Critical properties of a system near a defect of "fractional" dimensionality

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The properties of a system at a second-order phase transition in the vicinity of an inhomogeneous defect whose inhomogeneity is of the form $\lambda(r) = \lambda/r^\alpha$ are investigated. It is shown that the effective dimensionality of the defect varies smoothly with α, which leads to a local change in the critical behavior. For a certain value of α, the critical exponents exhibit a local violation of universality near the defect. Conditions are formulated for the appearance of anomalous behavior of the system correlation functions.

1. INTRODUCTION

Let us investigate a d-dimensional system described by the Hamiltonian

$$H=H_0+\lambda H_1=\int \varepsilon(\mathbf{r})\,d\mathbf{r}+\int \lambda\omega(\mathbf{r})\,d\mathbf{r} \tag{1}$$

with λ a small parameter. The energy density $\varepsilon(\mathbf{r})$ and perturbation density $\omega(\mathbf{r})$ are local quantities that depend on the microscopic variables which describe the system.

Suppose that the Hamiltonian H_0 leads to a phase transition at a temperature $T = T_c$. The theory of similarity[1] is based on the assumption that near a critical point there exists a set of strongly fluctuating fields $A_j(\mathbf{x})$. When we perform a similarity transformation the quantities A_j transform according to the following rule:

$$A_j(\lambda\mathbf{x})=\lambda^{-\Delta_j}A_j(\mathbf{x}). \tag{2}$$

The scaling dimensions Δ_j completely determine the critical exponents of the system, which in turn describe the behavior of thermodynamic quantities near the critical point.

According to the operator-algebra hypothesis,[2,3] any local quantity $A(x)$ can be represented in the form of a linear combination of the quantities $A_j(\mathbf{x})$:

$$A(\mathbf{x})=\sum_j a_jA_j(\mathbf{x}). \tag{3}$$

This implies that the set $\{A_j(\mathbf{x})\}$ is complete, i.e., the following reduction relations hold:

$$A_i(0)A_j(\mathbf{x})=\sum_k C_{ij}{}^k(\mathbf{x})A_k(0). \tag{4}$$

Our problem consists of determining how the scaling dimensions Δ_j change as we switch on the perturbation λH_1. This problem was investigated in Ref. 1; the results of this investigation led the authors to conclude that when the quantity $\omega(x)$ is expanded in a series of the form (3), the scaling dimension of the perturbation Δ_ω is determined by the most singular operator in the expansion. For $\Delta_\omega < d$ and if the presence of the perturbation changes the symmetry of the system, then switching on the perturbation leads to a change in the critical exponents of the system. For $\Delta_\omega > d$, the critical exponents remain unchanged. For the case $\Delta_\omega = d$, switching on the perturbation leads either to scale-invariant behavior of the correlation functions or to continuous dependence of the critical exponents on the microscopic parameters of the perturbation. Just such a situation obtains in the eight-vertex Baxter model.[4,5] The exact solution to this model demonstrated for the first time the possibility of a violation of the universality hypothesis; the explanation of this interesting fact was given by Kadanoff and Wegner.[6]

In Ref. 7, using the example of the two-dimensional Ising model with a line defect, the critical properties of a system near a defect with dimensionality $d' \leqslant d$ were investigated. The conclusions formulated above also remain valid in this case: in particular, the condition that there appear a continuous dependence of the critical exponents will be weakened for $\Delta_\omega = d' \leqslant d$. For $d' < d$ the symmetry of the system necessarily changes as a consequence of the breaking of translational invariance. Nevertheless, violation of the universality hypothesis will, as previously, be an exceptional event. This is because the existence of a quantity with integral dimensionality d' in the algebra should be anticipated only in the case where the system has a special symmetry; in the general case there is no such operator in the algebra. This fact is evident from the results of numerical calculations of the critical exponents of model systems.[1]

In this paper we will investigate the critical properties of a system near a defect where the microscopic parameter λ of the perturbation is a function of the coordinates $\lambda = \lambda(\mathbf{r})$; we show that as we vary the form of the function $\lambda(\mathbf{r})$, we can ensure that the relation $\Delta_\omega = d'$ is effectively fulfilled. In this case, the system will exhibit critical behavior near the defect which is nonuniversal.

