energy sound waves. The sound waves which have negative energy are those which propagate opposite the flow in the coordinate system of the fluid; their energy becomes negative under condition (15), i.e., if they are traveling downstream in the laboratory system. We thus conclude from the phase-matching condition that the condition for an instability is [cf. (16)]

$$c_T = u - s_e(1+k_\perp^2/k^2)^{1/2} > 0.$$

This condition can hold if

$$u > c_T + s_e. \qquad (18)$$

A slow wave propagating downstream may thus be unstable. The growth rate of this instability (Appendix 2) is

$$\frac{\gamma_{rad}}{\omega} = \frac{\pi}{4}\frac{\rho_e}{\rho_i}\frac{c_T^2(c_T-u)^2}{a_i^4}k^2R^2.$$

The threshold flow velocity for this instability is given by (18).

5. EXPLOSIVE INSTABILITY OF NEGATIVE-ENERGY WAVES

A specific nonlinear instability, an explosive instability, occurs in a system which contains waves with energies of different signs. This instability was first studied in Ref. 10 in the particular case of waves with random phases. It was later studied for a "triplet" of coherent waves in Ref. 11, which is the paper which proposed the term "explosive instability." A distinctive feature of an explosive instability is that the amplitudes of the interacting waves reach infinitely large values in a finite time. This assertion is of course slightly formal in nature: Higher-order nonlinear processes will limit the growth of the amplitude to a finite level.

In analyzing nonlinear processes, in particular, three-wave processes, it is convenient to assume that the sign of the frequency corresponds to the sign of the energy. When this approach is taken, the condition for an explosive instability for a three-wave process can be written in the form

$$\omega_1+\omega_2+\omega_3=0, \quad k_1+k_2+k_3=0, \quad |m_1|\pm|m_2|\pm|m_3|=0, \tag{19}$$

where the indices 1, 2, 3 refer to the three interacting waves. Since we are considering oscillations with $m=0$, ± 1, it follows from the last relation in (19) that either all three of the waves must have $m=0$, or two of them must have $m=\pm 1$, and the third $m=0$.

We can show that conditions (19) hold, so an explosive instability is possible in the interaction of one slow wave ($m=0$) and two bending waves ($m=\pm 1$). We assign a subscript T to quantities referring to a slow wave, and b to quantities referring to bending waves. Correspondingly, we replace (19) by the following conditions:

$$\omega_T+\omega_{b+}+\omega_{b-}=0, \qquad k_T+k_{b+}+k_{b-}=0. \tag{20}$$

The $+$ and $-$ with the subscript b correspond to waves which are traveling downstream and upstream.

It is simple to verify that conditions (20) are compatible if $k_T>0$ and if the following inequality holds:

$$u>\frac{c_T}{1+\eta}+\left(a^2\eta-c_T^2\frac{\eta(\eta^2+3\eta+3)}{1+\eta^2}\right)^{1/2}. \tag{21}$$

Inequality (21) is the condition for an explosive instability.

If only a single wave (e.g., a T wave with $k_T>0$) has been excited in the system at the origin on the time scale, and if the amplitudes of the two other waves are determined by thermal noise, then the amplitudes of these two waves will grow exponentially in the initial stage of the evolution of the explosive instability. It is clear from dimensionality considerations that the typical growth rate is $k_T v_{T\sim}$ in order of magnitude, where $v_{T\sim}$ is the velocity amplitude of the boundary of the tube in the slow oscillations. After a time on the order of several times the reciprocal of the growth rate, at which the amplitudes of all three waves have become equal in order of magnitude, the amplitudes begin a power-law growth, as has been established elsewhere,[10,11] in accordance with

$$v_\sim\sim(t_0-t)^{-1},$$

where t_0 is the time of the "explosion," which also is equal in order of magnitude to $(k_T v_{T\sim})^{-1}$ in our case.

6. HYDRODYNAMIC INSTABILITY OF BENDING OSCILLATIONS

As was shown in Sec. 2, a new instability arises in the system if the threshold flow velocity $u_c^b=a(1+\eta)^{1/2}$, determined by (5), is exceeded. This instability is related in nature to the instability of a tangential discontinuity in MHD. As we have already mentioned, this is a gross instability, in the sense that if the threshold is exceeded by an amount of order unity the growth rate becomes on the order of the frequency itself, and the growth distance becomes comparable to the wavelength. Under conditions such that this instability occurs, namely under the condition

$$u>a(1+\eta)^{1/2},$$

the subtler dissipative and nonlinear instabilities discussed in Secs. 3–5 fade to a status of minor importance.

