Effects of parity nonconservation in nonlinear atomic spectroscopy

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The effects of parity nonconservation are considered in atoms located in the field of a strong electromagnetic wave. It is shown that if the weak interaction between the electrons and the nucleus is taken into account, the quasi-energy level of an atom with electron angular momentum $j = 1/2$ in the field of a linearly polarized wave splits along the projection of the angular momentum on the direction of propagation of the wave. In fields $\sim 10^7$ V/cm this splitting in the case of Cs and Tl atoms lies between 10^{-1} and 1 Hz. The resonance situation is considered for the $6p_{1/2}$, $6p_{3/2}$ transition in Tl, and it is shown that the splitting of this case comes to 10^2 Hz in fields $\sim 10^6$ V/cm. The influence of hyperfine structure on the effects that have been considered is studied. Constraints are found on field intensity and laser frequency, imposed by the condition that no noise be produced by line broadening.

1. In this work the effects of parity nonconservation are considered for atoms placed in the field of a strong electromagnetic wave (laser), when these effects arise from taking into account the weak interaction of the electrons with the nucleus. The basic idea consists of examination of the correlation $\boldsymbol{\nu}\cdot\mathbf{j}$, where $\boldsymbol{\nu}$ is the unit wave vector and $\mathbf{j}$ is the angular momentum of the atom. This correlation is, of course, well known; in the case of emission or absorption by polarized atoms it must lead to dependence of the probabilities on the direction of emission of the photon. This effect applies, however, to the region of linear phenomena in optics.

What is new in our work is that we examine an intrinsically nonlinear effect, the splitting of quasi-energy levels of an atom in the field of an intense laser wave. This effect is similar to a well-known effect, the Stark splitting of levels with angular momentum $j = 1/2$ in the field of a circularly polarized wave. Our effect is strictly electrodynamic and is described by correlations $\mathbf{s}_{\text{ph}}\cdot\mathbf{j}$, where $\mathbf{s}_{\text{ph}}$ is the spin of the photon. In parity nonconservation resulting from the weak interaction the splitting of levels with $j = 1/2$ occurs for arbitrary laser polarization. Since this interaction is extremely small, direct measurement of such an effect is evidently difficult to realize. It is possible, however, to place the atom in an additional constant magnetic field $\mathbf{H}_0$. There then appears an addition to the Zeeman splitting, involving the correlation $\boldsymbol{\nu}\cdot\mathbf{n}_0$, (where $\mathbf{n}_0 \equiv \mathbf{H}_0/|\mathbf{H}_0|$), which can be measured with the methods of magnetic resonance.

Quantitative calculations show that in fields $E \sim 10^7$ V/cm the splitting of quasi-energy levels of the heavy atoms amounts to $\Delta E^{(P)} \sim 10^{-1}$–1 Hz (see Sec. 9). In the resonance situation the splitting increases ($\Delta E^{(P)}_{\text{res}} \sim 10^1$–$10^2$ Hz for a field intensity $E \sim 10^6$ V/cm), since the resonance effect is linear with respect to field intensity, whereas the nonresonance effect is quadratic. The resonance situation appears, however, more vulnerable in the sense of noise (see Sec. 10).

2. We now examine the possible correlations of the atomic angular momentum $\mathbf{j}$ with the various vectors in the field of an intense electromagnetic wave. The presence of such correlations in the expression for quasi-energy also indicates a splitting of atomic quasilevels according to the sign of the corresponding projection of $\mathbf{j}$.

We represent the field of the wave in the form

$$\begin{aligned}\mathbf{E}(\mathbf{r},t) &= \frac{E}{2}\left(\mathbf{e}e^{-i\omega(t-(\boldsymbol{\nu},\mathbf{r})/c)}+\mathbf{e}^{*}e^{i\omega(t-(\boldsymbol{\nu},\mathbf{r})/c)}\right),\\ \mathbf{H}(\mathbf{r},t) &= \frac{E}{2}\left(\mathbf{e}_H e^{-i\omega(t-(\boldsymbol{\nu},\mathbf{r})/c)}+\mathbf{e}_H^{*}e^{i\omega(t-(\boldsymbol{\nu},\mathbf{r})/c)}\right),\end{aligned} \tag{1}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field intensities, E and ω are the amplitude and frequency of the field, and the unit vector $\mathbf{e}_H$ is determined by the equality $\mathbf{e}\times\mathbf{e}_H = \boldsymbol{\nu}$. Since the condition $a/\lambda \ll 1$ is satisfied, where a is the characteristic dimension of the atom and λ is the laser wavelength, we shall drop the factors $e^{i\mathbf{k}\cdot\mathbf{r}}$ in Eq. (1) in what follows.

First we examine the parity-allowed correlation $\mathbf{e}_H\cdot\mathbf{j}$. In the case of a constant field this correlation leads to the Zeeman effect. In the case of an alternating field it is accompanied by the oscillating factor $e^{i\omega t}$ and vanishes when averaged over time. The same may be said of the correlation $\mathbf{e}\cdot\mathbf{j}$, which is moreover forbidden by both P- and T-parity. The correlation $i(\mathbf{e}\times\mathbf{e}^{*})\cdot\mathbf{j} = i(\mathbf{e}_H\times\mathbf{e}_H^{*})\cdot\mathbf{j} = \mathbf{s}_{\text{ph}}\cdot\mathbf{j}$ does not drop out in averaging over time and gives the above-mentioned Stark splitting. It is absent for a linearly polarized wave. In the case of linear polarization there remains a single combination that does not vanish in the averaging over time, $(\mathbf{e}\times\mathbf{e}_H)\cdot\mathbf{j} = \boldsymbol{\nu}\cdot\mathbf{j}$; and this is the odd parity correlation that interests us.

