Magnetoelastic excitation of inhomogeneous magnetization waves in ferromagnets by a uniform time-dependent magnetic field

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Spatially nonuniform magnetoelastic oscillations (standing waves) are analyzed for the case of a normally magnetized ferromagnetic plate near the magnetic spin-flip transition. The local and average magnetic susceptibilities in a uniform time-dependent field are calculated with allowance for elastic and spin relaxation. Conditions are found for excitation of trigonometric and hyperbolic waves and some new resonances are predicted. Amplification of NMR by elastomagnetic waves is considered.

INTRODUCTION

When a magnetoelastic interaction is present, one finds that a uniform time-dependent magnetic field $\mathbf{h}(t)$ can excite spatially nonuniform oscillations in the magnetization $\mathbf{M}$ of a ferromagnetic plate which appear as a component of the coupled magnetoelastic waves. Unlike ordinary spin-wave resonance, excitation can occur even in the absence of surface magnetic anisotropy, because the magnetoelastic coupling depends only on the boundary conditions. It should be stressed that such magnetoelastic waves [excited by a uniform field $\mathbf{h}(t)$] can be generated at frequencies both above and below the gap ω_0 in the spin-wave spectrum. In either case, a uniform field can excite acoustic waves when the frequency of the field $\mathbf{h}(t)$ divided by the plate thickness takes on certain values.

A systematic theory of magnetoelastic wave excitation under these conditions was first developed by Tiersten in Ref. 1. However, the magnetoelastic effects associated with spontaneous symmetry breaking[2] were neglected there and in subsequent work. These effects give rise to a magnetoelastic gap in the spectrum of the quasimagnon (spin-wave) mode and to a softening of the quasiphonon (acoustic) mode near the magnetic spin-flip transition. These effects are insignificant far from the transition and can legitimately be neglected there, but near it they can produce qualitatively new behavior. One of our goals in the present paper is to examine this in more detail.

It is also of interest to calculate and study in detail the magnetic susceptibility of a ferromagnetic plate in a field $\mathbf{h}(t)$ for various frequency ranges (this was not done in Ref. 1). This will enable us to find the conditions needed for excitation and elucidate the role played by trigonometric and hyperbolic functions, respectively. In our case the trigonometric and hyperbolic waves have a distinctive resonance where their spectral branches cross (this occurs at an imaginary frequency, since the wave vector for the hyperbolic waves is imaginary).

By considering the spontaneous symmetry breaking, we succeed in finding a precise condition for the frozen lattice model to be valid; this model has been used previously to analyze magnetoelastic oscillations (see the reviews in Refs. 2, 3). The critical film thickness, below which the magnetoelastic gap in the spin-wave spectrum disappears, can be found by analyzing the susceptibility of a film as the thickness tends to zero.

The paper closes with a discussion of how the magnetoelastic waves influence the NMR properties of a film; we will see that they provide another channel, in addition to uniform forced oscillations in the magnetization, through which the magnetic field can affect the nuclear spins.

ENERGY AND GROUND STATE

We consider a ferromagnetic plate bounded by the planes $z = L$ and $z = -L$, with $\mathbf{z}\|[001]$. The crystal structure is cubic, and the magnetization $\mathbf{M}_0$ and the wave vector $\mathbf{k}$ of the oscillations are also parallel to the z axis: $\mathbf{M}_0\|\mathbf{H}_0\|\mathbf{k}\|\mathbf{z}$. The total energy density (magnetic, elastic, and magnetoelastic) is then given by

$$\begin{aligned}F(\mathbf{r}) = {} & \frac{1}{2}E'a^2M_0^{-2}\frac{\partial\mathbf{M}}{\partial z}\frac{\partial\mathbf{M}}{\partial z} + K_1M_0^{-2}(M_x^2M_y^2+\ldots)\\ & +2\pi M_z^2-\mathbf{MH}\\ & +\frac{1}{2}C_{11}(e_{xx}^2+\ldots)+2C_{44}(e_{xy}^2+\ldots)+C_{12}(e_{xx}e_{yy}+\ldots)\\ & +B_1M_0^{-2}(M_x^2e_{xx}+\ldots)+2B_2M_0^{-2}(M_xM_ye_{xy}+\ldots). \quad (1)\end{aligned}$$

Here the dots indicate terms which are obtained from the first term in the parentheses by cyclic permutation of the indices x, y, z; $\mathbf{H}\equiv\{h_xe^{-i\omega t}, h_ye^{-i\omega t}), H_0\}$ is the magnetic field, which includes in addition to $\mathbf{H}_0$ the uniform alternating field $\mathbf{h}e^{-i\omega t}$; $e_{\alpha\beta} = (\partial u_\alpha/\partial x_\beta + \partial u_\beta/\partial x_\alpha)/2$ is the deformation tensor, where u is the displacement vector. The significance of the various constants is clear.

