Superradiance, superluminescence, and self-excitation in an optical cavity

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A theory of cooperative decay of polyatomic systems in a cavity is developed. It is shown qualitatively that the nature of changes in the parameters of pulses is different in the case of superradiant and superluminescent media as the Q factor of the cavity is varied. The criteria of superradiance are proposed. It is shown that the intensity of superradiance pulses can be increased considerably and their duration can be reduced at a given pumping rate.

1. INTRODUCTION

In studies of generation of coherent radiation in an excited medium enclosed by an optical cavity it is assumed that the influence of the processes of collectively induced polarization on the dynamics of generation can be ignored compared with the processes of a stimulated amplification. Therefore, a theory of superradiance in which these processes are important is usually developed independently of a theory of lasers. However, the situation has now changed significantly. This has been due to firstly to a significant increase in the range of media in which superradiance has been observed experimentally (see, for example, Ref. 1). Secondly, the methods for excitation of superradiant media have increased in number and have approached traditional laser methods. In the very first experiments resulting in the observation of superradiance the excitation has been provided by the one- or two-photon resonant transitions in atoms and molecules, whereas lately nonresonant radiation has been used for the purpose of excitation.[2–4] Finally, recent experiments[5] have revealed a transition from superradiance to superluminescence in the same sample, a $KCl{:}O_2^-$ crystal, as its temperature was increased from 10 to 27 K.

All this new evidence makes it necessary to review the theory of lasing in an optical cavity by introducing in a consistent manner an allowance for the processes of collective spontaneous emission. This is important not only from the fundamental point of view, but also because of interesting opportunities for optimization of the generation of coherent radiation pulses. Superradiant effects may have a considerable influence on the development of methods of generation of coherent short-wavelength radiation,[6] so that the ability to optimize the parameters of superradiant pulses by introduction of feedback is also important from this point of view. The fundamental difference between the experiments on superradiance in gaseous and solid media is that in the latter case the reflection at the boundaries of the active volume is very important. This circumstance has been ignored. However, the reported investigations have shown that the reflection alters greatly the parameters of superradiance pulses. Our investigations of the dependences of the parameters of such pulses on the reflection coefficients of mirrors have shown a qualitative difference between their behavior in the cases of superradiance and superluminescence. This has made it possible to suggest a method for experimental determination of the generation regime.

2. EQUATIONS FOR THE DESCRIPTION OF GENERATION DYNAMICS

The system of reduced Maxwell-Bloch equations for slowly varying amplitudes of the vector potential $A(x,t)$, the density of the current representing the resonant transition $j(x,t)$, and the density of the difference between the populations $R(x,t)$ is[1]

$$\left(\frac{\partial}{\partial t}+(-1)^n c\frac{\partial}{\partial x}\right)A_n(x,t)=-i\frac{2\pi}{\varkappa}[j_n(x,t)+j_{0n}(x,t)],$$

$$\frac{\partial j_n(x,t)}{\partial t}+\frac{1}{2T_2}j_n(x,t)=\frac{i}{\hbar c}|m|^2R(x,t)A_n(x,t), \qquad (1)$$

$$\frac{\partial R(x,t)}{\partial t}+\frac{R(x,t)-R_e(x,t)}{T_1}=\frac{2i}{\hbar c}\sum_{n=1}^{2}(j_nA_n^*-j_n^*A_n),$$

where $n=1$ and 2 labels the waves traveling to the left and right,

$$A(\mathbf{r},t)=\sum_{n=1}^{2}\{A_n(x,t)\exp[i(\omega_0t-\varkappa_n\mathbf{r})]+\text{c.c.}\},$$

$$j_n(x,t)=\frac{m}{V_1}\sum_{i\in V_1(x)}\langle\sigma_{i+}\rangle\exp[i(\varkappa_n\mathbf{r}_i-\omega_0t)],$$

