Transverse electron focusing spectroscopy of the electron–phonon interaction

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It is shown theoretically that a new method for the spectroscopy of the electron–phonon interaction in metals can be based on transverse focusing of nonequilibrium electrons [V.S. Tsoĭ, JETP Lett. **19**, 70 (1974)] that have experienced strong electron–phonon relaxation in the dirty vicinity of an emitter contact. The results are used to explain recent experiments that have revealed the phonon structure of a transverse focusing line [P. C. van Son, H. van Kempen, and P. Wyder, Phys. Rev. Lett. **58**, 1567 (1987); V. V. Andrievskii, E. I. Ass, and Yu. F. Komnik, JETP Lett. **47**, 124 (1988)].

1. INTRODUCTION

The method of trajectory focusing of electron fluxes in metals subjected to transverse[1] and longitudinal[2] magnetic fields is essentially ballistic and extremely sensitive to carrier relaxation. The use of point contacts for the injection of electron beams into a metal makes it possible to focus high-energy carriers with a controlled amount of the excess energy. These two circumstances determine the potentialities of the focusing method in studies of the processes of relaxation of "hot" electrons in conductors. Already the early experiments on longitudinal focusing have revealed nonlinear dependences of the signal on the emitter current, attributed to the energy dependence of the mean free path of carriers[3] and to the thermal heating of electron fluxes.[4] The possibility of investigating the electron–phonon interaction (EPI) by the method of transverse electron focusing was first pointed out in Ref. 5 and the influence of weak EPI processes on the transverse electron focusing was investigated theoretically in Ref. 6. An investigation of transverse focusing of a spatially localized energy distribution of nonequilibrium carriers, formed as a result of relaxation of a primary monoenergetic beam, was described in Ref. 7. If the trajectory radius is energy-dependent, this distribution is transformed into a spatially inhomogeneous one which can be determined by investigating the dependence of the collector voltage on the applied magnetic field. However, this procedure meets with difficulties due to the extremely short mean free paths of the high-energy electrons in the usual metals ($l \sim 10^{-5}$ cm). In semimetals of the bismuth type the low density of states and the high permittivity increase considerably the mean free paths.[8] This is clearly the reason for the observation of nonlinear behavior in the case of transverse electron focusing reported in Ref. 9.

In spite of the obvious attraction of the method of transverse focusing in investigations of inelastic relaxation of accelerated electrons, the first major progress has been made in studies of the intrinsic electrical conductivity of extremely small point contacts, less than the characteristic length of the electron energy loss. Point-contact spectroscopy[10–12] is now recognized worldwide as a simple and effective method for investigating relaxation processes in solids.

Very recently new experiments were reported which have continued the investigation of the transverse focusing of "hot" electrons begun in Ref. 5. A reduction in the amplitude of the electron focusing signal as the emitter injection current increases in silver and a maximum of the focusing signal at excess electron energies equal to a characteristic phonon energy were reported in Ref. 13. Such a dependence of the amplitude of the electron focusing line on the emitter voltage was convincingly explained in Ref. 13 by the role of the electron–phonon relaxation destroying the ballistic nature of the cyclotron motion of electrons. The most interesting results were obtained recently in a study of effects nonlinear in the emitter current and observed in the case of transverse focusing of electrons in bismuth.[14] Andrievskiĭ *et al.*[14] observed a reproducible structure of maxima in the derivative of the electron focusing signal with respect to the emitter current, which could be compared readily with multiple and combination frequencies of relaxation phonons in bismuth. These experiments demonstrated convincingly the potentialities of the electron focusing method in studies of phonon relaxation in metals and semimetals.

We shall develop a theory of relaxation phenomena in the course of transverse focusing of electrons in a metal. Our formulation of the problem is distinguished by the need to allow for a strong spatial inhomogeneity of the contamination, giving rise to a difference between the rates of relaxation processes occurring in the emitter and in the cyclotron trajectory in the interior of a metal. Experiments reported in Ref. 15 yielded directly the l_i representing elastic scattering of electrons in an emitter contact, and it was shown that this length was three orders of magnitude less than the corresponding length in a bulk metal. It therefore follows that the two parameters L/l_{ep} and $b/\lambda_\varepsilon \equiv b/(l_i l_{ep})^{1/2}$ [L is the distance between the emitter and the collector, b is the size of the emitter contact (see Fig. 2 below), and l_{ep} is the mean free path in the case of the electron–phonon scattering], reflecting the role of the relaxation processes along a trajectory and at a contact can be in an arbitrary ratio, giving rise to two types of relaxation nonlinearities. We shall consider separately these relaxation mechanisms and show that the effects reported in Ref. 13 are largely due to the trajectory relaxation of electrons, whereas the phonon structure of the electron focusing signal observed in Ref. 14 is associated with the discrete nature of the phonon relaxation of carriers directly in an emitter contact. The relationships obtained in the present study demonstrate that it should be possible to reconstruct the EPI function from the dependence of the electron focusing signal on the emitter current, which would provide the basis for a new cyclotron method for the spec-

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troscopy of the EPI in conductors. A special feature of this method, based on detection of the contribution of small electron groups focused in the collector, is the high directionality which makes it possible to study relaxation of specific electron states.

In studies of nonlinear effects in transverse focusing of electrons the problem of the nature of nonequilibrium electrons injected by a point contact becomes of primary importance. Investigations of the geometric nonlinearity effect[16,17] have shown that electrons injected into bismuth are characterized by a considerable (of the order of the Fermi energy ε_F) excess energy, but this energy is several times (and sometimes by an order of magnitude) smaller than the maximum possible energy eV (V is the voltage across the emitter). On the other hand, there is no thermal broadening of the peak. Investigations of the Shubnikov oscillations of the emitter resistance[18] have demonstrated directly the low degree of thermal heating of an electron gas in a junction. When these data are taken as a whole, we find that the processes of electron–phonon relaxation in an emitter may be very significant, whereas reabsorption of the phonons emitted by electrons in a contact is weak.[1)] Consequently, we shall consider a model in which the emitter is a dirty channel (Fig. 1):

$$l_i \ll b \ll 2d \tag{1}$$

(b is the channel diameter and $2d$ is its length), where the electron–phonon relaxation length λ_ε is arbitrary and there are no phonon reabsorption processes. The emission of phonons in this channel determines the profile of the energy dependence of the distribution function of carriers injected from the dirty channel into the pure metal. The electron focusing signal makes it possible to determine this electron energy distribution and to reconstruct the energy dependence of the inelastic relaxation length λ_ε. The hypothesis of a low transparency D of a tunnel barrier ($D \ll l_i/2d$) in the plane of contact between two conductors (heterocontact emitter structure[20] was used, for example, in the experiments reported in Refs. 1, 5, 9, 14, 15, 17, and 18) has made it possible to postulate an abrupt change in the electrical potential in the heterojunction plane and thus simplify greatly the mathematical analysis of the problem without distortion of the principal features of the effect.

A calculation of the electrical potential φ at a measuring contact P as a function of the magnetic field $\mathbf{H}$ and the voltage V applied to an emitter A will be made using the

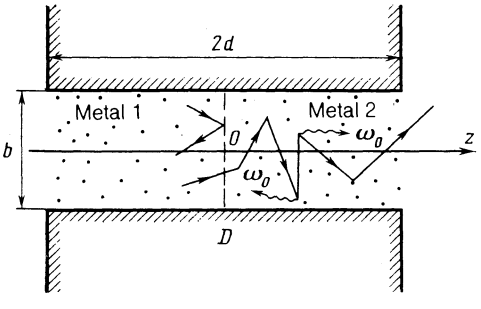

FIG. 1. Model of the emitter: a long "dirty" channel with a tunnel barrier characterized by a low transparency D.