Because of the magnetoelastic interaction (the terms B_1 and B_2 in (1)), spontaneous deformations $e_{\alpha\beta}^{(0)}$ are present in the ground state, which is obtained by minimizing the total energy $\int F(\mathbf{r})dV$; their explicit form is given, e.g., in Ref. 3. The ground state is stable if

$$\tilde{H}\equiv H_0-4\pi M_0+2K/M_0\geqslant 0, \quad (2)$$

where $K = K_1 + B_1^2/(C_{11}-C_{12}) - B_2^2/2C_{44}$ is the magnetic anisotropy constant, after renormalization for the magnetostriction. Equality in (2) holds at the spin-flip transition, at which the state with $\mathbf{M}_0\|\mathbf{z}$ becomes unstable. We will seek solutions having the form of small oscillations $\Delta\mathbf{M}\equiv\{M_x, M_y\}$ and $\Delta e_{\alpha\beta}\equiv\{(\partial u_x/\partial z, \partial u_y/\partial z\}$ about the ground state which are generated by the field $\mathbf{h}e^{-i\omega t}$, with allowance for the boundary conditions.

BOUNDARY CONDITIONS AND THE SUSCEPTIBILITY: GENERAL SOLUTION

Because of the assumed symmetry of the system (and hence of the ground state), the problem reduces to solving

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the coupled equations of motion

$$\dot{\mathbf{M}}=\gamma\left[\mathbf{M}\frac{\delta F}{\delta \mathbf{M}}\right]-\frac{r}{M_0}[\mathbf{M}\dot{\mathbf{M}}],$$
$$\rho\ddot{\mathbf{u}}=\frac{\partial}{\partial z}\left(\frac{\partial F}{\partial(\partial \mathbf{u}/\partial z)}\right)+\eta_{44}\frac{\partial^2\dot{\mathbf{u}}}{\partial z^2} \qquad (3)$$

for the cyclic variables $M_\pm = M_x \pm iM_y$ and $u_\pm = u_x \pm iu_y$. Here r and η_{44} are the damping parameters for the magnetic and elastic subsystems, respectively. We first find a solution for zero damping ($r - \eta_{44} = 0$) and then indicate how the result changes when $r\neq 0$ and $\eta_{44}\neq 0$.

Linearizing in the cyclic variables, we find that Eqs. (3) reduce to the inhomogeneous system

$$(\pm\omega+\omega_0-\omega_E a^2D^2)M_\pm+\gamma B_2 Du_\pm=\gamma M_0 h_\pm,$$
$$(B_2/\rho)DM_\pm+(\omega^2+\omega_D{}^2a^2D^2)M_0u_\pm=0 \qquad (4)$$

Here $D\equiv\partial/\partial z$, $\omega_E = E'/M_0$ is the exchange frequency, $\omega_D \equiv s/a = (C_{44}/\rho a^2)^{1/2}$ is a characteristic frequency comparable to the Debye frequency for the elastic subsystem (s is the speed of sound), and

$$\omega_0=\tilde{\omega}_0+\omega_{\mathrm{ME}} \qquad (5)$$

is the ferromagnetic resonance frequency (uniform precession frequency); $\tilde{\omega}_0 = \gamma\tilde{H}$ and $\omega_{\mathrm{ME}} = \gamma B_2{}^2/M_0C_{44}$. We have $\tilde{\omega}_0 = 0$ and $\omega_0 = \omega_{\mathrm{ME}}$ at the spin-flip transition.

Equations (4) must be solved subject to suitable boundary conditions. We will assume that there is no surface magnetic anisotropy[1] and no surface strain; these give the boundary conditions

$$DM_\pm|_{z=\pm L}=0, \qquad (6)$$

and

$$(C_{44}Du_\pm+B_2M_\pm/M_0)_{z=\pm L}=0, \qquad (7)$$

respectively. Note that we specifically consider the condition (6), under which a field $\mathbf{h}e^{-i\omega t}$ with $\mathbf{h}$ = const cannot excite nonuniform oscillations $M_\pm$ in a purely magnetic system. If such waves are generated in our case, it follows that they must be due entirely to the magnetoelastic interaction.