$$R(x,t)=\frac{1}{V_1}\sum_{i\in V_1(x)}\langle\sigma_{i3}\rangle,$$

m is a matrix element of the resonant transition current, V_1 is a physically small averaging volume, $|\varkappa_n|=\varkappa=\omega_0/c$, $\varkappa_n=(-1)^n\varkappa\mathbf{l}_x$, $\mathbf{l}_x$ is a unit vector along the x axis, T_2 and T_1 are the transverse and longitudinal relaxation times, $R_e(x,t)$ is the equilibrium value of the population difference density, and $j_{0n}(x,t)$ is the density of the current of spontaneous transitions. Introduction of $j_0(x,t)$ makes it possible to specify the following homogeneous initial conditions for $A(x,t)$ and $j(x,t)$:

$$A_n(x,0)=0,\quad j_n(x,0)=0,\quad R(x,0)=N/V, \qquad (2)$$

where N is the total number of resonating atoms and V is the volume of the medium. The boundary conditions for a medium with mirrors characterized by an amplitude reflection coefficient r_1 at the left-hand end of the medium and r_2 at the right-hand end are

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$$A_2(-L/2, t)=r_1A_1(-L/2, t), \quad A_1(L/2, t)=r_2A_2(L/2, t), \quad (3)$$

where L is the length of the active medium.

We shall introduce dimensionless variables $x' = x/L$ and $t' = t/\tau$, where $\tau = L/c$ is the time taken by a photon to cross a sample (from now on we shall omit the primes) and we shall also use the following functions

$$a_n(x,t)=A_n(x,t)\left(\frac{2\pi\hbar c^2}{\omega_0}\frac{N}{V}\right)^{-1/2},$$

$$p_n(x,t)=-i\frac{2\pi}{\varkappa}\tau j_n(x,t)\left(\frac{2\pi\hbar c^2}{\omega_0}\frac{N}{V}\right)^{-1/2},$$

$$\rho(x,t)=R(x,t)(N/V)^{-1}.$$

Then, the system of equations (1) becomes

$$[\partial/\partial t+(-1)^n\partial/\partial x]a_n=p_n+p_{0n},$$

$$\frac{\partial p_n}{\partial t}+\alpha p_n=\beta a_n\rho, \quad \frac{\partial \rho}{\partial t}=-4\sum_{n=1}^{2}a_np_n, \quad (4)$$

and the initial boundary conditions can be written in the form

$$a_n(x,0)=p_n(x,0)=0, \quad \rho(x,0)=1, \quad (5)$$

$$a_2(-1/2,t)=r_1a_1(-1/2,t), \quad a_1(1/2,t)=r_2a_2(1/2,t). \quad (6)$$

We shall now consider the spontaneous decay of fully excited resonant systems in time intervals much shorter than the spontaneous decay time T, so that in the system (4) the relaxation term of the third equation of the system (1) is omitted. The system (4) depends on two dimensionless parameters:

$$\alpha=\frac{\tau}{2T_2}, \quad \beta=\frac{2\pi|m|^2}{\hbar\omega}\frac{N}{V}\tau^2. \quad (7)$$

Using the relationship between the spontaneous decay time T and the square of the modulus of the matrix element of the transition current in the form

$$1/T=4\omega|m|^2/3\hbar c^3,$$

we can reduce the expression for β to

$$\beta=\frac{3\lambda^2}{8\pi}\frac{N}{V}\frac{\tau}{T}L=\frac{\tau}{\tau_c}, \quad (8)$$

where τ_c is the coherence time (see, for example, Ref. 1).

The parameters α and β represent the nature of the dynamics of decay of a polyatomic system. If $\alpha \ll 1$ we have superradiance, whereas for $\alpha \gg 1$ we have superluminescence, and if $\alpha \sim 1$ the regime is intermediate. The parameter β determines the ratio of the length of the medium to the characteristic size of the coherence region $L_c = c\tau_c$. The ratio of the parameters α and β governs the round-trip gain in the case of total inversion of the medium:

$$\mu_0L=\frac{\beta}{\alpha}=\frac{3\lambda^2}{4\pi}\frac{N}{V}\frac{T_2}{T}L, \quad (9)$$

where μ_0 is the gain at resonance. In fact, under steady-state amplification conditions it follows from Eq. (4) that if $\rho(x,t) = \rho_0$, then

$$\partial a/\partial x = (\beta/\alpha)\rho_0 a(x,t).$$

If the inversion is complete, then $\rho_0 = 1$ and $\mu_0 L = \beta/\alpha$.

