Vortex interaction with a twinning boundary in a superconductor

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The Ginzburg-Landau equations are used to analyze some mechanisms for the interaction of vortices with a twinning boundary. It is shown that under certain conditions the pinning force may depend nonmonotonically on temperature.

1. The compounds $Y_1Ba_2Cu_3O_{7-\delta}$, which are superconducting at high temperatures, are known to have a pronounced twinning structure. The twinning occurs during the structural transition from the tetragonal to the orthorhombic phases, which occurs at high temperatures well above the superconducting transition T_c. The presence of twinning planes was found by Khaĭkin and Khlyustikov[1] to increase T_c in tin, and a similar increase was subsequently noted in other crystals (In, Nb, etc.[2,3]). The question thus arises of how twinning planes alter the superconducting properties in high-temperature superconductors. A square-root temperature dependence $H_{c2} \sim (T - T_c)^{1/2}$ near the Curie point T_c was reported in Ref. 4; this dependence is characteristic for localized superconductivity near a twinning plane. Two discontinuities in the heat capacity for a single crystal were observed in Ref. 5, a large one at 89 K and a smaller one at 93 K. This was interpreted in Ref. 6 as indicating two superconducting transitions, one to a state with localized superconductivity (on the twinning plane) at 93 K, the other to a bulk superconducting state at 89 K.

In any event, twinning boundaries are planar defects. If such defects are numerous enough (separated by a mean distance of $\sim$300–1000 Å), they should greatly influence the pinning of Abrikosov vortices. Anisotropic pinning of a type consistent with pinning at twinning boundaries has in fact been observed in several experiments in which the critical current was measured.[7,8] In what follows we will consider some vortex-boundary interaction mechanism from the standpoint of the Ginzburg-Landau equation.

2. We consider vortices parallel to a twinning plane (more precisely, oriented along the C axis). The Ginzburg-Landau functional neglecting any effects of the twinning plane, can be written in the form[9]

$$F=\int\left\{\frac{(\mathbf{B}-\mathbf{H})^2}{8\pi}+\frac{1}{4m}\left|\left(\nabla-\frac{2ie}{c}\mathbf{A}\right)\psi\right|^2+a|\psi|^2+\frac{b}{2}|\psi|^4\right\}d^3r. \quad (1)$$

Since we will be interested in type II superconductors with Ginzburg-Landau parameter $\varkappa \gg 1$, the self magnetic field of the vortex can be neglected.

Equation (1) assumes that the superconducting properties are isotropic in the ab plane; if this is not the case, the factor $1/4m$ in the gradient term must be replaced by the effective mass tensor. The principal axes of this tensor will then be oriented differently on either side of the twinning plane, and its components will change discontinuously across the plane. However, it is easy to show by explicit calculation that this does not cause vortex pinning; this is because the symmetry of the twinning plane ensures that nothing is changed upon reflection in the plane. If the twin is isolated, one can make a linear change of coordinates which leaves the boundary fixed but takes the effective mass tensor into a multiple of the unit tensor (i.e., for which the problem becomes isotropic). In a symmetric twin we can use two such transformations applied symmetrically with respect to the twinning boundary. This leaves the boundary fixed while the mass tensor goes over to the same scalar tensor in both twins. The boundary thus no longer figures in the problem, i.e., the vortex energy is independent of the distance from the boundary and hence there is no pinning. We note that this argument is valid only if the anisotropy can be described by an effective mass tensor. Although this is the case where the Ginzburg-Landau theory applies, this assumption may be incorrect at low temperatures. In this case the pinning associated with anisotropy should decrease rapidly with heating. In what follows we consider some pinning mechanisms for a superconductor isotropic in the ab plane.

3. The transport and superconducting properties vary in a thin layer near the twinning boundary. If the layer width is assumed much smaller than the coherence length $\xi(T)$, then the boundary acts only through the boundary conditions of the Ginzburg-Landau equations. In the general case (assuming, however, symmetry about the boundary) these conditions are of the form[10,11]

$$\left(\frac{\partial\psi}{\partial x}\right)_1=L_{11}\psi_1+L_{12}\psi_2,$$
$$\left(\frac{\partial\psi}{\partial x}\right)_2=-L_{12}\psi_1-L_{11}\psi_2.$$