2. MODEL OF A DEFECT WITH "FRACTIONAL" DIMENSIONALITY

Let us investigate a d-dimensional system near a critical point determined by the Hamiltonian (1). We will assume that the perturbation is caused by the presence of a homogeneous defect of dimensionality d^*. In this case the perturbation itself has dimensionality $d'(d^* < d')$. We will define the function $\lambda(\mathbf{r})$ in the following way:

$$\lambda(\mathbf{r})=\lambda z(\mathbf{r}),$$
$$z(\mathbf{r})=[(x_{d^*+1})^2+(x_{d^*+2})^2+\ldots+(x_{d'})^2]^{-\alpha/2}, \tag{5}$$

where $x_1,\ldots,x_d$ are axes of a Cartesian coordinate system. The coordinate axes $x_1,\ldots,x_d$ are chosen in such a way that axes $x_1,\ldots,x_{d'}$, span the subspace of the perturbation in which the defect subspace with axes $x_1,\ldots,x_{d^*}$ is embedded. The integration in the first term of (1) is carried out in a d-dimensional space, while the second term involves a d'-dimen-

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sional subspace integration. The subspace of dimension d' will be assumed to be homogeneous and isotropic in the absence of the defect. In Eq. (5), λ is by assumption small, so that is valid to expand for small λ.

Let us illustrate the model under study here with several examples. For $d^*=0$, $d'=d=3$ we are dealing with a point defect in a three-dimensional lattice. Here the case $\alpha\to\infty$ corresponds to an intrinsic point defect which for $\alpha\to 0$ "smears out" into a defect with $d'=3$. For $d^*=1$, $d'=d=3$ we have a line defect whose dimensionality changes from $1(\alpha\to\infty)$ to $3(\alpha\to 0)$. We also can investigate the case $d'<d$ in this unified scheme.

Therefore, the initial point of our investigation is a Hamiltonian of the following form:

$$H=H_0+\lambda H_1$$
$$=\int \varepsilon(\mathbf{r})\,dx_1\ldots dx_d+\lambda\int \omega(\mathbf{r})\left\{\sum_{i=d^*+1}^{d'} x_i^2\right\}^{-\alpha/2} dx_1\ldots dx_{d'}. \quad (6)$$

The integration over each Cartesian coordinate x_1 is limited from below by the lattice constant, which we take equal to 1.

In the absence of the perturbation, the system exhibits a phase transition at the temperature T_c. In this case, relations derived from the theory of scale invariance and the operator algebra (4) are valid for the correlation functions of the system. In addition, we will assume that these correlators are translation-invariant and isotropic in the space of the perturbation.

The basis for our study of the model formulated above is the method developed by Polyakov for use in quantum field theory.[8] A discussion of this method was given in Ref. 1, where it was applied to the theory of critical phenomena; the case $d^*=d'=d$ was treated in this reference.

3. CALCULATION OF THE CORRELATION FUNCTION

Let us investigate the two-point order parameter correlation function

$$G(R,\lambda)=\langle\langle\varphi(0)\varphi(\mathbf{R})\rangle\rangle=\frac{\mathrm{Sp}\{\exp[-\beta(H_0+\lambda H_1)]\varphi(0)\varphi(\mathbf{R})\}}{\mathrm{Sp}\{\exp[-\beta(H_0+\lambda H_1)]\}}, \quad (7)$$

where the system Hamiltonian is defined by Eq. (6). Near a critical point of the unperturbed system we expand Eq. (7) in a power series in λ:

$$G(R,\lambda)=\sum_n \frac{(-\lambda)^n}{n!}\int \langle\langle\varphi(0)\varphi(\mathbf{R})\omega(\mathbf{r}_1)\ldots\omega(\mathbf{r}_n)\rangle\rangle \cdot\prod_{i=1}^{n} z(\mathbf{r}_i)\,d\mathbf{r}_1\ldots d\mathbf{r}_n, \quad (8)$$

where the double angle brackets denote the irreducible part of the multi-point correlation function. The integration in (8) is carried out over all $\mathbf{r}_i$ in the space with the perturbation dimension d'.