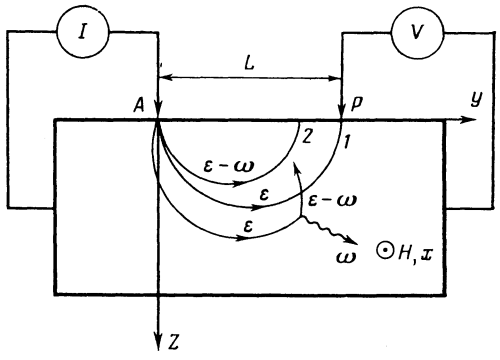

FIG. 2. Schematic representation of the observation of electron focusing (I is the source of the current and V is a voltmeter).

standard geometry of the transverse electron focusing effect (Fig. 2).

2. FORMULATION OF THE PROBLEM AND THE COMPLETE SYSTEM OF EQUATIONS

The complete system of equations describing the problem consists of the kinetic equation for the nonequilibrium electron distribution function $n(\mathbf{r},\mathbf{p})$

$$\mathbf{v}\frac{\partial n}{\partial \mathbf{r}}+\left(e\mathbf{E}+\frac{e}{c}[\mathbf{vH}]\right)\frac{\partial n}{\partial \mathbf{p}}=I_i(n)+I_{ep}(n) \tag{2}$$

and the electrical neutrality equation

$$\int d\mathbf{p}[n(\mathbf{r},\mathbf{p})-f_0(\varepsilon)]=0. \tag{3}$$

Here, e, $\mathbf{r}$, and $\mathbf{p}$ are the charge, coordinate, and momentum of an electron; $\epsilon(\mathbf{p})$ and $\mathbf{v}=\partial\varepsilon/\partial\mathbf{p}$ are the energy and velocity; $f_0(\varepsilon)$ is the Fermi distribution function; $\mathbf{E}$ and $\mathbf{H}$ are the intensities of electric and magnetic fields. The collision integral $I_i(n)$ describing the elastic scattering of electrons by impurities, assumed to be isotropic, is

$$I_i(n)=\frac{1}{(2\pi)^3}\int\limits_{\varepsilon(\mathbf{p})=\varepsilon(\mathbf{p}')}\frac{dS_{\mathbf{p}}}{v}W_{|\mathbf{p}-\mathbf{p}'|}[n(\mathbf{p})-n(\mathbf{p}')], \tag{4}$$

where $\hbar=1$ and $W_{|p|}/2\pi$ is the square of the modulus of the matrix element of the elastic scattering, governing the transport elastic relaxation time:

$$\tau_i^{-1}(\varepsilon)=\int\limits_{\varepsilon(\mathbf{p})=\varepsilon(\mathbf{p}')=\varepsilon}\frac{d\mathbf{S}_{\mathbf{p}'}}{(2\pi)^3 v}W_{|\mathbf{p}-\mathbf{p}'|}[1-\cos(\hat{\mathbf{p}}\hat{\mathbf{p}}')]. \tag{5}$$

Inelastic EPI processes are included in the collision integral

$$\begin{aligned}I_{ep}(n)=\sum_\alpha\int\frac{d\mathbf{q}}{(2\pi)^3}w_{\mathbf{q}}^{\alpha}\{&[n(\mathbf{p}+\mathbf{q})[1-n(\mathbf{p})](N_{\mathbf{q}}^{\alpha}+1)\\&-n(\mathbf{p})[1-n(\mathbf{p}+\mathbf{q})]N_{\mathbf{q}}^{\alpha}]\delta(\varepsilon(\mathbf{p}+\mathbf{q})-\varepsilon(\mathbf{p})-\omega_{\mathbf{q}}^{\alpha})\\&+[n(\mathbf{p}-\mathbf{q})[1-n(\mathbf{p})]N_{\mathbf{q}}^{\alpha}-n(\mathbf{p})[1-n(\mathbf{p}-\mathbf{q})](N_{\mathbf{q}}^{\alpha}+1)]\\&\cdot\delta(\varepsilon(\mathbf{p}-\mathbf{q})-\varepsilon(\mathbf{p})+\omega_{\mathbf{q}}^{\alpha})\}.\end{aligned} \tag{6}$$

Here, the summation is carried out over the numbers α of the branches of the phonon spectrum; w_q^α is the square of the modulus of the matrix element of the EPI. The system of equations (2)–(3) should generally be supplemented by the kinetic equation for the determination of the phonon distribution function $N_{\mathbf{q}}^{\alpha}$ of phonons with the dispersion law ω_q^α. However, we shall consider only the case of low temperatures and assume that the phonons are in equilibrium [at $T=0$ we have to substitute $N_q^\alpha=0$ in Eq. (6)].

The distribution functions of electrons $n_{v_n} < 0$ incident on the surface of a metal $\mathbf{r} = \mathbf{r}_s$ and of electrons reflected by this surface $n_{v_n} > 0$ are related by the boundary condition

$$n_{v_n>0}(\mathbf{p}) = n_{v_n<0}(\tilde{\mathbf{p}}) + \int_{v_n<0} d\mathbf{p}' P(\mathbf{p},\mathbf{p}')\,[n(\mathbf{p}') - n(\tilde{\mathbf{p}})], \quad (7)$$

which automatically ensures that there is no flow of the current (v_n is the velocity component normal to the surface). The momenta $\mathbf{p}$ and $\tilde{\mathbf{p}}$ satisfy the conditions of specular reflection, which conserve the energy of a carrier $\varepsilon(\mathbf{p}) = \varepsilon(\tilde{\mathbf{p}})$ and of the projection of its momentum $\mathbf{p}_t = \tilde{\mathbf{p}}_t$ on a plane which is in contact with the surface at the point $\mathbf{r} = \mathbf{r}_s$. The integral operator describes the diffuseness of the elastic processes of the scattering of electrons by the outer surface of a metal.

If the inequality (1) is satisfied, the function describing the distribution of electrons n in the emitter can be regarded as dependent only on one coordinate z and, without violating the generality of the solution of the problem, the condition which must be satisfied by the function $\mathbf{n}(z,\mathbf{p})$ on the surface of a contact between the metals $z = 0$ can be written in the form

$$n_{v_z>0}(z=+0,\ \mathbf{p}) = (1-D)\,n_{v_z<0}(z=+0,\ \tilde{\mathbf{p}}) + D n_{v_z>0}(z=-0,\ \tilde{\tilde{\mathbf{p}}}), \quad (8)$$

where $D(\mathbf{p})$ is the probability of electron tunneling across the heterojunction at $z = 0$. The momentum $\mathbf{p}$ is related to the momentum $\tilde{\mathbf{p}}$ of electrons incident on the $z = 0$ boundary and of electrons transmitted by this boundary $\tilde{\tilde{\mathbf{p}}}$ under conditions that the energy $[\varepsilon(\mathbf{p}) = \varepsilon(\tilde{\mathbf{p}}) = \varepsilon(\tilde{\tilde{\mathbf{p}}})]$ and the component of the momentum $\mathbf{p}_t = \tilde{\mathbf{p}}_t = \tilde{\tilde{\mathbf{p}}}_t$ tangential to the $z = 0$ boundary be conserved.