If in addition the atom is placed in an external (sufficiently strong) magnetic field $\mathbf{H}_0$, the electron angular momentum $\mathbf{j}$ is quantized in the direction $\mathbf{n}_0$ of this field i.e., there appears a Zeeman splitting $\mathbf{j}\cdot\mathbf{n}_0$. If one contemplates an experiment of the magnetic resonance type of the Zeeman sublevels of the atom, then it is necessary to observe the frequency shift of such a resonance for a change in the sign of $\boldsymbol{\nu}\cdot\mathbf{n}_0$, which may be obtained with a change in the direction of $\boldsymbol{\nu}$ or $\mathbf{n}_0$ upon reversal.

It is necessary to take into account the fact that experiments for observing the effect of parity nonconservation are carried out not with an individual atom but with a medium comprised of atoms. In this case a linearly polarized wave, as it passes through the medium, acquires a circular-polarization contribution (degree of ellipticity χ) due to the very same parity-nonconservation effects. Its magnitude is defined by the equation[1]

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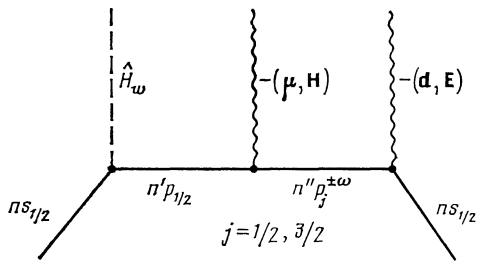

FIG. 1.

$$\chi = \frac{\pi l}{\lambda} \operatorname{Im}(n_+ - n_-),$$

where l in the length of the path, λ the wavelength, and $n_\pm$ the refraction coefficients for right- left- polarized waves. Thus in the non-resonance situation this contribution is extremely small in view of the smallness of absorption coefficients $\operatorname{Im} n_\pm$. In the case of resonance $\chi \sim lP_\rho W_0$, where P is the degree of circular polarization of the emission in the corresponding transition, W_0 is the probability of this transition, and ρ is the density of the medium. The generated ellipticity of the wave can lead to additional splitting of states with $j = 1/2$. If, however, the density of the medium is sufficiently small (as, for example, in atomic beams), then it is possible to neglect this additional splitting.[1])

3. From the form of the correlation $\mathbf{v}\cdot\mathbf{j}$ itself it follows that for its onset it is necessary to take into account the interaction of the atom with the electric and magnetic field of the wave, as well as the weak interaction that violates parity. In Fig. 1 there is represented one of the Feynman diagrams describing the effect that has been studied. The solid line in Fig. 1 represents the atomic electron, and the labels on this line designate the various states of the electron: $ns_{1/2}$ is the level that has been investigated; $n'p_{1/2}$ is the set of levels added to $ns_{1/2}$ by the weak interaction $\hat{\mathbf{H}}_\omega$; $n''p_j^{+\omega}(n''p_j^{-\omega})$ is the first (first negative) quasi-energy harmonic of state $n''p_j$; and $\boldsymbol{\mu}$ and $\mathbf{d}$ are the operators of the magnetic and electric dipole moments of the atom. It is necessary to add to this diagram all possible diagrams obtained as a result of interchange of interactions and permutation of corresponding intermediate states. The Feynman diagrams look similar in the case where the basic states of the atom are not $ns_{1/2}$ but $np_{1/2}$.

A picture of the splitting of level $nl_{1/2}$ $(l = 0, 1)$ is shown in Fig. 2. The magnitude of splitting $\Delta E^{(P)}$ is defined by the matrix element of the weak interaction (see Sec. 4).

In the case where the atom is placed in an additional constant magnetic field $\mathbf{H}_0$, the picture of the splitting of levels has the form shown in Fig. 3. Represented by the solid lines is the Zeeman splitting, of order of magnitude equal to αH_0 in atomic units (α is the fine-structure constant). Shown by the dashed lines is the shift of Zeeman sublevels for a change in the sign of $\mathbf{v}\cdot\mathbf{n}_0$.

4. The effective Hamiltonian of the weak interaction of the electron with the nucleus has the form (in atomic units)[1]

$$\hat{H}_w = \frac{G\alpha^2}{\sqrt{2}} Zg\mathbf{s}\{\mathbf{p}, \delta(\mathbf{r})\}_+, \tag{2}$$

where $\mathbf{s}$, $\mathbf{p}$, and $\mathbf{r}$ are the spin, momentum, and coordinate of the electron, $\{\ldots\}_+$ signifies the anticommutator, G is the

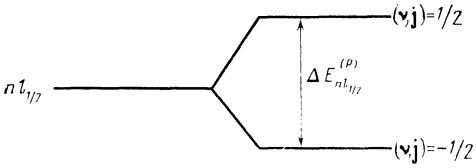

FIG. 2.

Fermi constant, Z the nuclear charge, and g is a factor of the order of unity for heavy atoms. We take into account for the atom only the part of the effective Hamiltonian that is not dependent on the spin of the nucleus; it gives the main contribution to the phenomena under consideration.

Inasmuch as the operator of Eq. (2) does not mix states with different projections of the electron angular momentum $\mathbf{j}$, it is sufficient, in order to take account of the weak interaction in our problem, to insert in all wave functions an admixture of states of opposite parity with the same value of projection of angular momentum. The coefficients of mixing, by this interaction, of $ns_{1/2}$ and $n'p_{1/2}$ states for the heavy atoms are cited in Ref. 1:

$$i\eta_{nn'} = \frac{\langle n'p_{1/2}|\hat{H}_w|ns_{1/2}\rangle}{E_{ns_{1/2}} - E_{n'p_{1/2}}} = \frac{i}{E_{n'p_{1/2}} - E_{ns_{1/2}}} \frac{G\alpha^2 Z^3 gR}{2^{3/2}\pi(\nu_s\nu_p)^{3/2}}, \tag{3}$$

where R is the relativistic intensification factor, and ν_s and ν_p are the effective principal quantum numbers of states $ns_{1/2}$ and $n'p_{1/2}$.