It follows from dispersion relation (4) that unstable perturbations propagate upstream:

$$\mathrm{Re}(\omega/k)=u/(1+\eta)>0.$$

Accordingly, if an upstream flow of surrounding plasma "blows over" a certain length of the tube, the bending oscillations excited here will subsequently propagate upstream. This instability is remarkable because it may be regarded as an important agent for exciting oscillations in regions far from a convection zone. It is usually assumed that the excitation of oscillations of magnetic tubes involves an oscillatory motion of the point at which the tube intersects the photosphere, caused by a time-dependent convection in the photosphere. Oscillations of magnetic tubes which are excited by convective motions undoubtedly do exist, but the frequency of these oscillations is on the order of the reciprocal of the timescale for a change in the structure of the granulation pattern, i.e., on the order of $1/\tau\sim 10^{-2}$–$3\cdot 10^{-3}\,\mathrm{s}^{-1}$. This is a very low frequency, and it makes it a difficult matter to use such oscillations to explain energy transport out of the photosphere into the upper part of the solar chromosphere.

The instability described above leads to the existence of another oscillation-excitation mechanism, which does not require motions at the base of the magnetic tube and which may act even far from the convection zone. The frequency of the oscillations of the tube is of course totally unrelated to the reciprocal of the timescale for a change in the structure of the granulation pattern; it may be much greater than $1/\tau$.

7. CONCLUSION

We have shown that when there is a relative motion of the plasmas inside and outside a magnetic tube the system will acquire a rich picture of effects, not seen in a plasma at rest. We have classified these effects.

In the first place, bending waves with a negative energy may arise in a system of this sort, and the presence of dissipative processes may cause an instability of these waves. In particular, a dissipative instability may result from a collisionless absorption of bending oscillations in an Alfvén-resonance layer within the tube. The specific type of dissipative instability is related to the emission of sound waves into the space around the tube (in a system without a flow, this process would result in radiative damping of the bending oscillations[2]). In principle, an instability can occur in two cases: when the bending wave has a positive energy and the emitted sound wave has a negative energy; or vice versa.

In a sense, dissipative instabilities are "weak": Their growth rate is usually small in comparison with the frequency. A gross instability of bending oscillations (with a growth rate on the order of the frequency), analogous to an instability of a tangential discontinuity, arises as the velocity of the relative flow is increased further. In the coordinate system in which the plasma in the tube is at rest, the unstable waves travel in the direction of the external flow. This mechanism for the excitation of bending oscillations may play an important role in energy transport in the solar atmosphere.

We have separately analyzed the three-wave processes in which bending and axisymmetric "slow" oscillations of the tube interact in a situation in which the former have a negative energy. We have identified the conditions for the occurrence of a nonlinear explosive instability.

The effects described here should play an important role in the dynamics of various processes in the solar atmosphere, in particular, in the transport of energy from the lower atmosphere to the upper atmosphere, in the buildup and release of energy, in the evolution of the magnetic fields, and in phenomena associated with the solar wind.

APPENDIX 1. EQUATION OF SMALL OSCILLATIONS OF A MAGNETIC TUBE

The linearized system of equations of single-fluid MHD for the case in which the matter has an unperturbed velocity is

$$\rho\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{u}\nabla)\mathbf{v}+(\mathbf{v}\nabla)\mathbf{u}=-\nabla p+\frac{1}{4\pi}\{[\mathrm{rot}\,\mathbf{b},\mathbf{B}]+[\mathrm{rot}\,\mathbf{B},\mathbf{b}]\},$$

$$\frac{\partial \mathbf{b}}{\partial t}=\mathrm{rot}[\mathbf{vB}]+\mathrm{rot}[\mathbf{ub}], \qquad (1.1)$$

$$\frac{\partial\,\delta\rho}{\partial t}+\mathrm{div}\,\rho\mathbf{v}+\mathrm{div}\,\delta\rho\mathbf{u}=0,\qquad \frac{\partial \delta F}{\partial t}+(\mathbf{v}\nabla)F+(\mathbf{u}\nabla)\delta F=0.$$

Here $F=p\rho^{-\gamma}$, and $\mathbf{v}$, $\mathbf{b}$, $\delta\rho$, and δp are the perturbations of the velocity, the magnetic field, the density, and the pressure. These equations should be supplemented with the condition for equilibrium of the magnetic tube in its unperturbed state:

$$p(r)+B^2(r)/8\pi=p_e, \qquad (1.2)$$

where p_e is the plasma pressure outside the tube.

