Effect of infrared radiation on the critical current of a Josephson tunnel junction

S. V. Lempitskiĭ

Moscow Institute of Steel and Alloys
(Submitted 24 May 1988)
Zh. Eksp. Teor. Fiz. **94**, 331–340 (November 1988)

The behavior of a superconducting tunnel junction irradiated by electromagnetic waves at a frequency higher than the reciprocal electron "tunneling time" is investigated. It is shown that the exponential enhancement of the external field inside the barrier should cause a large change of the critical current of the junction. The critical current can either increase or nonmonotonically decrease with increase of the irradiation power, depending on the direction of the alternating electric field.

1. INTRODUCTION

Research into the influence of periodic perturbations on the probabilities of various classically forbidden tunneling processes has been attracting interest of late.[1-5] It has been shown that an alternating field $\mathscr{E}\cos\omega t$ can cause a noticable increase of a low probability of below-barrier passage. This is due to absorption of external-field photons by the tunneling particle, resulting in a decrease of the difference. which enters in the argument of the tunnel exponential, between the particle energy E and the barrier height V. In the limit $\omega\tau \gg 1$, where $\tau \sim [2m/(V-E)]^{1/2}a$ is the effective "time" of motion of a particle of mass m under a barrier of thickness a, the tunneling probability even in rather weak field increases in proportion to $\exp[(\mathscr{E}/\tilde{\mathscr{E}})\exp\omega\tau]$ is the characteristic internal field). Effects of this type take place in hopping conduction in semiconductors,[3] in field emission,[4] in the decay of Josephson current states,[5] and in other cases (see Ref. 2).

The present paper is devoted to an analysis of the influence of an alternating electromagnetic field on the critical current of a superconducting tunnel junction. The field frequency ω is assumed high compared with the reciprocal passage time τ^{-1} and low compared with the height of the rectangular barrier $V_0-\mu$ (μ is the chemical potential, $V_0-\mu \sim 0.1$–1 eV), corresponding to the infrared region of the spectrum. The calculation is based on a microscopic method developed for the investigation of a superconductor-semiconductor-superconductor system,[6,7] with explicit account taken of the finite dimensions of the tunnel barrier. A specific feature of this siutation is coherence of the superconducting electrons that pass through the barrier. Corresponding to the usual (one-particle) tunneling in an alternating field is a process in which the electron increases its energy on starting its motion through the barrier by capture a certain number of external-field photons and emerges from below the barrier having this new energy (the optimum number of absorbed quanta is of the order of $(\mathscr{E}/\tilde{\mathscr{E}})\exp\omega\tau$, Ref. 2). In the case of Josephson tunneling, however, the electron entering the opposite bank should again be in the condensate, i.e., have an energy equal to the initial one. This means that the photons absorbed under the barrier, each of which increases the passage probability amplitude by a factor $\exp\omega\tau$, are emitted by the electron in the final stage of below-barrier motion. Thus, in this situation there are more acts of interaction between the electron and the radiation, and the characteristic parameter is here $(\mathscr{E}/\tilde{\mathscr{E}})^2\exp\omega\tau$. Although this quantity is small compared with the parameter $(\mathscr{E}/\tilde{\mathscr{E}})\exp\omega\tau$ of the one-particle process, a strong change of the critical current of the junction should be observed in fields of amplitude $\mathscr{E} \gtrsim \tilde{\mathscr{E}}\exp(-\omega\tau/2)$ even in our case.

It turns out that, depending on the polarization, an external alternating field can influence differently the properties of the junction. When the electric field is directed along the junction (from one electrode to the other), the critical current decreases nonmonotonically. If, on the other hand, the electric-field vector lies in the junction plane, superconductivity stimulation in the system is possible, i.e., an exponential increase of the junction critical current with increase of the irradiation power.