5. We write the interaction of the atom with the electromagnetic field of the laser wave in the form

$$\hat{W}(t) = \frac{1}{2}(\hat{V}e^{-i\omega t} + \hat{V}^+ e^{i\omega t}), \tag{4}$$

where

$$\hat{V} = -E((\mathbf{d}, \mathbf{e}) + (\boldsymbol{\mu}, \mathbf{e}_H)). \tag{5}$$

For a description of the behavior of an atom in such a field it is necessary to utilize a quasi-energetic formalism. In the case where it is possible to assume the field of the wave to be weak compared to the atomic field, it is possible to construct for the quasi-energies a perturbation theory analogous to time-independent perturbation theory.[2] If there is no degeneracy in the quasi-energies (i.e., in the absence of resonance), the correction of first order in E becomes zero on averaging over time. For second-order perturbation theory in E we have

$$\Delta E_n^{(2)}(\omega) = \frac{1}{4}\sum_{n_1}\left\{\frac{|\langle n|\hat{V}|n_1\rangle|^2}{E_n - E_{n_1} - \omega} + \frac{|\langle n|\hat{V}^+|n_1\rangle|^2}{E_n - E_{n_1} + \omega}\right\}. \tag{6}$$

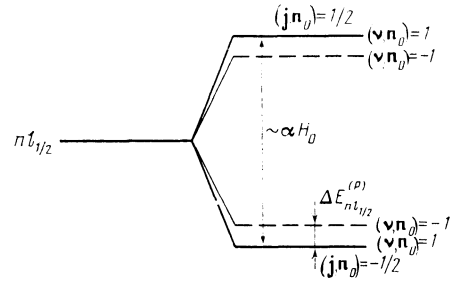

FIG. 3.

In the case of resonance, for example, of the level nl_j with level $n'l'_{j'}$, perturbation theory is no longer applicable, and it is necessary to diagonalize the operator $\hat{W}$ over states nl_j and $n'l'^{-\omega}_{j'}$. More accurately, subject to diagonalization is that part of operator $\hat{W}$ which mixes these states, namely $-\mathbf{d}\cdot\mathbf{E}$ for resonance of levels $ns_{1/2}$ and $n'p_j$ $(j=1/2, 3/2)$, and $-\boldsymbol{\mu}\cdot\mathbf{H}$ for resonance of levels $ns_{1/2}$, $n's_{1/2}$ and np_j, $n'p_{j'}$ $(j,j'=1/2, 3/2)$. The remaining part is taken into account, as formerly, by perturbation theory. From this it is evident that the greatest intensification of the effect arises in the resonance of levels ns, $n's$ or np, $n'p$, since in this case the smallness connected with the presence of the operator $-\boldsymbol{\mu}\cdot\mathbf{H}$ which contains the extraneous factor α is compensated.

Assuming a linearly polarized wave, i.e., real vectors $\mathbf{e}$ and $\mathbf{e}_H$, it is possible to set $\hat{V}=\hat{V}^+$ and rewrite Eq. (6) in the form

$$\Delta E_{nljm}(\omega) = \frac{E^2}{2}\sum_{n_1l_1j_1m_1}\frac{|\langle nljm|\,(\mathbf{e},\mathbf{d})+(\boldsymbol{\mu},\mathbf{e}_H)\,|n_1l_1j_1m_1\rangle|^2(E_{nlj}-E_{n_1l_1j_1})}{(E_{nlj}-E_{n_1l_1j_1})^2-\omega^2}; \tag{7}$$

here m and m_1 are the projections of the atomic angular momentum on the quantization axis. We break Eq. (7) into two parts:

$$\Delta E_{nljm}(\omega)=\Delta E^{(0)}_{nljm}(\omega)+\Delta E^{(P)}_{nljm}(\omega), \tag{8}$$

where the correction $\Delta E^{(0)}_{nljm}$ contains the usual dynamic polarizability and magnetic susceptibility

$$\Delta E^{(0)}_{nljm}(\omega)=\frac{E^2}{2}\sum_{n_1l_1j_1m_1}\left\{\frac{E_{nlj}-E_{n_1l_1j_1}}{(E_{nlj}-E_{n_1l_1j_1})^2-\omega^2}\times(|\langle nljm|\,(\mathbf{e},\mathbf{d})\,|n_1l_1j_1m_1\rangle|^2+|\langle nljm|\,(\mathbf{e}_H,\boldsymbol{\mu})\,|n_1l_1j_1m_1\rangle|^2)\right\}, \tag{9}$$

while the correction $\Delta E^{(P)}_{nljm}$ arises only if spatial parity is not conserved

$$\Delta E^{(P)}_{nljm}(\omega)=\frac{E^2}{2}\sum_{n_1l_1j_1m_1}\left\{\frac{E_{nlj}-E_{n_1l_1j_1}}{(E_{nlj}-E_{n_1l_1j_1})^2-\omega^2}(\langle nljm|\,(\mathbf{e},\mathbf{d})\,|n_1l_1j_1m_1\rangle\times\langle n_1l_1j_1m_1|\,(\mathbf{e}_H,\boldsymbol{\mu})\,|nljm\rangle+\text{к. с.})\right\}. \tag{10}$$

The correction $\Delta E^{(P)}_{nljm}$ can also be expressed by means of the odd-parity polarizability tensor β_{ik} introduced in Ref. 3:

$$\Delta E^{(P)}_{nljm}(\omega)=-\frac{E^2}{4}e_ie_{H_k}(\beta_{ik}+\beta_{ik}^*), \tag{11}$$

where

$$\beta_{ik}=\sum_{n_1l_1j_1m_1}\left\{\frac{\langle nljm|\,d_i\,|n_1l_1j_1m_1\rangle\langle n_1l_1j_1m_1|\,\mu_k\,|nljm\rangle}{E_{n_1l_1j_1}-E_{nlj}-\omega}+\frac{\langle nljm|\,\mu_k\,|n_1l_1j_1m_1\rangle\langle n_1l_1j_1m_1|\,d_i\,|nljm\rangle}{E_{n_1l_1j_1}-E_{nlj}+\omega}\right\}. \tag{12}$$

In Ref. 3 macroscopic effects were examined, arising from the passage of electromagnetic waves through an isotropic medium. In the resolution of β_{ik} into irreducible parts only the scalar part of this tensor $(1/3)\,\mathrm{Tr}\,\beta\cdot\delta_{ik}$ gave a contribution to these effects. For us, on the other hand, there remains only its vector part (see the next section).