We consider a model of the magnetic tube which is axisymmetric in the unperturbed state and which is homogeneous along the axis (which coincides with the z axis in a cylindrical coordinate system). In other words, we assume that the unperturbed density $\rho(r)$, the unperturbed pressure $p(r)$, and the unperturbed magnetic field $\mathbf{B}(r)$ depend only on the radius. We also assume that the shear flow is directed along the z axis: $\mathbf{u}=\{0,0,u(r)\}$. All of the perturbed quantities are assumed to be proportional to $\exp(-i\omega t+im\varphi+ikz)$. For such perturbations, we find the following from the first equation of system (1.1):

$$-i(\omega-ku)\rho v_r=-\frac{\partial}{\partial r}\left(\delta p+\frac{b_zB}{4\pi}\right)+\frac{ikB}{4\pi}b_r,$$

$$-i(\omega-ku)\rho v_\varphi=\frac{1}{r}\frac{\partial}{\partial\varphi}\left(\delta p+\frac{b_zB}{4\pi}\right)+\frac{ikB}{4\pi}b_\varphi, \qquad (1.3)$$

$$-i(\omega-ku)\rho v_z=-ik\delta p-\rho v_r\frac{\partial u}{\partial r}-\frac{kB}{4\pi(\omega-ku)}v_r\frac{\partial B}{\partial r}.$$

From the second equation of system (1.1) we find

$$b_r=-\frac{kB}{\omega-ku}v_r,$$

$$b_\varphi=-\frac{kB}{\omega-ku}v_\varphi, \qquad (1.4)$$

$$-i(\omega-ku)b_z=-\frac{1}{r}\frac{\partial}{\partial r}rBv_r-\frac{imB}{r}v_\varphi+b_r\frac{\partial u}{\partial r}.$$

In writing the last equation we used $\mathrm{div}\,\mathbf{b}=0$. The third and fouth equations in (1.1) take the following forms, respectively:

$$-i(\omega-ku)\delta\rho+\frac{1}{r}\frac{\partial}{\partial r}r\rho v_r+\frac{im}{r}\rho v_\varphi+ik\rho v_z=0,$$

$$-i(\omega-ku)(\delta p-s^2\delta\rho)+v_r\left(\frac{\partial p}{\partial r}-s^2\frac{\partial\rho}{\partial r}\right)=0, \qquad (1.5)$$

where $s^2=\gamma p/\rho$ is the sound velocity.

To put system of equations (1.3), (1.4), (1.5) in a compact and graphic form, we express all the perturbed quantities in terms of v_r, v_φ, and the perturbation of the total pressure, $\delta P=\delta p+b_zB/4\pi$. Carrying out the appropriate calculations, we find the following system of equations for small oscillations of the magnetic tube for the case in which there is a flow of matter:

$$i\delta P=\rho\frac{\Omega^2(s^2+a^2)-k^2s^2a^2}{\Omega^2-k^2s^2}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{v_r}{\Omega}\right)+\frac{im}{r}\frac{v_\varphi}{\Omega}\right],$$

$$\frac{\partial\delta P}{\partial r}=i\rho(\Omega^2-k^2a^2)\frac{v_r}{\Omega}, \qquad (1.6)$$

$$\frac{im}{r}\delta P=i\rho(\Omega^2-k^2a^2)\frac{v_\varphi}{\Omega}.$$

Here $a=(B^2/4\pi\rho)^{1/2}$ is the Alfvén velocity, and $\Omega=\omega-ku$.

System (1.6) describes all types of small oscillations of a magnetic tube. In the present paper we are considering only the $m=\pm1$ dipole mode, which corresponds to bending oscillations, and the $m=0$ axisymmetric mode, with a phase velocity $c_T=as(a^2+s^2)^{-1/2}$ which corresponds to slow ("varicose") oscillations.