The substitution

$$M_\pm=\frac{\gamma M_0h_\pm}{\pm\omega+\omega_0}+iM_0\psi_\pm, \qquad (8)$$

$$Du_\pm=-\frac{iM_0}{\gamma B_2}(\pm\omega+\omega_0-\omega_Ea^2D^2)\psi_\pm \qquad (9)$$

reduces the inhomogeneous system (4) with homogeneous boundary conditions (6), (7) to the homogeneous equation

$$\left\{\omega_E(aD)^4-\left(\pm\omega+\omega_0-\frac{\omega^2\omega_E}{\omega_D{}^2}\right)(aD)^2-\frac{\omega^2(\pm\omega+\omega_0)}{\omega_D{}^2}\right\}\psi_\pm=0 \qquad (10)$$

with inhomogeneous boundary conditions

$$-(\pm\omega+\tilde{\omega}_0-\omega_Ea^2D^2)\psi_\pm|_{z=\pm L}=i\frac{\omega_{\mathrm{ME}}\gamma h_\pm}{\pm\omega+\omega_0}. \qquad (11)$$

We may discard the $\pm$ sign in front of ω in (10) and suppress the $\pm$ subscripts on ψ [and also on h in Eq. (11)] by formally setting $\psi\equiv\psi_-$, $h\equiv h_-$ for $\omega>0$ (right-polarized oscillations) and $\psi\equiv\psi_+$, $h\equiv h_+$ for $\omega<0$ (left-polarized oscillations). Inserting

$$\psi=Ce^{ikz}$$

in (10), we then get a dispersion equation which can be solved to give two solutions for $k^2 = k^2(\omega)$:

$$2\omega_E(ak_{1,2})^2=\omega-\tilde{\omega}_0+\frac{\omega^2\omega_E}{\omega_D{}^2}\pm\left[\left(\omega-\tilde{\omega}_0-\frac{\omega^2\omega_E}{\omega_D{}^2}\right)^2+4\frac{\omega^2\omega_E\omega_{\mathrm{ME}}}{\omega_D{}^2}\right]^{1/2}. \qquad (12)$$

The general solution for ψ has the form

$$\psi=C_1e^{ik_1z}+C_2e^{-ik_1z}+C_3e^{ik_2z}+C_4e^{-ik_2z},$$

where the constants C_i are calculated from the boundary conditions (11). The final result for the forced component of the oscillations ψ, proportional to h, is found to be

$$\psi=i\left(\frac{\omega_{\mathrm{ME}}\gamma h}{\omega_0-\omega}\right)\frac{\cos k_1z/k_1\sin k_1L-\cos k_2z/k_2\sin k_2L}{A(k_1)-A(k_2)}, \qquad (13)$$

where

$$A(k)=[\omega-\tilde{\omega}_0-\omega_E(ak)^2]k^{-1}\operatorname{ctg}kL, \qquad (14)$$

and k_1,k_2 are given by Eqs. (12). Substituting this expression for ψ into (8) and (9) with $h\equiv h_-$, $\omega>0$ ($\psi\equiv\psi_-$), or with $h\equiv h_+$, $\omega<0$ ($\psi\equiv\psi_+$), we obtain the magnetization and deformation for the right- and left-polarized waves, respectively.

We see from Eqs. (8) and (13) that the first term in $M_\pm$ gives the homogeneous part of the magnetization, which has a resonance at the spin-flip transition frequency ω_0, while the second term (proportional to ψ) gives the inhomogeneous component. In addition to the resonance at ω_0, the latter has resonances at frequencies for which

$$A(k_1)=A(k_2). \qquad (15)$$

It has the crucial property that it vanishes when no magnetoelastic interaction is present ($\omega_{\mathrm{ME}} = 0$).

Whether the nonuniform oscillations are trigonometric or hyperbolic depends on the sign of k^2 in (12) for each of the terms in (13). It can be shown without difficulty that $k_1^2>0$ for all frequencies $-\infty<\omega<\infty$, whereas $k_2^2>0$ for $\omega>\omega_0$ and $k_2^2<0$ for $\omega<\omega_0$ (so that $-k_2^2\equiv f^2>0$, $\cos(k_2z)\equiv\cosh(fz)$, $k_2\sin(k_2L)\equiv-f\sinh(fL)$ and $k_2^{-1}\cos(k_2L)\equiv-f^{-1}\coth(fL)$.

In the first case ($\omega>\omega_0$), ψ is a superposition of two trigonometric waves $\cos(k_1z)$ and $\cos(k_2z)$, while in the second case ($\omega<\omega_0$) it is a combination of the trigonometric and hyperbolic waves $\cos(k_1z)$ and $\cosh(fz)$.