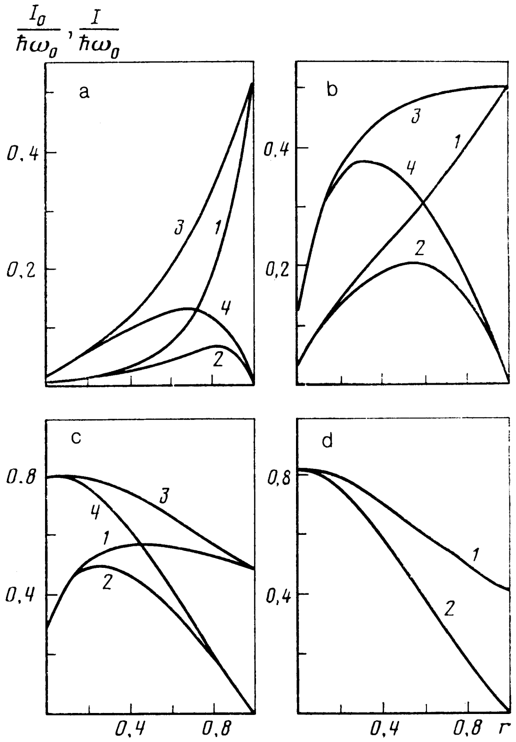

FIG. 1. Dependences of the field intensities I_0 inside a cavity (1,3) and I at the exit from the cavity (2,4) on r in the two cases when $r_1 = r_2 = r$ (curves 1 and 2) and $r_1 = 1$ and $r_2 = r$ (curves 3 and 4) for different values α and β: a) $\alpha = 10^{-3}$, $\beta = 10^{-1}$; b) $\alpha = 10^{-2}$, $\beta = 1$; c) $\alpha = 10^{-1}$, $\beta = 10$; d) $\alpha = 10$, $\beta = 10^3$.

3. DYNAMICS OF LASING IN A CAVITY

We shall consider the dependences of the maximum duration of the pulses, of the delay time (i.e., of the time taken to reach the maximum amplitude), and of the pulse duration on the reflection coefficients r_1 and r_2. As pointed out already, media with $\alpha \ll 1$ are regarded superradiant and those with $\alpha \gg 1$ are regarded as superluminescent. Figure 1 shows the dependence on r of the number of photons at the right-hand end of a sample at the moment of attainment of a maximum $n_{\max} = |a_2(\frac{1}{2},t_0)|^2$, where t_0 is the delay time for different values of α and β. Figure 1a corresponds to the values of $\alpha = 10^{-3}$ and $\beta = 10^{-1}$. Curves 1 and 2 in this figure apply to the case of a cavity with identical mirrors $r_1 = r_2 = r$, whereas curves 3 and 4 represent the case with different mirrors $r_1 = 1$ and $r_2 = r$. The curves labeled by odd numbers represent the dependence of the field intensity inside the cavity $I_0 = n_{\max}\hbar\omega_0$, whereas curves with even numbers represent the intensity at the exit from the cavity $I = I_0(1 - r^2)$. An increase in the value of r, which corresponds to an effective increase in the length of the medium, causes the intensity of the field inside the cavity to rise quadratically on increase in r. For $r = 1$ the value of I_0 reaches its maximum value $I_0^{\max} = 0.5$. For the system of equations (4) when we ignore the sources of spontaneous polarization, we find that

$$\bar{n}_1+\bar{n}_2+(1-r_2^2)\int_0^t n_2(1/2,t)\,dt+(1-r_1^2)\int_0^t n_1(-1/2,t)\,dt$$

$$=\frac{1-\bar{\rho}}{2},$$

where

$$\bar{n}_i = \int_{-1/2}^{1/2} n_i(x,t)\,dx, \qquad \bar{\rho}(t) = \int_{-1/2}^{1/2} \rho(x,t)\,dt,$$

and in the cavities with $r_1 = r_2 = 1$ the total number of photons in each of the waves reaches its maximum value $\bar{n} = \bar{n}_1 = \bar{n}_2 = 0.5$ in the case of total population inversion $\bar{\rho} = -1$. If the field amplitude does not vary greatly in the length of the sample, then n_1 and n_2 have the same values at the ends of the sample.

