Rates of deexcitation of the mesic molecule $dd\mu$ in the mesic molecular complex $[(dd\mu)dee]$

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The deexcitation of the mesic molecule $dd\mu$ from the weakly-bound $J = v = 1$ rotational-vibrational state to the $J = 1, v = 0$ state with conversion on an electron in the $[(dd\mu)dee]$ complex is examined. The deexcitation rate is calculated by perturbing theory in the interaction operator between electrons in the complex $[(dd\mu)dee]$ and the mesic molecule $dd\mu$. It is shown that including the contribution due to second-order terms representing virtual $E1$ transitions in the $dd\mu$ molecule leads to a significant cancellation of the contribution due to first order terms, which correspond to $E0$ transitions. The calculated rate $\lambda_{\text{dex}} = 0.22 \cdot 10^8\ \text{sec}^{-1}$ is much lower than the previously calculated $E0$ transition rate $\lambda_0(m) = 1.90 \cdot 10^8\ \text{sec}^{-1}$.

1. The measured[1–4] rate of production of the mesic molecules $dd\mu$ is a function of the rates of different processes that accompany μ-catalysis in deuterium,[2,5] including the rates of decay and stabilization of the mesomolecular complex $[(dd\mu)dee]$ that contains the $dd\mu$ molecule.[5,6] One of the stabilization channels is the deexcitation of the mesic molecule $dd\mu$ produced in the $J = 1, v = 1$ rotational-vibrational state in which the total nuclear spin is $I = 1$:

$$[(dd\mu)^{\bullet}_{J=v=1}\, dee] \to [(dd\mu)_{J'v'} de]^{+} + e. \tag{1}$$

The energy released in this reaction is transferred to the conversion electron. If we recall that the deuterons are identical, and use the nonrelativistic approximation, we find that the transition from the $J = 1, I = 1$ state can occur only from the state with odd J (Ref. 7–9), i.e., $(J = 1, v = 1) \to (J' = 1, v' = 0)$ is the only possible transition.

In this paper, we present a perturbation-theory calculation of the rate of deexcitation of the mesic molecule $dd\mu$, using only the monopole and dipole terms in the operator expansion for the interaction between the electrons in the molecular complex $[(dd\mu)dee]$ and the $dd\mu$ molecule, which is justified because the ratio of the dimensions of the mesic molecule to those of the complex is small.

In second-order perturbation theory, the contribution of the dipole term to the rate of the transition (1) from the $J = v = 1$ state tends to cancel the contribution due to the monopole state. This effect was previously examined in Refs. 10 and 11.

2. The rate of deexcitation of the mesic molecule $dd\mu$ in the process defined by (1) is given by (in the system of units in which $e = \hbar = 1$)

$$d\lambda_{dex} = 2\pi |t_{fi}|^2 \delta(E_f - E_i)\, d\Gamma_f. \tag{2}$$

where $E_i = \varepsilon_{11} + E_I$, $E_f = \varepsilon_{10} + q^2/2m_e$ are the total energies of the complex $[(dd\mu)dee]$ in the initial and final states, respectively, ε_{J_v} is the binding energy of the mesic molecule $dd\mu$, E_I is the binding energy of the electron in the ground state of the complex, $\mathbf{q}$ is the momentum of the conversion electron, $d\Gamma_f = d\mathbf{q}/(2\pi)^3$ is the number of final states of the Auger electron, and

$$|t_{fi}|^2 = \frac{1}{2J+1} \sum_{M_J, M_{J'}} |T_{fi}|^2. \tag{3}$$

In first-order perturbation theory, the matrix element T_{fi} is given by

$$T_{fi} = V_{fi}^{(\mathrm{I})} = \langle f | H_{int} | i \rangle, \tag{4}$$

$$V_{fi}^{(\mathrm{I})} = \int d\mathbf{R}\, d\mathbf{r}\, d\boldsymbol{\rho}\, \Psi^{(f)*}_{J'v'}(\mathbf{r}, \mathbf{R})\, \psi^{(f)*}(\boldsymbol{\rho})\, H_{int} \psi^{(i)}(\boldsymbol{\rho})\, \Psi^{(i)}(\mathbf{r}, \mathbf{R}),$$