2. GENERAL EXPRESSION FOR THE SUPERCONDUCTING TUNNEL CURRENT IN AN ALTERNATING FIELD

We investigate the properties of a Josephson junction in an external alternating field by the Keldysh method,[8,9] in which the superconducting system is described by a matrix of Green's functions

$$\hat{G}=\begin{pmatrix} G^R & G \\ 0 & G^A \end{pmatrix},$$

satisfying the equation

$$\Big[i\tau_z\hat{1}\frac{\partial}{\partial t}+(1/2m)(\partial/\partial\mathbf{r}-ie\mathbf{A}(\mathbf{r},t)\tau_z\hat{1})^2+\mu-V(z)$$
$$-e\varphi(\mathbf{r},t)+\check{\Delta}\hat{1}\Big]\hat{G}(\mathbf{r},\mathbf{r}',t,t')=\delta(\mathbf{r}-\mathbf{r}')\delta(t-t') \qquad (1)$$

and an analogous equation in $\mathbf{r}'$ and t'. Here τ_z is a Pauli matrix, $\mathbf{A}$ and φ are the field potentials, and $V(z)=V_0\theta(a^2-z^2)$ is the potential of the rectangular barrier (the homogeneous dielectric liner is located in the xy plane in the interval $-a<z<a$. In turn, $G^{R,A}$ and G are also matrices made up of ordinary Green's functions and Gor'kov functions F:

$$G=\begin{pmatrix} g_1 & F_1 \\ -F_2 & g_2 \end{pmatrix}. \qquad (2)$$

The order-parameter matrix is of the form

$$\check{\Delta}=\begin{pmatrix} 0 & \Delta(\mathbf{r},t) \\ -\Delta^*(\mathbf{r},t) & 0 \end{pmatrix} \qquad (3)$$

and is assumed to differ from zero only in the superconducting banks, while the current density is given by the equation

$$\mathbf{j}(\mathbf{r},t)=\frac{e}{4m}\mathrm{Sp}(1+\tau_z)\left(\frac{\partial}{\partial\mathbf{r}'}-\frac{\partial}{\partial\mathbf{r}}\right)G(\mathbf{r},\mathbf{r}',t,t)\big|_{\mathbf{r}\to\mathbf{r}'}. \qquad (4)$$

 0038-5646/88/112362-05$04.00

It follows from (1) that the Green's functions $G^{R,A}$ are connected with the corresponding normal-state functions $G_n{}^{R,A}$ by the equations

$$G^{R,A}(\mathbf{r},\mathbf{r}',t,t')$$
$$=G_n^{R,A}(\mathbf{r},\mathbf{r}',t,t')-\int d^3r_1\,dt_1 G_n^{R,A}(\mathbf{r},\mathbf{r}_1,t,t_1)\check{\Delta}(\mathbf{r}_1,t_1)$$
$$\times G^{R,A}(\mathbf{r}_1,\mathbf{r}',t_1,t'). \quad (5)$$

We assume for simplicity that the superconducting electrodes are made of the metal characterized by an order-parameter modulus Δ, and the temperature T is close to the critical temperature of the banks. It is then possible to calculate the superconducting current in only the first order in Δ. We assume the transverse dimensions of the junctions to be small enough, so that the situation is homogeneous in the xy plane and the current density has only a z component, which is convenient to calculate near the junction boundary (as $z\to a$). In view of the relative weakness of the alternating field, we can neglect the time dependence of the order parameter and regard the superconducting banks as being in equilibrium; the connection between the matrix G and the functions $G^{R,A}$ is then

$$G(t,t')=\int dt_1[G^R(t,t_1)f(t_1-t')-f(t-t_1)G^A(t_1,t')], \quad (6)$$

where $f(t-t_1)$ is the equilibrium "distribution function" whose Fourier transform is $\tanh(\varepsilon/2T)$. Taking into account all the foregoing, and also the relation

$$g^R_{1,2n}(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)=g^{A*}_{1,2n}(\mathbf{r}_2,\mathbf{r}_1,t_2,t_1) \quad (7)$$

we obtain from (4) for the superconducting-current density

$$j_s=\operatorname{Re}\frac{e}{m}\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial z'}\right)\int d^3r_1\,d^3r_2\,dt_1\,dt_2\,dt_3 f(t_3-t)$$
$$\times g_{1n}{}^R(\mathbf{r},\mathbf{r}_1,t,t_1)\Delta(z_1)$$
$$\times g_{2n}{}^R(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)\Delta^*(z_2)g_{1n}{}^R(\mathbf{r}_2,\mathbf{r}',t_2,t_3)\,|_{z\to z'}, \quad (8)$$

and the time average of this expression is what of interests us.