In the case of superluminescence systems the dependence $n_{max}(r)$ is of the form shown in Fig. 1d; here curve 1 describes the intensity $I_0 = n_{max}\hbar\omega_0$ inside the cavity, whereas curve 2 represents $I = I_0(1 - r^2)$. These dependences are practically identical in the cases when $r_1 = r_2 = r$ and also when $r_1 = 1$ and $r_2 = r$. We can see that the amplitude of the signal has its maximum for $r = 0$ and falls monotonically on increase in r.

This different behavior of the maximum intensity of a pulse due to variation of r is readily understood on the basis of physical considerations. In fact, we are dealing with two interacting subsystems: the field and the medium. Since the medium is resonant, the central frequencies of the oscillations coincide but not the half widths because of the difference of the characteristic decay increments: the field leaves the cavity in a time $\tau/(1 - r)$, whereas the rate of irreversible relaxation of the medium is $1/T_2$. The interaction between these two subsystems is strongest when the relaxation rates in question are equal: $(1 - r)/\tau \approx T_2^{-1}$. On the other hand, in the case of superradiant systems we have $\max T_2^{-1} \ll \tau^{-1}$, so that an increase in r results in equalization of the relaxation rates and, consequently, increases the strength of the interaction. However, in the case of superluminescence systems we have $\tau^{-1} \ll T_2^{-1}$ and an increase in r weakens the interaction. This conclusion is supported also by the nature of the dependences of the delay time t_0 (Fig. 2) and of the pulse duration τ_p (Fig. 3) on r. In the case of superradiance we find that t_0 and τ_p decrease strongly on increase in r, whereas in the case of superluminescence the value of t_0 is practically independent of r, but the pulse duration increases strongly in the range $r > 0.7$.

In superluminescent media a pulse is formed during a transit time of a photon across a sample (more accurately, in a time approximately equal to $L/2c$). The difference between the populations then vanishes and the field ceases to interact with the medium. An increase in the pulse duration is due to an increase in $\tau/(1 - r)$, which is the time that the field spends in the cavity. In the case of superradiant media we find that a resonant exchange of the energy between the field and the medium occurs even when $r_1 = r_2 = 1$, because the difference between the populations varies from $+1$ to -1. An increase in r increases the strength of the interaction between the field and the medium until the relaxation time of the field leaving the cavity reaches $\sim\tau_c$. A further increase in r does not increase the rate of exchange, but simply increases the field amplitude.

It follows from Fig. 1 that classification of the systems simply on the basis of the parameter α is far too rough. An important parameter is β, which determines the ratio of the length of the medium to the coherence length L_c. The length L_c determines the maximum size of the region within which the phase-locked emission of light by atoms is possible. If $L > L_c$, the processes of stimulated amplification begin to predominate over the processes of collective decay and the decay dynamics becomes more and more superluminescent in nature. It is clear from Fig. 1 that an increase in β shifts the maximum of the amplitude of a pulse leaving the cavity from the range $r = 0.8$–0.9 to the range $r = 0$–0.2. If we fix α,

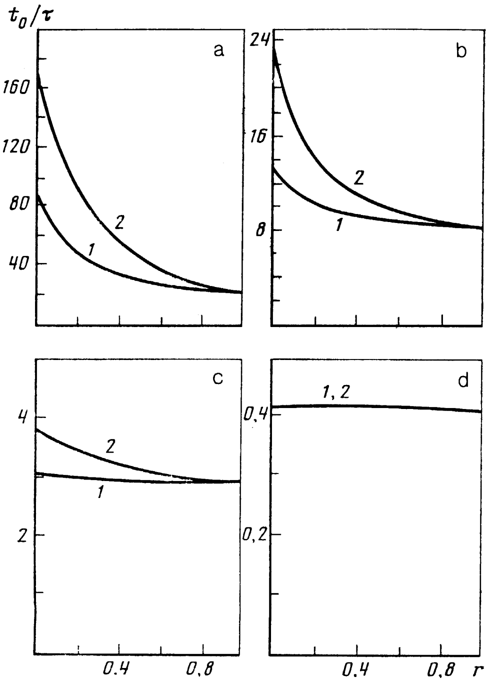

FIG. 2. Dependences of the delay time t_0 on r for two cases when $r_1 = 1$ (1) and $r_1 = r_2$ (2) and the values of α and β are the same as in Fig. 1.

