Theory of coherent frequency conversion via ultrashort pulses in resonant media

L. A. Bol'shov, N. N. Yelkin, V. V. Likhanskiĭ, and M. I. Persiantsev

I. V. Kurchatov Atomic Energy Institute

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We investigate the coherent propagation of short light pulses in a three-level V-configuration medium with arbitrary level populations (i.e., "nonzero temperature"), using both analytical and numerical methods. We show that for unequal populations of the upper levels simultons—i.e., multifrequency self-induced transparency pulses—cannot exist. The only stable pulse which forms in this case is a single-frequency 2π pulse which propagates accompanied by a transition to a minimum difference of the original populations. We point out that experimental realization of a coherent frequency conversion scheme using such light pulses does not require additional preparation of the medium and can be implemented over a wide range of parameters of the resonant medium.

1. Resonant multilevel media have been the objects of intense study in the last few years (see, e.g., Ref. 1 and the references cited therein), because of certain interesting features of the coherent interaction between short light pulses which propagate in them. From a practical point of view, the use of such media in their parametric and Raman modes promises to provide efficient frequency conversion and fixed-length laser pulses.[1] For problems of laser isotope separation, resonant photochemistry, and nonlinear spectroscopy, special interest attaches to the generation of multifrequency self-induced transparency pulses, the so-called simultons.[2] By applying mathematical methods derived from the inverse scattering problem (ISP),[3] it is possible to investigate the nonlinear dynamics of formation and interaction of simultons in considerable detail.[1] However, there are gaps in our understanding of these systems. Exact soliton solutions can be constructed only for the case of coupled resonant transitions with equal oscillator strengths. Moreover, up until now all inferences concerning the propagation and interaction of simultons have applied to the case of a medium at "zero temperature", i.e., all resonant levels are assumed to be originally in their ground states. The only exception is the case investigated in Ref. 2 involving propagation of a simulton in a three-level medium with a "cascade" configuration (i.e, a nonintegrable system). Here the existence of simultons is possible only when additional preparation of the system, e.g., a special ratio of the original level populations, is provided. Of course, this formulation of the problem is just as "artificial" as the case of zero temperature.

The purpose of this paper is to fill in some of these gaps. We will investigate here a V-configuration medium with an arbitrary ratio of oscillator strengths for the transitions and arbitrary initial populations of all levels. In Sec. 2 we will investigate the propagation of a single-frequency 2π pulse in such a medium and show that it is stable at the transition $g\to a$ with the smaller of the two original population differences: $n_g - n_a < n_g - n_b$. However, propagation of a 2π pulse at the transition $g\to b$ with the larger original population difference is unstable against conversion into radiation at the $g\to a$ transition frequency. (The subscripts g, a, b, denote respectively the ground state level and the two working levels; see Fig. 1 below: $n_{g,a,b}$ are the original populations of the corresponding levels, which are subject to, e.g., the Boltzmann distribution.)

In Sec. 3 we will obtain exact nonlinear solutions for the case of equal oscillator strengths using ISP methods, which describe the complete transformation of a 2π pulse of frequency ω_b to a 2π pulse of frequency ω_a (for $n_g > n_a > n_b$); we will also show that the existence of simultons is not possible in such a medium. We will compare these solutions which describe collision and exchange of photons between simultons in a zero temperature medium.

In Sec. 4 we present results of numerical calculations which confirm the analytical conclusions, and also which allow us to study the stages of nonlinear interaction in situations which are not amenable to a fully analytical treatment.

2. The interaction of light pulses whose length is considerably shorter than all relaxation times in the problem with a resonant three-level V-configuration medium is described by the following equations:[1]

$$\frac{\partial c_g}{\partial t}=i(\Omega_a c_a+\Omega_b c_b),\qquad \frac{\partial c_{a,b}}{\partial t}=i\Omega_{a,b}^{*}c_g, \tag{1}$$

$$i\left(\frac{\partial}{\partial x}+\frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_{a,b}=\frac{2\pi N}{\hbar c}\mu_{a,b}^{2}\omega_{a,b}c_g c_{a,b}^{*}. \tag{2}$$