6. In Eqs. (9) and (10) it is possible to sum over m_1 and explicitly separate out the dependence on m.

We examine an expression of the general form

$$\sum_{n_1l_1j_1m_1}\frac{\langle nljm|\,(\mathbf{e}_A,\mathbf{A})\,|n_1l_1j_1m_1\rangle\langle n_1l_1j_1m_1|\,(\mathbf{e}_B,\mathbf{B})\,|n'l'jm'\rangle}{E_{nlj}-E_{n_1l_1j_1}\pm\omega} \equiv X^{j(\pm)}_{nlm,n'l'm'}, \tag{13}$$

where $\mathbf{A}$ and $\mathbf{B}$ are arbitrary vector operators. Utilizing standard transformations taking account of the Wigner-Eckart theorem

$$\langle nljm|A_\mu|n_1l_1j_1m_1\rangle=\frac{C^{jm}_{j_1m_1,1\mu}}{(2j+1)^{1/2}}\langle nlj\|\mathbf{A}\|n_1l_1j_1\rangle, \tag{14}$$

where $\langle nlj\|\mathbf{A}\|n_1l_1j_1\rangle$ is the reduced matrix element, and of the Clebsch-Gordan[4] coefficients $C^{jm}_{j_1m_1,1\mu}$, we obtain

$$\begin{aligned}X^{j(\pm)}_{nlm,n'l'm'}=&-\frac{(\mathbf{e}_A,\mathbf{e}_B)}{6(2j+1)}\sum_{j_1=j,\,j\pm1}M^{j_1(\pm)}_{A,B;nlj,\,n'l'j}\delta_{mm'}\\&-\frac{i\,[\mathbf{e}_A,\mathbf{e}_B]^{m-m'}}{4j(j+1)(2j+1)}\\&\times\begin{pmatrix}-[(j+m)(j-m+1)/2]^{1/2} & \text{for } m=m'+1\\ m & \text{for } m=m'\\ [(j-m)(j+m+1)/2]^{1/2} & \text{for } m=m'-1\end{pmatrix}\\&\times(M^{j(\pm)}_{A,B;nlj,n'l'j}+(j+1)M^{j-1(\pm)}_{A,B;nlj,n'l'j}\\&-jM^{j+1(\pm)}_{A,B;nlj,n'l'j})-\frac{1}{4j(j+1)(2j+1)}\\&\times\begin{cases}e_A^1e_B^1[(j+m-1)(j+m)(j-m+1)(j-m+2)]^{1/2} & \text{for } m=m'+2\\ (e_A^1e_B^0+e_A^0e_B^1)[(j+m)(j-m+1)/2]^{1/2}(1-2m) & \text{for } m=m'+1\\ (e_A^0e_B^0-{}^1/_3(\mathbf{e}_A,\mathbf{e}_B))(3m^2-j(j+1)) & \text{for } m=m'\\ (e_A^{-1}e_B^0+e^0{}_Ae_B^{-1})[(j-m)(j+m+1)/2]^{1/2}(1+2m) & \text{for } m=m'-1\\ e_A^{-1}e_B^{-1}[(j-m-1)(j-m)(j+m+1)(j+m+2)]^{1/2} & \text{for } m=m'-2\end{cases}\\&\times\left(M^{j(\pm)}_{A,B;nlj,n'l'j}-\frac{j}{2j+3}M^{j+1(\pm)}_{A,B;nlj,\,n'l'j}-\frac{j+1}{2j-1}M^{j-1(\pm)}_{A,B;nlj,\,n'l'j}\right).\end{aligned} \tag{15}$$

Here there is introduced the notation:

$$M^{j_1(\pm)}_{A,B;nlj,n'l'j}\equiv 2\sum_{n_1l_1}\frac{\langle nlj\|A\|n_1l_1j_1\rangle\langle n'l'j\|B\|n_1l_1j_1\rangle^*}{E_{n_1l_1j_1}-E_{nlj}\mp\omega}; \tag{16}$$

a^μ is the contravariant cyclic component of the vector $\mathbf{a}$ (the system of coordinates is chosen so that the component $a^0=a_z$ is the projection on the axis of quantization).

In agreement with Eqs. (9) and (10), here only the quantities diagonal in m and m' of Eq. (13) interest us. The first term in Eq. (15) vanishes for $\mathbf{e}_A = \mathbf{e}$, $\mathbf{e}_B = \mathbf{e}_H$, and for $j = 1/2$ the third term likewise vanishes. On the other hand, the second term does not vanish and gives an explicit dependence on the sign of the projection m. To be specific we consider a configuration of electrons in which the one valence electron is in state $nl_{1/2}$ ($l = 0, 1$). Then it is possible to write Eq. (10) in the form

$$\Delta E^{(P)}_{nl\,1/2\,m} = -\frac{E^2 \nu m^0}{3} I_{nl\,1/2}, \tag{17}$$

where

$$I_{nl_{1/2}} = \sum_{\substack{n_1 \\ l_1=0,\,1}} \mathrm{Im}\left(\langle nl_{1/2} \| \mathbf{d} \| n_1 l_{1\,1/2}\rangle \langle nl_{1/2} \| \boldsymbol{\mu} \| n_1 l_{1\,1/2}\rangle^*\right) \times \frac{\Delta E_{n_1 l_1\,1/2}}{\Delta E^2_{n_1 l_1\,1/2} - \omega^2} - \frac{1}{2} \sum_{n_1} \mathrm{Im}\left(\langle nl_{1/2} \| \mathbf{d} \| n_1 p_{3/2}\rangle \langle nl_{1/2} \| \boldsymbol{\mu} \| n_1 p_{3/2}\rangle^*\right) \times \frac{\Delta E_{n_1 p_{3/2}}}{\Delta E^2_{n_1 p_{3/2}} - \omega^2} \tag{18}$$