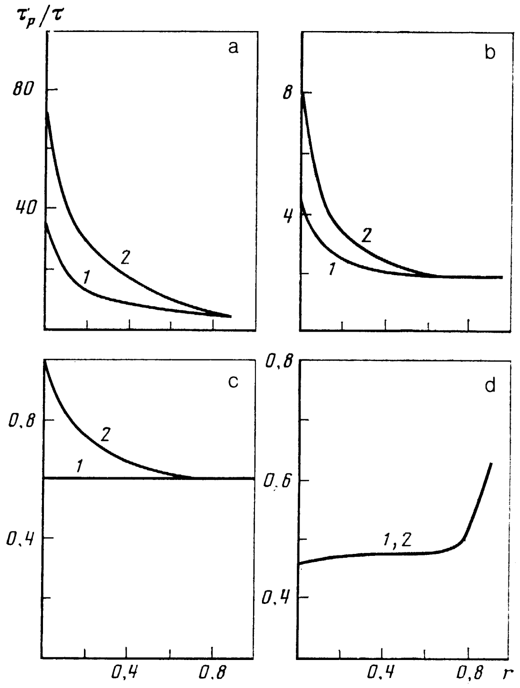

FIG. 3. Dependence of the pulse duration τ_p on r.

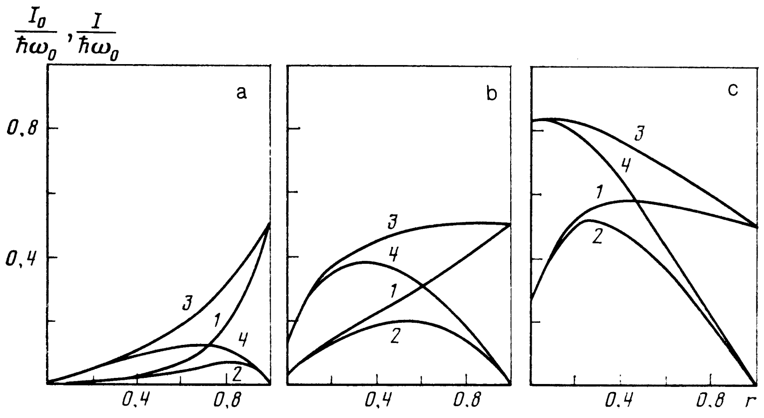

FIG. 4. Dependences of the intensities I_0 of the field inside a cavity (1,3) and I at the exit from the cavity (2,4) on r in the cases when $r_1 = r_2$ (curves 1 and 2) and $r_1 = 1$ (curves 3 and 4) for different values of β and fixed $\alpha = 10^{-2}$: a) $\beta = 10^{-1}$; b) $\beta = 1$; c) $\beta = 10$.

then even in the case when $\alpha \ll 1$, we can vary the parameter β and modify the regime from superradiance decay to superluminescence. This is demonstrated in Fig. 4, which shows the dependence of the nature of the dynamics of decay on the value of the parameter β for a fixed value $\alpha = 10^{-2}$. We can see that although $\alpha \ll 1$, the dynamics of decay for $\beta = 10$ and $r_1 = 1$ is typical of superluminescent media.

Figure 5 illustrates the dependence of the nature of decay dynamics on the parameter α when β is fixed and the two parameters are in the range corresponding to superradiant media. We can see that a change in α by two orders of magnitude does not alter the nature of the decay dynamics. The chain curves in Figs. 5a and 5c correspond to the case of self-excitation of superradiant media. In fact, in this case we have $\mu_0 L = \beta/\alpha = 1$, i.e., the system is below the superradiance threshold in the case when $r_1 = r_2 = 0$. An increase in the cavity Q factor ensures that the generation threshold $r_1 r_2 \exp(\mu_0 L) = 1$ is exceeded for $r_1 = r_2 = 0.5$; a further increase in r results in a quadratic rise of the intensity of the field inside the cavity on increase in the effective length of the medium $L_{\text{eff}} = L/(1-r)$.