Here the x coordinate is normal to the boundary, and the subscripts 1 and 2 denote values of ψ and $\partial\psi/\partial x$ along the different directions from the boundary. The coefficients L_{11} and L_{22} must be found from the microscopic equations. First, however, it is helpful to recast these conditions in the slightly different form

$$\left(\frac{\partial\psi}{\partial x}\right)_1=\frac{\psi_2-\psi_1}{\alpha}+A\psi_1,$$
$$\left(\frac{\partial\psi}{\partial x}\right)_2=\frac{\psi_2-\psi_1}{\alpha}-A\psi_2 \quad (2)$$

in which the physical significance of the coefficients A and α is highlighted. The coefficient A determines the increase in T_c at the boundary,[12,13] while α corresponds to the barrier transparency and can be calculated microscopically.[14] For low transparency we have

$$\alpha=7\zeta(3)v_0/3\pi^3T_c\int_0^1\cos\theta D(\cos\theta)\,d(\cos\theta),$$

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where v_0 is the Fermi velocity and D the transmission coefficient. In the opposite limiting case of high transparency,

$$\alpha=\pi^3 v_0\int_0^1 \cos^3\theta R(\cos\theta)d(\cos\theta)/28\zeta(3)T_c$$

(the last formula has been written for the pure case, and R is the reflection coefficient). For high transparency we have $\alpha\to 0$, and we see from (2) $\psi_2\to\psi_1$, with $\partial\psi_1/\partial x = A\psi_1$ small.

We note that α is proportional to the coherence length ξ_0 and is independent of temperature. Also, $\partial\psi/\partial x\sim\psi/\xi(T)$, where $\xi(T)\propto\tau^{-1/2}$, $\tau=(T_c-T)/T_c$. The dimensionless coefficient α/ξ thus tends to zero as $T\to T_c$, i.e., the barrier transparency becomes less important near T_c. In the opposite limit $\alpha/\xi\gg 1$, the vortex interacts with the boundary as well as with the surface of the specimen. In this case it is again helpful to introduce a time-independent dimensionless parameter $\beta=\alpha/\xi\tau^{1/2}$ characterizing the transparency. We will consider below the case when $\alpha/\xi=\beta\tau^{1/2}\ll 1$, which always holds for temperatures near the critical point, and which for high transparencies is valid whenever the Ginzburg-Landau equations apply. Since twinning boundaries in crystals are usually quite transparent,[15] the assumption that β is small is resonable. In this case the discontinuity in the order parameter at the boundary is small and the latter can be handled using perturbation theory.

Depending on its sign, the coefficient A in (2) either decreases the order parameter (as at an interface with a normally conducting metal) or increases it near the boundary, giving rise to a localized superconductivity at temperatures above the superconducting transition up to a value T_g, where $(T_g-T_c)/T_c=\tau_0=A^2\eta/4m$; in Eq. (1) we have $a=-\tau/\eta$, where we assume that $\tau_0\ll 1$. The second situation is of greatest interest, and we consider it below. At temperatures $\tau\gg\tau_0$ the order parameter increases slightly near the boundary, so that perturbation theory again applies. Under these conditions, the two pinning mechanisms (due to transparency and to changes in T_c) may be analyzed separately.

4. We first analyze how boundary transparency affects the vortex pinning. In calculating the vortex-boundary interaction energy, it is helpful to modify the Ginzburg-Landau functional (1) so as to obtain the boundary conditions (2). This can be done by replacing the factor $1/4m$ in (1) by $C(x)/4m$, where $C(x)=1$ outside the boundary and is very small in a narrow region close to it, so that

$$\int_1^2\frac{dx}{C(x)}=\alpha.$$

Indeed, near the boundary we need retain only the term containing the large derivative in the Ginzburg-Landau equations:

$$\frac{\partial}{\partial x}\left(\frac{C(x)}{4m}\frac{\partial\psi(x)}{\partial x}\right)=0.$$

Integrating, we get

$$\frac{\partial\psi}{\partial x}=\left(\frac{\partial\psi}{\partial x}\right)_1\frac{1}{C(x)}.$$

Integrating again across the entire boundary, we obtain (2)

$$\psi_2-\psi_1=\left(\frac{\partial\psi}{\partial x}\right)_1\int_1^2\frac{dx}{C(x)}=\left(\frac{\partial\psi}{\partial x}\right)_1\alpha=\left(\frac{\partial\psi}{\partial x}\right)_2\alpha.$$