where $\psi^{(i,f)}(\boldsymbol{\rho})$ and $\Psi^{(i,f)}(\mathbf{r},\mathbf{R})$ are, respectively, the wave functions of the conversion electron and the mesic molecule in the initial and final state, $\boldsymbol{\rho}$ is the positron vector of the electron, measured from the center of mass of the mesic molecule, $\mathbf{R}$ is the radius vector joining the nuclei of the mesic molecule, and $\mathbf{r}$ is the positron vector of the negative muon, measured from the center of the segment R. The formula given by (3) has been averaged over the projection M_J of the orbital angular momentum J in the initial state, and summed over the projection M'_J in the final state.

It will be sufficient to evaluate the rate (2) and the corresponding transition matrix element (3) for the deexcitation of the mesic molecule $dd\mu$ in the analog of the hydrogen atom:

$$[(dd\mu)_{11} e] \to (dd\mu)_{10} + e. \tag{5}$$

The required rate λ_{dex} of process (1) can be expressed in terms of the deexcitation rate $\lambda_{\text{dex}}^{(a)}$ of the process (5) as follows:[12,13]

$$\lambda_{dex} = \varkappa \lambda_{dex}^{(a)}, \tag{6}$$

where $\varkappa$ is the ratio of the electronic densities near the nucleus in the hydrogen molecule H_2 and in the hydrogen atom H (Refs. 12 and 13):

$$\varkappa = 1.45. \tag{7}$$

The wave functions of the conversion electron in (5) in the initial and final states have the form[14]

$$\psi^{(i)}(\boldsymbol{\rho}) = \psi_{1s}(\rho) = (m_e^3/\pi)^{1/2} e^{-m_e \rho}, \tag{8}$$

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$$\psi^{(f)}(\rho)=\left(\frac{\pi}{2}\right)^{1/2}\frac{1}{q}\sum_l i^l(2l+1)e^{-i\sigma_l}R_{ql}(\rho)P_l(\cos\theta), \quad (9)$$

$$R_{ql}(\rho)=C_{ql}\frac{(2q\rho)^l}{(2l+1)!}e^{-iq\rho}F(l+1+i\eta, 2l+2, 2iq\rho),$$

$$C_{ql}=2[\eta(1-e^{-2\pi\eta})]^{-1/2}\prod_{s=1}^{l}(s^2+\eta^2), \quad \eta=m_e/q,$$

$$R_{ql}(\rho)\underset{\rho\to\infty}{\sim}\left(\frac{2}{\pi}\right)^{1/2}\frac{1}{\rho}\sin\left(q\rho+\eta\ln 2q\rho-\frac{\pi l}{2}+\sigma_l\right). \quad (10)$$

The wave functions of the mesic molecule $dd\mu$ in the bound state (Jv) were calculated in the one-level approximation using the adiabatic method of the three-body problem[15]:

$$\Psi_{Jv}^{(i,f)}(\mathbf{r},\mathbf{R})=\Phi_{1s\sigma_g}(\mathbf{r};R)\varphi_{Jv}^{(i,f)}(\mathbf{R}), \quad (11)$$

where $\Phi_{1s\sigma_g}$ is the even solution of the problem of two centers of force,[16] normalized by the condition

$$\int d\mathbf{r}\,\Phi_{1s\sigma_g}^2(\mathbf{r};R)=1.$$

The expression given by (11) does not take into account excited states in the motion of the muon. Their contribution can be neglected when the deexcitation rates are calculated with the required precision.[8]

The wave functions $\varphi_{Jv}^{(i,f)}(\mathbf{R})$ describe the relative motion of the nuclei:

$$\varphi_{Jv}^{(i,f)}(\mathbf{R})=\frac{1}{R}\chi_{Jv}^{(g)}(R)Y_{JM_J}(\theta,\phi), \quad (12)$$

and are normalized by the condition

$$\int dR[\chi_{Jv}^{(g)}(R)]^2=1.$$

The transitions in the mesic molecule $dd\mu$ occur under the influence of the perturbation[7-9]

$$H_{int}=\sum_i\left(\frac{e_i}{\rho}-\frac{e_i}{|\rho-\mathbf{R}_i|}\right), \quad (13)$$

where e_i and R_i are, respectively, the charge and the distance of the two deuterons and of the meson from the center of mass of the mesic molecule.