Here $C_{g,a,b}$ are probability amplitudes for the level occupations, $\Omega_{a,b}=\mu_{a,b}E_{a,b}/2h$ are the Rabi frequencies for the transitions $g\to a$ and $g\to b$ respectively, $E_{a,b}$ are the envelopes of the pulses, $\mu_{a,b}$ and $\omega_{a,b}$ are the dipole moments and frequencies of the corresponding transitions, and N is the concentration of resonant particles. The initial conditions which reflect a nonzero medium temperature are manifested in the original populations of the levels a and b:

$$\begin{aligned}&|c_g(t=0,\ x)|^2=n_g<1,\\ &|c_a(t=0,\ x)|^2=n_a\neq 0,\\ &|c_b(t=0,\ x)|^2=n_b\neq 0.\end{aligned} \tag{3}$$

Physically this implies that the medium consists of particles of three kinds: I—particles in the ground state (the number of which is Nn_g); II—particles originally excited to level (whose number is Nn_a); III—particles originally excited to level b (whose number is Nn_b). The particles of each type act on the field and contribute to the polarization of the medium; this contribution must be taken into account when solving Eqs. (2) in a field.

We study the problem of stability of a propagating soli-

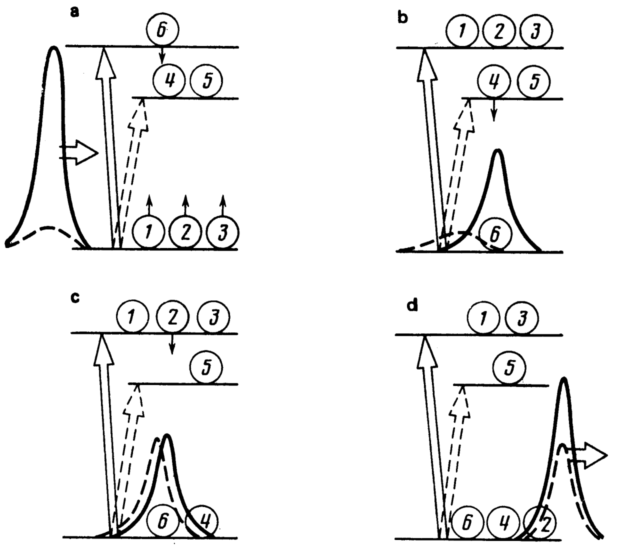

FIG. 1. Interaction scheme: (a) a 2π pulse E_b (solid curve) and a weak signal E_a (dashed) enter into interaction with a partially excited medium; (b) the transition $g\to b$ absorbed part of the energy of the 2π pulse to form a population inversion at the transition $g\to a$; (c) and nucleus E_a removes the inversion with the transition $g\to a$, decreasing thereby the energy which the transition $b\to g$ can convert into radiation E_b; (d) the amplified Stokes pulse and remained "pump" cease to interact, leaving behind an energy surplus in the form of excitation of the level b.

ton E_b at the transition $g\to b$ in the presence of a weak field E_a at the frequency of the other transition $g\to a$. Without loss of generality we will assume a natural state for the medium, i.e., $n_g > n_a > n_b$ for $\omega_b > \omega_a$. When $E_a = 0$, there is a solution for E_b in the form of a 2π pulse of the following form:

$$\Omega_a=0,\quad \Omega_b=-(i/\tau_p)\operatorname{sech}\tau, \tag{4}$$

$$\begin{pmatrix} c_g \\ c_a \\ c_b \end{pmatrix} = n_g^{1/2}\begin{pmatrix} \operatorname{th}\tau \\ 0 \\ \operatorname{sech}\tau \end{pmatrix}_{\mathrm{I}},\quad n_a^{1/2}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{\mathrm{II}},\quad n_b^{1/2}\begin{pmatrix} \operatorname{sech}\tau \\ 0 \\ -\operatorname{th}\tau \end{pmatrix}_{\mathrm{III}}. \tag{5}$$

Here $\gamma = (t - x/v)/\tau_p$; γ_p is the pulse length, a free parameter which determines the amplitude and velocity of propagation of the pulse:

$$c/v-1=\Omega_0^2\tau_p^2(n_g-n_b), \tag{6}$$

where $\Omega_0 = (2\pi N\mu_b{}^2\omega_b/\hbar)^{1/2}$ is the so-called cooperative frequency.