The local value of $\omega(\mathbf{r})$ can be expanded in a series with respect to the fundamental components of the operator algebra (3):

$$\omega(\mathbf{r})=\sum_j c_j A_j(\mathbf{r}).$$

We will confine ourselves here to the most singular operator, identifying it from here on as the perturbation operator $\omega(\mathbf{r})$. The most singular operator is the one with the smallest scaling dimension Δ_ω. It is possible to distinguish the various types of reduction relations (4) as a function of the specific type of perturbation operator $\omega(\mathbf{r})$. Let us discuss two of the most important. We limit ourselves to the most singular operator on the right side of (4):

$$\varphi(0)\omega(\mathbf{r})=a_1\varphi(0)|\mathbf{r}|^{-\Delta_\omega},$$
$$\omega(0)\omega(\mathbf{r})=b_1\omega(0)|\mathbf{r}|^{-\Delta_\omega}; \quad (9)$$

$$\omega(0)\omega(\mathbf{r})=a_2\varepsilon(0)|\mathbf{r}|^{-2\Delta_\omega+\Delta_\varepsilon},$$
$$\omega(0)\varepsilon(\mathbf{r})=b_2\omega(0)|\mathbf{r}|^{-\Delta_\varepsilon}, \quad (10)$$

In the first case the perturbation operator $\omega(\mathbf{r})$ behaves like the energy density operator $\varepsilon(\mathbf{r})$, in the second case it behaves like the order parameter $\varphi(\mathbf{r})$. For $d^*=d'=d$ such perturbations do not change the system symmetry, and consequently the critical exponents also do not change. Therefore such perturbations were not discussed in Ref. 1. In the present paper we assume $d^*<d'$ or $d'<d$. In this case the system symmetry must necessarily change as a result of breaking of translation invariance. Therefore perturbations with the reduction relations (9) and (10) are of the most interest.

The change in the correlation function caused by switching on the perturbation has the following form to first order in λ:

$$\delta G^{(1)}(R,\lambda)=-\lambda\int \langle\langle\varphi(0)\varphi(\mathbf{R})\omega(\mathbf{r})\rangle\rangle z(\mathbf{r})\,d\mathbf{r}. \quad (11)$$

For definiteness we will direct the vector $\mathbf{R}$ orthogonal to the defect subspace d^*. Depending on the configuration of the points 0, $\mathbf{R}$, $\mathbf{r}$, there exist three regions with different contributions to the variation of the correlation function (8):

(a) $|\mathbf{R}|>|\mathbf{r}|$

(b) $|\mathbf{R}|>|\mathbf{R}-\mathbf{r}|$

(c) $|\mathbf{r}|, |\mathbf{R}-\mathbf{r}|>R$

Let us first investigate the contribution from the region (a) to the integral (8). We apply relation (9) to the product $\varphi(0)\omega(\mathbf{r})$:

$$\delta G^{(1)}_{(a)}(R,\lambda)\approx-\lambda G(R,0)\int |\mathbf{r}|^{-\Delta_\omega}z(\mathbf{r})\,d\mathbf{r}. \quad (12)$$

Substituting (9) into (12) and evaluating the integral so obtained in a spherical coordinate system, we obtain an estimate of the contribution from region (a). We then calculate this integral separately for the different values d^* and d'; in all cases we obtain the following estimate:

$$\delta G^{(1)}_{(a)}(R,\lambda)\sim-\lambda G(R,0)R^{d_{\text{eff}}-\Delta_\omega}, \quad (13)$$

where

$$d_{\text{eff}}=\max\{d'-\alpha,d^*\}. \quad (14)$$

The contributions from (b) and (c) are estimated analogously; in order of magnitude they coincide with (13).

In this way we arrive at the conclusion that the perturbations we are investigating, i.e., with inhomogeneities of the form $z(\mathbf{r}) = |\mathbf{r}|^{-\alpha}$, change the critical properties of the system near the center of the defect; these changes are analogous to a perturbation with dimensionality d_{eff}.