To calculate the current from (8) it is convenient to change to a Fourier representation of the Green's function with respect to the two times and to the transverse coordinates $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$:

$$g(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)$$
$$=\int\frac{d^2p\,d\varepsilon_1\,d\varepsilon_2}{(2\pi)^4}\exp\{i[\varepsilon_2t_2-\varepsilon_1t_1+\mathbf{p}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)]\}g(\mathbf{p},z_1,z_2,\varepsilon_1,\varepsilon_2) \quad (9)$$

(since the system is homogeneous in the junction plane, the Green's functions depend only on the difference $\boldsymbol{\rho}_1-\boldsymbol{\rho}_2$). Taking into account the relation

$$g_{2n}{}^R(\mathbf{p},z_1,z_2,\varepsilon_1,\varepsilon_2)=g_{1n}{}^{R*}(-\mathbf{p},z_1,z_2,-\varepsilon_1,-\varepsilon_2), \quad (10)$$

we rewrite (8) in the form

$$j_s=\operatorname{Re}\frac{e}{m}\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial z'}\right)\int\frac{d^2p}{(2\pi)^2}dz_1\,dz_2\frac{d\varepsilon\,d\varepsilon'\,d\varepsilon_1\,d\varepsilon_2}{(2\pi)^4}\operatorname{th}\frac{\varepsilon'}{2T}$$
$$\times\exp[i(\varepsilon'-\varepsilon)t]g(\mathbf{p},z,z_1,\varepsilon,\varepsilon_1)\Delta(z_1)g^*(-\mathbf{p},z_1,z_2,-\varepsilon_1,-\varepsilon_2)$$
$$\times\Delta^*(z_2)g(\mathbf{p},z_2,z',\varepsilon_2,\varepsilon')\,|_{z\to z'}, \quad (11)$$

where $g\equiv g^n_{1n}$, (we drop henceforth the indices 1, n, and R).

The function $g(\mathbf{p},z_1,z_2,\varepsilon_1\varepsilon_2)$ satisfies the equations

$$[\varepsilon_{1,2}-\hat{H}_0(z_{1,2})]\,g(\mathbf{p},z_1,z_2,\varepsilon_1,\varepsilon_2)$$
$$-\left\{\begin{array}{l}\int U(z_1,\varepsilon_1-\varepsilon')g(\mathbf{p},z_1,z_2,\varepsilon',\varepsilon_2)\dfrac{d\varepsilon'}{2\pi}\\ \int g(\mathbf{p},z_1,z_2,\varepsilon_1,\varepsilon')U(z_2,\varepsilon'-\varepsilon_2)\dfrac{d\varepsilon'}{2\pi}\end{array}\right\}$$
$$=2\pi\delta(\varepsilon_1-\varepsilon_2)\,\delta(z_1-z_2), \quad (12)$$

where $\hat{H}_0(z)=(1/2m)(-d^2/dz^2+p^2)-\mu+V(z)$ is the Hamiltonian of the perturbed problem and U is the perturbation potential describing the electron interaction with the external field [its explicit form in any particular situation follows from Eq. (1)]. The solution of Eqs. (12) can be written as

$$g(\mathbf{p},z_1,z_2,\varepsilon_1,\varepsilon_2)=2\pi\delta(\varepsilon_1-\varepsilon_2)g_p(z_1,z_2,\varepsilon_1)$$
$$+\int g_p(z_1,z',\varepsilon_1)U(z',\varepsilon_1-\varepsilon')$$
$$\times g(\mathbf{p},z',z_2,\varepsilon',\varepsilon_2)\frac{d\varepsilon'}{2\pi}dz'=2\pi\delta(\varepsilon_1-\varepsilon_2)g_p(z_1,z_2,\varepsilon_2)$$
$$+\int g(\mathbf{p},z_1,z',\varepsilon_1,\varepsilon')U(z',\varepsilon'-\varepsilon_2)g_p(z',z_2,\varepsilon_2)\frac{d\varepsilon'}{2\pi}dz', \quad (13)$$