and $\Delta E_{n_1 l_1 j_1} = E_{n_1 l_1 j_1} - E_{nl(1/2)}$. The parity-allowed reduced matrix elements in Eq. (18) are known[5]; calculation of the forbidden elements gives:

$$\langle ns_{1/2} \| \boldsymbol{\mu} \| n' p_{1/2}\rangle = -i\alpha \sqrt{\frac{2}{3}}\, \eta_{nn'},$$

$$\langle ns_{1/2} \| \boldsymbol{\mu} \| n' p_{3/2}\rangle = i \frac{\alpha}{\sqrt{3}}\, \eta_{nn'},$$

$$\langle ns_{1/2} \| \mathbf{d} \| n' s_{1/2}\rangle = i \sqrt{\frac{2}{3}} \left(\sum_{n''} \eta_{n'n''} D_{ns_{1/2},\, n'p_{1/2}} - \sum_{n''} \eta_{nn''} D_{n's_{1/2},\, n''p_{1/2}} \right),$$

$$\langle np_{1/2} \| \mathbf{d} \| n' p_{1/2}\rangle = i \sqrt{\frac{2}{3}} \left(\sum_{n''} \eta_{n''n'} D_{n''s_{1/2},\, np_{1/2}} - \sum_{n''} \eta_{n''n} D_{n''s_{1/2},\, n'p_{1/2}} \right), \tag{19}$$

$$\langle np_{1/2} \| \mathbf{d} \| n' p_{3/2}\rangle = -i \frac{2}{\sqrt{3}} \sum_{n''} \eta_{n''n} D_{n''s_{1/2},\, n'p_{3/2}},$$

where

$$D_{ns_{1/2},\, n'p_j} \equiv \int_0^\infty R_{ns_{1/2}}(r)\, R_{n'p_j}(r)\, r^3\, dr, \tag{20}$$

R_{nl_j} is the radial part of the wave function, and $\eta_{nn'}$ are the mixing coefficients (3). We note that for the hydrogen atom the quantity $I_{nl(1/2)}$ is calculated exactly in Ref. 6.

In the presence of an added magnetic field the only defined direction in which the angular momentum $\mathbf{j}$ can be quantized is given by the vector $\boldsymbol{\nu}$ (see Sec. 2). With this choice of axis of quantization $\nu^0 = 1$ and Eq. (17) gives the splitting of level $nl_{1/2}$:

$$\Delta E^{(P)}_{nl\,1/2} = \frac{E^2}{3} I_{nl\,1/2}. \tag{21}$$

In the presence of an added magnetic field $\mathbf{H}_0$ the axis of quantization has the direction of $\mathbf{n}_0$, so that $\nu^0 = \boldsymbol{\nu}\cdot\mathbf{n}_0$ and the frequency shift of the magnetic resonance upon replacement of $\boldsymbol{\nu} = \mathbf{n}_0$ by $\boldsymbol{\nu} = -\mathbf{n}_0$ is equal to

$$\Delta E^{(P)\,M.\Gamma.}_{nl\,1/2} = \frac{2}{3} E^2 I_{nl\,1/2}. \tag{22}$$

7. We now examine separately the resonance situation in which any one of the denominators in $I_{nl(1/2)}$ becomes zero. In this case it is necessary to utilize the analog of time-independent perturbation theory in the presence of degeneracy. We consider as an example the resonance of an $np_{1/2}$ level with an $np_{3/2}$ level. Then it is possible to disregard all the intermediate states except $np_{3/2}$, and it is necessary for us to diagonalize the matrix of the Hamiltonian taken over states $np_{1/2}$ and $np^{-\omega}_{3/2}$. We designate the deviation from resonance by δ. Then, directing the x axis along vector $\mathbf{e}$ and the y axis along $\mathbf{e}_H$, we obtain a matrix (see Table I) in which there is introduced the notation

$$F \equiv -\frac{\sqrt{3}}{2} \mathrm{Im}\left(\langle np_{1/2} \| \mathbf{d} \| np_{1/2}\rangle\right). \tag{23}$$

With this choice of axes the matrix breaks in two and is immediately diagonalized. For $\delta \ll \alpha E$ the six eigenvalues of this matrix have the form

$$\lambda_{1,2} = E_{np_{1/2}}$$

$$\lambda_{3-6} = E_{np_{1/2}} \pm \frac{\alpha E}{6\sqrt{2}} \pm \frac{EF}{6\sqrt{2}}. \tag{24}$$

Thus the odd-parity splitting in this case comes to

$$\Delta E^{(P)}_{\mathrm{res}\,(np_{1/2},\, nP_{1/2})} = EF/3\sqrt{2}. \tag{25}$$

It is evident that, unlike the nonresonance situation, the effect is proportional to first order in the field. Moreover, in Eq. (25) the previously mentioned smallness connected with operator $-\boldsymbol{\mu}\cdot\mathbf{H}$ is not present.

8. Equations (21), (22), and (25) pertained to a spinless nucleus. Consideration of the hyperfine structure (HFS) of the levels leads to some changes, with the results different for integer and half-integer spins of the nucleus.