For $d_{\text{eff}} < \Delta$ the correction $\delta G^{(1)}$ under study is small compared to $G(R,0)$, because all distances are by assumption large compared to unity. In this case the critical exponents of the system remain unchanged.

For $d_{\text{eff}} > \Delta$ the expansion parameter (8) becomes large. The expansion itself is difficult to augment by any further analysis. In this case the critical exponents change. This implies either that the phase transition near the defect belongs to another universality class or that the temperature T_c for this region is not critical. A schematic state diagram is given in Fig. 1. In region I the critical exponents of the unperturbed system coincide with those of the perturbed system, while in region II they do not coincide.

The most interesting case is $d_{\text{eff}} = \Delta_\omega$. In this case we can calculate the change in the critical exponents to linear order in λ. In order to do this, it is necessary to sum the leading logarithms in the expansion (8). For the case under discussion, i.e., $d_{\text{eff}} = \Delta_\omega$, we will concentrate our attention on the more restrictive case $0 < \alpha < d' - d^*$, i.e.,

$$d_{\text{eff}} = d' - \alpha > d^* . \tag{15}$$

The case $d_{\text{eff}} = d^* = \Delta_\omega > d' - d^*$ is analogous to the case $\alpha = 0$, which was previously discussed in Ref. 7.

Thus,

$$\Delta_\omega - d' + \alpha = 0. \tag{16}$$

The basic contribution to (8) is given by regions of values of the arguments $0, \mathbf{R}, \mathbf{r}_1, \dots, \mathbf{r}_n$, which are pairwise close to one another.[1] By assumption, the vector $\mathbf{R}$ is orthogonal to the defect subspace; therefore the largest contribution is given by those configurations in which only the arguments $0, \mathbf{r}_1, \dots, \mathbf{r}_n$ approach one another. So as to carry out the calculations in an easily-visualized way, we will introduce a graphical illustration of the terms of the expansion (8). Following Ref. 1, we will associate circles with the quantities $\varphi(\mathbf{r})$ and crosses with the quantities $\omega(\mathbf{r})$. We enclose within the oval those pairs of points separated by distances which are smaller than all other distances; applying the relation (9) of the operator algebra to these pairs, we carry out the integration over one of the arguments. Two typical configurations are shown in Fig. 2. Let us compute the contribution from configuration (a):

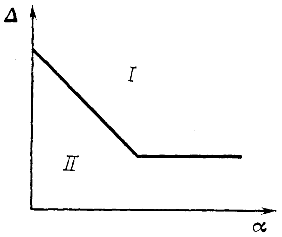

FIG. 1. Phase diagram for the critical behavior of a system near a defect of "fractional" dimension.

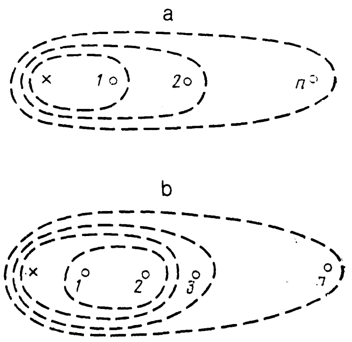

FIG. 2. Graphical illustration of the terms in the expansion (8).

$$(-\lambda)^n \int \langle\langle \varphi(0)\varphi(\mathbf{R})\omega(\mathbf{r}_1)\dots\omega(\mathbf{r}_n)\rangle\rangle \prod_{i=1}^{n} z(\mathbf{r}_i)\,d\mathbf{r}_1\dots d\mathbf{r}_n$$

$$\approx (-\lambda)^n a_1 \int \langle\langle \varphi(0)\varphi(\mathbf{R})\omega(\mathbf{r}_2)\dots\omega(\mathbf{r}_n)\rangle\rangle \prod_{i=1}^{n} z(\mathbf{r}_i)\,d\mathbf{r}_2\dots d\mathbf{r}_n$$

$$\cdot \int_1^{|r_2|} \frac{dr_1}{|r_1|^{\Delta_\omega}} z(\mathbf{r}_1)\,d\mathbf{r}_1$$