from which follows an expansion of the functions g in powers of the external field. Here $g_p(z_1,z_2,\varepsilon)$ functions corresponding to the stationary state of the system and determined from Eq. (12) with $U=0$. They are of the form

$$g_p=\begin{cases}W^{-1}\psi_1(z_1)\psi_2(z_2), & z_2<z_1\\ W^{-1}\psi_1(z_2)\psi_2(z_1), & z_2>z_1\end{cases},$$
$$W=(\psi_1'\psi_2-\psi_2'\psi_1)/2m, \quad (14)$$

where $\psi_{1,2}$ are linearly independent solutions of the Schrödinger equation $(\hat{H}_0-\varepsilon)\psi_{1,2}=0$. The functions ψ_1 and ψ_2 should be taken to be the solutions of the problem of barrier penetration by a particle incident from the left and right, respectively; this ensures the required analytic properties of the Green's functions:

$$\psi_1=\begin{cases}e^{ikz}+Ae^{-ikz}, & z<-a\\ Be^{\varkappa z}+Ce^{-\varkappa z}, & -a<z<a,\\ De^{ikz}, & z>a\end{cases}$$
$$\psi_2=\begin{cases}e^{-ikz}+Ae^{ikz}, & z>a\\ Be^{-\varkappa z}+Ce^{\varkappa z}, & -a<z<a,\\ De^{-ikz}, & z<-a\end{cases}$$

$$A=-i\frac{D}{2}\left(\frac{k}{\varkappa}+\frac{\varkappa}{k}\right)\operatorname{sh}2\varkappa a,\qquad B=\frac{D}{2}e^{ika-\varkappa a}\left(1+\frac{ik}{\varkappa}\right),$$

$$C=\frac{D}{2}e^{ika+\varkappa a}\left(1-\frac{ik}{\varkappa}\right),$$

$$D(\varepsilon,p)=\frac{2e^{-2ika}}{2\operatorname{ch}2\varkappa a+i\left(\dfrac{\varkappa}{k}-\dfrac{k}{\varkappa}\right)\operatorname{sh}2\varkappa a}$$

$$k\equiv k(\varepsilon,p)=[2m(\varepsilon+i0+\mu-p^2/2m]^{1/2},$$
$$\varkappa\equiv\varkappa(\varepsilon,p)=[2m(V+p^2/2m-\mu-\varepsilon-i0)]^{1/2}. \quad (15)$$

The functions defined by (14) satisfy the relation

$$\left(\frac{\partial}{\partial z'}-\frac{\partial}{\partial z}\right)g_p(z,z_1,\varepsilon)g_p(z_2,z',\varepsilon)\,|_{z\to z'}$$

$$=2mg_p(z_2,z_1,\varepsilon)\left[\theta(z_1-z)-\theta(z_2-z)\right]. \qquad (16)$$

With the aid of this expression and Eqs. (13), in which the action of the alternating perturbation is taken into account only in the junction region $-a<z<a$. we obtain the relation

$$\int\frac{d\varepsilon}{2\pi}\left(\frac{\partial}{\partial z'}-\frac{\partial}{\partial z}\right)g(\mathbf{p},z,z_1,\varepsilon,\varepsilon_1)g(\mathbf{p},z_2,z',\varepsilon_2,\varepsilon)\,|_{z,z'\to a}\,\mathrm{th}\frac{\varepsilon}{2T}$$
$$=2mg(\mathbf{p},z_2,z_1,\varepsilon_2,\varepsilon_1)\left[\mathrm{th}\frac{\varepsilon_1}{2T}\theta(z_1-a)-\mathrm{th}\frac{\varepsilon_2}{2T}\theta(z_2-a)\right], \qquad (17)$$

which is a generalization, to include the nonstationary case of the corresponding expressions of Refs. 6 and 7, and which are needed for the calculation of the superconducting current using Eq. (11).

