Quantum and dissipative effects in dislocation dynamics

R. A. Vardanyan and Yu. A. Osip'yan

Institute of Solid-State Physics, Academy of Sciences of the USSR, Chernogolovka, Moscow Province
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An analysis is made of the dynamics of dislocations governed by fluctuation-induced overcoming of point obstacles distributed at random in the bulk of a metal. It is shown that an elementary event of fluctuation-induced detachment of a dislocation from a pinning center depends strongly on the stochastic dynamics of the center. A study is made of low-temperature anomalies of temperature-independent behavior of the plasticity of materials and also of the superconducting softening effect in a metal. The temperature at which temperature-independent behavior of the yield stress appears is related to quantum fluctuations of a dislocation or a pinning center, depending on the ratio of the masses of the atoms of the matrix and of the point defect. In the case of sufficiently heavy obstacles this temperature is given by the frequency of a quasilocal mode and is of the order of tens of kelvin. On the other hand, the softening of a metal as a result of a phase transition to the superconducting state is attributed to the exponential dependence of the velocity of dislocations on the viscosity determined by quasiparticle excitations.

1. INTRODUCTION

In developing a physical theory of the strength and plasticity of solids we have to exhibit the mechanisms of microscopic processes limiting the dislocation dynamics, because it is this which determines the process of plastic deformation of a crystal. The difficulties encountered in experimental and theoretical investigations of the dynamic behavior of dislocations are due to the great variety of factors influencing their mobility. Under a given external load the deformation conditions depend on the type of a crystal, the crystal structure of the lattice, nature of the moving dislocation, and on numerous mechanisms of the interaction both with the microstructure of the stress field (determined by the characteristics of the real material and by the lattice defects) as well as with various quasiparticle excitations. In discussing this problem it is usual to separate the main dislocation drag mechanisms into two groups (for a review see, for example, Ref. 1). The first group deals with the effects of barriers created at a local obstacle such as impurities, point defects, radiation damage, etc. or by the Peierls potential relief, which is an unavoidable concomitant of the periodic structure of the lattice. As is well known, such barriers are overcome by thermal or quantum fluctuations. The second group comprises dynamic dissipative processes of the interaction between dislocations and elementary excitations in a crystal, primarily phonons and electrons. This type of interaction is viscous and at low dislocation velocities the drag force is a linear function of the velocity, the coefficient of proportionality representing the viscosity of the quasiparticle gas.

This arbitrary division has been used for a long time and seems very natural. However, in the interpretation of experimental results the contributions of fluctuation and dissipative effects to the physical mechanics of a crystal are frequently nonadditive. This is manifested particularly in low-temperature investigations of the plastic properties of metals.

The first investigations of mechanical properties of solids at helium temperatures were made over fifty years ago by Meissner, Polanyi, and Schmid.[2] They found that the plasticity of cadmium and zinc single crystals was maintained down to 4.2 and 1.2 K, respectively, and that they deformed at stresses 10^2–10^3 times less than for an ideal crystal. The rate of creep of these materials was found to be independent of temperature. At present there are many experimental data demonstrating low-temperature temperature-independent behavior of plastic properties of a wide range of metals.

The increase in the plasticity of metal as a result of a phase transition to the superconducting state discovered in 1968 is very interesting.[3,4] Numerous subsequent experiments described in Refs. 5 and 6 have established that this superconducting softening effect may reach 40%. A review of experimental and theoretical investigations of these low-temperature anomalies of the plasticity of metals was made by Startsev.[7]

According to the current theoretical ideas, the temperature-independent behavior of the plasticity of metals is a manifestation of the quantum effects in the elementary event by which a dislocation overcomes obstacles, i.e., it is due to deviation from the classical thermal-fluctuation Arrhenius law. We shall restrict ourselves to the case when the lattice hinders the motion of a dislocation only via local obstacles.

The first theoretical investigation of the role of quantum effects in low-temperature plasticity of metals was made by Mott.[8] He calculated the probability of overcoming of an obstacle by a part of a dislocation adjoining directly a pinning center. Mott assigned an effective mass to this part and regarded it as a free particle; in this way he was able to use the quantum-mechanical tunnelling effect to explain the experimental creep rate.[1)] The characteristic temperatures at which this mechanism can become dominant are ~ 1 K. However, subsequent experiments have shown that temperature-independent effects in the plasticity of metals occur at temperatures right up to several tens of kelvin. Further development of the theory is due to Leibfried,[11] who analyzed the force exerted by a dislocation on a defect pinning it and showed that the probability of overcoming this local obstacle by fluctuations is largely determined by zero-point vibrations of dislocation segments adjoining the pinning center. The quantum motion of dislocations across a local obstacle was considered in Refs. 12–14 and in the case of strong coupling it was found that the characteristic tempera-

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ture for the appearance of the quantum effects is the Debye temperature. However, as shown in Ref. 15, a more realistic model of weak pinning gives a temperature ~ 1 K for predominance of quantum fluctuations. Mathematically this difference reduces to inclusion of more general boundary conditions imposed on the field of dislocation displacements at the pinning point and, therefore, to a correct allowance for the density of the vibrational states of the segments which represent distributed systems. A similar result was obtained later by Labusch[16] who used the same model. Therefore, a satisfactory theoretical interpretation of the temperature at which temperature-independent anomalies in the low-temperature plasticity of metals appear is still lacking.