We observe finally that according to Eqs. (3), the damping can be treated by making the replacements

$$\tilde{\omega}_0\to\tilde{\omega}_0-i\omega r,\quad \omega_D{}^2\to\omega_D{}^2(1-i\omega\eta_{44}/C_{44}) \qquad (16)$$

in Eqs. (4), (5) and then in Eqs. (8)–(14). In this case the resonance condition (15) applies to the real part $\mathrm{Re}A(k)$.

DISPERSION CURVES AND THE AVERAGE SUSCEPTIBILITY

Figure 1 shows the curves $k_{1,2}(\omega)$ found using (12) near and at the spin-flip transition (a: $\tilde{\omega}\gtrsim\omega_{\mathrm{ME}}$, and b: $\tilde{\omega}_0=0$,

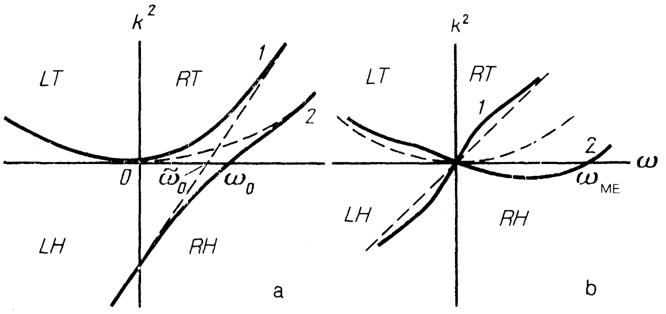

FIG. 1. Dispersion curves $k^2_{1,2} = k_2^{1,2}(\omega)$ near (a) and at (b) the transition point ($\tilde{\omega}_0 \gtrsim \omega_{ME}$ and $\tilde{\omega}_0 = 0$, respectively). The left- and right-polarized waves are labeled L and R, respectively; T, H stand for trigonometric and hyperbolic.

$\omega_0 = \omega_{ME}$, respectively). Frequencies $\omega > 0$ and $\omega < 0$ correspond to right- and left-polarized waves. The curves in the upper and lower halfplanes ($k_2 \equiv -f^2 > 0$ and < 0, respectively) correspond to trigonometric and hyperbolic waves. The dashed lines show the dispersion curves for noninteracting waves ($\omega_{ME} = 0$). We note that hyperbolic waves are generated only below the ferromagnetic resonance frequency, $\omega < \omega_0$.

We will now analyze the resonance between the inhomogeneous trigonometric and hyperbolic waves, given by Eq. (15), in somewhat greater detail for several frequency ranges. The average susceptibility

$$\chi = \frac{1}{2Lh}\int_{-L}^{L} M(z)\,dz = \frac{\gamma M_0}{\omega_0-\omega}\left\{1+\frac{\omega_{ME}(k_1^2-k_2^2)}{Lk_1^2k_2^2[A(k_1)-A(k_2)]}\right\} \tag{17}$$

can be calculated from Eqs. (8) and (13) ($\chi \equiv \chi_-$ for $\omega > 0$ and $\chi \equiv \chi_+$ for $\omega < 0$).

We first simplify this result in the limit of very thick films, for which $|Lk_{1,2}| \ll 1$ (for thick plates, $L \to \infty$, the inhomogeneous term in (17) vanishes). Expanding $A(k_1)$-$A(k_2)$ as a series in these variables, we find from (17) that

$$\chi = \frac{\gamma M_0}{\omega_0-\omega}\left\{1+\frac{\omega_{ME}}{\tilde{\omega}_0-\omega-\omega_E(aL)^2k_1^2k_2^2/3}\right\} = \frac{\gamma M_0}{\tilde{\omega}_0-\omega-\omega_{ME}[1-(\pi^2/12)(\omega^2/\omega_L^2)]^{-1}}. \tag{18}$$

Here

$$\omega_L = \omega_D(\pi a/2L) \tag{19}$$

is the minimum frequency for transverse standing elastic waves in a plate of thickness $2L$. If

$$\omega \ll \omega_L \tag{20}$$

Eq. (18) gives

$$\chi \approx \gamma M_0/(\tilde{\omega}_0-\omega). \tag{18'}$$

This result coincides with the homogeneous susceptibility (no magnetoelastic gap); thus in the present case the gap disappears and the frozen lattice model[3] breaks down for ferromagnets of finite dimensions when (20) is satisfied.