For slow oscillations, system (1.6) reduces to a single equation for v_r. Eliminating δP from (1.6), and noting that w have $v_\varphi=0$ in the slow (axisymmetric) oscillations, we find

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\rho\frac{(a^2+s^2)(\Omega^2-k^2c_T^2)}{\Omega^2-k^2s^2}\frac{1}{r}\frac{\partial}{\partial r}r\frac{v_r}{\Omega}\right]$$

$$+\rho(\Omega^2-k^2a^2)\frac{v_r}{\Omega}=0. \qquad (1.7)$$

For the bending oscillations in the long-wave limit, the system (1.6) reduces to Eq. (10) in the case of an inhomogeneous tube; in the case of a homogeneous tube, it reduces to Eq. (1), where the displacement vector $\mathbf{v}=d\xi/dt$ is introduced in place of the velocity. As has already been mentioned, the $m=0$ mode also corresponds to torsional oscillations of the magnetic tubes and oscillations with changes in the plasma pressure and the magnetic pressure which occur in phase (an analog of fast magnetosonic waves). However, longitudinal flows of matter have only a slight effect on the former, while the latter have a very high frequency ($\sim a/R$), even in the case $kR\ll1$, so they are rapidly damped through the emission of sound waves into the surrounding medium. The high-order azimuthal modes ($m=\pm2,\pm3,\ldots$) are of little interest since they have only a slight effect on the "global" characteristics of tubes.

APPENDIX 2. EMISSION OF SOUND WAVES INTO THE EXTERNAL SPACE

We write the density perturbation in the sound wave outside the tube in the form

$$\delta\rho=\cos(m\varphi)[\tfrac{1}{2}f(r)\exp(-i\omega t+ikz)+\text{c.c.}]. \qquad (2.1)$$

The value $m=0$ corresponds to slow oscillations, and

$m = 1$ to plane-polarized bending oscillations. The function f satisfies the equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{df}{dr}\right)-\frac{m^2}{r^2}f-k^2f+\frac{(\omega-ku)^2}{s_e^2}f=0.$$

A solution of this equation corresponding to sound waves which go out from the tube is

$$f(r)=AH_m^{(1)}(k_\perp r), \tag{2.2}$$

where

$$k_\perp=[s_e^{-2}(\omega-ku)^2-k^2]^{1/2}=\frac{k}{s_e}[(v_\varphi-u)^2-s_e^2]^{1/2}. \tag{2.3}$$

The emission of sound waves is of course possible only if the expression in the radical in (2.3) is nonnegative. The conditions under which this is actually the case were formulated in Sec. 4.

At large distances from the tube ($k_\perp r \gg 1$), solution (2.2) has the asymptotic behavior

$$f=A\left(\frac{2}{\pi k_\perp r}\right)^{1/2}\exp\left(ik_\perp r-\frac{im\pi}{2}-i\frac{\pi}{4}\right). \tag{2.4}$$

For $k_\perp r \gg 1$, solution (2.1) with f as in (2.4) is approximately a plane-wave solution. Correspondingly, we calculate the energy density of a (locally) plane sound wave of the type

$$\delta p=\frac{1}{2}\delta p_0\cos(m\varphi)\exp(-i\omega t+ikz+ik_\perp r)+\text{c.c.} \tag{2.5}$$

We find

$$W_s=\frac{\rho_e s_e^2}{2}\left|\frac{\delta\rho_0}{\rho_e}\right|^2\frac{(k^2+k_\perp^2)^{1/2}s_e-ku}{(k^2+k_\perp^2)^{1/2}s_e}\cos^2 m\varphi. \tag{2.6}$$

The quantity W_s is negative under condition (15). Using relation (2.4), we find

$$W_s=\frac{s_e}{\pi k_\perp r}\frac{|A|^2}{\rho_e}\frac{\varkappa s_e-ku}{\varkappa}\cos^2 m\varphi; \tag{2.7}$$

here $\varkappa=(k^2+k_\perp^2)^{1/2}$. The energy flux from a unit length of the tube,

$$Q=2\pi r\langle W_s\rangle_\varphi k_\perp s_e/\varkappa,$$

is

$$Q=\frac{2s_e^2|A|^2}{\rho_e}\frac{\varkappa s-ku}{\varkappa^2}\cdot\begin{cases}1, & m=0,\\ 1/2, & m=1.\end{cases} \tag{2.8}$$

The problem is now one of expressing the coefficient A in terms of the tube oscillation amplitude. For this purpose, we consider the solution (2.2) near the tube boundary in the case $k_\perp r \ll 1$. In slow oscillations, we have here[4]

$$f=\left(1+\frac{2i}{\pi}\ln\frac{Ck_\perp r}{2}\right), \tag{2.9}$$

where C is Euler's constant. It can be seen from Eqs. (1.1) for an external region without a magnetic field that the density perturbation is related to the radial component of the displacement of the fluid, ξ_r, by

$$\xi_r=-\frac{s_e^2}{\rho_e(\omega-ku)^2}\frac{\partial\delta\rho}{\partial r}. \tag{2.10}$$