To calculate the energy change due to boundary effects, we need to consider how the order parameter changes near the boundary. Writing $\psi=\psi_0+\delta\psi$, where ψ_0 is the order parameter for the vortex in the absence of the twinning boundary, and taking the variation of the modified functional (1), we obtain

$$\delta F=\int 2\left(a\psi_0^*\delta\psi+b|\psi_0|^2\psi_0^*\delta\psi-\frac{1}{4m}\delta\psi\nabla^2\psi_0^*\right)d^3r$$
$$-\int dy\,dz\int\frac{(C(x)-1)}{4m}\nabla\psi_0^*\nabla(\delta\psi)\,dx. \qquad (3)$$

The first integral in this expression is equal to zero by virtue of the unperturbed Ginzburg-Landau equations. To evaluate the second integral, we use the fact that the region of integration and the value of C inside it are small. The energy per unit length of vortex is then found to be

$$\delta F=-\frac{1}{2m}\int_{-\infty}^{+\infty}\frac{\partial\psi_0(r)}{\partial x_0}(\delta\psi)\Big|_1^2 dy=-\frac{\alpha}{2m}\int_{-\infty}^{+\infty}\left|\frac{\partial\psi_0(r)}{\partial x_0}\right|^2 dy. \qquad (4)$$

Here the y coordinate is along the boundary and $r=(x_0^2+y^2)^{1/2}$ is the distance from the boundary to the center of the vortex, located at $(0, x_0)$. The vortex solution $\psi_0=(|a|/b)^{1/2}\rho(r)\exp(i\varphi)$ for the order parameter must be substituted into (4) [here $\rho(r)$ is the normalized absolute value of the order parameter: $\rho(\infty)=1$, and φ is its phase]. We calculate the force acting on the vortex by expressing a and b in terms of the critical field $H_c^2=4\pi a^2/b$ and the coherent length $\xi=(ma)^{1/2}/2$:

$$f_1=-\frac{\partial\delta F}{\partial x_0}=\frac{\beta H_c^2\xi^3\tau^{1/2}}{\pi}\frac{\partial}{\partial x_0}\int_0^\infty\left[\left(\frac{d\rho}{dr}\right)^2\frac{x_0^2}{r^2}+\frac{\rho^2y^2}{r^4}\right]dy. \qquad (5)$$

Far from the boundary $(\xi\ll x_0\ll\lambda)$ this is equal to

$$f_1=-\beta H_c^2\xi^3\tau^{1/2}/4x_0^2.$$

The pinning force is strongest at distances $x_0\sim\xi$; to calculate it we must find the vortex solution $\rho(r)$ numerically. The result is

$$|f_1|_{max}=0.40\frac{\beta H_c^2\xi\tau^{1/2}}{\pi},\qquad x_0=0.9\xi.$$

We note that the force is always attractive. The critical current stripped by the vortex from the boundary is equal to

$$j_{c1}=c|f_1|_{max}/\Phi_0=0.33j_0\beta\tau^{1/2},\quad \beta\tau^{1/2}\ll 1. \qquad (6)$$

Here Φ_0 is the flux quantum, c the speed of light, and j_0 the depairing current in the superconductor. We note that the temperature dependence $j_{c1}\sim\tau^2$ differs from that of j_0 by a factor $\tau^{1/2}$. This is because the barrier transparency has less influence on the superconducting properties as $T\to T_c$.

5. We now study how the higher order parameter near the twinning boundary effects the vortex pinning. Boundary conditions of the type (2) with $\alpha=0$ can be obtained[12,13] by adding the term

$$-\int \gamma\delta(x)\,|\psi|^2 d^3r, \text{ where } \gamma=\frac{A}{2m}=\left(\frac{\tau_0}{\eta m}\right)^{1/2},$$

to the Ginzburg-Landau functional. Calculating the vortex energy as before, we get

$$\delta F=-\gamma\int|\psi_0(r)|^2\delta(x)\,dx\,dy.$$

The force acting on the vortex is equal to

$$f_2=\left(\frac{\tau_0}{\tau}\right)^{1/2}\frac{H_c^2\xi}{\pi}\frac{\partial}{\partial x_0}\int_0^\infty \rho^2(x_0^2+y^2)\,dy; \qquad (7)$$

for distances $\xi\ll x_0\ll\lambda$ we have

$$f_2=\left(\frac{\tau_0}{\tau}\right)^{1/2}\frac{H_c^2\xi^3}{2x_0^2}.$$

The maximum force is

$$|f_2|_{max}=0.67\left(\frac{\tau_0}{\tau}\right)^{1/2}\frac{H_c^2\xi}{\pi},\qquad x_0=1{,}2\xi.$$