Linearizing the system (1), (2) with respect to small perturbations $\tilde{\Omega}_{a,b}$, $\tilde{C}_{g,a,b}$ of the solutions (4), (5), we obtain

$$\frac{\partial u_a}{\partial\tau}-\frac{\partial u_a}{\partial\xi}=\frac{i}{n_g-n_a}\varkappa^{-2}(c_g\tilde{c}_a{}^*+\tilde{c}_g c_a{}^*),$$
$$\frac{\partial u_b}{\partial\tau}-\frac{\partial u_b}{\partial\xi}=\frac{i}{n_g-n_b}(c_g\tilde{c}_b{}^*+\tilde{c}_g c_b{}^*), \tag{7}$$

$$\frac{\partial\tilde{c}_g}{\partial\tau}-\tilde{c}_b\operatorname{sech}\tau=i(u_a c_a+u_b c_b),$$
$$\frac{\partial\tilde{c}_a}{\partial\tau}=iu_a{}^*c_g,\quad \frac{\partial\tilde{c}_b}{\partial\tau}+\tilde{c}_g\operatorname{sech}\tau=iu_b{}^*c_g. \tag{8}$$

For convenience we have transformed here to the new variables τ, $\xi = (1 - v/c)x/v\tau_p$, and $u_{a,b} = \Omega_{a,b}\tau_p$. The parameter $\varkappa^2$ in (7) represents the ratio of oscillator strengths for the two transitions, i.e., $\varkappa^2 = \mu_b{}^2\omega_b/\mu_a{}^2\omega_a$.

Let us seek a solution for u_a in the form $u_a = u(\tau)\exp(\gamma\xi)$. After solving (8) with respect to $\tilde{c}_g$, $\tilde{c}_a$ for each of the three kinds of particles and performing some transformations, we obtain an eigenvalue problem

$$\hat{L}u=\gamma u \tag{9}$$

for the integrodifferential operator

$$\hat{L}u=\frac{\partial}{\partial\tau}u-\varkappa^{-2}\Big[A\operatorname{th}\tau\int_{-\infty}^{\tau}u\operatorname{th}\tau'\,d\tau' +B\operatorname{sech}\tau\int_{-\infty}^{\tau}u\operatorname{sech}\tau'\,d\tau'\Big], \tag{10}$$

where $A = (n_g - n_a)/(n_g - n_b)$, $B = (n_b - n_a)/(n_g - n_b)$. The spectrum of the operator $\hat{L}$ for $B = 0, A = 1$ (i.e., $n_a = n_b$) and $\varkappa^2 = 1$ was investigated earlier see Ref. 1) in the context of the problem of transverse instability of simultons. Analogously, we can show that for $B = 0$ but $\varkappa\neq 1$ there is an isolated discrete level $\gamma_0 = 0$ in the spectrum of $\hat{L}$ which corresponds to the eigenfunction

$$u_0(\tau,\ \xi)=\varepsilon_0\operatorname{sech}^{1/\varkappa}\tau\qquad(\varepsilon_0=\text{const}). \tag{11}$$

In particular, this implies that propagation of a single-frequency 2π pulse in a three-level medium with identical populations of the upper levels is neutrally stable against the appearance of an accompanying weak signal of the form (11) at the frequency of the other transition. As we will see below, the requirement that the populations of the upper levels be equal is critical for the existence of simultons in a medium with arbitrary $\varkappa$.

For moderately small differences in the original level populations a and b, i.e., $|n_a - n_b|\ll 1$, $(n_g - n_b)\sim 1$, we can make use of standard perturbation theory and seek corrections to $\gamma_0 = 0$ with respect to the small parameter B. After multiplying Eq. (9) by the solution $\bar{u}_0$ of the unperturbed associated equation and integrating with respect to τ from $-\infty$ to $+\infty$, we obtain

$$2\gamma\int_{-\infty}^{+\infty}\bar{u}_0^2\,d\tau+\varkappa^{-2}B\left\{\int_{-\infty}^{+\infty}\bar{u}_0\operatorname{sech}\tau\,d\tau\right\}^2=0. \tag{12}$$

From this we find

$$\gamma=G(\varkappa)\left[(n_a-n_b)/(n_g-n_b)\right], \tag{13}$$

where

$$G(\varkappa)=[(4\pi)^2/2^{4/\varkappa}]\Gamma(2/\varkappa)\Gamma^{-4}(1/2\varkappa).$$

Thus, when a 2π pulse propagates at one of the transitions of the three-level medium a perturbation develops at the other transition as follows:

$$u_a(\tau,\ \xi)=\varepsilon_0[\operatorname{sech}^{\nu}\tau+O(B)+\ldots]\exp(\gamma\xi), \tag{14}$$

where γ is the growth rate given by (13), ε_0 is the amplitude of the original "seed" signal at the frequency ω_a, $\nu\equiv\varkappa^{-1}A^{1/2}$, and $O(B)$ denotes terms of order $|n_b - n_a|/(n_g - n_b)\ll 1$.