In a monochromatic field of frequency ω, when $U(z,\omega)=u(z)[\delta(\omega)+\delta(-\omega)]$, the Green's function is given by

$$g(\mathbf{p},z_1,z_2,\varepsilon_1,\varepsilon_2)=2\pi\sum_l g_{\mathbf{p}}^{(l)}(z_1,z_2,\varepsilon_1)\delta(\varepsilon_2-\varepsilon_1-l\omega). \qquad (18)$$

Substituting this expansion in (11) and using (17), we obtain for the time-averaged current

$$j_s=4e\Delta^2\,\mathrm{Re}\int_{-\infty}^{-a}dz_2\int_a^{\infty}dz_1e^{i\chi}\int\frac{d^2p}{(2\pi)^2}\frac{d\varepsilon}{2\pi}\mathrm{th}\frac{\varepsilon}{2T}$$
$$\times\sum_l g_{\mathbf{p}}^{(l)}(z_1,z_2,\varepsilon)g_{-\mathbf{p}}^{(-l)*}(z_1,z_2-\varepsilon), \qquad (19)$$

where χ is the order-parameter phase difference between the banks. For z_1 and z_2 lying in different superconducting electrodes $(z_2<-a, z_1>a)$ the coordinate dependence of the Green's functions follows from relations (13)–(15):

$$g(\mathbf{p},z_1,z_2,\varepsilon_1,\varepsilon_2)$$
$$=g(\mathbf{p},a,-a,\varepsilon_1,\varepsilon_2)\exp\left[ik(\varepsilon_1,p)(z_1-a)-ik(\varepsilon_2,p)(z_2+a)\right]. \qquad (20)$$

We can now integrate with respect to the coordinates and the energy ε in (19), which takes then the form

$$j_s=j_c\sin\chi,$$
$$j_c=\frac{e\Delta^2\mu}{m}\int\frac{d^2p}{(2\pi)^2}\sum_l\frac{\mathrm{th}(l\omega/2T)}{l\omega}g_{\mathbf{p}}^{(l)}(a,-a,0)$$
$$\times g_{-\mathbf{p}}^{(-l)*}(a,-a,0). \qquad (21)$$

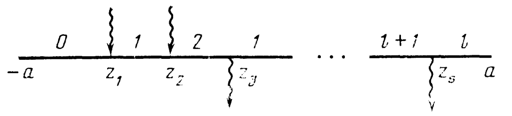
The Josephson-current density in an alternating field is thus determined by Eq. (21), and to use this equation we must calculate the values of $g_{\mathbf{p}}^{(l)}(a,-a,0)$. These parameters describe the probabilities of the processes whereby a particle arriving at the barrier with zero energy (relative to μ) tunnels under the influence of an alternating perturbation and leaves with an energy $l\omega$. In the absence of an external field, only the term with $l=0$ remains in the sum, $g_{\mathbf{p}}^{(0)}$ coincides with the function g_p defined in accordance with (14) and (15), and expression (21) goes over into the known equation[6,10]

$$j_c=j_{c0}=\frac{e\Delta^2}{4T}\int\frac{d^2p}{(2\pi)^2}\,|D^2(0,p)|, \qquad (22)$$

where $|D^2(\varepsilon,p)|$ is the transparency of the barrier to an electron with total energy $\varepsilon+\mu$ and a transverse momentum $\mathbf{p}$.

3. CRITICAL CURRENT OF JOSEPHSON JUNCTION IN AN INFRARED-RADIATION FIELD

Let us substitute in Eq. (13) the expansion (18) for Green's functions in a periodic field. We obtain then for the amplitudes $g_p^{(l)}$, which determine the critical current of the junction in accordance with (21), a perturbation-theory series in which the sth order term is a sum of all possible diagrams with s-field diagrams of the form



(23)

(a similar diagram technique for discrete states is described in Ref. 11). Here each horizontal line with index r corresponds to a Green's function $g_p(z_i,z_{i+1},r\omega)$ defined by expressions (14) and (15); each field vertex $u(z_i)$, depending on the direction of the arrow, decreases or increases r by 1 (corresponding to emission or absorption of a field photon); integration is carried out over the intermediate coordinates z_i.