This force is repulsive, and the critical current is

$$j_{c2}=0.56\left(\frac{\tau_0}{\tau}\right)^{1/2}j_0. \qquad (8)$$

The quantities f_2 and j_{c2} are expressible in terms of measurable parameters, and the results can be compared quantitatively with experiment. We note that j_{c2} varies more slowly with temperature ($\propto\tau$) than does j_0. For $\tau\lesssim\tau_0$ the critical current j_{c2} exceeds the depairing current, and perturbation theory does not apply. For $\tau\gg\tau_0$ there is a slight increase in the order parameter at the boundary. For $\tau\sim\tau_0$, this increase is comparable to the order parameter far from the boundary. For $\tau\ll\tau_0$, the order parameter at the boundary is much larger than far away from it, hence it becomes very difficult for a vortex to penetrate the boundary. At these temperatures the critical current may be comparable to the depairing current.

We now examine the combined effects of the two pinning mechanisms. One force f_1 is attractive while the other f_2 is repulsive, and they have different temperature dependences: $f_1\propto\tau^2$, $f_2\propto\tau$. Thus, a vortex that is attracted to the boundary at low temperatures will be repelled by it at higher temperatures. Although the two forces have the same spatial dependence $\propto x_0^{-2}$ far from the boundary, these dependences differ at distances $\sim\xi$, so that the maximum pinning force

$$f_{pin}=\max|f_1(x)+f_2(x)|$$

does not vanish. The temperature curve $f_{\max}(T)$ [and also $j_c(T)$] should be nonmonotonic with a discontinuous slope. The total pinning force can be written as

$$f_{pin}=\left\{\frac{H_c^2}{\tau^2}\frac{\xi\tau^{1/2}}{\pi}\frac{\tau_0}{\beta}\right\}\tilde f\left(\frac{\tau\beta}{\tau_0^{1/2}}\right). \qquad (9)$$

The factor in braces does not depend on temperature; the entire temperature dependence of the pinning force is described by the universal function

$$\tilde f(t)=t\max\left|\frac{\partial}{\partial x_0}\left(t\xi^2\int_0^\infty\left(\left|\frac{d\rho}{dr}\right|^2\frac{x_0^2}{r^2}+\frac{\rho^2y^2}{r^4}\right)dy+\int_0^\infty\rho^2(r)\,dy\right)\right|, \qquad (10)$$

which is plotted in Fig. 1. The discontinuity occurs at $\tau_1=1.86\tau_0^{1/2}/\beta$ and the maximum lies at $\tau_2=1.14\tau_0^{1/2}/\beta$.

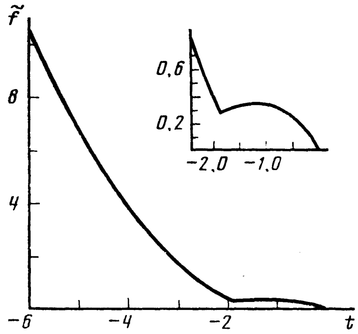

FIG. 1. Plot of the function $\tilde f(t)$ giving the temperture dependence of the pinning force: $t=\beta(T-T_c)/\tau_0^{1/2}T_c$.

As already noted, all of these formulas are valid for $\tau\gg\tau_0$ (under our assumptions $\beta\tau_0^{1/2}\ll1$, so that $\tau_1,\tau_2\gg\tau_0$).

We remark that if an external force (due, e.g., to a transport current) acts on the vortex and there is a temperature above which attraction to the boundary is replaced by repulsion, then it is easy to see that upon heating, the vortex will cross the boundary and end up on the side where the external and repulsive forces point in the same direction, i.e., it will escape across the boundary. This cannot happen when the temperature is decreased: the force exerted by the boundary on the vortex will then always point opposite to the external force.

6. It is not yet known whether a localized superconducting state actually forms on twinning boundaries in high-temperature superconductors, or whether the boundaries merely influence the charge carrier transport. Decorating techniques can be used to elucidate the influence of twinning planes on vortex pinning. It was found experimentally[16] that vortices are attracted to the boundaries at 4 K. It would be of interest to verify whether the attraction is replaced by repulsion at higher temperatures, and also to measure the temperature dependence of the critical current for niobium twins,[3] for which the existence of localized superconductivity may be regarded as well established.

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