The factor $G(\varkappa)$ in Eq. (13) for the growth rate decreases monotonically as $\varkappa$ increases. For large values of $\varkappa$ (i.e., a weak Stokes transition with $\mu_a{}^2\omega_a \ll \mu_b{}^2\omega_b$), we have $\gamma \propto \varkappa^{-3}$. This behavior of the growth rate has a simple explanation. First, the gain coefficient for a weak signal in the pulse field E_b is proportional to $\varkappa^{-2}$ [see Eq. (7)]; secondly, the effective amplification length decreases as $\varkappa^{-1}$, since the pumping pulse is shorter than the evolving perturbation by a factor of $\varkappa$. In the opposite limit, i.e., the case of strong Stokes transitions, $\varkappa \to 0$ and $G(\varkappa)$ behaves like $\varkappa^{-3/2}$, despite the fact that the amplification takes place over the entire length of the pulse, which would seem to imply that the growth rate ought to increase as $\varkappa^{-2}$. The fact is that in this case there is a competition between the amplification effects of the weak signal in the field of the 2π pulse, the dispersive group velocity, and the absorption of the amplified radiation outside the interaction region.

The case $\varkappa^2 = 1$ admits an exact solution to the problem (9) even when the inequality $|n_a - n_b|/(n_g - n_b) \ll 1$ is not fulfilled. In this case Eq. (13) for the growth rate of the weak signal u_a simplifies:

$$\gamma = (n_a - n_b)/(n_g - n_b), \tag{15}$$

and the perturbation itself has the form $u_a(\tau,\xi) = \exp(\gamma\xi)$ sech τ, or in dimensional variables

$$u_a(t,x) = \exp[\Omega_0{}^2\tau_p(n_a - n_b)x/c]\,\mathrm{sech}\,\frac{t - x/v}{\tau_p}. \tag{16}$$

Let us discuss the physical reasons for the exponential growth of the perturbation at the frequency ω_a in the field of the 2π pulse of frequency ω_b. Equation (5) shows that when a soliton propagates at the transition $g \to b$ the population of the lower level changes in the following way:

$$|c_g(\tau,\ \xi)|^2 = n_g\,\mathrm{th}^2\,\tau + n_b\,\mathrm{sech}^2\,\tau. \tag{17}$$

The maximum depletion of the g level takes place at the center of the 2π pulse, i.e., $\tau = 0$, and equals $|C_g|^2_{\min} = n_b$. Consequently, for $n_a > n_b$ the region of the 2π pulse constitutes an inverted layer for radiation of frequency ω_a whose width is

$$l_p = v\tau_p \approx c/\Omega_0{}^2\tau_p. \tag{18}$$

From this it follows that over the length of the 2π pulse the Stokes signal is amplified by a factor of $\exp(n_a - n_b)$.

It is interesting to trace through the dynamics of energy exchange between radiation and the medium in such a process (Fig. 1). During propagation of a 2π pulse the medium absorbs $n_g - n_b$ quanta of frequency ω_b [see Fig. 1(b)]. Then, once it is found in the inverted layer, the Stokes signal removes $n_a - n_b$ quanta of frequency ω_a [see Fig. 1(c)]. The population of the levels at this time is $|C_g|^2 = n_a$, $|C_a|^2 = n_b$, $|C_b|^2 = n_g$. Finally, the transition $b \to g$ can convert only $n_g - n_a$ quanta to radiation of frequency ω_b [see Fig. 1(d)]. Thus the energy of the 2π pulse decreases during each elementary interaction event by the quantity $\Delta\varepsilon = \hbar\omega_b(n_a - n_b)$, one part of which

$$\Delta\mathscr{E}_{rad} = \hbar\omega_a(n_a - n_b),$$

is converted into radiation at frequency ω_a while the other,

$$\Delta\mathscr{E}_{med} = \hbar(\omega_b - \omega_a)(n_a - n_b),$$

is consumed in creating an inversion between levels b and a; thus, $|C_b|^2_{\text{fin}} = n_g$, $|C_a|^2_{\text{fin}} = n_b$. Obviously the analysis we have carried out up to this point, which is linear in the field u_a, cannot address the question of the final results of the interaction. However, it is clear that a high-frequency 2π pulse in a three-level medium with the natural distribution of initial populations is unstable relative to transformation into radiation of lower frequency; in this case the upper level is populated while the intermediate level is depleted.