The problem can therefore be separated into two parts: 1) determination of the distribution function $n(z,\mathbf{p})$ for a dirty point contact; 2) calculation of the distribution function of electrons $n(\mathbf{r},\mathbf{p})$ in a pure metal, satisfying the effective boundary condition in the $z = d$ plane:

$$n_{v_z>0}(\mathbf{R},\mathbf{p}) = \left[q(\mathbf{p})\, n_{v_z<0}(\mathbf{R},\mathbf{p}) + \int_{v_z<0} d\mathbf{p}'\, P(\mathbf{p},\mathbf{p}')\, n(\mathbf{R},\mathbf{p}') \right] \cdot \theta(\mathbf{R}\in S_k) + n_{v_z>0} \quad (z=d,\mathbf{p})\,\theta(\mathbf{R}\in S_k), \quad (9)$$

where

$$q(\mathbf{p}) = 1 - \int_{v_z<0} d\mathbf{p}'\, P(\mathbf{p},\mathbf{p}'),$$

$\mathbf{R}$ is a two-dimensional vector in the $z = s$ plane; S_k is the set of vectors lying in the emitter plane ($z = d, x^2 + y^2 < b^2/4$). Knowing the function $n(\mathbf{r},\mathbf{p})$, we can use the electrical neutrality equation to calculate the distribution of the potential on the surface of a sample.

If a tunnel barrier is characterized by a low transparency $D \ll l_i/d \ll 1$, then the Boltzmann equation (2) can be linearized with respect to a small (proportional to D) correction f to the Fermi distribution function f_0 $[n(\mathbf{r},\mathbf{p}) = f_0(\varepsilon) + f(\mathbf{r},\mathbf{p})]$. In the case of a dirty metal (emitter) the function f can be represented as an expansion in powers of the short elastic electron relaxation time $\tau_i(\varepsilon)$ (Ref. 21):

$$f(z,\mathbf{p}) = \bar{f}(z,\varepsilon) - \tau_i(\varepsilon) v_z \frac{\partial \bar{f}}{\partial z} + \dots,$$

$$\bar{f}(z,\varepsilon) = \frac{\langle f\rangle_\varepsilon}{\langle 1\rangle_\varepsilon}, \qquad \langle \dots\rangle_\varepsilon = \frac{1}{4\pi^3}\int_{\varepsilon(\mathbf{p})=\varepsilon} d\mathbf{p}\dots \quad (10)$$

The function $\bar{f}(z,\varepsilon)$ averaged over the direction of the momenta satisfies the equation

$$\frac{v(\varepsilon)}{3} l_i(\varepsilon) \frac{\partial^2 \bar{f}}{\partial z^2} = \bar{f}(\varepsilon,z) \int_0^{|\varepsilon-\varepsilon_F|} d\omega\, g(\omega, \varepsilon - \omega\,\mathrm{sign}(\varepsilon-\varepsilon_F)) - \int_0^\infty d\omega [g(\omega,\varepsilon+\omega)\bar{f}(z,\varepsilon+\omega)\theta(\varepsilon-\varepsilon_F) + g(\omega,\varepsilon-\omega)\bar{f}(z,\varepsilon-\omega)\theta(\varepsilon_F-\varepsilon)], \quad (11)$$

where

$$g(\omega,\varepsilon\pm\omega) = \frac{1}{\langle 1\rangle_\varepsilon}\sum_\alpha \langle\langle w^\alpha_{\mathbf{p}-\mathbf{p}'}\delta(\omega-\omega^\alpha_{\mathbf{p}-\mathbf{p}'})\rangle_\varepsilon\rangle_{\varepsilon\pm\omega}, \quad (12)$$

$l_i(\varepsilon) = v(\varepsilon)\tau_i(\varepsilon)$, and $v(\varepsilon)$ is the modulus of the electron velocity.

The boundary conditions at the heterojunction $z = 0$ and at the boundary between pure and dirty metals $z = d$ can be obtained from Eq. (8) and from the condition of continuity of the distribution function $f(\mathbf{r},\mathbf{p})$ in the $z = d$ plane by using a diffuse expansion of Eq. (10) and averaging over the electron momenta. These boundary conditions are of the form (see also Ref. 15)

$$\frac{l_i}{\bar{D}}\frac{\partial \bar{f}}{\partial z}\Big|_{z=0} = -1, \quad (13)$$

$$\bar{f}(d,\varepsilon) + \frac{1}{2} l_i \frac{\partial \bar{f}}{\partial z}\Big|_{z=d} = 0. \quad (14)$$

Using the diffuse expansion of Eq. (10), we can easily show that the boundary condition given in Eq. (13) in fact means that the current (proportional to the transparency D) injected in the $z = 0$ plane is independent of the elastic scattering length of electrons l_i. This condition is physically self-evident in the limit of low transparency of a tunnel barrier $D \ll l_i/d \ll 1$ assumed above. It should also be noted that the boundary condition (14) ensures continuity of the electric current at the boundary between pure and dirty metals if we allow for the fact that in the pure metal this current is governed by the function $\bar{f}$, whereas in the dirty region the current is expressed in terms of the gradient combination $l_i \partial f/\partial z$.

It is not possible to obtain the solution of the problem for an arbitrary relationship between the lengths of elastic and inelastic electron scattering, and also between them and the geometric dimensions b, d, and L describing the experimental setup. Therefore, we shall consider the most interesting limiting cases.

3. TRAJECTORY RELAXATION IN THE CASE OF TRANSVERSE ELECTRON FOCUSING

Let us assume that the length $\lambda_\varepsilon = (l_i l_{ep})^{1/2}$, representing the inelastic relaxation of carriers in a dirty metal is

considerably greater than the emitter channel length and the value of l_{ep} is comparable with the distance L between the emitter and collector. In this case we can simplify Eq. (11) by dropping the electron–phonon collision integral and the solution of this equation satisfying the boundary conditions of Eqs. (13) and (14) is

$$\bar{f}_0(z,\varepsilon)=\bar{D}\left(\frac{1}{2}+\frac{d-z}{l_i}\right). \tag{15}$$

If in the case of a pure metal the elastic mean free path is $l_i \gg L$, then in the emitter–collector region the carriers move along ballistic trajectories, which may have kinks due to the electron–phonon interaction. The main contribution to the maxima of the focusing lines is made by the group of effective electrons representing a small fraction $\sim(|eV|/\varepsilon_F)^{1/2}(b/L)^{3/2}\varkappa$ of all the emitter-injected carriers[16] [$\varkappa=\min((b/L)^{1/2},(|eV|/\varepsilon_F)^{1/2})$]; here and later we shall assume that $|eV|/\varepsilon_F \ll 1$. Since in our problem the nonequilibrium part f of the carrier distribution function is a "sharp" function of the momentum, and since we are planning later to analyze the amplitudes of the maxima of an electron focusing line, we can ignore the incoming terms[2)] in the collision integrals of Eqs. (4) and (6) and write down the solution of Eq. (2) for a pure metal in the form

$$f(\mathbf{r},\mathbf{p})=F(\mathbf{r}-\mathbf{r}(t))\exp\left\{-\int_{\lambda}^{t}\nu_{\mathbf{p}'}\,dt'\right\}-\varphi(\mathbf{r})\frac{\partial f_0}{\partial\varepsilon}$$

$$+\int_{\lambda}^{t}dt'\nu_{\mathbf{p}'}\frac{\partial f_0}{\partial\varepsilon}\exp\left\{-\int_{t'}^{t}dt''\nu_{\mathbf{p}''}\right\}\varphi(\mathbf{r}+\mathbf{r}(t')-\mathbf{r}(t)), \tag{16}$$

where

$$\mathbf{r}(t)=\int^{t}\mathbf{v}(t')\,dt',$$

t is the time of motion along a trajectory in a magnetic field, and $\lambda(\mathbf{r},\mathbf{p})\leqslant t$ is the time at which an electron is last reflected by the surface of a sample;

$$\nu_p=\tau_i^{-1}(\varepsilon)+\nu_{ep}(\mathbf{p}),$$
$$\nu_{ep}(\mathbf{p})=\sum_{\alpha}\int_{0}^{|\varepsilon-\varepsilon_F|}d\omega\langle w^{\alpha}_{\mathbf{p}-\mathbf{p}'}\delta(\omega-\omega^{\alpha}_{\mathbf{p}-\mathbf{p}'})\rangle_{\varepsilon(\mathbf{p}')=\varepsilon}; \tag{17}$$