Since the superconducting transition is known to affect only the state of the electron subsystem, it follows that the change (reduction in the flow stress) in the plasticity as a result of the superconducting softening effect can be explained by the sensitivity of the process of thermal activation to a reduction in the electron damping of a dislocation. The increase in the plasticity was interpreted in Refs. 17 and 18 in the spirit of the Kramers theory[19] in terms of the way the preexponential factor in the probability for fluctuation-induced overcoming of a center depends on the viscosity governed by the contribution of electrons in the normal state. This example is a sufficient demonstration of the nonadditivity of the fluctuation and dissipative mechanisms contributing to the process of dislocation drag. However, direct experimental measurements of the ratio of the velocities of a dislocation in superconducting and normal phases give $w_S/w_N \sim 7\times 10^4$ (Ref. 20), which is in clear conflict with the predictions of the theories of Refs. 17 and 18, according to which we should have $w_S/w_N \sim 20$. Such a difference by three or four orders of magnitude demands a further refinement of the models of dislocation motion. An alternative (to Refs. 17 and 18) mechanism for the influence of the viscosity λ on the fluctuation mobility of dislocations was put forward in Ref. 21. Development of the model of Ref. 13 demonstrated that in the low-temperature (quantum) range the enhancement of the influence of the viscosity on fluctuations of the temperature T in the Arrhenius law could be replaced with $(\frac{1}{4})\omega_D(1 + 2\lambda/\pi\omega_D)$, where ω_D is the Debye frequency. We can easily show that for realistic values of $\lambda \lesssim 10^{11}$ s^{-1} (Refs. 1 and 21) we can ensure the observed value of w_S/w_N only if we satisfy a fairly stringent condition $\omega_D < 10$ K. Here we propose a new mechanism for thermal-fluctuation motion of dislocations which makes it possible to allow for the quantum and dissipative effects in a unified manner.

2. DYNAMIC DESCRIPTION OF A DISLOCATION

To study the dynamics of a dislocation we use a chain model[22] which is equivalent (in the long-wavelength approximation) to the model of a string. Under the influence of a shear stress τ which exceeds the Peierls stress τ_P a dislocation is pressed against a system of local obstacles lying in a glide (slip) plane $z = 0$; it assumes a static configuration which depends on the positions and nature of the pinning centers (point defects) as well as on the applied force $b\tau$, where b is the Bürgers vector. We shall assume that point centers are distributed uniformly throughout the crystal and consider the simplest model, in which a dislocation moving in the x direction is separated along y into segments with the average length l. The position of an atom along the dislocation is $y = na$, where a is the period along the chain and $n = \pm 1, \pm 2, \ldots, \pm N/2 = l/a$. We are interested in the process in which a dislocation overcomes a specific obstacle located at a point $y = 0$, and we construct the following potential along the direction of motion of the dislocation

$$V(x) = \begin{cases} \zeta x^2; & |x| \leqslant x_c \\ 0; & |x| > x_c \end{cases}; \quad x_c = (V_0/\zeta)^{1/2}, \tag{1}$$

so that the equation of motion [or of the displacement field $u(y,t) \equiv u_n(t)$] can be written in the form

$$m\ddot{u}_n + 2m\lambda\dot{u}_n + \beta(2u_n - u_{n-1} - u_{n+1}) = b\tau a - 2\zeta u_n \delta_{n0} + F_n(t). \tag{2}$$

Here, m is the mass of an atom in the chain; λ is the kinematic viscosity[23]; β is the quasielastic force coefficient; and $F_n(t)$ is the fluctuation force which generally has, in accordance with the fluctuation–dissipation theorem,[24] the following statistical properties:

$$\langle F_n(t)\rangle = 0, \tag{3}$$

$$\langle F_n(r) F_{n'}(t+\tau)\rangle = 4m\lambda T K(\tau)\delta_{nn'},$$

$$K(\tau) = \int_{-\omega_c}^{\omega_c} \frac{d\omega}{2\pi} \exp(i\omega\tau) K(\omega), \quad K(\omega) = \frac{\hbar\omega}{2T} \coth\frac{\hbar\omega}{2T}. \tag{4}$$

Here the angular brackets denote the average of an ensemble of realizations of a random quantity. In the classical limit of high temperatures, assuming that $T \gg \hbar\omega$, we find that $K(\omega) = 1$, and the random force $F_n(t)$ represents the ordinary thermodynamic white noise. In the opposite limiting case of low temperatures, when $T \ll \hbar\omega$, and $K(\omega) = |\omega|/2T$, there are quantum fluctuations due to the zero-point vibrations. The frequency ω_c is the cutoff parameter, which removes the divergence of the energy of a noise source. The white noise is an idealization, valid only if the space–time scale is sufficiently large. When the correlation scales are $x < a$ and $t < \omega_c^{-1}$, the fluctuations are, generally speaking, nonlocal and time-dependent.[25] The use of the approximation represented by Eq. (4) makes it possible to consider the model analytically on the basis of the generally accepted[13,18,21,26] theory of Gaussian fluctuations.