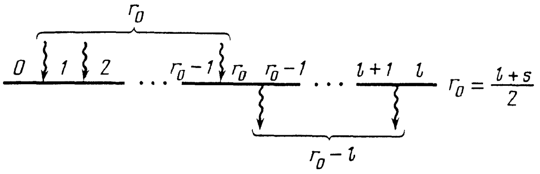
We consider a junction of low transparency, $\varkappa a\gg 1$. The below-barrier electron momentum $\varkappa$ in the junction is assumed here to be small compared with the Fermi momentum $k_F\gtrsim k$ in the metal. On the other hand, if the semiclassical condition $\omega\ll\varkappa v$ is met ($v=\varkappa/m$ is the effective below-barrier velocity) the distances $\sim v/\omega$, over which photon absorption takes place and which make the main contributions to the integrals in expressions (23), exceed significantly the tunneling length $\varkappa^{-1}$. In this case the Green's functions (14) and (5) have for $a-|z_{1,2}|\gg\varkappa^{-1}$ the form

$$g_p(z_1,z_2,r\omega)=(-1/v)\exp(-\varkappa_r|z_1-z_2|),$$

$$\varkappa_r=\left[2m(V_0+p^2/2m-\mu-r\omega)\right]^{1/2}, \qquad (24)$$

and $g(p)$ differs from (24) by a factor $2i\varkappa/k$ when one of the coordinates coincides with $\pm a$.

Each decrease or increase of r by 1 causes a corresponding increase or decrease of the damping factor $\varkappa_r$ of the Green's function (24) by an amount ω/v. On this basis it can be verified that the main contribution to the sth order term is made by diagrams of the type



(25)

in which the elctron first absorbs r_0 photons, and then emits r_0-l photons (l and s have obviously the same parity). Since the absorption, from the standpoint of increasing the tunneling probablity, is most favored at the start of the below-barrier motion, and the emission at the end, the intermediate variables z_i in (25), with $1 \leqslant i \leqslant r_0$, are close to $-a$, while the z_i with $i > r_0$ are close to a. The terms of order s that differ from (25) are relatively small by virtue of the condition $s\exp(-\omega\tau) \ll 1$, which is met for substantial $s(\tau = 2a/v$ is the "time" of electron motion below the barrier). As a result, taking into acount the smoothness of the variation of the perturbation potential $u(z)$ for $-a<z<a$, we can obtain with the aid of (13), (24), and (25) an analytic expression for the amplitude of tunneling with a net energy change $l\omega$:

$$g_p^{(l)}(a,-a,0)=g_p(a,-a,0)\exp[\omega\tau l\theta(l)]$$
$$\times[-u(-a\,\mathrm{sgn}(l))/v]^{|l|}$$
$$\times\sum_{n=0}^{\infty}[u(a)u(-a)/v^2]^n\exp(n\omega\tau)B_{0,n+|l|}B_{0,n},$$
$$g_p(a,\ -a,\ 0)=(4\varkappa_0 m/k^2)\exp(-2\varkappa_0 a), \tag{26}$$

where

$$B_{0,n}=\int_0^\infty\int_0^\infty\ldots\int_0^\infty\exp(-\varkappa_0x_1-\varkappa_1|x_1-x_2|-\ldots-\varkappa_{n-1}|x_{n-1}-x_n|$$
$$+\varkappa_nx_n)\,dx_1\,dx_2\ldots dx_n\approx\left(\frac{v}{\omega}\right)^n\frac{1}{n!},$$
$$\frac{n\omega}{\varkappa v}\ll 1,\qquad B_{0,0}=1. \tag{27}$$

(The integrals in (27) are calculated by making the substitutions $y_1 = x_1, y_2 = x_2 - x_1 \ldots, y_n = x_n - x_{n-1}$.)