For $R_i\leqslant\rho\ll 1$ the perturbation (13) has the following form in the monopole approximation[7,9]:

$$H_{int}=H_{int}^{(M)}=\frac{2\pi}{3}Q^{(M)}(\mathbf{r},\mathbf{R})\delta(\rho), \quad (14)$$

$$Q^{(M)}=R_1^2+R_2^2-R_\mu^2=\frac{1}{2}R^2-\frac{2(2M_d^2-M_\mu^2)}{(2M_d+M_\mu)^2}r^2, \quad (15)$$

where M_d and M_μ are, respectively, the mass of the deuterium nucleus and of the muon. For $\rho\gg R_i$, the operator (13) is given by the following expression in the dipole approximation[7-9]:

$$H_{int}=H_{int}^{(D)}=-\mathbf{d}\rho/\rho^3, \quad (16)$$

$$\mathbf{d}=-(\mathbf{R}_1+\mathbf{R}_2-\mathbf{R}_\mu)=-\left(1+\frac{M_\mu}{2M_d+M_\mu}\right)\mathbf{r}. \quad (17)$$

According to the definition given by (4), the perturbation (16) does not induce dipole transitions between bound states of the mesic molecule $dd\mu$ because the wave functions $\Psi_{Jv}^{(i,f)}(\mathbf{r},\mathbf{R})$ given by (11), which describe these states, have the same parity g (in the nonrelativistic approximation) under inversion of the coordinates of the meson $\mathbf{r}\to-\mathbf{r}(\Psi(\mathbf{r},\mathbf{R})=\Psi(-\mathbf{r},\mathbf{R}))$ (Refs. 7 and 8). The $E0$ transitions, which correspond to the interaction operator (14), are thus the only nonzero transitions in first-order perturbation theory (4).

The matrix element (4) is given by the following expression in the monopole approximation after integration with respect to the coordinates ρ of the electron:

$$V^{(I)}=\delta_{M_JM_{J'}}v^{(I)}(q), \quad (18)$$

where

$$v^{(I)}(q)=\frac{2\pi}{3}I(q)\langle Q^{(M)}\rangle, \quad (19)$$

$$I(q)=\psi^{(f)}(0)\psi^{(i)}(0)=\left[\frac{2m_e^3\eta}{1-e^{-2\pi\eta}}\right]^{1/2}, \quad (20)$$

$$\langle Q^{(M)}\rangle=\int d\mathbf{R}\,d\mathbf{r}\,\Psi_{J'v'}^{(f)*}Q^{(M)}\Psi_{Jv}^{(i)}=\frac{1}{2}J_1-\frac{2(2M_d^2-M_\mu^2)}{(2M_d+M_\mu)^2}J_2, \quad (21)$$

$$J_\mu(R)=\int r^2\Phi_{1s\sigma_g}^2(\mathbf{r};R)\,d\mathbf{r},$$

$$J_1=\int R^2\chi_{Jv}^{(i)}(R)\chi_{J'v'}^{(f)}(R)\,dR, \quad (22)$$

$$J_2=\int\chi_{Jv}^{(i)}(R)\chi_{J'v'}^{(f)}(R)J_\mu(R)\,dR.$$

Integrating with respect to the electron momenta $\mathbf{q}$ in (2), we obtain

$$\lambda_{dex}^{(0)}=\frac{4\pi}{9}m_eq_0I^2(q_0)|\langle Q^{(M)}\rangle|^2 \quad (23)$$

where

$$q_0=[2m_e(|\varepsilon_{10}|-|\varepsilon_{11}|-|E_I|)]^{1/2}, \quad m^{-1}=M_\mu^{-1}+M_d^{-1}. \quad (24)$$