The criterion for overcoming of an obstacle is

$$u_0(t) \geqslant x_c. \tag{5}$$

The coefficient $\chi = 2\zeta/\beta$ represents how the dislocation is pinned at the point $n = 0$. In the case of strong pinning we have $\chi \to 1$ (ζ is large) and $u_0(t) \to 0$.

Equation (2) is based on the assumption that the obstacle has an infinite mass. The solution of the problem of overcoming of a local obstacle by dislocation as a result of fluctuations at high temperatures (corresponding to the range of validity of the Arrhenius law) was obtained for this case in Ref. 26 and, allowing for quantum oscillation, also in Refs. 15 and 16.

A point center of finite mass μ is elastically coupled to the crystal matrix and the coupling is represented by the coefficient $\varkappa = \mu\omega_0^2$, so that fluctuation motion $v(t)$ of such a pinning center is possible and this motion is described by

$$\mu\ddot{v} + 2\mu\nu\dot{v} + \varkappa v(t) = f(t), \tag{6}$$

where $f(t)$ is a steady-state random force acting on an obstacle and characterized by a zero average

$$\langle f(t)\rangle=0 \tag{7}$$

and by the correlation function

$$\langle f(t)f(t+\tau)\rangle=4\mu\nu TK(\tau), \tag{8}$$

where ν is the viscosity of the obstacle. In the same approximation (4) postulating large-scale fluctuations, we shall assume that the forces $F_n(t)$ and $f(t)$ are uncorrelated. For simplicity, we shall assume that the coefficients λ and ν are identical and equal to γ. This assumption simplifies the calculations, but does not affect the basic principles of the problem discussed here.

When we allow for stochastic motion of the pinning center, the fluctuation dynamics of a dislocation can be described by two coupled differential equations

$$m\ddot{u}_n+2m\gamma\dot{u}_n+\beta(2u_n-u_{n-1}-u_{n+1})=b\tau a-2\zeta(u_n-v)\delta_{n0}+F_n(t),$$

$$\mu\ddot{v}+2\mu\gamma\dot{v}+\varkappa v=2\zeta(u_0-v)+f(t). \tag{9}$$

We shall describe the displacement of a dislocation by a sum:

$$u_n(t)=u_n^s+\xi_n(t). \tag{10}$$

Then, in the weak-pinning approximation characterized by $\chi\ll 1$ the static displacement of an obstacle can be ignored and the sag of a dislocation under the action of an applied stress τ is obtained by imposing on the ends of the segments the periodic boundary conditions

$$u_{-N/2}=u_{N/2} \tag{11}$$

and assuming that $(2a/\chi l)\ll 1$, this gives

$$u_n^s=u_0^s[1+{}^1/_4\chi l(2|n|/N-4n^2/N^2)], \tag{12}$$

where

$$u_0^s=bl\tau/2\zeta. \tag{13}$$

We shall seek $\xi_n(t)$ in the form

$$\xi_n(t)={}^1/_2N^{-1/2}\sum_q(Q_qe^{iqna}+\text{c.c.}),\quad Q_q=Q_{-q}^+, \tag{14}$$

where the sum is calculated using all the permissible values of the wave vector

$$q=2\pi g/Na,\quad g=0,\pm1,\ldots,+N/2, \tag{15}$$

deduced from Eq. (11). Substituting in the first equation of Eqs. (9) the above expansion for $\xi_n(t)$, multiplying it by $N^{-1/2}\exp(-iqn'a)$, summing over n, and assuming that

$$\sum_n\exp[i(q-q')na]=N\delta_{q,q'}\ ,$$

we find that, instead of Eq. (9), we now have

$$\ddot{Q}_q+2\gamma\dot{Q}_q+\omega^2(q)Q_q=-N^{-1/2}\tilde{\omega}^2(u_0-v)+m^{-1}F_q(t), \tag{16}$$

where

$$\ddot{v}+2\gamma\dot{v}+\omega_0^2v(t)=m\mu^{-1}\tilde{\omega}^2(u_0-v)+\mu^{-1}f(t),$$

$$\tilde{\omega}^2=2\zeta/m;\quad F_q(t)=N^{-1/2}\sum_nF_n(t)\exp(-iqna), \tag{17}$$

and the dispersion law of dislocation vibrations is given by the expression

$$\omega(q)=\omega_m\sin(qa/2),\quad \omega_m=(4\beta/m)^{1/2}. \tag{18}$$

It should be noted that the deviation of the dispersion law of dislocation vibrations from the linear one $\omega(q)=sq$, where s is the velocity of sound, is sensitive at short wavelengths to the selection of the model and in the case of dislocations it may be more complex. However, as shown below in Sec. 5, the effects of the temperature-independent behavior of the plasticity, and of the influence of the viscosity on quantum fluctuations are determined by the low-frequency part of the spectrum of vibrations of a dislocation: $\omega\sim\tilde{\omega}^2/\omega_m\ll\omega_m$.