To continue the calculation of the superconducting current we must specify the actual coordinate dependence of the external-perturbation potential $u(z)$. In a Josephson junction with massive electrodes the field of amplitude $\mathscr{E}$ is usually perpendicular to the junction plane:

$$u(z)=e\mathscr{E}z/2\quad(u(-a)=-u(a)). \tag{28}$$

We obtain then from (26) and (27) for the Green's functions $g_p^{(l)}$

$$g_p^{(l)}(a,\ -a,\ 0)$$
$$=g_p(a,\ -a,\ 0)J_l[(e\mathscr{E}a/\omega)\exp(\omega\tau/2)]\exp(l\omega\tau/2) \tag{29}$$

(J_l is a Bessel function). It follows from (29) that the products $g_p^{(l)}g_{-p}^{(-l)*}$ in Eq. (21) for the current are of equal order of magnitude. Therefore, in view of the high photon energy ω compared with the temperature T, one can neglect the terms with $l \neq 0$ in the sum of Eq. (21). For the critical current density of the tunnel junction we arrive ultimately at the expression

$$j_c=j_{c0}J_0^2[(e\mathscr{E}a/\omega)\exp(\omega\tau/2)], \tag{30}$$

where

$$j_{c0}=\frac{e(V_0-\mu)^{3/2}m^{1/2}}{2^{1/2}\pi T\mu a}\exp\{-4[2m(V_0-\mu)]^{1/2}a\}$$

is the critical current density of the non-irradiated junction [see Eq. (22)]. Thus, in an infrared field at a given polarization of the periodic field, the critical junction current executes oscillations with diminishing amplitude.

Another situation arises when one of the superconducting electrodes is so thin that the radiation perpendicularly incident on the junction plane reaches the barrier region. An expression for $u(z)$ can then be easily obtained by describing the action of the wave by the vector potential $A \sim \omega^{-1}\mathscr{E}$:

$$u(z)=(e\mathscr{E}/\omega)^2/2m\quad(u(-a)=u(a)). \tag{31}$$

In this case the perturbation is quadratic in the external field, and in all the foregoing equations ω must be taken to mean double the radiation frequency ω' (since the electrons passing under the barrier have mainly low transverse momenta, the linear term in the Hamiltonian, which is proportional to **p·A**, is significant only in the region of very weak fields, when the increment to the critical current is relatively small). In analogy with the preceding case, we get for the critical current in the junction

$$j_c=j_{c0}I_0^2[(\mathscr{E}/\mathscr{E}_0)^2\exp\omega'\tau],\quad \mathscr{E}_0=(2\omega')^{3/4}m^{1/4}/e, \tag{32}$$

where I_0 is a Bessel function of imaginary argument.

4. DISCUSSION OF RESULTS

A Josephson tunnel junction should thus respond strongly to radiation of frequency ω higher than the reciprocal time τ^{-1} of electron motion under the barrier. The alternating-field amplitude is multiplied in Eqs. (30) and (32) for the critical junction current density by the large factor $\exp(\omega\tau/2)$. Such an effective amplification of an external action is similar in nature to other cases of tunneling in an alternating field—the below-barrier tunneling probability is increased by photon capture. The tunneling of Cooper pairs under the influence of radiation includes more interactions with the field than single-particle processes, for after the end of passage through the barrier the superconducting electrons should have the same energy as at the beginning. The optimal number of photons absorbed by the electron and then returned to the field can be estimated, for a sufficiently high irradiation power, from the condition that the product of the square of the small probability amplitude for absorption (emission) of n photons $(u/\omega)^n/n!$ be a maximum and that the gain of the tunneling probability amplitude $\exp(n\omega\tau)$ be large. This number of photons is

$$n_0\sim\frac{u}{\omega}\exp(\omega\tau/2). \tag{33}$$

Accordingly, the influence of the external field turns out in our case to be weaker than for single-particle tunneling.

The character of the action of radiation on the critical current of the junction depends substantially on the direction of the electric-field vector. When the field is directed along the "motion path" of the particle below the barrier, the contributions to the superconducting current from acts of passage with participation of various numbers of photons weaken each other (the series (26) for the passage amplitude turns out to be of alternating sign). As a result, when the irradiation power is increased the critical current of the junction does not have a monotonic behavior, and its envelope has a power-law decrease [Eq. (30)]. Note that a seemingly similar relation holds also in the case of low frequencies

$\omega\tau = \ll 1$, viz., $j_c = j_{c0} |J_0(4e\mathscr{E} a/\omega|$ (Ref. 10),but the scale of variation of j_c in our case is substantially smaller.