$F(\mathbf{r}-\mathbf{r}(t))$ is an arbitrary function of the characteristic which is found using the boundary condition of Eq. (9) (see, for example, Ref. 22). Without analyzing the procedure for solving the equation of electrical neutrality and separating the nonmonotonic part $\tilde{\varphi}(\mathbf{H})$ of the dependence of the potential at the measuring contact on the magnetic field (see Refs. 23 and 24), which describes the electron focusing line, we give the final result:

$$\tilde{\varphi}(L,H)=\sum_{n=1}^{\infty}\varphi^{(n)}(L,H)=\frac{1}{e}\sum_{n=0}^{\infty}\int_{\varepsilon_F}^{\varepsilon_F+eV}d\varepsilon\,\bar{f}(z=d,\varepsilon)A_n(\varepsilon), \tag{18}$$

where

$$A_n(\varepsilon)=\frac{1}{\langle 1\rangle_{\varepsilon}}\left\langle\theta(L-\mathbf{n}\Delta\mathbf{R}(\mathbf{p})\in S_k)\left(1+2\frac{v_z}{v}\right)\right.$$

$$\left.\cdot q^{n-1}(\mathbf{p})\exp\left\{-n\int_{0}^{T_\lambda}dt'\nu_{\mathbf{p}'}\right\}\right\rangle_{\varepsilon,v_z>0}, \tag{19}$$

$\Delta\mathbf{R}(\varepsilon,T_\lambda\ p_x)$ is the displacement of an electron along the surface of a conductor in a time T_λ between two successive collisions with the surface, and p_x is the projection of the momenta along the magnetic field direction. The function $A_n(\varepsilon)$ is the partial contribution made to the focusing signal by electrons of energy ε, which are scattered in the bulk and on the surface of the metal; A_n assumes its maximum value after n specular reflections by the boundary carriers that can reach the collector having experienced an extremal displacement $\Delta\mathbf{R}_{extr}(\varepsilon)$ for a given energy ε. Each of the terms $\varphi^{(n)}(\mathbf{L},\mathbf{H})$ in the sum of Eq. (18) describes the amplitude and profile of the nth transverse electron focusing line.

If $|eV|/\varepsilon_F < b/L \ll 1$, when the geometric nonlinearity effects are absent,[15,16] we find that in the main approximation in terms of the small parameter b/L the position of the nth electron focusing maximum on the scale of the magnetic field H_n can be found from the condition

$$\mathbf{n}\Delta\mathbf{R}_{extr}(\varepsilon,T_{extr},P_x^{extr};H_n)=L \quad \text{for} \quad \varepsilon=\varepsilon_F, \tag{20}$$

where $T_{extr}(\varepsilon_F;p_x^{extr})$ is the time of motion of a carrier with the Fermi energy along a trajectory with an extremal displacement $\Delta\mathbf{R}_{extr}$. Using Eqs. (18) and (19), we can easily show that the ratio of the first and second derivatives (with respect to the emitter voltage V) of the amplitude $\varphi_{\max}$ of the nth electron focusing line is

$$(\partial^2\varphi^{(n)}_{max}/\partial V^2)\,(\delta\varphi^{(n)}_{max}/\partial V)^{-1}=eT_{extr}G_{extr}(eV). \tag{21}$$

The function $G_{extr}(\omega)$, defined by the expression

$$G_{extr}(\omega)=\frac{1}{T_{extr}}\int_{0}^{T_{extr}}dt\sum_{\alpha}\langle w^{\alpha}_{\mathbf{p}-\mathbf{p}'}\delta(\omega-\omega^{\alpha}_{\mathbf{p}-\mathbf{p}'})\rangle_{\varepsilon(\mathbf{p}')=\varepsilon_F}, \tag{22}$$

is the "cyclotron" EPI function representing the probability of electron–phonon scattering for a selected group of electrons characterized by the extremal size of the electron orbit in the momentum space. It should be stressed that in Eq. (22) the angular brackets represent averaging with respect to $\mathbf{p}'$ within the limits of the Fermi surface, whereas $\mathbf{p}$ is the momentum on an extremal trajectory which depends on the duration of motion t. The relationship given by Eq. (21) demonstrates that the cyclotron method can be used to investigate the EPI of certain electron states.

When electrons with an excess energy $|eV| \gtrsim \omega_D$ (ω_D is the phonon Debye frequency) are focused in a metal, the derivatives occurring in Eq. (21) are small because the inelastic electron–phonon mean free path l_{ep} is short. Although Eq. (21) is finite only in the limit $l_{ep}\to 0$, in practice it is very difficult to determine and the transverse electron focusing is effective as a method for the spectroscopy of the EPI only in the initial part of the spectrum where $l_{ep}(eV) \gtrsim L$. In the case of semimetals such as bismuth, theoretical[8] and experimental[9,15] investigations have shown that even at excess electron energies $|eV| > \omega_D$ the mean free path l_{ep} is fairly long ($l_{ep} \approx 10^{-2}-10^{-3}$ cm), so that focusing experiments can be used to reconstruct completely the "cyclotron" EPI function of Eq. (22). If $b/L \ll |eV|/\varepsilon_F \ll 1$, the phonon nonlinearities of the electron focusing signal are manifested against the background of the geometric nonlin-

earity effect which shifts the focusing line (when the polarity of the emitter voltage is such as to retard the electrons) and also gives rise to an additional term $a_n'(\varepsilon_F + eV)/a_n(\varepsilon_F + eV)$, which describes the change in the amplitude of the peak and occurs on the right-hand side of Eq. (21).