3. The deexcitation rate (23) calculated in first-order perturbation theory is of order $(a_m/a_e)^4$. It follows from Refs. 10 and 11 that the rate of transition from the state with low binding energy $|\varepsilon_{11}|\ll|E_I|$ contains a similar contribution due to the interaction (16) in second-order perturbation theory. This means that the total rate for process (5) is determined by the transition matrix element (4) of the form

$$T_{fi}=V_{fi}^{(I)}+V_{fi}^{(II)}, \quad (25)$$

$$V_{fi}^{(II)}=\sum_{nN}\frac{\langle f|H_{int}^{(D)}|nN\rangle\langle nN|H_{int}^{(D)}|i\rangle}{\varepsilon_{11}+E_I-\varepsilon_N-E_n}, \quad (26)$$

where the sum is evaluated over all the states of the $dd\mu$ molecule and the atomic complex $[(dd\mu)e]$ with energies ε_N and E_n, respectively.

The main contribution to the sum over the electron states in (26) is provided[10] by continuum states with characteristic electron momenta $q\gg 1$ (in atomic units). This condition enables us to describe the motion of the electron by the plane wave

$$\psi_q(\rho)=e^{iq\cdot\rho} \quad (27)$$

rather than the wave function (9).

As already noted, matrix elements of the form $\langle J'=0;2|\mathbf{d}|J=1\rangle$, which corresponded to $E1$ transitions between the bound states of the mesic molecule $dd\mu$, are all zero, so

that the index N in (26) labels the $J' \equiv L = 0; 2$ states only in the continuous spectrum of the $dd\mu$ molecule, i.e., the system $d\mu + d$. The wave function describing this system in states to which the transitions takes place has the form

$$\Psi_k(\mathbf{r}, \mathbf{R}) = \Phi_{2p\sigma_u}(\mathbf{r}; R)\varphi_k(\mathbf{R}),$$
$$\varphi_k(\mathbf{R}) = \frac{1}{R}\chi_{Lk}^{(u)}(R)\, Y_{LM_L}(\theta, \phi), \qquad (28)$$
$$k^2 = 2M\varepsilon_N, \quad M = M_d/2, \quad L = 0; 2,$$

where $\Phi_{2p\sigma u}$ is the odd solution of the problem of two centers of force,[16] and the function $\chi_{Lk}^{(u)}(R)$ is normalized by the asymptotic condition

$$\chi_{Lk}^{(u)}(R) \underset{R\to\infty}{\sim} (2/\pi)^{1/2} \sin(kR - \pi L/2 + \delta_u) \qquad (29)$$

and is evaluated in Ref. 17 and 18.

In view of (27) and (28), the expression given by (26) reduces to

$$V_{fi}^{(II)} = -\int dk \langle J'=1, M_J'; v'=0 | d_\alpha | LM_L\rangle \langle LM_L | d_\gamma | J=1,$$
$$M_J; v=1\rangle Q_{\alpha\gamma}(k), \qquad (30)$$

where

$$Q_{\alpha\gamma}(k) = 2m_e \int \frac{d\mathbf{q}'}{(2\pi)^3} \frac{\langle \mathbf{q}|\rho_\alpha/\rho^3|\mathbf{q}'\rangle \langle \mathbf{q}'|\rho_\gamma/\rho^3|1S\rangle}{(q')^2 + \tilde{k}^2}, \qquad (31)$$

$$\tilde{k}^2 = 2m_e(|\varepsilon_{11}| + |E_I| + k^2/2M), \quad q' = (2m_e E_n)^{1/2}, \qquad (32)$$

and the sum is evaluated over the twice repeated indices $L = 0; 2$ and $\alpha,\gamma = x,y,z$. The quantity $Q_{\alpha\gamma}(k)$ is evaluated in the Appendix and is given by

$$Q_{\alpha\gamma}(k) = A\delta_{\alpha\gamma} + Bq_\alpha q_\gamma/q^2. \qquad (33)$$

The explicit expressions for $A \equiv A(k)$ and $B \equiv B(k)$ are also given in the Appendix.