Therefore, an investigation of the fluctuation dynamics of a dislocation reduces to an analysis of a system of stochastic ordinary differential equations.

3. ANALYSIS OF CORRELATION FUNCTIONS

In studying how a dislocation detaches from a fluctuating obstacle, it is important to consider their relative motion and, in contrast to the case of a static obstacle with an infinite mass, the criterion for overcoming the obstacle is now

$$\eta(t)=u_0(t)-v(t)\geqslant x_c. \tag{19}$$

We shall determine the correlation functions or moments $\eta(t)$ and $\dot{\eta}(t)$ using the spectral theory of stationary random functions. Using the Wiener–Khinchin theorem,[27] we shall introduce spectral expansions of fluctuating quantities employing the usual Fourier expansion formulas. For example,

$$Q_{q\omega}=\int_{-\infty}^{\infty}Q_q(t)\exp(i\omega t)\,dt. \tag{20}$$

Then, $u_{0\omega}=N^{-1/2}\sum_qQ_{q\omega}$ and v_ω can be found from Eq. (16):

$$u_{0\omega}=\frac{1}{N^{1/2}}\sum_q\frac{F_{q\omega}}{m[\omega^2(q)-\omega^2-2i\gamma\omega]}-\frac{\tilde{\omega}^2}{N}\sum_q\frac{1}{\omega^2(q)-\omega^2-2i\gamma\omega}(u_0-v)_\omega, \tag{21}$$

$$v_\omega=\frac{f_\omega}{\mu(\omega_0^2-\omega^2-2i\gamma\omega)}+\frac{m}{\mu}\frac{\tilde{\omega}^2}{\omega_0^2-\omega^2-2i\gamma\omega}(u_0-v)_\omega. \tag{22}$$

Subtracting Eq. (22) from Eq. (21), we find that the quantity η_ω of interest to us is described by

$$\eta_\omega=\frac{1}{A_\omega}\left[\frac{1}{N^{1/2}}\sum_q\frac{F_{q\omega}}{m[\omega^2(q)-\omega^2-2i\gamma\omega]}-\frac{f_\omega}{\mu(\omega_0^2-\omega^2-2i\gamma\omega)}\right] \tag{23}$$

where

$$A_\omega=1+\frac{\tilde{\omega}^2}{N}\sum_q\frac{1}{\omega^2(q)-\omega^2-2i\gamma\omega}+\frac{m}{\mu}\frac{\tilde{\omega}^2}{\omega_0^2-\omega^2-2i\gamma\omega}. \tag{24}$$

Allowing for the uncorrelated nature of $F_{q\omega}$ and f_ω, we apply the flucutation–dissipation theorem[24]

$$(F^2)_\omega=(f^2)_\omega=2\gamma\hbar\omega\,\mathrm{cth}(\hbar\omega/2T), \tag{25}$$

which in the case of the mean square of a fluctuating quantity

$$\sigma^2=\langle\eta^2(t)\rangle=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\eta_\omega^2 \tag{26}$$

yields

$$\langle\eta^2(t)\rangle=\frac{\hbar}{m\tilde{\omega}^2}\int_{-\infty}\frac{d\omega}{4\pi i}\coth\frac{\hbar\omega}{2T}\left(\frac{T_\omega^R}{1+T_\omega^R}-\frac{T_\omega^A}{1+T_\omega^A}\right) \tag{27}$$

where the retarded (advanced) function is

$$T_\omega^{R(A)}=\frac{\tilde{\omega}^2}{N}\sum_q\frac{1}{\omega^2(q)-\omega^2\mp2i\gamma\omega}+\frac{m}{\mu}\frac{\tilde{\omega}^2}{\omega_0^2-\omega^2\mp2i\gamma\omega}. \tag{28}$$

If we regard the integrand in Eq. (27) as a function of a complex variable in accordance with the Cauchy theorem, we find that the expression for $\langle\eta^2(t)\rangle$ can be represented by a sum of the Matsubara frequencies ω_n, i.e., by a sum of residues at points

$i\omega_n=2\pi inT/\hbar$,

which are governed by simple poles of the denominator $\coth(\hbar\omega/2T)$. We thus obtain

$$\langle\eta^2(t)\rangle=\frac{T}{m\tilde{\omega}^2}\sum_{n=-\infty}^{\infty}\frac{T(i\omega_n)}{1+T(i\omega_n)}. \tag{29}$$