$$\approx (-\lambda)^n a_1 S_{d'd^*}(\alpha) \int \langle\langle \varphi(0)\varphi(\mathbf{R})\omega(\mathbf{r}_1)\dots\omega(\mathbf{r}_{n-1})\rangle\rangle$$

$$\prod_{i=1}^{n-1} z(\mathbf{r}_i)\ln|r_1|\,d\mathbf{r}_1\dots d\mathbf{r}_{n-1}, \tag{17}$$

where $S_{d'd^*}(\alpha)$ is a numerical coefficient which arises in calculating the intergral

$$\int_1^{|\mathbf{r}_1|} \frac{d\mathbf{r}}{|\mathbf{r}|^{\Delta_\omega}} z(\mathbf{r}) = \ln|\mathbf{r}_1| S_{dd^*}(\alpha). \tag{18}$$

For $d^* = 0$, $S_{d'0}(\alpha) = S_{d'}$ is the surface of a sphere of unit radius in the d'-dimensional space. Substituting (5) into (18), for the other typical values of d' and d^* we find

$$S_{21}(\alpha) = 2^{1+\alpha}\pi\Gamma(1-\alpha)\left[\Gamma\left(1-\frac{\alpha}{2}\right)\right]^{-2},$$
$$S_{31}(\alpha) = 2^{\alpha}\pi^2\Gamma(2-\alpha)\left[\Gamma\left(\frac{3-\alpha}{2}\right)\right]^{-2}, \tag{19}$$
$$S_{32}(\alpha) = 4\pi(1-\alpha)^{-1},$$

where $\Gamma(\alpha)$ is the gamma function. Calculation of the contribution (b) shown in Fig. 2 is analogous to (18):

$$(-\lambda)^n \int \langle\langle \varphi(0)\varphi(\mathbf{R})\omega(\mathbf{r}_1)\dots\omega(\mathbf{r}_n)\rangle\rangle \prod_{i=1}^{n} z(\mathbf{r}_i)\,d\mathbf{r}_1\dots d\mathbf{r}_n$$

$$\approx (-\lambda)^n b S_{d'} \int \langle\langle \varphi(0)\varphi(\mathbf{R})\omega(\mathbf{r}_1)\dots\omega(\mathbf{r}_{n-1})\rangle\rangle$$

$$\cdot \prod_{i=1}^{n-1} z(\mathbf{r}_i)\,d\mathbf{r}_1\dots d\mathbf{r}_{n-1}. \tag{20}$$

The expression in (20) is logarithmically small compared to that in (17). From this it is clear that the contribution to the leading logarithm approximation is given only by those configurations in which the quantity $\varphi(0)$ is present in operator pairs which approach each other. Continuing the procedure for calculating (17), in the leading logarithm approximation we obtain the contribution of the nth term in the expansion (8):

$$G^{(n)}(R,\lambda)=[-\lambda a_1 S_{d'd^*}(\alpha)]^n(\ln|\mathbf{R}|)^n G(R,0). \quad (21)$$

For the correlation function we are led to the following result:

$$G(R,\lambda)=\sum_n \frac{1}{n!}[-\lambda a_1 S_{d'd^*}(\alpha)]^n(\ln R)^n G(R,0)$$
$$=R^{-\lambda a_1 S_{d'd^*}(\alpha)}G(R,0), \quad R=|\mathbf{R}|. \quad (22)$$

Thus, the critical exponent of the order parameter correlation function near the defect center is found to be a continuous function of the microscopic parameter λ. In this case, to first order,

$$\Delta_\varphi=\Delta_\varphi^0+\lambda a_1 S_{d'd^*}(\alpha). \quad (23)$$

Analogous calculations for the correlation function $\langle\langle\omega(0)\omega(\mathbf{R})\rangle\rangle$ lead to the result