We consider the level $nl_{1/2}$ in an atom in which the spin of the nucleus is equal to I. The electron angular momentum $j = 1/2$ is not quantized in the field of an electromagnetic wave. Taking account of HFS, however, the sublevels with total angular momentum $F \neq 0$ or $1/2$ are found to be quantized in the direction of the electric field. This quantization occurs because of the tensor term in Eq. (15). At the same time each of the HFS sublevels is split in the square M_F^2 of the projection of the total angular momentum along the electric field. This splitting is of the order of

$$\Delta E \sim E^2 \frac{\Delta E^{\mathrm{HFS}}}{\mathrm{Ry}}. \tag{26}$$

Without taking the HFS into account the quantization axis was in the direction of the vector $\boldsymbol{\nu}$. Therefore the diagonal matrix element of the perturbation matrix of Eq. (15)

TABLE I.

	$n\bar{p}^{\omega}_{3/2}, {}^{3}/_{2}$	$n\bar{p}^{\omega}_{3/2}, {}^{1}/_{2}$	$n\bar{p}^{\omega}_{3/2}, -{}^{1}/_{2}$	$n\bar{p}^{\omega}_{3/2}, -{}^{3}/_{2}$	$np_{1/2}, {}^{1}/_{2}$	$np_{1/2}, -{}^{1}/_{2}$
$n\bar{p}^{\omega}_{3/2}, {}^{3}/_{2}$	$E_{np_{1/2}}+\delta$	0	0	0	$\frac{iE}{2\sqrt{6}}\left(\frac{\alpha}{2}+F\right)$	0
$n\bar{p}^{\omega}_{3/2}, {}^{1}/_{2}$	0	$E_{np_{1/2}}+\delta$	0	0	0	$\frac{iE}{6\sqrt{2}}\left(\frac{\alpha}{2}+F\right)$
$n\bar{p}^{\omega}_{3/2}, -{}^{1}/_{2}$	0	0	$E_{np_{1/2}}+\delta$	0	$\frac{iE}{6\sqrt{2}}\left(\frac{\alpha}{2}-F\right)$	0
$n\bar{p}^{\omega}_{3/2}, -{}^{3}/_{2}$	0	0	0	$E_{np_{1/2}}+\delta$	0	$\frac{iE}{2\sqrt{6}}\left(\frac{\alpha}{2}-F\right)$
$np_{1/2}, {}^{1}/_{2}$	$-\frac{iE}{2\sqrt{6}}\left(\frac{\alpha}{2}+F\right)$	0	$-\frac{iE}{6\sqrt{2}}\left(\frac{\alpha}{2}-F\right)$	0	$E_{np_{1/2}}$	0
$np_{1/2}, -{}^{1}/_{2}$	0	$-\frac{iE}{6\sqrt{2}}\left(\frac{\alpha}{2}+F\right)$	0	$-\frac{iE}{2\sqrt{6}}\left(\frac{\alpha}{2}-F\right)$	0	$E_{np_{1/2}}$

gave the energy correction (10), to allow for weak interactions, while the off-diagonal elements were equal to zero.

Since allowance for the HFS changes, the axis of quantization is changed (it is now in the direction of **E**, i. e., orthogonal to **ν**), the diagonal elements of Eq. (15) now, conversely, become equal to zero and the off-diagonal ones differ from zero only for $M_F' = M_F \pm 1$ [we recall that the parity nonconserving effect arises only because of the vector term in Eq. (15)].

Thus it is necessary for us to carry out a diagonalization of the vector term in Eq. (15) over states which remain degenerate after diagonalization of the tensor term of Eq. (15), i.e., over states $\pm M_F$. If F is an integer, the matrix to be diagonalized is found to be identically equal to zero, since the states to be mixed do not satisfy the condition $M_F' = M_F \pm 1$. The splitting due to weak interactions does not exist in this instance. If F is half-integer, the very same thing occurs for all sublevels except $M_F = \pm 1/2$. In the latter case a splitting appears of the very order as in the absence of HFS.[2)]

Turning on the field $\mathbf{H}_0$ substantially changes this picture. We consider initially the case where the field $\mathbf{H}_0$ can destroy the HFS of level $nl_{1/2}$ completely (in heavy atoms the necessary fields for this are $H_0 \gtrsim 10^4$–10^5G). Allowance for the HFS causes then each of the Zeeman components of level $nl_{1/2}$ to split along the projection of the nuclear spin on the magnetic field, such that this splitting is of the order of the HFS splitting of level $nl_{1/2}$. A change of the sign of $\mathbf{ν}\cdot\mathbf{n}_0$ shifts the entire system of sublevels as a whole by an amount $\Delta E^{(P)}$, in anology with Fig. 3. At the same time, from the point of view of possible experiments, the situation apparently does not change as compared with that in the absence of HFS.

However, disruption of the HFS is not obligatory for observation of the effect. It is sufficient only to destroy the Stark splitting [Eq. (26)] of the hyperfine components of level $nl_{1/2}$, and this already occurs in fields $H_0 \gtrsim 1$ G. In this case there arises a Zeeman splitting of a level with definite F into states with definite projections of the total angular momentum M_F in the direction of the field $\mathbf{H}_0$. We will assume that the field $\mathbf{H}_0$ is in the direction of the vector **ν** or in the opposite direction. Then the axis of quantization is again directed along **ν**, and only the diagonal elements differ from zero in the vector term of the perturbation matrix (15). Upon sign reversal of $\mathbf{ν}\cdot\mathbf{n}_0$ each of the Zeeman levels shifts by an amount proportional to M_F and $\Delta E^{(P)}$ (Fig. 4). In Fig. 4 the level scheme for one sign of $\mathbf{ν}\cdot\mathbf{n}_0$ is designated by solid lines, and the other by dashed lines. A frequency shift of the magnetic resonance will be observed between Zeeman sublevels with $\Delta M_F = \pm 1$.