In the low-temperature limit $T\to0$ (i.e., at temperatures lower than all the characteristic energies) allowing for

$$\frac{1}{N}\sum_q\left(u^2+2\gamma|u|+\omega_n^2\sin^2\frac{qa}{2}\right)^{-1}\quad[(u^2+2\gamma|u|)(u^2+2\gamma|u|+\omega_m^2)]^{-1/2} \tag{30}$$

we can go over from summation to integration in Eq. (29):

$$\langle\eta^2(t)\rangle=\frac{1}{\pi m\tilde{\omega}^2}\int_0^\infty du\,\frac{\mu^{-1}m(\omega_0^2+u^2+2\gamma u)^{-1}+[(u^2+2\gamma u)(u^2+2\gamma u+\omega_m^2)]^{-1/2}}{\tilde{\omega}^{-2}+\mu^{-1}m(\omega_0^2+u^2+2\gamma u)^{-1}+[(u^2+2\gamma u)(u^2+2\gamma u+\omega_m^2)]^{-1/2}}. \tag{31}$$

Equations (29) and (31) are derived assuming that the kinematic viscosities of a string and an obstacle are the same and amount to 2γ. In general, the expressions retain their initial form apart from the replacement $\gamma\to\lambda$ in the first term of Eq. (28) and $\gamma\to\nu$ in the second term.

Since in the case of stationary random functions, we have[27]

$$\sigma_1^2=\langle\dot{\eta}^2(t)\rangle=\int\frac{d\omega}{2\pi}\omega^2\eta_\omega^2,$$

it follows that the mean square of the rate of change of the fluctuating quantity we can readily obtain expressions analogous to Eqs. (29) and (31), which are convenient for analyzing the correlation functions in the limits of high and low temperatures.

4. PROBABILITY OF A FLUCTUATION TRANSITION

Before we calculate the values of the variances $\langle\eta^2(t)\rangle$ and $\langle\dot{\eta}^2(t)\rangle$, we shall consider the values of the frequencies characterizing the model and the approximations which allow us to greatly simplify the procedure and thus identify the range of validity of our results. In connection with the problem of thermal-fluctuation detachment of a dislocation from a harmonically vibrating obstacle, it was pointed out in Ref. 15 that for typical values of the crystal parameters, we have the inequality

$$\tilde{\omega}^2\ll\omega_m^2, \tag{32}$$

where ω_m is the limit of the spectrum of dislocation oscillators (Debye frequency). This inequality, equivalent to

$$\zeta\ll\beta\sim\varkappa, \tag{33}$$

implies that the pinning center is highly compliant, i.e., that the obstacle–dislocation coupling is weak compared with the rigidity of a string as a whole. This assumption underlies the formulation of the problem. The coefficient representing the rigidity β of the string is of the same order of magnitude as the rigidity of the matrix (or of the coupling of an obstacle to the matrix), $\varkappa$, i.e.,

$$m\omega_m^2\sim\mu\omega_0^2. \tag{34}$$

Hence it is clear that, depending on the ratio of the masses of an obstacle and the lattice atoms, the frequency ω_0 of vibrations of an obstacle can have various values. If a local obstacle is a heavy impurity ($\mu\gg m$), we have a quasilocal mode $\omega_0\equiv\omega_Q<\omega_m$. In the other limiting case, when $\mu\ll m$, we have a local mode $\omega_0\equiv\omega_L>\omega_m$ (Ref. 29).

The system we are considering consists of a central dislocation oscillator, coupled to a fluctuating pinning center, is characterized by a system of linear coupled differential equations. The response of such a system to an input in the form of the Gaussian noise is Gaussian.[27] In our case this response is a two-dimensional normal process $\{\eta(t),\dot{\eta}(t)\}$. The joint probability density (distribution function) of a random quantity $\eta(t)$ and of $\dot{\eta}(t)$, uncorrelated with that quantity, is therefore

$$W_2(\eta,\dot{\eta})=(2\pi\sigma\sigma_1)^{-1}\exp[-(\eta-\langle\eta\rangle)^2/2\sigma^2-\dot{\eta}^2/2\sigma_1^2], \tag{35}$$

where the mathematical expectation η [because of $\langle v(t)\rangle=\langle\xi(t)\rangle=0$] according to Eq. (12), is

$$\langle\eta(t)\rangle=\langle u_0^s\rangle=\tau x_c/2\tau_c. \tag{36}$$

Here $\tau_c=\zeta x_c/bl$ is the critical shear stress.

The two-dimensional distribution function of Eq. (35) is simply the Wigner function which describes the joint distribution of the probabilities of the coordinate and momentum.[30] It appears naturally in semiclassical studies of the stochasticity of a dynamic system described by a Hamiltonian which is a quadratic function of the coordinate and momentum. The Wigner representation is based on general principles which apply both in quantum and classical mechanics, and it is particularly convenient in discussing a system in which one of two interacting systems is classical and the other quantum. It is this case, which is encountered in studying how quantum fluctuations arise in the dynamics of a dislocation, which is regarded as a classical system.