$$\Delta_\omega=\Delta_\omega^0+\lambda b_1 S_{d'd^*}(\alpha). \quad (24)$$

According to investigations presented earlier,[1] for $d'=d^*=\Delta_\omega$ we should expect that in general there will be a violation of scale invariance in the system; a continuous dependence of the critical exponents on the parameter λ is expected only for $b_1=0$, for which case scale invariance is preserved. In the case we investigated i.e., $d'-\alpha=\Delta_\omega$ ($d'\neq d^*$) the system exhibits a continuous dependence of the critical exponents on the parameter λ independent of whether $b_1=0$ or $b_1\neq 0$. However, in the general case of $b_1\neq 0$, in addition to a continuous dependence of the critical exponents on the parameter λ we should also expect a breaking of scale invariance, because the critical exponents (23) and (24) are not related to each other. This can indicate, in particular, that the temperature T_c is not critical for the region near a defect of "fractional" dimensionality.

If the behavior of the perturbation operator $\omega(\mathbf{r})$ is analogous to the order parameter, in particular if the correlation function of an odd number of operators $\omega(\mathbf{r})$ equals zero for temperatures $T\geqslant T_c$, then as operators approach each other the reduction is implemented according to the rule (10). In this case, the expansion (8) for the correlation functions contains only even terms:

$$G_\omega(R,\lambda)=\sum_n \frac{\lambda^{2n}}{(2n)!}\int\langle\langle\omega(0)\omega(\mathbf{R})\omega(\mathbf{r}_1)\ldots\omega(\mathbf{r}_{2n})\rangle\rangle \cdot\prod_{i=1}^{2n} z(\mathbf{r}_i)\,d\mathbf{r}_1\ldots d\mathbf{r}_n. \quad (25)$$

Here the first-order and leading logarithm contributions to the correlation function for the configuration shown in Fig. 2 are determined by their sum:

$$G_\omega^{(n)}(R,\lambda)=\lambda^{2n}a_2 b_2 S^2_{d'd^*}(\alpha)\left\{\frac{1}{\Delta_\varepsilon-\Delta_\omega}+\frac{1}{\Delta_\varepsilon-\Delta_\omega+\alpha}\right\}$$
$$\cdot\int\langle\langle\omega(0)\omega(\mathbf{R})\omega(\mathbf{r}_1)\ldots\omega(\mathbf{r}_{2n-2})\rangle\rangle\prod_{i=1}^{2n-2} z(\mathbf{r}_i)\ln|\mathbf{r}_1|\,d\mathbf{r}_1\ldots d\mathbf{r}_{2n-2}. \quad (26)$$

Continuing this procedure, we obtain the following expression for the correlation function near a defect:

$$G_\omega(R,\lambda)\sim R^{-\Delta_\omega'-\Delta_\omega}, \quad (27)$$

where

$$\Delta_\omega=\Delta_\omega^0-\lambda^2 a_2 b_2 S^2_{d'd^*}(\alpha)\left\{\frac{1}{\Delta_\varepsilon^0-\Delta_\omega^0}+\frac{1}{\Delta_\varepsilon^0-\Delta_\omega^0+\alpha}\right\}. \quad (28)$$

Thus, even in this case the critical exponents for the order parameter correlation function are functions of the microscopic parameter λ. Further calculations show that to first order in λ^2 the scaling dimension of the energy density operator is unchanged.

4. VIOLATION OF SCALE INVARIANCE

So far, we have shown that for $d_{\rm eff}=\Delta_\omega$ the system exhibits a local nonuniversality of the critical indices. If in addition to this the condition $\Delta_\omega=d^*$ i.e., $\alpha=d'-d^*$, is satisfied, then we have combination of the two conditions for anomalous behavior of the correlation function. Let us see what this leads to. Let

$$\Delta_\omega=d^*, \quad \alpha=d'-d^*,$$

carrying out the reduction of the operators according to (9). The contribution to the leading logarithm approximation will, as previously, be determined by the configuration shown in Fig. 2(a). In comparison, the configuration shown in Fig. 2(b) will be logarithmically small. Computing the integral (18) for $\alpha=d'=d^*$, we obtain

$$\int_1^{|\mathbf{r}_1|}\frac{d\mathbf{r}}{|\mathbf{r}|^{\Delta_\omega}}z(\mathbf{r})=\frac{1}{2}\ln^2|\mathbf{r}_1|S_{d'}', \quad (29)$$