Let us further consider the frequencies for which

$$|(\omega-\tilde{\omega}_0)/\omega| \gg |\omega\omega_E/\omega_D^2|, \tag{21}$$

which excludes an interval of length $\lesssim 10^{-2}$ in the immediate vicinity of the frequency $\tilde{\omega}_0$, which according to Fig. 1 corresponds to magnetoacoustic resonance. (The case $\omega \approx \tilde{\omega}_0$ will be considered separately.) We let the plate thickness $2L$ be arbitrary and consider three frequency intervals satisfying (21).

1. $\omega < \tilde{\omega}_0$ (this includes negative frequencies). In this case, using (21) and (12) we find that

$$(ak_1)^2 = \omega^2(\omega_0-\omega)/\omega_D^2(\tilde{\omega}_0-\omega) > 0,$$
$$(ak_2)^2 \equiv -(af)^2 = -(\tilde{\omega}_0-\omega)/\omega_E - \omega^2\omega_{ME}/\omega_D^2(\tilde{\omega}_0-\omega) \approx -(\tilde{\omega}_0-\omega)/\omega_E < 0. \tag{22}$$

Since $f \gg k_1$ and $|A(k_1)| \gg |A(k_2)|$, we obtain from (17) that

$$\chi = \frac{\gamma M_0}{\omega_0-\omega}\left\{1+\frac{\omega_{ME}}{Lk_1(\tilde{\omega}_0-\omega)\operatorname{ctg} Lk_1}\right\}. \tag{23}$$

We thus have a resonance of trigonometric waves at frequencies satisfying

$$ak_1 = \frac{|\omega|}{\omega_D}\left(\frac{\omega_0-\omega}{\tilde{\omega}_0-\omega}\right)^{1/2} = (2n+1)\frac{\pi a}{2L}, \tag{24}$$

where $n = 0, 1, 2, \ldots$. We note that (24) does not apply if n is so large that (21) is violated.

Equation (24) gives the quasi-acoustic mode for coupled magnetoelastic waves; the mode frequency is seen to be dimensionally quantized (the plate thickness $2L$ must be an integral multiple of the number of halfwaves).

In the present case, Eq. (23) for χ again describes both right-polarized ($\chi \equiv \chi_-, \omega > 0$) and left-polarized waves ($\chi \equiv \chi_+$, with ω replaced by $-\omega$).

2. $\omega > \omega_0$. In this case k_1 and k_2 must be interchanged as compared with case 1; Eqs. (12) thus gives [cf. (22)]

$$(ak_1)^2 \approx (\omega-\tilde{\omega}_0)/\omega_E > 0,$$
$$(ak_2)^2 \approx \omega^2(\omega-\omega_0)/\omega_D^2(\omega-\tilde{\omega}_0) > 0. \tag{25}$$

An approximate expression for χ again follows from (23) with k_1 replaced by k_2, and the values k_2, corresponding to the resonance frequencies, by the analog of (24):

$$ak_2 \approx \frac{\omega}{\omega_D}\left(\frac{\omega-\omega_0}{\omega-\tilde{\omega}_0}\right)^{1/2} = (2n+1)\frac{\pi a}{2L}. \tag{26}$$

3. $\tilde{\omega}_0 < \omega < \omega_0 \equiv \tilde{\omega}_0 + \omega_{ME}$. For frequencies in this narrow range which still continue to satisfy (21), Eq. (25) remains valid for k_1^2 and k_2^2, except that now

$$k_2^2 \equiv -f^2 < 0.$$

Replacing k_1 by k_2 and k_2 by $-if$, we find from (23) that

$$\chi = \frac{\gamma M_0}{\omega_0-\omega}\left\{1-\frac{\omega_{ME}}{(\omega-\tilde{\omega}_0)Lf\operatorname{cth} Lf}\right\}, \tag{27}$$

where

$$Lf = \frac{\pi\omega}{2\omega_L}\left(\frac{\omega_0-\omega}{\omega-\tilde{\omega}_0}\right)^{1/2}.$$

Hyperbolic waves are thus excited, and there is no thickness-dependent resonance.

For thin films ($Lf \ll 1$) Eq. (27) leads to the previous result (18), as it must, while for thick plates (coth $(Lf) \to 1$) we have

$$\chi \approx \frac{\gamma M_0}{\omega_0-\omega}\left\{1-\frac{2\omega_{ME}\omega_L}{\pi\omega[(\omega_0-\omega)(\omega-\tilde{\omega}_0)]^{1/2}}\right\}.$$

For frequencies in the interval $\tilde{\omega}_0<\omega<\omega_0$ considered, in all three cases the susceptibility for a film in a uniform field is less than for a thick plate because of the magnetoelastic interaction. It is even possible for χ to change sign, so that the dispersion curve goes from positive to negative. This occurs, for example, in the thin-film limit described by Eq. (18).