Accordingly, if we write the displacement of the tube boundary in the slow oscillations in the form

$$\xi_r=\frac{1}{2}\xi_0\exp(-i\omega t+ikz)+\text{c.c.}, \tag{2.11}$$

then by using Eqs. (2.1), (2.9), and (2.10) we find

$$A=-\frac{i\pi}{2}\frac{R\rho_e(\omega-ku)^2}{s_e^2}\xi_0. \tag{2.12}$$

In other words, for slow oscillations we have

$$Q_T=\frac{\pi^2\rho_e}{2}v_\varphi(v_\varphi-u)^2k^3R^2|\xi_0|^2.$$

We now consider bending oscillations. Using the well-known expansion of the Hankel function $H_1^{(1)}(k_\perp r)$ at $k_\perp r \ll 1$, we find from (2.2) the following result for the region close to the tube:

$$f\approx-\frac{2iA}{\pi k_\perp r}. \tag{2.13}$$

Writing the radial component of the displacement of tube in the bending oscillations in the form

$$\xi_r=\cos\varphi\left[\frac{1}{2}\xi_0\exp(-i\omega t+ikz)+\text{c.c.}\right]$$

(ξ_0 means the amplitude of the excursion of the tube axis from its unperturbed position), we find from (2.1), (2.10), and (2.13),

$$A=-\frac{i\pi}{2}\frac{k_\perp R^2\rho_e(\omega-ku)^2}{s_e^2}\xi_0.$$

We then find that the energy flux from a unit length of the tube is

$$Q_b=\frac{\pi^2\rho_e}{4}v_\varphi(v_\varphi-u)^2[(v_\varphi-u)^2-s_e^2]\frac{k^5R^4|\xi_0|^2}{s_e^2}.$$

Let us find the energy of the slow and bending oscillations for a unit length of the tube. For the bending oscillations, the corresponding result follows directly from Eq. (2):

$$W_b=\frac{\pi R^2}{4}|\xi_0|^2k^2\left[(\rho_i+\rho_e)\frac{\omega^2}{k^2}+a^2\rho_i-u^2\rho_e\right].$$

In the case of the slow oscillations, we need to carry out some calculations. For these oscillations we find

$$W_T=\left\langle\frac{\rho\delta v_\parallel^2}{2}+\frac{\gamma p}{2}\left(\frac{\delta\rho}{\rho}\right)^2+\frac{\delta B^2}{8\pi}\right\rangle\pi R^2,$$

where the angle brackets mean an average over a wavelength. In writing this expression we allowed for the circumstance that the transverse velocity of the plasma motion inside the tube is very small in comparison with the longitudinal velocity. From the equations of motion, the continuity equations, and the frozen-in condition we find, inside the tube,

$$\delta v_\parallel=\frac{s_i}{\rho}\delta\rho,\quad \delta B=B\frac{\xi}{2R},\quad \delta\rho=\frac{a^2}{2s_i^2}\frac{\xi}{R},$$

where ξ is the radial displacement of the boundary of the fluid. Specifying ξ as in (2.10), we then find

$$W_T=\frac{\pi|\xi_0|^2}{8}\frac{\rho_i a^2(a^2+s_i^2)}{s_i^2}.$$

Using the expression $\gamma=Q/2W$, we can now find the instability growth rates. For the bending oscillations we find

$$\frac{\gamma_b^{rad}}{\omega}=\pi k^2R^2\frac{(v_\varphi-u)^2[(v_\varphi-u)^2-s_e^2]}{2s_e^2v_\varphi[(1+\eta)v_\varphi-u]},$$

and for the slow oscillations we find

$$\frac{\gamma_T^{rad}}{\omega}=\frac{\pi k^2R^2}{4}\frac{\rho_e}{\rho_i}\frac{c_T^2(c_T-u)^2}{a^4}.$$

[1)] That negative-energy waves might exist in a nonequilibrium medium was originally pointed out by Kadomtsev *et al.*[8] With regard to negative-energy waves in hydrodynamics, we refer the reader to Ref. 9, for example.

[2)] This conclusion is a consequence of our assumption that there is no magnetic field outside the tube. In the general case, an instability mechanism of this sort would be possible.

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