Using (33) and the definitions given by (11), (12), and (28) together with the Wigner-Eckart theorem[19]

$$\langle JM_J | d_\mu | LM_L\rangle = C_{LM_L, 1\mu}^{JM_J} \frac{\langle J \| \mathbf{d} \| L\rangle}{(2J+1)^{1/2}}, \qquad (34)$$

where d_μ are the cyclic components of the vector $\mathbf{d}(d_\pm = \mp(d_x \mp idy)\sqrt{2}, d_0 = d_z)$ and $C_{LM_L,1\mu}^{JM_J}$ are the Clebsch-Gordan coefficients, we find that (30) takes the form

$$V_{fi}^{(II)} = \delta_{M_J M_J'} v_{M_J}^{(II)}(q), \qquad (35)$$

$$v_{M_J}^{(II)}(q) = \frac{1}{3}\int dk \Big\{ [A + B(1-M_J^2)] D_0 + \Big[A + \frac{B}{10}(4 - M_J^2)\Big] D_2 \Big\}. \qquad (36)$$

where

$$D_L = \langle J' \| \mathbf{d} \| L\rangle \langle L \| \mathbf{d} \| J\rangle, \qquad (37)$$

and, according to the theorem (34), the reduced matrix elements $\langle J \| \mathbf{d} \| L\rangle$ are given by

$$\langle J \| \mathbf{d} \| L\rangle = (\delta_{L0} - 2^{1/2}\delta_{L2}) \int dR\, \chi_{Jv}^{(g)} \chi_{Lk}^{(u)} D_{gu}(R),$$
$$D_{gu}(R) = \frac{\mathbf{R}}{R}\int d\mathbf{r}\, \Phi_{1s\sigma_g}(\mathbf{r}; R)\, \mathbf{d}\, \Phi_{2p\sigma_u}(\mathbf{r}; R), \qquad (38)$$
$$\langle L \| \mathbf{d} \| J\rangle = -\langle J \| \mathbf{d} \| L\rangle, \quad L = 0; 2, \quad J = 1.$$

The matrix element (3) corresponding to the transition (5) is given by the following expression when (18) and (35) are taken into account:

$$|t_{fi}|^2 = \frac{1}{2J+1} \sum_{M_J M_J'} |V_{fi}^{(I)} + V_{fi}^{(II)}|^2$$
$$= |V_{fi}^{(I)}|^2 \sum_{M_J} \frac{1}{3} \Big| 1 + \frac{v_{M_J}^{(II)}(q)}{v^{(I)}(q)} \Big|^2. \qquad (39)$$

If we go to second-order in perturbation theory and integrate over the electron momenta, we find that the expression for the deexcitation rate (2) becomes

$$\lambda_{dex}^{(a)} = \frac{1}{3}\lambda_{dex}^{(0)} \sum_{M_J} \Big| 1 + \frac{v_{M_J}^{(II)}(q_0)}{v^{(I)}(q_0)} \Big|^2, \qquad (40)$$

where $\lambda_{dex}^{(0)}$ is given by (23).

When the quantities $v_{M_J}^{(II)}(q)$ were evaluated, the wave functions describing the motion of the electron in the continuum were chosen in the form of the plane waves (27), whereas $v^{(I)}(q)$ was calculated using the exact Coulomb functions of the hydrogen atom, given by (9). Since the main contribution to the integral $Q_{\alpha\gamma}$ (31) is provided by values of ρ defined by $\rho \lesssim q^{-1} \ll 1$ atomic units (see Appendix), the substitution

$$\psi_q^{(c)}(0) = [2\pi\eta/(1 - \exp(-2\pi\eta))]^{1/2}$$

with $\psi_q(0) = 1$ introduces the following uncertainty into $v_{fi}^{(II)}$:

$$\sim [|E_I|/(|\varepsilon_{10}| - |\varepsilon_{11}|)]^{1/2} \approx 0.3.$$

The quantity $\bar{v}^{(I)}(q)$ calculated from (19) with $\psi^{(f)}(0) = 1$ substituted in (20) is subject to a similar uncertainty. It follows that the value of the rate $\lambda_{dex}^{(a)}$ can be improved by writing (40) in the form

$$\lambda_{dex}^{(a)} = \lambda_{dex}^{(0)} \sum_{M_J} \frac{1}{3} \Big| 1 + \frac{v_{M_J}^{(II)}(q_0)}{\bar{v}^{(I)}(q_0)} \Big|^2. \qquad (41)$$