We now examine a resonance situation in the presence of HFS. As an example of the influence of HFS on the resonance effect we consider resonance of $np_{1/2}$ and $np_{3/2}$ (nuclear spin $I = 1/2$). In this case, in fields $E \lesssim 10^6$ V/cm (this constraint follows from the condition that there be no noise due to line broadening, see Sec. 10) the quantity $\alpha E/6\sqrt{2}$ from Eq. (24) is found to be much less than the HFS splitting of level $np_{1/2}$, but greater than HFS splitting of level $np_{3/2}$. Hence it is necessary to examine separately the resonance of level $np_{3/2}$ with each HFS $np_{1/2}F$ sublevel for $F = 0$ and 1. Of greatest interest is the case of resonance of $np_{3/2}$ with $np_{1/2}1$. For determination of the structure of the levels it is necessary initially to diagonalize the matrix of the operator $-\boldsymbol{\mu}\cdot\mathbf{H}$ ($\mathbf{H}$ is the field of the wave) over the wave functions of the states $np_{1/2}1$ and $np_{3/2}{}^{-\omega}$, and then take into account the HFS of level $np_{3/2}$. The resultant picture is the following (Fig. 5). The left side of Fig. 5 corresponds to the quasi-energy level scheme of the atom without taking account of the interaction with the magnetic field of the wave $-\boldsymbol{\mu}\cdot\mathbf{H}$; the right side depicts the resonance levels when taking this interaction into account. In Fig. 5 the HFS splittings of levels $np_{1/2}$ and $np_{3/2}$ are designated by $\Delta_{1/2}$ and $\Delta_{3/2}$. As already stated, it is assumed that

$$\Delta_{3/2} \ll \frac{\alpha E}{6\sqrt{2}} \ll \Delta_{1/2}. \tag{27}$$

In contrast to the nonresonance case, quantization axis can now be defined for both correlations $(\mathbf{j}\cdot\mathbf{E})^2$ and $|\mathbf{j}\cdot\mathbf{H}|$. Estimates show that for $E \lesssim 10^6$ V/cm the angular momentum is quantized in the direction $\mathbf{e}_H$ of the magnetic field of the wave. A weak splitting does not arise here for the very same reason as also in the nonresonance situation (the axis of quantization is directed orthogonal to **ν**). For observation of the effect it is necessary to turn on an external magnetic field $\mathbf{H}_0$ that sets a new direction of quantization. For this it is sufficient to take a field $H_0 \sim 10^2$ G that mixes only levels split by a magnitude of order of $\Delta_{3/2}$, but not of αE. We examine the group of levels S in Fig. 5. Simultaneous diagonalization of HFS interactions and of $-\boldsymbol{\mu}\cdot\mathbf{H}_0$ over these states leads to the appearance of three levels instead of two (see Fig. 6). Upon change of the sign of $\mathbf{ν}\cdot\mathbf{n}_0$ the upper and lower levels of this triplet are shifted by $\Delta E_{\text{res}}^{(P)}$ due to the weak interaction. This shift is designated by dashed lines in Fig. 6. Also presented are calculations by diagonalization, of the dependence of the energy levels and of the quantity $\eta \equiv \Delta E_{\text{res}}^{(P)}/\Delta \tilde{E}_{\text{res}}^{(P)}$ on the field H_0. The quantity

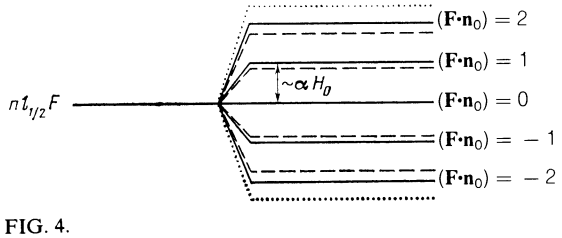

FIG. 4.

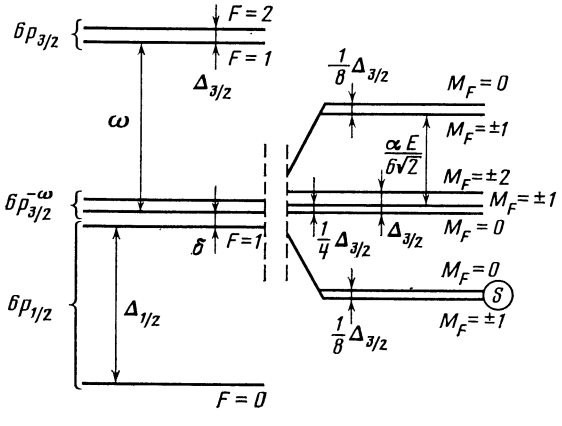

FIG. 5.

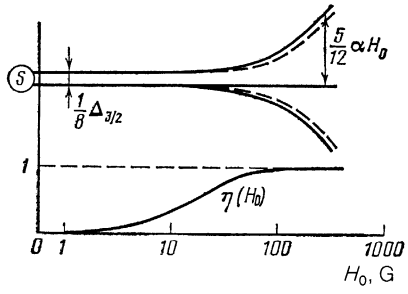

FIG. 6.

$\Delta\tilde{E}^{(P)}_{\rm res} = EF/6\sqrt{2}$ [see Eq. (24)] is the maximum possible amount of level shift.

9. We now find the numerical values of the splitting for atoms Cs and Tl. For this we assume that in the nonresonance case the laser frequency satisfies the condition $\omega < \Delta E_{n'l'j'}$, which is necessary for the absence of noticeable line broadening (see the next section). Leaving in the sum over n_1 and in (18) the terms that according to Ref. 1 givethe main contribution, and inserting the values, borrowed from the same reference, of the mixing coefficients and radial integrals, we find in the nonresonance situation, for a field intensity $E \approx 10^7$ V/cm,

$$\Delta E^{(P)}_{6s_{1/2}}(\mathrm{Cs}) \approx 10^{-1}\,\mathrm{Hz},$$

$$\Delta E^{(P)}_{6p_{1/2}}(\mathrm{Tl}) \approx 1\ \mathrm{Hz}. \qquad (28)$$

In the case of resonance we obtain for a field intensity $E \approx 10^6$ V/cm

$$\Delta E^{(P)}_{\mathrm{res}(6s_{1/2},\,7s_{1/2})}(\mathrm{Cs}) \approx 10\ \mathrm{Hz},$$

$$\Delta E^{(P)}_{\mathrm{res}(6p_{1/2},\,6p_{1/2})}(\mathrm{Tl}) \approx 10^2\ \mathrm{HZ}. \qquad (29)$$