4. INELASTIC RELAXATION OF ELECTRONS IN THE EMITTER

1. We shall first consider weak relaxation of electrons through interaction with phonons in the emitter when $\lambda_\varepsilon \gg d$. We shall also assume that relaxation of carriers traversing a ballistic trajectory is unimportant, i.e.,

$$L/\min(l_i, l_{ep}) \ll d/\lambda_\varepsilon \ll 1 \tag{23}$$

and in the expression for the partial amplitude A_n given by Eq. (19) we can substitute $\nu_p = 0$. When the inequality of Eq. (23) is satisfied, the electron–phonon collision integral in Eq. (11) can be allowed for using perturbation theory. The inelastic correction $\bar{f}_1$ to the distribution function $\bar{f}_0$ of Eq. (15) has the following value for the emitter aperture

$$\bar{f}_1(d,\varepsilon) = -\frac{3}{4}\frac{d^2}{v}\left\{\bar{D}(\varepsilon) l_i^{-1}(\varepsilon) \int_0^{\varepsilon-\varepsilon_F} d\omega\, g(\omega, \varepsilon-\omega) - \int_0^{\varepsilon_F+eV-\varepsilon} d\omega g(\omega, \varepsilon+\omega)\bar{D}(\varepsilon+\omega) l_i^{-1}(\varepsilon+\omega)\right\}. \tag{24}$$

We shall consider the specific case when $eV > 0$. If the electron distribution function in the emitter is substituted in the form of the sum $\bar{f}_0(d,\varepsilon) + \bar{f}_1(d,\varepsilon)$ into the expression for the potential across the measuring contact given by Eq. (18), the inelastic correction to the electron focusing signal at the nth maximum is given by

$$\Delta\varphi_n^{max} \approx \frac{1}{e}\int_{\varepsilon_F}^{\varepsilon_F+eV} d\varepsilon A_n(\varepsilon)\bar{f}_1(d,\varepsilon). \tag{25}$$

Similarly, if $eV/\varepsilon_F \ll b/L \ll 1$, the second derivative of the amplitude of the electron focusing line with respect to the emitter voltage is

$$\frac{\partial^2\varphi_n^{max}}{\partial V^2} = \frac{e}{2}\bar{D}(\varepsilon_F)\frac{\partial A_n}{\partial \varepsilon_F}\left[1 - \frac{3}{2}\frac{d^2}{l_i(\varepsilon_F)v_F} g(eV) eV\right] \tag{26}$$

and contains the EPI function $g(\omega)$ [$g(\omega) \equiv g(\omega, \varepsilon_F)$, $v_F = v(\varepsilon_F)$]. The "background" independent of V can easily be separated on the basis of the signal amplitude beyond the phonon spectrum ($eV > \omega_D$). It should be pointed out that the function $g(\omega)$ does not contain the geometric form factor, which appears in the theory of point-contact spectroscopy;[11,12] this is due to the fact that the dependence of the amplitude of the electron focusing line on the shape of a contact is much weaker than the corresponding dependence of the resistance.

2. If $\lambda_\varepsilon \gtrsim d$, then perturbation theory cannot be applied to the electron–phonon collision integral. It is not possible to solve the integrodifferential equation (11) for arbitrary assumptions about the EPI function. We shall consider a model in which an allowance is made for the interaction with phonons of specific frequency; i.e., the EPI function is

$$g(\omega, \varepsilon) = w\nu(\varepsilon)\delta(\omega - \omega_0), \tag{27}$$

where $\nu(\varepsilon)$ is the density of electron states on the $\varepsilon(\mathbf{p}) = \varepsilon$ surface. Such a model describes well the process of, for example, inelastic relaxation of carriers by interaction with optical and intervalley phonons in semimetals, because in these materials the interaction of electrons with phonons is "localized" in small regions of the momentum space $\Delta q \sim p_F \ll q_D$ (p_F is the Fermi momentum in a semimetal and q_D is the Debye momentum of phonons). In this connection we should mention that experimental studies of the nonlinear effects in the case of electron focusing reported in Refs. 5, 9, 14, and 17 were carried out on bismuth.

Since the maximum energy of the electrons injected in the channel is $\varepsilon_v = \varepsilon_F + eV$, it follows that the distribution function $\bar{f}$ satisfies the obvious relationship $\bar{f}(\varepsilon > \varepsilon_v) \equiv 0$. On the other hand, if $\varepsilon < \varepsilon_v$, we have $\bar{f} \neq 0$. Therefore, at the boundary of the nonequilibrium band, $\varepsilon = \varepsilon_v$, there is an abrupt change in the distribution function $\Delta f_0 \equiv \bar{f}(\varepsilon_v - 0)$. The magnitude of the change is calculated in the Appendix [see Eq. (A2)] where it is also shown that a discrete structure of the energy relaxation process results in reproduction of this discontinuity at energies $\varepsilon_n = \varepsilon_v - n\omega_0$. The exact expression for the discontinuity $\Delta f_n \equiv \Delta f(\varepsilon_n)$ is fairly cumbersome [see Eq. (A6)]. However, it can be simplified when the inelastic relaxation length of electrons in the emitter $\lambda(\varepsilon)$ depends weakly on the energy. If the characteristic change $\Delta\lambda \approx \omega_0(\partial\lambda(\varepsilon)/\partial\varepsilon)$ in the relaxation length $\lambda(\varepsilon)$ satisfies the inequality

$$\Delta\lambda/\lambda(\varepsilon) \ll 1, \tag{28}$$

the ratio of the discontinuities of the distribution function at $z = d$ is given by the expression

$$\frac{\Delta f_n}{\Delta f_{n-1}} \approx \left\{\frac{\mathrm{ch}[(d/\lambda(\varepsilon_v))(1+\Delta\lambda/\lambda(\varepsilon_v))]}{\mathrm{ch}[d/\lambda(\varepsilon_v)]} - 1\right\}\frac{\lambda(\varepsilon_v)}{2\Delta\lambda}, \tag{29}$$

$$\lambda(\varepsilon) = 3w\nu(\varepsilon)v^{-1}(\varepsilon)l_i^{-1}(\varepsilon).$$

It is clear from Eq. (29) that the energy structure of the distribution function depends strongly on the parameter d/λ_ε. In the limit of weak electron–phonon relaxation the largest discontinuity of the distribution function is Δf_0 and is due to the energy edge of tunnel injection at $\varepsilon_v = \varepsilon_F + eV$. Successive phonon replicas of the discontinuity are proportional to powers of the parameter $(d/\lambda_\varepsilon)^2$ [see Eq. (29)], which reflect the low probability of the processes of successive emission of phonons in a channel. The most probable are one-phonon scattering processes discussed in subsection 1 of the present section. In the case of strong electron–phonon relaxation ($d/\lambda_\varepsilon \gg 1$) the number of nonequilibrium electrons falls exponentially with energy, so that discontinuities of the distribution function are strongest at lower energies. Therefore, the dependence of the jump Δf_n on its number is an important qualitative criterion of the intensity of the electron–phonon relaxation process at a point contact.

Substituting the expression for the function $\bar{f}$ at $z = d$ into Eq. (18), and differentiating with respect to the voltage V, we obtain

$$\frac{\partial\varphi}{\partial V} = \sum_{k=1}^{\infty}\sum_{n=0}^{N} A_k(\varepsilon_n)\Delta f_n, \qquad \varepsilon_n = \varepsilon_F + eV - n\omega_0. \tag{30}$$

If $b/L \ll \omega_0/\varepsilon_F < eV/\varepsilon_F < 1$, the maxima of the derivative of Eq. (30), which appear in fields defined by the relationship

$$k\Delta R_{y\,extr}(\varepsilon_n,\ H_{k,n})=L, \tag{31}$$

are separated on the magnetic field scale and their positions make it possible to find the phonon frequency ω_0. The relationship (31) reflects the spatial separation of the cyclotron orbits corresponding to different values of ε_n (trajectories 1 and 2 in Fig. 2). The existence of specific phonon relaxation energies is the result of discrete selection of the magnetic fields $H_{k,n}$ ensuring focusing of the relevant electrons in the collector. The ratio of the amplitudes of the derivative of the focusing signal at neighboring minima in the range $\omega_0 \ll \varepsilon_n$ is (Fig. 3)

$$\left(\frac{\partial\varphi}{\partial V}\right)_{H=H_{k,n}}\left(\frac{\partial\varphi}{\partial V}\right)^{-1}_{H=H_{k,n-1}}\approx\frac{\Delta f_n}{\Delta f_{n-1}}. \tag{32}$$

In accordance with Eq. (29), the approximate equality (32) can provide direct information on the energy dependence of the inelastic electron scattering length λ_ε.