We point out that Eq. (23) for χ, with k_1 replaced by k_2 from (25), can be specialized to the spin-flip transition, at which $\tilde{\omega}_0=0$ ($\omega_0=\omega_{ME}$). The same holds for the case when $k_2^2 \equiv -f^2<0$, for which χ is of the form (27) (with $\tilde{\omega}_0=0$).

Finally, let us consider the magnetoelastic resonance region $\tilde{\omega}_0\approx\omega$, where the spin and elastic branches of the spectrum cross. If we assume that the interacton is strong enough so that

$$\omega_{ME}\gg\tilde{\omega}_0^2\omega_E/\omega_D^2$$

(this is always the case close enough to the spin-flip frequency), we find from Eqs. (12) and (17) that

$$k_1^2\approx-k_2^2\equiv f^2\approx(\omega_0^2\omega_{ME}/\omega_D^2\omega_E)^{1/2}a^{-2},$$

$$\chi \approx \frac{\gamma M_0}{\omega_{ME}}\left\{1+\frac{2\omega_{ME}}{\omega_E Lf(af)^2[\operatorname{ctg} Lf-\operatorname{cth} Lf]}\right\}. \qquad (28)$$

The distinctive feature of this case is that the trigonometric and hyperbolic excitations enter on an equal footing and their contributions to χ are identical. They also have a distinctive resonance when the equality

$$\operatorname{ctg} Lf=\operatorname{cth} Lf \qquad (29)$$

is satisfied. This equation is easily solved graphically; there are infinitely many roots at intervals of π/L on the f-axis. The first (smallest) root occurs when $Lf>\pi$, for which coth Lf is nearly equal to unity. We may thus approximate (29) as $\cot(Lf)=1$, which has the roots $Lf=(n+1/4)\pi$, $n=1,2,3,\ldots$. The wavelength λ of the corresponding trigonometric component [the first term in the numerator in (13)] of the nonuniform resonant excitations satisfies

$$2L/\lambda=n+{}^1/_4.$$

Since in this case the frequency is already specified by the condition $\omega=\tilde{\omega}_0$, we see from (29) that resonance can be achieved only by varying both ω and $\tilde{\omega}_0$. For a specified $\omega=\tilde{\omega}_0$ there are certain thicknesses for which (29) is satisfied.

RESONANT SUSCEPTIBILITY WITH DAMPING

A detailed description of the resonance naturally requires a consideration of the damping. This is easily done by making replacements analogous to (16) to the expressions for k_1 and k_2, separating the real and imaginary parts, and substituting the resulting values into the expressions for χ.

In what follows we present results for χ, with damping included, near the resonance frequencies in a form uniformly valid for all frequencies satisfying (15) (including $\omega<0$), except for the interval $\tilde{\omega}_0<\omega<\omega_0$. The final result is

$$\chi=\frac{\gamma M_0}{\omega_0-\omega}\left\{1+4\omega_{ME}\tilde{\omega}_L^2/\pi^2(\tilde{\omega}_0-\omega)|\omega|\right.$$
$$\left.\times\left[(2n+1)\tilde{\omega}_L-|\omega|-\frac{i}{2}\Delta\omega\right]\right\}, \qquad (30)$$

where

$$\tilde{\omega}_L=\omega_L[(\tilde{\omega}_0-\omega)/(\omega_0-\omega)]^{1/2}, \qquad (31)$$

and the quantity

$$\Delta\omega=\omega^2[\eta_{44}/C_{44}+r\omega_{ME}/(\tilde{\omega}_0-\omega)(\omega_0-\omega)]$$

describes the damping and gives the width of the resonance line when $|\omega|\ll\tilde{\omega}_0$. This formula is obtained by substituting into Eq. (23) for χ (there are two cases, $\omega<\tilde{\omega}_0$ and $\omega>\omega_0$) the values for k_1, k_2 found as indicated above from Eqs. (22) and (25); one then expands $\cot(Lk_1)$ or $\cot(Lk_2)$ near the resonance frequencies given by Eqs. (24) or (26). To first order we need consider the damping only in the resonant factor $\cot(Lk_1)$ [or $\cot(Lk_2)$] in the denominator in (23) and retain terms linear in the damping parameters r and η_{44}.