Since $v_{M_J}^{(II)}(q) = v_{-M_J}^{(II)}(q)$, it is clear from (36) and (6) that the final expression for the rate deexcitation in the $dd\mu$ molecule is

$$\lambda_{dex} = \lambda_0^{(m)} P_v, \qquad (42)$$

where

$$\lambda_0^{(m)} = \varkappa \lambda_{dex}^{(0)}, \qquad (43)$$

$$P_v = \frac{1}{3}\Big(1 + \frac{v_0^{(II)}(q_0)}{\bar{v}^{(I)}(q_0)}\Big)^2 + \frac{2}{3}\Big(1 + \frac{v_1^{(II)}(q_0)}{\bar{v}^{(I)}(q_0)}\Big)^2. \qquad (44)$$

4. The figure shows the reduced matrix element $\langle Jv \| \mathbf{d} \| Lk\rangle$ (38) as a function of k and of the orbital angular momentum $L = 0; 2$ associated with the relative motion of the mesic atom $d\mu$ and the nucleus d.

The deexcitation rate λ_{dex} of the mesic molecule $dd\mu$, calculated from (42)–(44), is given by

$$\lambda_{dex} = 0.22 \cdot 10^8 \text{ sec}^{-1}. \qquad (45)$$

The following table gives the numerical values of intermediate quantities (in the system of units in which