Generalization of the result (30) to the case of two phonon frequencies ω_{01} and ω_{02} presents no fundamental difficulties, but it does lead to very cumbersome analytic expressions. We can show that the derivative $\partial\varphi/\partial V$ has extrema in fields $H_{k,ij}$ which are related to the combination frequencies $i\omega_{01}+j\omega_{02}$ by

$$k\Delta R_{y\,extr}(\varepsilon_F+eV-(i\omega_{01}+j\omega_{02}),\ H_{k,ij})=L,\quad i,j=0,1,2,\ldots \tag{33}$$

It is this system of maxima of the derivative with respect to the emitter current representing the electron focusing signal that was reported in bismuth in Ref. 14.

We now consider the case of an extremely strong inelastic relaxation of electrons in the channel, when the following inequality is satisfied:

$$\lambda_\varepsilon \ll b \ll d. \tag{34}$$

In this case the processes of multiphonon relaxation are concentrated in the direct vicinity of the tunnel injector at $z=0$. Electrons crossing the tunnel barrier acquire abruptly an excess energy eV, which directly after the passage of the barrier falls to a value $\Delta\varepsilon = eV - N\omega_0$ ($N=[eV/\omega_0]$) because of phonon emission. Consequently, all the electrons injected into a band of energies $[\varepsilon_F,\varepsilon_F+eV]$ are concentrated in an energy interval $[\varepsilon_F,\varepsilon_F+eV-N\omega_0]$ where the threshold electron–phonon relaxation mechanism is impossible. Further transport of charge in the channel occurs subject to conservation of the electron energy and is described by the diffusion equation $\partial^2\bar{f}/\partial z^2=0$. The boundary condition at $z=0$ describing multiphonon relaxation reflects conservation of a partial flux of electrons with the specific energy as a result of a relaxation transition accompanied by the emission of a phonon of frequency ω_0 (Ref. 26). This condition, in combination with the injection condition of Eq. (14), leads to

$$-\nu(\varepsilon)v(\varepsilon)l_i(\varepsilon)\frac{\partial\bar{f}}{\partial z}\Big|_{z=0} = \sum_{n=0}^{N}\nu(\varepsilon+n\omega_0)v(\varepsilon+n\omega_0)\bar{D}(\varepsilon+n\omega_0)\cdot\theta(\varepsilon_F+eV-n\omega_0-\varepsilon). \tag{35}$$

The boundary condition at the other end of the channel, where $z=d$, is given by Eq. (13), exactly as before. The solution of the problem is a function

$$\bar{f}(z,\varepsilon)=\left(\frac{1}{2}+\frac{d-z}{l_i(\varepsilon)}\right)\sum_{k=0}^{N}\theta(\varepsilon_F+eV-k\omega_0-\varepsilon)\bar{D}(\varepsilon+k\omega_0)\cdot\frac{\nu(\varepsilon+k\omega_0)v(\varepsilon+k\omega_0)}{\nu(\varepsilon)v(\varepsilon)}. \tag{36}$$

Using this function in Eq. (18) for the potential on the measuring contact and differentiating $\varphi(V)$ with respect to the emitter voltage, we obtain

$$\frac{\partial\varphi}{\partial V}=\sum_{n=1}^{\infty}A_n(\varepsilon_F+eV-N\omega_0)\bar{D}(\varepsilon_F+eV)\cdot\frac{\nu(\varepsilon_F+eV)v(\varepsilon_F+eV)}{\nu(\varepsilon_F+eV-N\omega_0)v(\varepsilon_F+eV-N\omega_0)}. \tag{37}$$

The derivative in Eq. (37) is a maximum in fields $H_n(\varepsilon_F+eV-N\omega_0)$ satisfying Eq. (20) when $\varepsilon=\varepsilon_F+eV-N\omega_0$. An increase in the voltage shifts the maximum of $\partial\varphi/\partial V$ toward higher magnetic fields right up to the value $V=V^*$, such that $eV^*-N\omega_0=\omega_0$. At $V=V^*$ the maximum disappears in a magnetic field $H_n(\varepsilon_F+\omega_0)$, but simultaneously a maximum $H_n(\varepsilon_F)$ in the magnetic field appears. As V is increased, maxima appear at $H=H_n(\varepsilon_F)$ shift along the magnetic field scale to $H=H_n(\varepsilon_F+\omega_0)$, and disappear at $\partial\varphi/\partial V$ periodically.

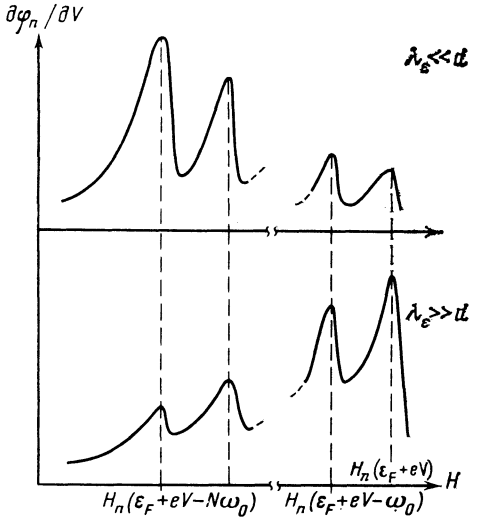

FIG. 3. Qualitative dependence of the electron focusing signal $\partial\varphi/\partial V$ on the magnetic field in the case when $b/L \ll \omega_0/\varepsilon_F < eV/\varepsilon_F \ll 1$ under conditions of strong ($\lambda_\varepsilon \ll d$) and weak ($\lambda_\varepsilon \gg d$) electron–phonon relaxation in the emitter. The fields corresponding to the maxima $H_n(\varepsilon_F+eV-k\omega_0)=H_{n,k}$ satisfy Eq. (31).

5. TRANSVERSE FOCUSING UNDER CONDITIONS OF TRAJECTORY AND EMITTER ELECTRON–PHONON RELAXATION

In the preceding section we demonstrated that the electron–phonon relaxation process in the emitter results in a

periodic rise of the signal $\partial\varphi/\partial V$ associated with redistribution of the electron energies. On the other hand, allowance for the electron relaxation on a cyclotron trajectory of motion from the emitter to the collector reduces the value of $\partial\varphi/\partial V$ as the excess carrier energy eV increases. Therefore, we can expect the simultaneous action of these two factors to give rise to a nonmonotonic dependence of the electron focusing signal on the emitter voltage. This is particularly interesting in connection with the experimentally detected[13] nonmonotonic behavior of $\partial\varphi/\partial V$ as a function of V. We shall consider the limiting case when

$$\lambda_\varepsilon \gg d, \quad l_{ep} \lesssim L. \tag{38}$$

These inequalities correspond to strong relaxation along a ballistic trajectory (see Sec. 3) and also make it possible to allow for the relaxation process in the emitter using perturbation theory (see Sec. 4.1). Calculations fully analogous to those in Secs. 3 and 4 yield

$$\frac{\partial\varphi_n^{max}}{\partial V} = \frac{1}{2}\bar{D}(\varepsilon_F+eV)A_n(\varepsilon_F+eV)$$
$$\cdot\left[1-\frac{3}{2}\frac{d^2}{l_i(\varepsilon_F+eV)v(\varepsilon_F+eV)}\int_0^{eV}d\omega\, g(\omega)\right]$$
$$+\frac{3}{4}\bar{D}(\varepsilon_F+eV)\frac{d^2}{l_i(\varepsilon_F+eV)}$$
$$\cdot\int_0^{eV}\frac{d\omega}{v(\varepsilon_F+eV-\omega)}g(\omega)A_n(\varepsilon_F+eV-\omega), \tag{39}$$