3. Before turning to the exact solution of the system (1), (2) with initial conditions (3), we recall that in a medium with $\varkappa = 1$ at zero temperature it is possible for simultons to exist—i.e., multifrequency solitons, all of whose components have the identical propagation velocity v and envelope sech $[(t - x/v)\tau_p]$ (Refs. 1,2). Moreover, even for $\varkappa \neq 1$ it is possible in a zero-temperature medium to form a combined multifrequency pulse[1] which possesses a well-defined margin of stability.

Let us now investigate the nonzero initial conditions (3) and construct the exact soliton solution. For $\varkappa = 1$, this solution should describe the dynamics of a two-frequency pulse, while for $n_a - n_b \to 0$ it should degenerate into the usual simulton. After using ISP techniques, whose applicability to a three-level system we have discussed in a number of previous papers,[1] it is not difficult to show that such a solution has the form

$$\Omega_{a,b} = -\frac{2i}{\tau_p}\alpha_{a,b}[e^{\tau_{a,b}} + e^{-\tau_{a,b}}(|\alpha_{a,b}|^2 + |\alpha_{b,a}|^2 e^{\mp 2gx})]^{-1}, \tag{19}$$

where

$$\tau_{a,b} = (t - x/v_{a,b})/\tau_p,\ v_{a,b} = c\,[1 + \Omega_0{}^2\tau_p{}^2(n_g - n_{a,b})]^{-1},$$
$$g = \Omega_0{}^2\tau_p(n_b - n_a)/c$$

[Compare this to the linear increment (16)]; the constants $\alpha_{a,b}$ and also the length τ_p are determined by the boundary conditions at the entrance to the medium. Expression (19) shows that the condition for existence of simultons is equality of the initial populations of the upper levels, i.e., $n_a = n_b$. Otherwise the two-frequency pulse is unstable against conversion to a single-frequency 2π pulse at the transition for which the initial population of the upper level is larger (the more "transparent" transition). We note that the instability of simultons has no threshold, i.e., a "seed" of radiation at frequency ω_a of arbitrarily small amplitude and area will cause degeneration of any two-frequency soliton into a frequency 2π pulse. The conversion length is

$$l \sim g^{-1} = \frac{c}{\Omega_0{}^2\tau_p(n_a - n_b)} \approx l_p\frac{n_g - n_b}{n_a - n_b}, \tag{20}$$

where $l_p = v_a\tau_p$ is the size of the 2π pulse in the medium. In this case fulfillment of the usual requirement on the initial area

$$(\vartheta_a{}^2 + \vartheta_b{}^2)^{1/2} > \pi, \tag{21}$$

provides a natural threshold for soliton formation, where

$$\vartheta_{a,b} = \int_T \Omega_{a,b}(x=0,t)\,dt,$$

T is the time interval during which radiation is present in the medium at both frequencies simultaneously.

It is interesting to compare the solution (19) with solutions which describe the collision of simultons in the zero-temperature medium:

$$\Omega_{a,b}=-i\{\beta_{a,b}|\alpha|T_1\,\mathrm{ch}(\tau_1-\varphi_1)+\alpha_{a,b}|\beta|T_2\,\mathrm{ch}(\tau_2-\varphi_2)$$
$$-(\alpha'_{a,b}\beta_{a,b})^{1/2}(\alpha\beta)^{1/2}T_{12}[\mathrm{ch}(\tau_1-\chi_1^{(a,b)})$$
$$+\mathrm{ch}(\tau_2-\chi_2^{(a,b)})]\}\Big\{|\alpha||\beta|T_1T_2\,\mathrm{ch}(\tau_1$$
$$-\varphi_1)\,\mathrm{ch}(\tau_2-\varphi_2)-(\alpha\beta)T_{12}{}^2\,\mathrm{ch}^2\Big(\frac{\tau_1+\tau_2}{2}-\varphi_{12}\Big)\Big\}^{-1}. \quad (22)$$