4. CONCLUSIONS

The above investigation leads to the following conclusions.

1. Superradiance in a cavity is a new method for gener-

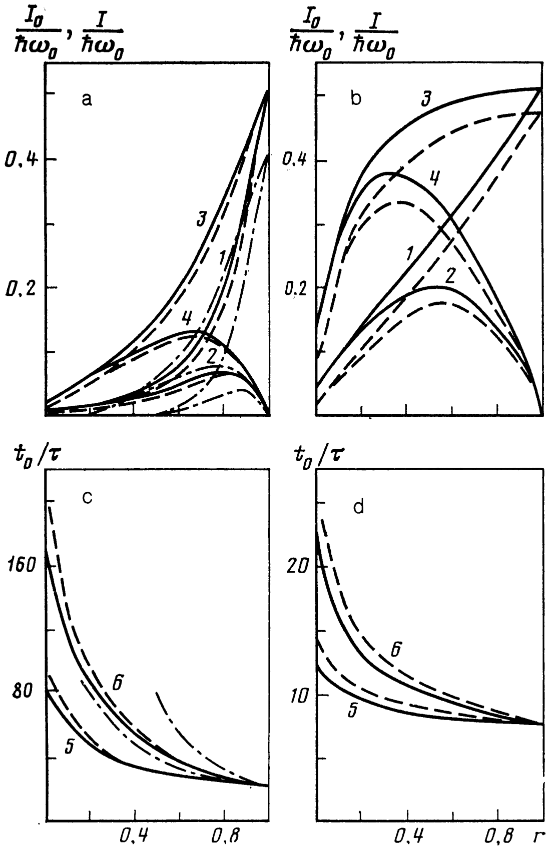

FIG. 5. Dependences of the intensities I_0 of the field inside a cavity (1,3) and I at the exit from the cavity (2,4) on r in two cases $r_1 = r_2$ (curves 1 and 2) and $r_1 = 1$ (curves 3 and 4) (Figs. 5a and 5b) and dependences of the delay time t_0 on r when $r_1 = r_2$ (curve 5) and $r_1 = 1$ (curve 6) (Figs. 5c and 5d), plotted for different values of α and β: a), c) $\beta = 10^{-1}$, $\alpha = 10^{-3}$ (the dashed curve corresponds to $\alpha = 10^{-2}$ and the chain curve to $\alpha = 10^{-1}$); b), d) $\beta = 1$, $\alpha = 10^{-2}$ (the dashed curve corresponds to $\alpha = 10^{-1}$).

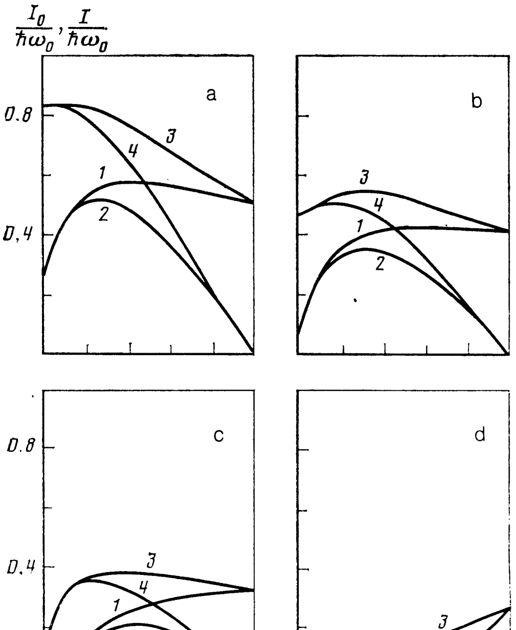

FIG. 6. Dependences of the field intensities I_0 inside a cavity (1,3) and I at the exit from the cavity (2,4) on r when $r_1 = r_2$ (curves 1 and 2) and $r_1 = 1$ (curves 3 and 4) obtained for different values of α and fixed $\beta = 10$: a) $\alpha = 10^{-2}$; b) $\alpha = 1$; c) $\alpha = 2.5$; d) $\alpha = 10$.