where the function $A_n(\varepsilon)$ is described by Eq. (19). The relationship (39) can be simplified greatly in the case of metals characterized by $eV\sim\omega_D \ll \varepsilon_F$ (it should be noted that the experiments reported in Ref. 13 were carried out using silver). The dominant dependence on the voltage V in $\partial\varphi_n^{max}/\partial V$ is governed by the energy dependence of the electron–phonon relaxation time l_{ep}. This dependence can be separated explicitly by measurement of a normalized quantity $(\partial\varphi_n^{max}(V)/\partial V)(\partial\varphi_n(0)/\partial V)^{-1}$. If $eV/\varepsilon_F \ll b/L$, we obtain

$$\frac{\partial\varphi_n^{max}(V)}{\partial V}\Big/\frac{\partial\varphi_n^{max}(0)}{\partial V}$$
$$\approx \exp\left[-\frac{L}{l_{extr}(eV)}\right]\left\{1-\frac{3}{2}\frac{d^2}{l_i(\varepsilon_F)l_{ep}(eV)}\right\}$$
$$+\frac{3}{2}\frac{d^2}{l_i(\varepsilon_F)l_{ep}(eV)}\exp\left[-\frac{L}{l_{extr}(eV-\varepsilon^*)}\right], \tag{40}$$

where

$$l_{extr}^{-1}(\varepsilon)=\frac{1}{v_F}\int_0^{\varepsilon}d\omega\, G_{extr}(\omega),$$
$$l_{ep}^{-1}(\varepsilon)=\frac{1}{v_{F_0}}\int_0^{\varepsilon}d\omega\, g(\omega),$$

and the EPI functions $G_{extr}(\omega)$ and $g(\omega)$ are defined by Eqs. (22) and (12).

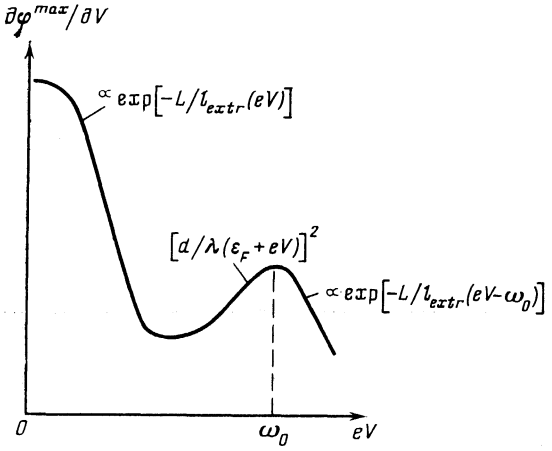

FIG. 4. Schematic representation of the dependence of the amplitude of the electron focusing signal $\partial\varphi^{max}/\partial V$ at its maximum on the emitter voltage V in the case of relaxation of electrons by interaction with phonons in the emitter and along a ballistic motion trajectory.

The interpolation relationship (40) yields the correct asymptotic expressions in the cases when $eV \gg \omega_0$ and $eV \ll \omega_0$, where ω_0 is the characteristic phonon frequency at which the EPI function $g(\omega)$ has a maximum. We then have

$$\varepsilon^*(eV)=\begin{cases} eV, & eV\ll\omega_0 \\ \omega_0, & eV\gg\omega_0 \end{cases}.$$

In the range of intermediate values of eV the relationship (40) describes qualitatively the dependence $\partial\varphi_n^{max}(V)/\partial V$ and approaches the exact description as $g(\omega)$ in Eq. (39) approaches the δ-function form. Only two terms in Eq. (40) contain exponential factors which depend on the relaxation length at shifted energies $l_{extr}(eV)$ and $l_{extr}(eV-\varepsilon^*)$. If $l_{extr}(\varepsilon)$ is a decreasing function of the energy, we reach the conclusion that in spite of the weakness of the relaxation in the emitter $(d/\lambda_\varepsilon \ll 1)$, the second term may be responsible for the considerable contribution to the EPI signal in the range of excess energies $eV\sim\omega_0$, where the relaxation-induced suppression of the first term is significant. This contribution depends nonmonotonically on the emitter bias V, increasing at low voltages $(eV\ll\omega_0)$ and falling exponentially in the range of high values of V $(eV>\omega_0)$. The energy dependence of the signal $\partial\varphi_n^{max}(V)/\partial V$ is shown qualitatively in Fig. 4. The dependence of the EPI signal $\partial\varphi_n^{max}(V)/\partial V$ obtained in this way is in good agreement with the experimental results of Ref. 13.

6. CONCLUSIONS

Investigations of the nonlinear electrical conductivity of metallic point contacts in the presence of a magnetic field[14,28] extend greatly the opportunities for studying the EPI in metals. Our results show that the method of transverse focusing of electrons makes it possible to study the electron–phonon relaxation processes. Such relaxation of high-energy electrons reduces the focusing signal representing the scattering of carriers by phonons in the course of motion on a cyclotron orbit. It is possible to study the EPI for selected electron groups characterized by extremal dimensions of the cyclotron orbits [Eq. (21)]. If we compare these results with the potentialities of point-contact spectroscopy in the absence of a magnetic field,[10–12] we reach the

conclusion that the method of transverse relaxation is highly directional and stresses the contribution of specific electron states to the relaxation process.

The ability to use the method of electron focusing by a magnetic field is important in the study of the electron–phonon relaxation processes which accompany successive emission of more than one phonon (multiphonon relaxation). At first sight this possibility is in conflict with the ballistic nature of the method (even a one-phonon process removes an electron from the number of those that reach the collector) and is realistic under conditions of spatial localization of the relaxation phenomena occurring in a strongly contaminated vicinity of the emitter contact. When one-phonon processes are important, the second derivative of the focusing signal with respect to the emitter voltage is related in a simple manner to the thermodynamic EPI function [Eq. (26)], which differs from the corresponding function for point contacts[11,12] by the absence of the transport form factor of the electron–phonon scattering process. This is due to the fact that the focusing method is not based on the transport effects, but on the distribution of the density of nonequilibrium electrons injected from the point contact. Our analysis shows that strong relaxation of electrons in the emitter is accompanied by successive emission of more than one phonon by "hot" electrons, giving rise to discontinuities in the energy distribution of carriers. Discrete relaxation by interaction with phonons gives rise to a system of maxima in the case of the first derivative of the amplitude of the transverse focusing line with respect to the voltage. Analysis of the resultant structure makes it possible to determine the energy dependence of the inelastic relaxation length and of the frequency of relaxation phonons in metals with low carrier concentrations.

Point electrical contacts, which are small strongly contaminated regions, allow transverse focusing of electrons by a magnetic field to be ulitized as a method for investigating spatially localized strongly nonequilibrium states of quasiparticles in a solid.