Here $\tau_{1,2}=(t-x/v_{1,2})/T_{1,2}$; T_1 and T_2 are the lengths of two independent simultons, $v_{1,2}=c(1+\Omega_0^2T_{1,2}{}^2)^{-1}$ are their velocities, $T_{12}=2(1/T_1+1/T_2)^{-1}$, and $\boldsymbol{\alpha}=(\alpha_a, \alpha_b)$ and $\boldsymbol{\beta}=(\beta_a, \beta_b)$ are vectors whose components are determined by the boundary conditions;

$$\varphi_1=\ln|\alpha|,\quad \varphi_2=\ln|\beta|,\quad \varphi_{12}=\ln(\alpha\beta)^{1/2},$$
$$\chi_{1,2}^{(a,b)}=\varphi_{12}\pm{}^1/_2\ln(\alpha_{a,b}/\beta_{a,b})$$

are phase constants.

Expression (22) is quite cumbersome, and for analysis it is convenient to use its asymptotic forms. Thus, for $t\to-\infty$ we have two separate simultons:

$$i\Omega_{a,b}=\frac{\alpha_{a,b}}{|\alpha|}T_1{}^{-1}A_{a,b}^-\,\mathrm{sech}(\tau_1-\varphi_1{}^-)+\frac{\beta_{a,b}}{|\beta|}T_2{}^{-1}\,\mathrm{sech}(\tau_2-\varphi_2{}^-), \quad (23)$$

where

$$A_{a,b}^-=\Big[1-\frac{T_{12}}{T_2}\frac{\beta_{a,b}}{\alpha_{a,b}}\frac{(\alpha\beta)}{|\beta|^2}\Big]\Big[1-\frac{T_{12}{}^2}{T_1T_2}\frac{(\alpha\beta)^2}{|\alpha|^2|\beta|^2}\Big]^{-1/2},$$
$$\varphi_1{}^-=\varphi_1+\frac{1}{2}\ln\Big[1-\frac{T_{12}{}^2}{T_1T_2}\frac{(\alpha\beta)^2}{|\alpha|^2|\beta|^2}\Big],$$
$$\varphi_2{}^-=\varphi_2-\frac{1}{2}\ln\Big[1-\frac{T_{12}{}^2}{T_1T_2}\Big]=\varphi_2+\ln\frac{T_1+T_2}{T_1-T_2}.$$

In the other asymptotic limit $t\to+\infty$ we also have two separate simultons, but with altered amplitudes:

$$i\dot{\Omega}_{a,b}=-\frac{\alpha_{a,b}}{|\alpha|}T_1{}^{-1}\,\mathrm{sech}(\tau_1-\varphi_1{}^+)$$
$$+\frac{\beta_{a,b}}{|\beta|}T_2{}^{-1}B_{a,b}^+\,\mathrm{sech}(\tau_2-\varphi_2{}^+), \quad (24)$$

where

$$B_{a,b}^+=\Big[1-\frac{T_{12}}{T_1}\frac{\alpha_{a,b}}{\beta_{a,b}}\frac{(\alpha\beta)}{|\alpha|^2}\Big]\Big[1-\frac{T_{12}{}^2}{T_1T_2}\frac{(\alpha\beta)^2}{|\alpha|^2|\beta|^2}\Big]^{-1/2},$$
$$\varphi_1{}^+=\varphi_1-\frac{1}{2}\ln\Big[1-\frac{T_{12}{}^2}{T_1T_2}\Big]=\varphi_1+\ln\frac{T_1+T_2}{T_1-T_2},$$
$$\varphi_2{}^+=\varphi_2+\frac{1}{2}\ln\Big[1-\frac{T_{12}{}^2}{T_1T_2}\frac{(\alpha\beta)^2}{|\alpha|^2|\beta|^2}\Big].$$

Equations (23), (24) show that during a collision the total number of quanta and propagation velocities for each simulton do not change; however, redistribution of the amplitudes of the separate frequency components takes place. This possibility was mentioned for the first time in Ref. 4. From the point of view of coherent frequency conversion using optical pulses there now arises the interesting possibility of formation of two-frequency solitons from a single-frequency one, and conversely, i.e., shifting of 2π pulses from one frequency to another, etc. It is noteworthy that after the collision each of the simultons carries with it information about the length and amplitude of the other.