where

$$S_3'=4\pi, \quad S_2'=2.$$

In the leading logarithm approximation, the nth term of the expansion (8) is written in the following form:

$$G^{(n)}(R,\lambda)=(-\lambda)^n\int\langle\langle\varphi(0)\varphi(R)\omega(\mathbf{r}_1)\ldots\omega(\mathbf{r}_n)\rangle\rangle\cdot\prod_{i=1}^{n} z(\mathbf{r}_i)\,d\mathbf{r}_1\ldots d\mathbf{r}_n$$
$$=(-\lambda)^n a_1 S_{d'}'\int\langle\langle\varphi(0)\varphi(\mathbf{R})\omega(\mathbf{r}_1)\ldots\omega(\mathbf{r}_{n-1})\rangle\rangle\cdot\prod_{i=1}^{n-1} z(\mathbf{r}_i)\frac{1}{2}\ln^2|\mathbf{r}_1|\,d\mathbf{r}_1\ldots d\mathbf{r}_{n-1}. \quad (30)$$

As a result of these recurrence calculations we obtain

$$G(R,\lambda)=\sum_{n=0}^{\infty}\left(\frac{-\lambda a_1 S'_{d'}}{2}\right)\frac{1}{n!}(\ln R)^{2n}G(R,0)$$

$$=R^{-(\lambda a_1 S'_{d'}\ln R)/2}G(R,0). \qquad (31)$$

Calculations are carried out analogously for the case where the perturbation operator behaves like an order parameter, and the reduction of operator pairs is accomplished according to Eq. (10):

$$G_\omega(R,\lambda)=G(R,0)\exp\{-\lambda^2 a_2 b_2 (S_{d'}')^2[(\Delta_\varepsilon^0-\Delta_\omega^0)^{-1}+(\Delta_\varepsilon^0-\Delta_\omega^0+\alpha)^{-1}]\ln^2 R\}. \qquad (32)$$

Therefore, in both cases the order parameter correlation functions exhibit non-scale-invariant behavior when the condition $\Delta_\omega = d^* = d' - \alpha$ holds.

5. CONCLUSION

Thus, the investigation presented here has shown that introducing a defect of "fractional" dimension into the system can cause a local change in the critical properties of the system. This situation obtains if the effective dimensionality of the perturbation (14) which causes this defect is larger than its scaling dimension ($d_{\text{eff}} > \Delta_\omega$). In the opposite case ($d_{\text{eff}} < \Delta_\omega$) the critical exponents of the system near the defect remain the same as they do far from it. An interesting situation arises for $d_{\text{eff}} = \Delta_\omega$: in this case, the local critical exponents are continuous functions of the microscopic perturbation parameter λ; this indicates that there is local violation of the universality hypothesis. Such a situation can occur in the general case, and not only for the case of a special symmetry—a characteristic of the defect models investigated earlier whose dimensionalities were integers.[1,4,7] In this case, if the perturbation operator is analogous to the energy density operator, then locally not only the universality hypothesis but also the similarity relation between critical exponents is violated. This may indicate, in particular, that the temperature T_c is not critical for the region around the defect.

If the perturbation operator is analogous to the order parameter, then the similarity relations, at least to linear order in λ^2, are not violated.

A very interesting situation is the case $\Delta_\omega = d^* = d' - \alpha$. Here the correlation functions of the system near the defect behave in a non-scale-invariant way, with an unusual dependence on distance. It is interesting to study the correlation functions of a two-dimensional Ising model with line and point defects with inhomogeneity exponents $\alpha = 1$, and also the n-vector models of magnets, in the context.[9-11] Study of these and other models allow us to better understand the way such situations arise, and also to identify by means of experimental observations the phenomenon of local nonuniversality of the critical exponents.

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[1)]This assertion can be justified based on rather plausible estimates of the contribution of various configurations, as was shown above for $\delta G^{(1)}$. However, strictly speaking, along with (2) and (4) this assertion is a third assumption of the theory proposed here.

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