The probability that a dislocation overcomes a local obstacle at a point $y = 0$ as a result of fluctuations in the direction of application of an external load τ is equal—in accordance with the criterion of Eq. (19)—to the probability of a random event $\eta(t) \geqslant x_c$, i.e., it is equal to the number of intersections ϑ of a given level x_c by a random function $\eta(t)$ per unit time. According to the theory of large excursions of random quantities[31], we find that

$$\vartheta = \int\int_0^\infty \dot{\eta}(t)\,\delta(\eta(t)-x_c)\,W_2(\eta,\dot{\eta})\,d\eta\,d\dot{\eta}.$$

Simple integration readily yields the required probability

$$\vartheta = \frac{\sigma_1}{2\pi\sigma}\exp\left[-\frac{(x_c-u_0^s)^2}{2\sigma^2}\right] = \frac{\sigma_1}{2\pi\sigma}\exp(-V^*/2\zeta\sigma^2), \qquad (37)$$

where

$$V^* = V_0\left(1-\frac{\tau}{2\tau_c}\right)^2 \qquad (38)$$

is the effective activation energy of the process.

5. TEMPERATURE-INDEPENDENT BEHAVIOR OF THE PLASTICITY AND THE SUPERCONDUCTING SOFTENING EFFECT

It is not always possible to calculate the variances σ and σ_1, so that we shall consider only various limiting cases. It is particularly interesting to discuss the case of a fixed obstacle ($\mu \to \infty$) in the limit of low viscosity $\gamma \to 0$. A calculation of σ^2 in accordance with Eqs. (29) and (31) in the limiting cases of high and low temperatures gives

$$\sigma^2 = \frac{1}{2\zeta}\begin{cases} T, & T \gg \dfrac{\tilde{\omega}^2}{2\pi\omega_m} \\ \dfrac{\tilde{\omega}^2}{2\pi\omega_m}\ln\left(\dfrac{\omega_m}{\tilde{\omega}}\right)^2 + O\left(\dfrac{2\pi T\omega_m}{\tilde{\omega}^2}\right), & T \ll \dfrac{\tilde{\omega}^2}{2\pi\omega_m}. \end{cases} \qquad (39)$$

Similarly, the variance σ_1^2 of the velocity is

$$\sigma_1^2 = \frac{1}{2m}\begin{cases} T, & T \gg \omega_m \\ \dfrac{\omega_m}{2}\left[1+\dfrac{2\pi^2}{3}\left(\dfrac{T}{\omega_m}\right)^2\right], & T \ll \omega_m \end{cases}. \qquad (40)$$

These values of the variance were obtained earlier[15,16,26] employing a model which ignores the fluctuation dynamics of obstacles in the continuum string approximation, confirming that the results obtained are insensitive to the selection of a specific dislocation model (as discussed at the end of Sec. 2). It follows from Eqs. (39) and (40) that at high temperatures the Arrhenius law applies

$$\vartheta = \frac{1}{2\pi}\left(\frac{\zeta}{m}\right)^{1/2}\exp\left(-\frac{V^*}{T}\right) \qquad (41)$$

and temperature-independent anomalies can be expected at temperatures $T < \chi\omega_m/8\pi \sim 1$ K, when the zero-point vibrations of dislocation oscillators become important. It should be noted that in the strong-coupling limit characterized by

$$\tilde{\omega}^2 \approx \omega_m^2 \qquad (42)$$

it follows from Eq. (29) that the deviation from the linear temperature dependence of the variance occurs at $T < \omega_m$. Such a result is obtained in Refs. 12–14. However, as pointed out above, such a strong coupling between a pinning center and a dislocation can hardly be regarded as a realistic interaction.

We shall assume that the obstacle has a finite mass and consider the limiting case when the second term is important in the expression for $T_\omega^{R(A)}$ given by Eq. (28). Ignoring in the weak-coupling limit the quantity $m\mu^{-1}\tilde{\omega}^2$ compared with ω_0, we can readily show that in the case of a low viscosity characterized by $\gamma \to 0$, we have

$$\eta_\omega^2 = \frac{\hbar}{2im\tilde{\omega}^2}\coth\frac{\hbar\omega}{2T}\lim_{\gamma\to 0}\left(\frac{T_\omega^R}{1+T_\omega^R} - \text{c.c.}\right)$$

$$= \frac{\pi\hbar}{\mu}\coth\frac{\hbar\omega}{2T}\,\delta(\omega^2-\omega_0^2).$$

Thus, we find that

$$\sigma^2 = \frac{\hbar}{2\mu\omega_0}\coth\frac{\hbar\omega_0}{2T}. \qquad (43)$$

Similarly, we obtain

$$\sigma_1^2 = \frac{\hbar\omega_0}{2\mu}\coth\frac{\hbar\omega_0}{2T}. \qquad (44)$$

This result has a clear physical meaning. In fact, in the case of high-Q oscillators characterized by $\gamma \ll \omega_0$, the spectral density of η_ω^2 in a small range $\Delta\omega \sim \gamma$ of frequencies $\pm\omega_0$ assumes large values, which are nevertheless finite because of $\langle f^2(t)\rangle \sim \gamma$. In full agreement with the fluctuation–dissipation theorem, the external action is replaced by white noise with spectral density equal to the spectral density of the input process at an eigenfrequency of the system. This is true also of a system of oscillators with variances described by Eqs. (39) and (40) and representing the sums of contributions of normal vibrations each of which has an "equivalent temperature" corresponding to its normal coordinate.[32]