We present as an example two curious schemes of nondegenerate interaction (the brackets here denote the relative amplitudes of the various frequency components of the simulton):

$$\frac{1}{2^{1/2}}\begin{pmatrix}1\\1\end{pmatrix}_b+\begin{pmatrix}0\\1\end{pmatrix}_M\to\frac{1}{D}\Big\{\begin{pmatrix}\tau_M\\\tau_b\end{pmatrix}_M+\frac{1}{2^{1/2}}\begin{pmatrix}\tau_M-\tau_b\\\tau_M+\tau_b\end{pmatrix}_b\Big\}, \quad (25)$$

$$\begin{pmatrix}0\\1\end{pmatrix}_b+\frac{1}{2^{1/2}}\begin{pmatrix}1\\1\end{pmatrix}_M\to\frac{1}{D}\Big\{\frac{1}{2^{1/2}}\begin{pmatrix}\tau_M-\tau_b\\\tau_M+\tau_b\end{pmatrix}_M+\begin{pmatrix}\tau_b\\\tau_M\end{pmatrix}_b\Big\}. \quad (26)$$

Here $D=(\tau_M^2+\tau_b^2)^{1/2}$, where τ_M and τ_b are respectively the lengths of the slow and fast simultons, i.e., $\tau_M>\tau_b$ (let us recall that the propagation velocity of a pulse is $v\propto\tau^{-2}$). For large differences in the lengths $\tau_M\gg\tau_b$ the interaction (25) gives rise to almost complete conversion of the slow 2π pulse to the other frequency. If, however, the fast single frequency pulse overtakes the slow two-frequency pulse, then the relative amplitudes of both pulses are left almost unchanged [scheme (26)]. We note that within the framework of the assumption that the medium is at zero temperature, or indeed when the original populations of the upper levels are equal, the transformations (25), (26) are invertible.

4. In Fig. 2. we present results of a numerical solution of Eqs. (1), (2) for two values of the parameter $\varkappa=(\mu_b^2\omega_b/\mu_a^2\omega_a)^{1/2}$, i.e., 0.7 and 1.5. Details of the computational method have been published previously (see Ref. 1). For both choices of $\varkappa$ the initial and boundary conditions were taken to be the same: $n_g(t=0)=0.7$, $n_a(t=0)=0.3$, $n_b(t=0)=0$; at the boundary $z=0$ a 2π pulse E_b enters the medium with a 1% (intensity) correction of radiation E_a. It is apparent from the figure that the pulse E_b is unstable against complete conversion into the pulse E_a. The rate of conversion agrees well with the linear estimate (13). In the region of interaction we observe a population inversion of levels a and b with respect to the scheme in Fig. 1. The length of the inversion zone left behind by the pulses in the medium depends both on the conversion growth rate and on the propagation velocity of the pulse in the medium. The length and peak value of the intensity of the Stokes pulse depend strongly on the value of $\varkappa$. For the stronger transition $g\to a$, $\varkappa<1$, the Stokes pulse is preserved because the cooperative frequency at this transition is large, i.e., $\tau_a/\tau_b\sim\varkappa$; its intensity exceeds the intensity of the "pump", i.e., $|E_a|^2/|E_b|^2\sim\varkappa^{-2}$, while the propagation velocity increases both because of the increase in intensity and because of the larger "transparency" of the transition, i.e., $n_g-n_a<n_g-n_b$. A further result of this is an increased pulse length in the medium, i.e., $l_a\sim l_b\varkappa^{-1}(n_g-n_b)/(n_g-n_a)$. For $\varkappa>1$ the situation is reversed: the Stokes pulse is slower and less intense. However, because of its low propagation velocity the transfer takes place over a shorter distance in the medium.

We should point out the close analogy between the interaction under study here and the coherent regime of simulated Raman scattering (SRS).[1] As in the latter, during the