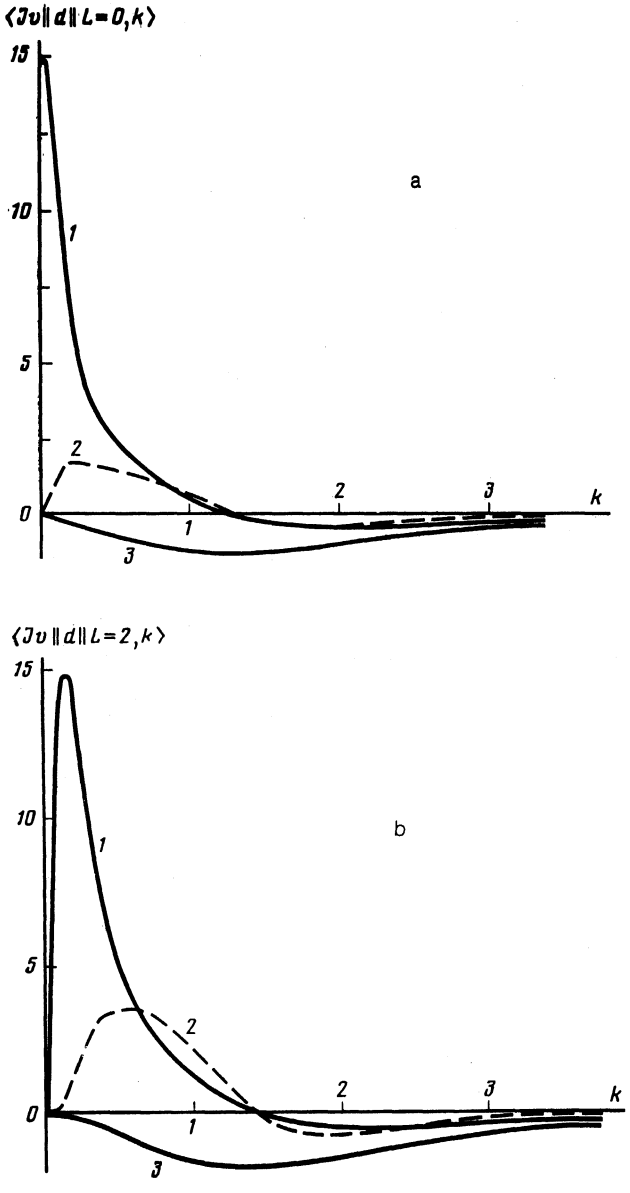

FIG. 1. The quantity $D_L(k)$ and the reduced matrix elements $\langle Jv|\mathbf{d}|Lk\rangle$ as functions of momentum k (in the system of units in which $e=\hbar=m=1$) for (a) $L=0$ (curve 1—$\langle J=1, v=1\|\mathbf{d}\|L=0, k\rangle$, and 2—$D_0(k)$, 3—$\langle J=1, v=0\|\mathbf{d}\|L=0, k\rangle$, and b) $L=2$ (1—$\langle J=1, v=1\|\mathbf{d}\|L=2, k\rangle$, 2—$D_2(k)$, 3—$\langle J=1, v=0\|\mathbf{d}\|L=2, k\rangle$).

$e=\hbar=m=1$), obtained during the calculation of the rate λ_{dex}, together with the rate $\lambda_0^{(m)}$ of the monopole transition $(J=1, v=1)\to(J=1, v=0)$, given by (43):

q_0	$\bar{v}^{(I)}$	$v_0^{(II)}$	$v_1^{(II)}$	$\lambda_0^{(m)}$
0.020	$-4.6\cdot10^{-4}$	$2.4\cdot10^{-4}$	$3.5\cdot10^{-4}$	$1.9\cdot10^{8}$ sec^{-1}

Comparison of the rates $\lambda_0^{(m)}$ and λ_{dex} (45) shows that the inclusion of the dipole term (16) in the expansion for the interaction operator (13) in second-order perturbation theory leads to a significant reduction in the deexcitation rate obtained when only the monopole term (14) is taken into account.

An analogous cancellation effect was noted when the correction for the finite size of the mesic molecule was introduced into the energy levels of complexes of the form $[(dd\mu)dee]$ in Ref. 10, and was calculated numerically in Ref. 11. According to these calculations, the first term in the matrix element T_{fi} (25) is almost completely canceled by the second if $|f\rangle=|i\rangle$ and $|\varepsilon_{Jv}|\ll E_I$. In this calculation of the deexcitation rates, $|f\rangle\neq|i\rangle$ and $|\varepsilon_{11}|\ll|E_I|$ but $|\varepsilon_{10}|\gg|E_I|$ ($\varepsilon_{11}=-1.96$ eV, $\varepsilon_{10}=-226.61$ eV, $E_I=-13.61$ eV), so that the cancelation is less complete.

5. The deexcitation rate λ_{dex} that we have calculated is important for calculations of the probability w of nuclear fusion in the mesic molecule $dd\mu$. This probability is given by[5,6]

$$w=\tilde{\lambda}_f/(\tilde{\lambda}_f+\Gamma)$$

where $\tilde{\lambda}_f=\lambda_{11}+\lambda_{\text{dex}}$ and $\Gamma\approx10^8$ sec^{-1} are, respectively, the rates of stabilization and decay of the molecular complex $[(dd\mu)dee]$ formed in the resonance reactions[5] ($\lambda_{11}=4.3\times10^8$ sec^{-1} is the rate of the nuclear reaction in the $J=v=1$ state of the $dd\mu$ molecule.[20]) Once we know the probability w, we can accurately determine the energy level ε_{11} of the weakly-bound state with $J=v=1$ from the measured rate of production of the $dd\mu$ molecules.

The main uncertainty in the calculated deexcitation rate is due to the use of the plane-wave description (27) for the electron state instead of the wave functions of the excited electronic states of the complex $[(dd\mu)dee]$. However, as noted in Refs. 12 and 13 and in the present paper, the use of (6) and (41) leads to a precision of about 10% for λ_{dex}. This is quite sufficient for calculations of the wave functions in the discrete and continuous spectra of the mesic molecule $dd\mu$ in the one-level approximation (11), (28) of the adiabatic method.[15]

The scheme presented above has a degree of generality and is valid for many problems in which the cancelation effect has to be taken into account when transition rates are calculated.