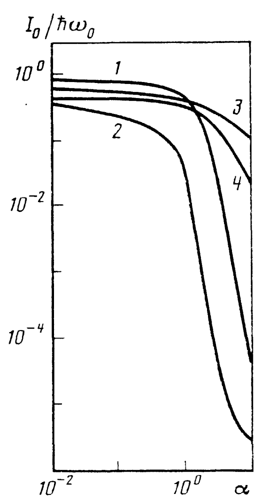

FIG. 7. Dependences of the field intensity I_0 inside a cavity on the parameter α plotted for fixed $\beta = 10$ and $r_1 = 1$ and $r_2 = 0$ (1), $r_1 = r_2 = 0$ (2), $r_1 = 1, r_2 = 0.5$ (3), and $r_1 = r_2 = 0.5$ (4).

ating short coherent radiation pulses. The use of optical cavities makes it possible to control the parameters of superradiance pulses providing an opportunity to increase the maximum intensity several tens of times and at the same time reduce by an order of magnitude the pulse duration.

2. Investigations of the dependence of the maximum intensity of the pulses on the reflection coefficients of the mirrors makes it possible to determine the nature of the lasing dynamics. Calculations indicate that pure superradiance occurs if $(\partial I_0/\partial r)_{r=0} > 0$ and $(\partial I_0/\partial r)_{r=1} \geqslant 0$, whereas superluminescence is obtained if $(\partial I_0/\partial r)_{r=0} < 0$ and $(\partial I_0/\partial r)_{r=1} < 0$. These criteria are applicable to systems above the threshold of pure superradiance or superluminescence, i.e., they can be applied when $\mu_0 L \geqslant \ln N$. In systems with $0 < \mu_0 L \leqslant 1$, we should observe self-excitation, i.e., crossing of the threshold on increase in r, whereas in systems with $1 < \mu_0 L < \ln N$ we can expect superradiance in low-gain media[7] (or the analogous superluminescence). In low-gain media the process of superradiance involves only some of the atoms and their number is $N_{\rm eff} = N[1 - (1/\mu_0 L)]$. An increase in r then increases $N_{\rm eff}$, which approaches N. Therefore, in the case of systems with $0 < \mu_0 L < \ln N$ the derivative $(\partial I_0/\partial r)_{r=0}$ is greater than zero simply because of an increase in the number of atoms that interact effectively with the field. The features of the dependences of I and I_0 on r mentioned above are illustrated for such systems in Fig. 6, which shows the dependences $I(r)$ and $I_0(r)$ plotted for different values of α when $\beta = 10$ is fixed (see also Fig. 1c). It is clear from these figures that the behavior of $I(r)$ and $I_0(r)$ ceases to depend on α even for $\alpha \leqslant 1$. If $\alpha \leqslant 10^{-1}$, these dependences are practically identical.

3. The fundamental difference between the parameters α and β is that α depends only on the parameters of the medium, whereas β depends on the parameters of the medium and of the parameters of the pump radiation, because N/V is the number density of the excited atoms. For a fixed value of β in open systems $(r_1 = r_2 = 0)$ the intensity decreases strongly on increase in α (Fig. 7). Therefore, we can obtain comparable values of the intensity for $\alpha \ll 1$ and $\alpha > 1$ if in the latter case we increase significantly the power density of the exciting radiation. The use of optical cavities equalizes largely the characteristic values of the parameters of the pulses if the systems with $\alpha > 1$ are above the self-excitation threshold defined by $r_1 r_2 \exp(\mu_0 L) = 1$.

4. In the case of systems below the self-excitation threshold when $r_1 = r_2 = 0$ the derivative $\partial I_0/\partial r$ may be greater than zero for all values of r. However, we can see from a comparison of Figs. 5a and 6d that in the case of superradiant media the dependence $I_0(r)$ is quadratic above the self-excitation threshold, whereas in the case of superluminescent media this dependence is very different from quadratic. This qualitative difference can be used in experimental identification of the lasing regime.

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