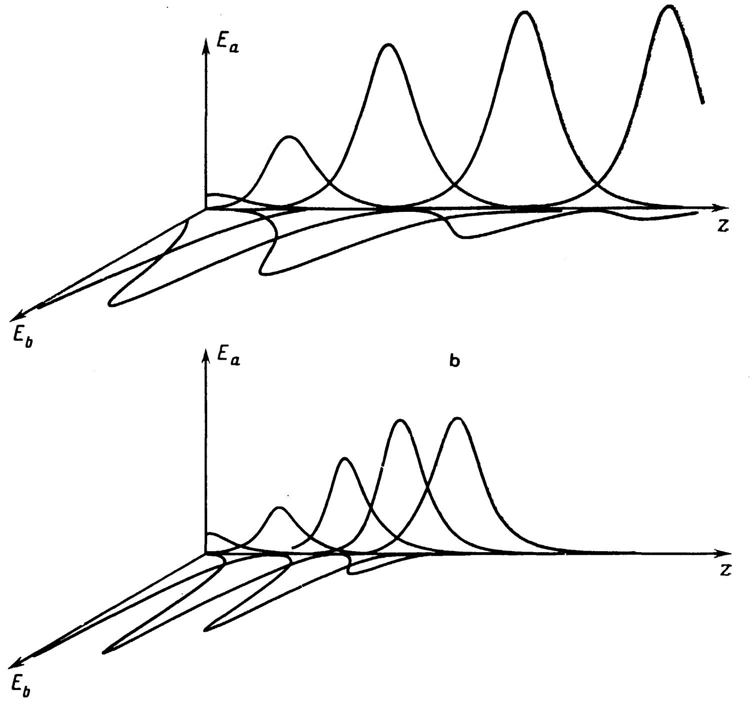

FIG. 2. Energy transfer of a 2π pulse E_b into a 2π pulse E_a in a medium with $\varkappa \neq 1$: (a)—$\varkappa = 0.7$, (b)—$\varkappa = 1.5$. The scale along the z-axis is based on units of the length of incoming pulse based on the transition $g \to a$ [the total length of the axis corresponds to 15 lengths of the pulse E_a (entering)]. The scale of the axis E_a corresponds to the value $\varkappa/\tau_a$ (entering) Ω_0 (i.e., the amplitude of a Stokes 2π pulse with a length equal to the length of the "pump" pulse). The curves in Figs. (a) and (b) are drawn for one and the same instants of time for identical boundary conditions and initial populations $n_g = 0.7$, $n_a = 0.3$, $n_b = 0$.

interaction the two pulses form a bound state, which brings about the complete conversion of energy into radiation at a lower frequency (for the natural level populations of the medium). The quantum efficiency of the process also comes to 100%, and the excess energy remains in the medium in the form of excitation of the upper level. The relationships between the Stokes parameters of the pulse and the "pump" as $\varkappa$ varies reinforce this analogy.

Thus we have shown that it is possible to have coherent frequency conversion of short light pulses in a three-level V-configuration medium. To obtain such a regime experimentally we need not fulfil any artificial conditions, such as, e.g., previous preparation of the medium or equality of the oscillator strengths of the two transitions. The numerical calculations show that typical conversion lengths amount to ~ 10 pulse lengths in the medium, less than the distance tranversed by both single-frequency and multifrequency solitons up to their decay due to transverse instability.[1] A natural limitation is imposed on the lengths of the pulses: they should not exceed the longitudinal and transverse relaxation times in the medium. From this, based on the ratio $\mu E\tau_p/\hbar \sim 2\pi$, there follow estimates of the pulse energy densities required to observe these coherent effects. In experiments on resonant interactions it is customery to use atomic vapors,[5,6] molecular gases,[7,8] and mixed crystals.[9] The relaxation times in these media are determined by various mechanisms. Thus, for atomic vapors ($\tau \lesssim 10^{-9}$–10^{-10} sec, $\mu \sim 10^{-18}$ cgs units) we require energy densities $I \sim 10$–$100\ \mu J/cm^2$, for molecular gases ($\tau \lesssim 10^{-8}$–10^{-9} sec, $\mu \sim 10^{-19}$ cgs units) $I \sim 0.1$–1 mJ/cm^2, and for condensed media ($\tau \lesssim 10^{-10}$–10^{-12} sec) $I \sim 0.1$–10 mJ/cm^2.

In conclusion we remark that the results of the present work also are relevant to the problem of simulation formulation. Thus, the combined propagation of multifrequency pulses on a length L requires preliminary matching of the populations of the upper levels to less than the quantity

$$(n_a - n_b) \leqslant G^{-1}(\varkappa)\, l_p/L$$

(l_p is the pulse length in the medium) for $\varkappa \sim 1$ or the use of a medium with a larger ratio of transition oscillator strengths, $\varkappa \gg 1$, for which $(G\varkappa) \ll 1$.

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