The resonance frequencies are given by

$$\tilde{\omega}_L(2n+1)=|\omega|. \qquad (32)$$

In particular, near the spin-flip transition ($\tilde{\omega}_0=0$) Eq. (32) gives the two roots

$$\omega\equiv\omega_-(n)=[(\omega_{ME}/2)^2+\omega_{Ln}^2]^{1/2}+\omega_{ME}/2,$$
$$\omega\equiv\omega_+(n)=-\{[(\omega_{ME}/2)^2+\omega_{Ln}^2]^{1/2}-\omega_{ME}/2\}, \qquad (33)$$

where

$$\omega_{Ln}=\omega_L(2n+1). \qquad (34)$$

The first root ω_- corresponds to resonance of right-polarized waves (for $h\equiv h_-$) and the second ($|\omega_+|$) to resonance of left-polarized waves ($h\equiv h_+$).

In the low-frequency region, for $|\omega|\ll\tilde{\omega}_0\neq0$ we have two roots of equal absolute value

$$\omega_-(n)=|\omega_+(n)|=\tilde{\omega}_{L0}(2n+1), \qquad (35)$$

which give identical frequencies for right- and left-polarized fields **h**.

We recall that if the susceptibilities $\chi_-\equiv\chi$ and $\chi_+\equiv\chi$ for $\omega>0$ and $\omega<0$, respectively, are known, the relations

$$\chi_{xx}=\chi_{yy}=\frac{1}{2}(\chi_++\chi_-),\qquad \chi_{xy}=-\chi_{yx}=\frac{1}{2i}(\chi_--\chi_+)$$

can be used to find the constants of the tensor $\chi_{\alpha\beta}$ for a linearly polarized field **h**.

We further remark that at a resonance corresponding to the nonuniform (magnetoelastic) part of the susceptibility $\Delta\chi\equiv\chi_{ME}=\chi_{ME'}+i\chi_{ME''}$, the latter may significantly exceed the homogeneous component. Thus if $\omega\ll\tilde{\omega}_0$, (30 gives

$$\chi_{ME}''/\chi_0=8\omega_{ME}\tilde{\omega}_{L0}^2Q(\omega)/\pi^2\tilde{\omega}_0\omega^2, \qquad (36)$$

where $\omega=\tilde{\omega}_{L0}(2n+1)$ is the resonance frequency, $Q(\omega)=(\Delta\omega/\omega)^{-1}$ is the quality of the plate, and $\chi_0=\gamma M_0/\omega_0$ is the homogeneous (dynamic) susceptibility. For a fixed ω [resonance is then achieved by changing $\tilde{\omega}_{L0}=\omega/(2n+1)$], (36) must be replaced by

$$\chi_{ME}''/\chi_0=8\omega_{ME}Q(\tilde{\omega}_0)/\pi^2(2n+1)^2\omega. \qquad (37)$$

Here $Q(\tilde{\omega}_0)=Q(\omega)\omega/\tilde{\omega}_0$ is the Q-factor at the frequency $\tilde{\omega}_0$.

Considering for example the fundamental ($n=0$) mode in yttrium-iron garnet (YIG), with $\omega_{ME}=8\cdot10^6\ s^{-1}$, and using the value $Q=2\cdot10^5$ at the frequency $\tilde{\omega}_0/2\pi\approx1$ GHz (Ref. 4), we obtain

$$\chi_{ME}''/\chi_0 \approx 1,3 \cdot 10^{12} \omega^{-1}. \tag{38}$$

This decreases as $(2n+1)^{-2}$ with increasing mode number. In the other case of fixed $\tilde{\omega}_{L0}$ (so that resonance is achieved by changing ω), (36) shows that the height of the peak decreases with n as $\omega^{-3} \sim (2n+1)^{-3}$.

MAGNETOELASTIC MODES AND NMR

We have seen that due to the magnetostatic interaction, nonuniform oscillations in **M** can be excited by a uniform field **h** at low frequencies (below the ferromagnetic resonance frequency ω_0), a fact which has important consequence for NMR studies. This is because the NMR frequency ω_η is generally much less than ω_0, and under these conditions the gain coefficient of the RF field,[5] which determines the intensity and is due to homogeneous magnetization oscillations ($\eta_0 = A\chi_0$, where A is the hyperfine interaction constant) can be much less than the gain associated with the inhomogeneous (magnetoelastic) oscillations: $\eta_{ME} = A\chi_{ME}$. In this case the NMR signal is due primarily to the magnetoelastic excitation channel. This is true in particular when the NMR frequency ω_η equals one of the resonance frequencies for the magnetoelastic oscillations in the plate, as considered above and described in general by Eq. (15). Thus for YIG ($\omega_\eta = 4\cdot 10^8\ \mathrm{s}^{-1}$) the estimate (38) for χ''_{ME}/χ_0 gives $|\eta_{ME}/\eta_0| \approx 3\cdot 10^3$.