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APPENDIX: DETERMINATION OF "PHONON JUMPS" OF THE DISTRIBUTION FUNCTION OF ELECTRONS IN THE EMITTER

In the case of relaxation due to interaction with dispersion-free phonons [Eq. (27)] the integrodifferential equation (11) is replaced by a system of differential–difference equations[27]:

$$\frac{\partial^2 \bar{f}(\varepsilon)}{\partial z^2} - \lambda_-^{-2}\bar{f}(\varepsilon)(1-\delta_{nN}) + \lambda_+^{-2}\bar{f}(\varepsilon+\omega_0)(1-\delta_{n0}) = 0 \quad \text{(A1)}$$

if

$$\varepsilon_F + eV - (n+1)\omega_0 \leqslant \varepsilon \leqslant \varepsilon_F + eV - n\omega_0, \quad n=0, 1 \ldots N,$$

where

$$\lambda_\pm^{-2}(\varepsilon) = 3w\nu(\varepsilon \pm \omega_0)v^{-1}(\varepsilon)l_i^{-1}(\varepsilon),$$

$N = [eV/\omega_0]$ is the maximum number of phonons which an electron can emit.

The magnitude of the first jump $\Delta f_0 = \bar{f}(\varepsilon_v - 0)$ of the electron distribution function can be found by direct solution of Eq. (A1) and the boundary of a band and, after allowance for the boundary conditions of Eqs. (13) and (14), this magnitude is given by

$$\Delta f_0 \equiv \bar{f}(\varepsilon_V - 0, z) = \bar{D}(\varepsilon_V)\left[\frac{1}{2}\operatorname{ch}\frac{d-z}{\lambda_-(\varepsilon_V)} + \frac{\lambda_-(\varepsilon_V)}{l_i(\varepsilon_V)}\operatorname{sh}\frac{d-z}{\lambda_-(\varepsilon_V)}\right] \times\left[\operatorname{ch}\frac{d}{\lambda_-(\varepsilon_V)} + \frac{l_i(\varepsilon_V)}{2\lambda_-(\varepsilon_V)}\operatorname{sh}\frac{d}{\lambda_-(\varepsilon_V)}\right]^{-1}. \quad \text{(A2)}$$

The nonlocal (with respect to the energy) nature of Eq. (A1) is responsible for the appearance of the jumps Δf_n of the distribution function at discrete energies $\varepsilon_n = \varepsilon_v - n\omega_0$ The magnitudes of these jumps Δf_n satisfy a recurrence relationship which can be derived using the Green's function $G(\varepsilon;z,z')$ of the operator $\hat{L} = \{\partial^2/\partial z^2 - \lambda_-^{-2}(\varepsilon)\}$ [see Eq. (A1) and the boundary conditions of Eqs. (13) and (14)]:

$$\Delta f_n(z) = \lambda_+^{-2}(\varepsilon_n)\int_0^d d\xi G(\varepsilon_n; z, \xi)\Delta f_{n-1}(\xi), \quad n=1,2\ldots N, \quad \text{(A3)}$$

where

$$G(\varepsilon; z, \xi) = \frac{\lambda_-}{\operatorname{ch}\frac{d}{\lambda_-} + \frac{l_i}{2\lambda_-}\operatorname{sh}\frac{d}{\lambda_-}} \times \begin{cases} \operatorname{ch}\frac{z}{\lambda_-}\left[\operatorname{sh}\frac{d-\xi}{\lambda_-} + \frac{l_i}{2\lambda_-}\operatorname{ch}\frac{d-\xi}{\lambda_-}\right], & 0 \leqslant z \leqslant \xi \\ \operatorname{ch}\frac{\xi}{\lambda_-}\left[\operatorname{sh}\frac{d-z}{\lambda_-} + \frac{l_i}{2\lambda_-}\operatorname{ch}\frac{d-z}{\lambda_-}\right] & \xi \leqslant z \leqslant d. \end{cases} \quad \text{(A4)}$$

It should be noted that since the functions Δf_0 and G are essentially positive, the jumps Δf_n of the function $\bar{f}(z,\varepsilon)$ are also positive.

The solution of the recurrence relationship of Eq. (A3) can be simplified greatly in the limit $l_i/\lambda_\varepsilon \ll 1$. In the zeroth approximation with respect to the parameter l_i/λ_s at all the internal points of the channel ($d - z \gg l_i$) we obtain an equation

$$\int_0^d d\xi G(\varepsilon_n; z\xi)\frac{\operatorname{sh}[(d-\xi)/\lambda_-(\varepsilon_k)]}{\operatorname{ch}(d/\lambda_-(\varepsilon_k))} = \alpha_{nk}\operatorname{sh}\frac{d-z}{\lambda_-(\varepsilon_k)} + \alpha_{kn}\operatorname{sh}\frac{d-z}{\lambda_-(\varepsilon_n)}, \quad \text{(A5)}$$

which makes it possible (as is readily shown) to write down the solution of the recurrence equation of Eq. (A3) at $z = d$ in the form

$$\Delta f_n = \bar{D}(\varepsilon_V)\frac{l_i(\varepsilon_n)}{2l_i(\varepsilon_V)}\prod_{s=1}^{n}\lambda_+^{-2}(\varepsilon_s) \cdot \sum_{k=0}^{n-1} B_k^{(n)}\frac{\operatorname{ch}^{-1}(d/\lambda_-(\varepsilon_n)) - \operatorname{ch}^{-1}(d/\lambda_-(\varepsilon_k))}{\lambda_-^{-2}(\varepsilon_n) - \lambda_-^{-2}(\varepsilon_k)},$$

$$\varepsilon_n = \varepsilon_F + eV - n\omega_0, \quad n=1, 2, 3\ldots \quad \text{(A6)}$$

The coefficients $B_k^{(n)}$ satisfy algebraic relationships

$$B_k^{(n+1)} = \alpha_{nk} B_k^{(n)}, \quad k \neq n; \quad B_n^{(n+1)} = \sum_{s=0}^{n-1} \alpha_{sn} B_s^{(n)}; \quad B_0^{(1)} = 1, \tag{A7}$$

$$\alpha_{ik} = \frac{\operatorname{ch}^{-1}(d/\lambda_-(\varepsilon_i))}{\lambda_-^{-2}(\varepsilon_i) - \lambda_-^{-2}(\varepsilon_k)}. \tag{A8}$$

It follows from Eq. (A8) that if λ_- is independent of energy, then $\alpha_{ik} \to \infty$. However, we can easily show that in the limit $\lambda_-(\varepsilon_i) \to \lambda_-(\varepsilon_k)$ the combination $\gamma = \alpha_{ik} + \alpha_{ki}$ remains finite. Consequently, the magnitude of the jump Δf_n of Eq. (A6) is governed by a sum

$$S_n = \sum_{k=0}^{n-1} B_k^{(n)},$$

which—as can be shown on the basis of Eq. (A7)—is equal to $S_n = \gamma^{n-1}$ and is also finite. If $eV \ll n\omega_0$, then in the functions which occur in Eq. (A6) we can ignore the dependence on the relaxation frequency $\omega_0(\lambda_+(\varepsilon) \equiv \lambda_-(\varepsilon) = \lambda_\varepsilon)$ and Δf_n is given by the expression

$$\Delta f_n \approx \frac{\bar{D}(\varepsilon_V)}{2} \operatorname{ch}^{-1} \frac{d}{\lambda(\varepsilon_V)} \left(\frac{d}{2\lambda(\varepsilon_V)} \operatorname{th} \frac{d}{\lambda(\varepsilon_V)} \right)^n,$$

$$\varepsilon_V = \varepsilon_F + eV. \tag{A9}$$

[1)]The possibility of a "bottleneck" in the processes of reabsorption of nonequilibrium phonons in semimetals is discussed in Ref. 19.

[2)]An analysis shows that allowance for these terms corresponds to the contribution of "ineffective" electrons to the amplitude of the focusing signal. This contribution determines the monotonic component of the electron focusing signal depending weakly on the magnetic field.

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