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APPENDIX

Before we can calculate the deexcitation rate, we must evaluate the integral $Q_{\alpha\gamma}(k)$ in (31). In view of (8) and (27), the matrix elements in the integrand are given by

$$\langle\mathbf{q}|\rho/\rho^3|\mathbf{q}'\rangle=4\pi i(\mathbf{q}'-\mathbf{q})/(\mathbf{q}'-\mathbf{q})^2, \tag{A1}$$

$$\langle\mathbf{q}'|\rho/\rho^3|1S\rangle\approx-4\pi i\mathbf{q}'\psi_{1s}(0)/(\mathbf{q}')^2. \tag{A2}$$

The integral then reduces to the form

$$Q_{\alpha\gamma}(k)=\frac{4m_e}{\pi^{3/2}}\int d\mathbf{q}'\,\frac{(q'-q)_\alpha q_\gamma'}{(\mathbf{q}'-\mathbf{q})^2(\mathbf{q}')^2[(\mathbf{q}')^2+\tilde{k}^2]}, \tag{A3}$$

whence, after integration over the directions of the momentum $\mathbf{q}'$, we have

$$Q_{\alpha\alpha}=\frac{8m_e}{\pi^{1/2}q}F_1(\beta),\quad \frac{q_\alpha q_\gamma}{q^2}Q_{\alpha\gamma}=\frac{4m_e}{\pi^{1/2}q}F_2(\beta), \tag{A4}$$

where

$$F_1(\beta)=\int_0^\infty\frac{dx}{x^2+\beta^2}\left(1+\frac{x^2-1}{2x}\ln\left|\frac{x+1}{x-1}\right|\right),\quad \beta=\frac{\tilde{k}}{q}. \tag{A5}$$

$$F_2(\beta)=\int_0^\infty\frac{dx(1-x^2)}{x^2+\beta^2}\left(1-\frac{x^2+1}{2x}\ln\left|\frac{x+1}{x-1}\right|\right), \tag{A6}$$

We write the integral $F_2(\beta)$ in the form

$$F_2(\beta)=(1+\beta^2)G(\beta)-G(0), \tag{A7}$$

$$G(\beta)=\int_0^\infty \frac{dx}{x^2+\beta^2}\left(1-\frac{x^2+1}{2x}\ln\left|\frac{x+1}{x-1}\right|\right). \tag{A8}$$

The expression for $G(0)$ was obtained from (A8) by the replacement $x\to x^{-1}$ and setting $\beta=0$.

Using the integrals

$$\int_0^\infty \frac{dx}{x^2+\beta^2}=\frac{\pi}{2\beta},\quad \int_0^\infty \frac{dx}{x}\ln\left|\frac{x+1}{x-1}\right|=\frac{\pi^2}{2}, \tag{A9}$$

$$\int_0^\infty \frac{dx}{x(x^2+\beta^2)}\ln\left|\frac{x+1}{x-1}\right|=\frac{\pi}{\beta^2}\arctan\beta,$$

listed in the tabulation given in Ref. 21, we obtain the following expressions for F_1 and F_2 in (A5) and (A6):

$$F_1(\beta)=\frac{\pi^2}{4}+\frac{\pi}{2\beta}\left(1-\frac{1+\beta^2}{\beta}\arctan\beta\right), \tag{A10}$$

$$F_2(\beta)=-\frac{\pi^2}{4}\beta^2+\frac{\pi(1+\beta^2)}{2\beta}\left(1-\frac{1-\beta^2}{\beta}\arctan\beta\right). \tag{A11}$$

Finally, the quantities A and B defined by (33) are given by

$$A=\frac{4m_e}{\pi^{1/2}q}\left(F_1-\frac{1}{2}F_2\right),\quad B=\frac{4m_e}{\pi^{1/2}q}\left(-F_1+\frac{3}{2}F_2\right). \tag{A12}$$

For $q=q_0=3.94$, $k=1.76\cdot 10^{-2}$, the numerical values of A and B (in atomic units) are $A=9.5$, $B=-4.2$.

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