It follows from Eqs. (37), (43), and (44) that

$$\vartheta = \frac{\omega_0}{2\pi}\exp\left(-\frac{2V^*}{\chi\hbar\omega_0\coth(\hbar\omega_0/2T)}\right). \qquad (45)$$

Comparing Eqs. (41) and (45), we can easily see that at high temperatures because of $\chi \ll 1$, we again have the Arrhenius law, Eq. (41), i.e., the dynamics of the transition is now influenced by thermal fluctuations of the dislocation oscillators and not those of the obstacle. At low temperatures such that $T < T_0$, where

$$\chi \frac{\hbar\omega_0}{2T_0} \coth \frac{\hbar\omega_0}{2T_0} = 1, \quad T_0 \approx \frac{1}{2} \chi\hbar\omega_0, \tag{46}$$

the dominant effect is due to quantum fluctuations, which are manifested experimentally as temperature-independent anomalies in the plasticity of materials. It is clear from Eqs. (45) and (46) that the higher the frequency of the vibrations of an obstacle ω_0 (i.e., the lower the mass of the obstacle), the easier it is to overcome and the higher is the temperature below which the quantum effects appear. The fact that experiments fail to reveal temperature-independent effects at high temperatures shows that such obstacles are ineffective compared with the heavy ones. This conclusion is important in analysis and identification of the role of various dopants in the dynamics of dislocations. It follows from Eqs. (39) and (46) that the intensity of quantum fluctuations of an obstacle predominates over the intensity of quantum fluctuations of a dislocation if

$$\frac{1}{4\pi} \ln\left(\frac{4}{\chi}\right) < \left(\frac{m}{\mu}\right)^{1/2}. \tag{47}$$

If the mass of an obstacle is so large or if the relative strength of the coupling is so low that the inequality of Eq. (47) is disobeyed, the plasticity of a material is governed entirely by the fluctuation dynamics of a dislocation when the obstacle is fixed. It should be pointed out that the experimentally observed temperature-independent anomalies of the plasticity of lead, cadmium, silver, zinc, copper and aluminum occur at temperatures (respectively 8, 14, 16, 26, 30, and 40 K) much lower than the Debye value. In full agreement with these observations, the temperature of Eq. (46) characterizing the effects associated with quantum fluctuations in a crystal with heavy obstacles is proportional to and much less than the Debye temperature.

We have considered so far only the limit of low viscosity. However, in the case of superconducting materials characterized by a high value of the electron–phonon interaction constant, we can also encounter high viscosity. Since we are interested in the softening of metals as a result of the transition from the normal to the superconducting state, we shall consider the range of fairly low temperatures. If we assume that the obstacle is fixed, so that $\mu \gg m$, we find that the variance governing the dissipative fluctuation dynamics of a dislocation is given by the following expressions, which apply to limiting cases:

$$\sigma^2 = \langle \eta^2(t) \rangle = \frac{1}{2\zeta} \frac{\chi\omega_m}{8\pi} \begin{cases} \ln(4/\chi) - 4\lambda/\chi\omega_m, & \lambda \ll \chi\omega_m/4 \\ \ln(\omega_m/\lambda) - \chi\omega_m/4\lambda, & \chi\omega_m/4 \ll \lambda \ll \omega_m \end{cases}. \tag{48}$$

Similarly when the inequality of Eq. (47) holds and the dynamics of a dislocation is governed by fluctuations of the obstacle the dependence of the variance of an obstacle on its viscosity ν is described by

$$\sigma_1^2 = \langle \eta^2(t) \rangle = \frac{1}{2\zeta} \frac{\chi\omega_0}{2} \begin{cases} 1 - \dfrac{4\nu}{\pi\omega_0}, & 2\nu \ll \omega_0 \\ \dfrac{\omega_0}{2\pi\nu} \ln \dfrac{2\omega_0}{\nu}, & \omega_0 \ll 2\nu \ll \omega_m \end{cases}. \tag{49}$$

It readily follows from Eqs. (48) and (49) that a strong reduction in the viscosity due to the transition of a metal from the normal to the superconducting state increases the plasticity of a material limited in the $\lambda = 0$ or $\nu = 0$ cases by the quantum fluctuations of the dislocation oscillators or the obstacle, depending on the mass ratio m/μ. Note that the superconducting softening effect is stronger in nonrigid materials containing light obstacles and thus exhibiting a stronger plasticity at low temperatures.

An exponential dependence of the probability of the transition on the viscosity can be explained by a major change in the ratio of the velocity of motion of dislocations w_S/w_N in transitions from the normal to the superconducting state. Using typical values for the parameters of a metal, we can readily see that when the inequality $\gamma_S/\gamma_N \ll 1$ holds we find from Eqs. (37), (48), and (49) that $w_S/w_N \sim 10^5$, i.e., we now obtain the value found experimentally.[20] In the strong-coupling limit corresponding to $\chi \to 1$, the variance (48) of the viscous–dissipation fluctuation of the dislocation agrees, to within logarithmic accuracy, with the results of Ref. 21. However, as already pointed out in the Introduction, in this case the experimentally observed value of the ratio w_S/w_N cannot be explained for any reasonable values of the parameters λ and ω_m.