10. We estimate possible noise. First, from Eqs. (28) and (29) there follows a condition imposed on the time variations of the magnetic field $\mathbf{H}_0$. The frequency shift of a resonance due to an instability ΔH_0 must not exceed $\Delta E^{(P)}$, whence

$$\Delta H_0[\mathrm{G}] \leqslant 10^{-6}\Delta E^{(P)}[\mathrm{Hz}]. \qquad (30)$$

This is the usual condition for experiments in search of the effects of parity nonconservation by the method of magnetic resonance. In principle it is possible to get rid of these limitations by measuring the frequency difference of the magnetic resonance for propagation of a laser ray through the atomic beam in opposite directions. With such an arrangement of the experiment the absolute magnitude of the constant magnetic field does not affect this difference and thereby condition (30) is eliminated.

There exists yet another limitation on the magnitude of a stray electric field $E_{\rm st}$, due to the correlation $(\boldsymbol{\nu}\times\mathbf{E}_{\rm st})\cdot\mathbf{j}$, not violating parity, but imitating the effect that has been examined. The corresponding constraint turns out to be weak enough:

$$E_{\rm st}[\mathrm{V/cm}] \leqslant 10\Delta E^{(P)}[\mathrm{Hz}], \qquad (31)$$

so that even for $\Delta E^{(P)} \sim 0.1$ Hz it gives $E_{\rm st} \lesssim 1$V/cm.

Further, it is necessary to impose a condition on the magnitude of the uncontrollable degree of ellipticity of the incident wave, since the correlation $\mathbf{s}_{\rm ph}\cdot\mathbf{j}$ mentioned in Sec. 1 in fact reduces to the correlation $\chi\,\boldsymbol{\nu}\cdot\mathbf{j}$ and also leads to an imitation of the effect. Estimates of Stark splitting in fields $E \sim 10^6$ V/cm lead to the condition

$$\chi \leqslant 10^{-9}\Delta E^{(P)}[\mathrm{Hz}]. \qquad (32)$$

Present day experiments enable one to reach,[1] at best, the constraint $\chi \leqslant 10^{-4}$, which is insufficient to separate the effect from the background noise. Utilization of totally unpolarized laser radiation may be the way out of the situation. The correlation $\boldsymbol{\nu}\cdot\mathbf{j}$ is insensitive to polarization; therefore the effects of parity nonconservation remain, whereas the correlation $\mathbf{s}_{\rm ph}\cdot\mathbf{j}$, leading to an imitation of the effect, is averaged out. The equations determining the frequency shift of the magnetic resonance due to weak interactions, and the numerical estimates of the magnitude of the effect, do not change.

Besides constraints (30)–(32), it is necessary to take into account the fact that in the field of a strong electromagnetic wave all levels acquire a width connected with the possibility of ionization by the field. In an experiment it is necessary to utilize such field intensity and laser frequency that this width is at the outside no greater than the effect being studied. From available results of calculations for the Cs atom,[2] it follows that the resonance situation means, for heavy atoms, the possibility of three- or four-photon ionization, and that the level width due to ionization in fields $E \sim 10^6$–10^7 V/cm exceeds considerably the splitting considered by us. Moreover, the resonance of levels ns, $n's$ or np, $n'p$ for $n = n'$ is very narrow due to smallness of the corresponding amplitude of the M 1-transition (this is forbidden). This inhibits the possible use of resonance for heavy atoms. The sole exception, apparently, is the $6p_{1/2}$, $6p_{3/2}$ resonance in Tl, where the M 1-transition is allowed and only seven-photon ionization is possible. For an intensity $E \sim 10^5$–10^6 V/cm the width of level $6p_{1/2}$, due to such ionization, must, according to estimates, not exceed the quantity $\Delta E^{(P)}_{\rm res}$ [see Eq. (29)]. In fact the level $6p_{3/2}$ has a natural width Γ, which remains practically unchanged in the field of the laser. This width, however, is not great ($\Gamma \sim 10^2$ Hz), since it is connected with the M 1-transition, and does not interfere with the observed effect. We emphasize that the large width of the $6p_{1/2}$, $6p_{3/2}$ resonance in the field of a laser (it is of order αE) does not mean broadening of the levels themselves.[7]

It is quite probable that most useful from an experimental point of view will prove to be the nonresonance effect. In this case, in order to utilize a large field intensity for these same constraints on width, it is necessary to employ relatively low-frequency lasers. Possibly suitable for the experiments being considered in use of an infrared CO_2-laser ($\omega = 944\ \mathrm{cm}^{-1}$, $E \sim 10^5$–10^6 V/cm in the cw mode[8]). For ionization of a Cs atom this requires something of the order of 30 photons, or 50 in the case of Tl.

A shortcoming of the experiments under consideration is the presence of an imprecisely known quantity in the final equations, the field intensity E. However, it is possible to determine this quantity indirectly from the Stark splitting of the levels in the field of circularly polarized laser radiation with the very same other parameters. There also exist other methods[2] for evaluation of the intensity E.

In principle, one can consider analogous experiments, in which yet another weak-interaction constant will be determined, connected with a term dependent on nuclear spin.

More useful for these experiments, apparently are the diatomic molecules. These questions, however, call for a separate investigation.

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[1)] In principle, one can speak of another experimental arrangement, viz., measurement of the degree of ellipticity arising on passage of a linearly polarized wave through a dense gaseous medium. It is possible to determine this degree directly from the Stark splitting of a level with $j = 1/2$ (see Sec. 1).

[2)] We note that a similar purely electrodynamic effect—vanishing of Zeeman splittings—should be observed in the case of an atom placed in a magnetic field crossedwith a strong electric field.

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