The large NMR gain from the magnetoelastic channel in YIG stems from the high value of φ for this material (the linewidth $\Delta\omega$ is small). However, the ratio $|\eta_{ME}/\eta_0| \approx 10^2$ remains quite large even for pure metallic iron at room temperature, for which the film thickness is less than the skin depth $\delta \approx 10^{-3}$ cm in this case $\Delta\omega$ and ω_{ME} are roughly 100 and 5 times larger than in YIG, respectively.

We note that when ω_η coincides with one of the magnetoelastic resonance frequencies [given, e.g., by (35)], one should observe an abrupt increase in acoustic generation at this frequency in addition to the NMR peak.

CONCLUSIONS

The magnetoelastic interaction thus provides an independent mechanism for excitation of spatially nonuniform magnetization waves in plates (films) even when no surface magnetic anisotropy is present. (Magnetic anisotropy could also be included in the analysis without any fundamental difficulty.) These waves are superpositions of trigonometric and hyperbolic waves, which contribute equally to the total susceptibility near the magnetoelastic resonance frequencies.

The existence of a magnetoelastic gap in the spin-wave spectrum is responsible for the presence of an interval $\tilde{\omega}_0 < \omega < \omega_0$ in with the specific frequency dependence of χ. For thick plates ($Lf > \pi$), we see from (27) that at these frequencies χ increases as $(\omega_0 - \omega)^{-3/2}$ for $\omega \to \omega_0$ and as $(\omega - \tilde{\omega}_0)^{-1/2}$ when $\omega \to \tilde{\omega}_0$. [We recall that Eq. (27) is valid only when (21) is satisfied.]

For thin films satisfying (20), the magnetoelastic gap disappears and the frozen lattice model[2,3] thus breaks down. According to this model, the homogeneous oscillations in **M** corresponding to the quasimagnon magnetoelastic mode do not alter the magnetostriction deformations $e_{\alpha\beta}$, which are frozen-in and remain equal to their spontaneous values $e_{\alpha\beta}^{(0)}$ in the ground state.

For frequencies $\omega < \tilde{\omega}_0$ and $\omega > \omega_0$ there should be a thickness-dependent resonance in ferromagnetic plates with intensity given by (30). Although there is some resemblance to spin-wave resonance, the fact that magnetoelastic waves are present at low frequencies $\omega \ll \omega_0$ should permit resonance to be observed (both directly and by means of NMR) in large single-crystal specimens as well as in the thin films where ordinary spin-wave resonance is typically found.

Evidence for a resonance of this type has been observed in Dy and Tb easy-plane rare-earth ferromagnets at microwave frequencies (cf. Ref. 6 and the literature cited therein), i.e., at frequencies an order of magnitude less than the magnetoelastic gap for these materials ($\omega_{ME} \approx 10^{12}\ \mathrm{s}^{-1}$). However, a more detailed analysis would require a theory and treats the case when $\mathbf{M}_0$ lies in the easy plane and also includes the skin effect.

We note in closing that evidence for some of the magnetoelastic effects considered above in NMR systems has already been observed in antiferromagnetic $FeBO_3$ with a weak ferromagnetic moment.[7,8] It would be of interest to observe and analyze such effects for ferromagnets also, particularly near the spin-flip transition [for example, resonances at the frequencies given by Eqs. (33) and (34)].

Magnetic excitation of ultrasound at the ferromagnetic resonance frequency ω_0 was recently detected[9] in amorphous Fe–B films (18–29 at. %B), which have a large magnetostriction ($\omega_{ME} = 1.7\cdot 10^8\ \mathrm{s}^{-1}$). This suggests that magnetoelastic mode resonance with frequencies $\omega < \omega_0$ and their associated NMR signals might also be observed in amorphous ferromagnets. For the film studied in Ref. 9 ($L \approx 1000$ Å), $\Delta\omega \equiv \gamma\Delta H \approx 7\cdot 10^8\ \mathrm{s}^{-1}$ at $\omega_0 \approx \tilde{\omega}_0 = 5.9\cdot 10^{10}\ \mathrm{s}^{-1}$; Eq. (37) thus gives $|\chi''_{ME}/\chi_0| = |\eta_{ME}/\eta_0| \approx 50/(2n+1)^2$ (the frequencies here are comparable to the NMR frequency for ^{57}Fe nuclei). The boundary conditions in the experiment in Ref. 9 differed from ours—the spins were free on one surface and fixed on the other. However, our formulas can be modified without difficulty to cover that case.

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