6. FINAL COMMENTS AND CONCLUSIONS

Our analysis is limited on the high-temperature side and also in respect of the applied stresses by the condition

$$(\tau_c - \tau)^2 \gg T\tau_c^2/V_0. \tag{50}$$

When this inequality is satisfied the temperature must be low compared with the effective height of the barrier to be overcome and this ensures, on the one hand, that we can use the quasistationary approximation in the calculation of the transition probability and, on the other, it allows us to ignore the nonlinear effects. The simplest among these effects are "relativistic" and they appear in the dynamics of a dislocation at velocities comparable with the velocity of sound in a crystal; another nonlinear effect is the breakdown of the Stokes law due to turbulence in the establishment of the viscosity of a quasiparticle gas.

If we consider the conditions for the oscillator approximation to be a valid description of the dynamics of a dislocation and its obstacle, we note that in view of the smallness of ω_m and T compared with the barrier height ζx_c^2, describing their interaction, the potential can be represented by the first term of its expansion at zero [Eq. (1)]. Since the difference $u_0 - v$ reaches a comparatively large value of $x_c \sim a$ when the vibrations are small and we ignore the anharmonic effects, we are justified in using the Gaussian distribution for the probability of such a large fluctuation, which is derived above.

The criteria for the appearance and predominance in lattice dynamics of local and quasilocal vibrations of isotopic defects are discussed sufficiently thoroughly in the monograph of Kosevich.[33] In contrast to a local mode, separated by a gap from a continuous spectrum of vibrations, which appears in the case of a light impurity, a quasilocal mode with $\omega_0 < \omega_m$ suffers from broadening $T \sim \omega_0^2/\omega_m$ because of the interaction with crystal vibrations. Therefore, as pointed out in Ref. 33, in view of the condition $\Gamma \ll \omega_0$, we can ignore the width of a quasistationary packet and consider only the single-mode approximation regarding the contribution of the quasilocal mode ω_0 to be considerably larger than the contribution of ordinary normal vibrations of a crystal with frequencies from the continuous spectrum. It should be noted that the neglect of quasilocal mode correlations with the motion of the rest of the lattice corresponds to the same approximation of large-scale fluctuations, valid in the long-wavelength case $\omega_0 \ll \omega_m$.

Another natural limitation is the validity of the description, in the continuum approximation, of a dislocation by a mathematical string which is assumed to be inextensible so that $(\partial u/\partial y) \ll 1$. According to Eqs. (12) and (13), at the point $y = 0$, where $\partial u/\partial y$ attains a maximum, this condition takes the form

$$\frac{\partial u}{\partial y} \sim \frac{x_c}{b} \frac{\tau}{\tau_c} \frac{\zeta}{\varkappa} \ll 1.$$

We can easily see that if $\tau \sim \tau_c$ and $x_c \sim b$, then $\zeta \ll \varkappa$, which means that a dislocation cannot be described by an inextensible string in describing how it becomes detached from a rigid obstacle.

The main conclusions derived from this model are as follows:

1) in studies of the fluctuation-induced overcoming of a pinning center by a dislocation we must allow for the stochastic dynamics of an obstacle due to its finite mass; since the probability of overcoming of a light obstacle is greater than that of a heavy one, the plastic properties of a material are governed by the system heavy obstacles;

2) the quantum effects in the case of light obstacles occur even at high temperatures, but the experimentally observed temperature-independent anomalies are manifested only at low temperatures because of the dominant role of heavy centers pointed out above;

3) the characteristic temperature corresponding to the onset of quantum fluctuations in real crystals is governed by the frequency of a quasilocal mode of the obstacle and temperature-independent anomalies occur at temperatures $T_0 \sim \chi\hbar\omega_0$, on the order of tens of kelvin;

4) at sufficiently low temperatures the probability of fluctuation-induced overcoming of an obstacle is governed by dissipative effects and depends exponentially on the viscosity determined by quasiparticle excitations;

5) the superconducting softening effect is limited by the finite yield stress, due to the quantum fluctuations of the dislocation of the obstacle, depending on the mass ratio m/μ;

6) the strong exponential dependence of the rate of the process on the viscosity accounts for the large ratio of velocities of single dislocations in a metal in the transition from the normal to the superconducting state.

Our analysis of low-temperature anomalies of the rates of the processes is based, by way of example, on the dynamics of dislocations governing plastic properties of materials. However, the predicted quantum and dissipation effects are fairly general and may be exhibited also by various chemical and biological systems.[34]

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[1)] The dominant role of quantum-mechanical representations, particularly of the tunnelling effect, in the low-temperature anomalies in martensitic phase transformations was first demonstrated by Lyubov and Osip'yan.[9] Subsequently a similar mechanism was proposed by Gol'danskiĭ[10] in the specific case of a theory of reaction rates in chemical and biological systems at low temperatures.

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