If the electric field is in the plane of the junction (this is possible when the wave is incident on a film electrode thinner than the depth of penetration of the radiation), the critical current can increase when the irradiation power is increased [Eq. (32)]; at high power the dependence is exponential:

$$j_c \infty j_{c0} \exp [2(\mathscr{E}/\mathscr{E}_0)^2 \exp \omega\tau].$$

To be sure, in a strong alternating field the superconductivity in the electrodes is queched[12]; for this quenching to set in already after a noticeable increase of the Josephson current, it is necessary to satisfy the condition

$$\frac{\omega}{T} \frac{l_{ne}^2}{a^2} \varkappa a\omega\tau \exp(-\omega\tau) \ll 1, \tag{34}$$

where l_{ne} is the inelastic mean free path of the electrons in the superconducting. The critical-current stimulation effect considered here can therefore be more readily observed in sufficiently thick junctions with low barriers (such as a sandwich with a semiconducting liner) and at a high electrode superconducting-transition temperature.

For an estimate of the accuracy of the results, we point out that, in the calculation of the coefficients $B_{0,n}$ in the expression for the tunneling amplitude $g_{\mathbf{p}}^{(l)}$ [see Eqs. (26) and (27)], increments of order $n\omega/\varkappa v \ll 1$ were neglected in each of the integrations. Since the characteristic number of absorbed photons is of the order n_0 [Eq. (30)], expression (30) is valid for $(e\mathscr{E} a/\omega)\exp(\omega\tau/2) < (\varkappa v/\omega)^{1/2}$, while Eq. (32) for $(\varkappa v/\omega)^{1/4} < (\mathscr{E}/\mathscr{E}_0)\exp(\omega\tau/2) < (\varkappa v/\omega)^{1/2}$ is only exponentially accurate (the right-hand inequality corresponds to the fact that the change of electron energy by tunneling remains small compared with the barrier height). We point out also that at high irradiation power the quantity τ in Eqs. (30) and (32) should strictly speaking denote the time corresponding to the energy $\varepsilon \sim n_0\omega$. Finally, we note that in our case it is difficult to use the semiclassical methods that are traditional for similar problems and are based on solution of the Hamilton-Jacobi equation, since the latter cannot as a rule be linearized in the greater part of the considered power interval.

Since the quasiparticle current in a tunnel junction varies much more rapidly than the superconducting current with increase of the alternating field amplitude (the tunnel resistance decreases like

$$R_N(\mathscr{E}) \approx R_N(0) \exp[-(e\mathscr{E} a/\omega) \exp \omega\tau]$$

if the electric field is directed from one elctrode to the other, while if the field is parallel to the junction plane we have

$$R_N(\mathscr{E}) \approx R_N(0) \exp[-(\mathscr{E}/\mathscr{E}_0)^2 \exp 2\omega\tau]),$$

it follows that one should expect the radiation to alter the form of the current-voltage characteristic strongly (more than in the case of low frequencies). Such a tendency, together with too abrupt a decrease of the critical current, was observed in numerous experiments carried out at high IR frequencies.[13,14] It is not clear, however, whether the point junctions investigated in these references were tunnel structures and whether the frequency employed met the conditions indicated above. At the same time, the effect predicted here can be of interest for a more detailed experimental investigation aimed at using it for IR radiation detection.

The author is deeply grateful to A. A. Abrikosov, B. I. Ivlev, and A. I. Larkin for a valuable discussion of the results.

[1]B. I. Ivlev and V. I. Mel'nikov, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 116 (1985) [JETP Lett. **41**, 142 (1985)].
[2]B. I. Ivlev and V. I. Mel'nikov, Zh. Eksp. Teor. Fiz. **90**, 2208 (1986) [Sov. Phys. JETP **63**, 1295 (1986)].
[3]V. N. V'yurkov and V. I. Ryzhiĭ, *ibid.* **78**, 1158 (1980) [**51**, 583 (1980)].
[4]M. Yu. Sumetskiĭ, Pis'ma Zh. Tekh. Fiz. **11**, 1080 (1985) [Sov. Tech. Phys. Lett. **11**, 448 (1985)].
[5]B. I. Ivlev and V. I. Mel'nikov, Zh. Eksp. Teor. Fiz. **89**, 2248 (1985) [Sov. Phys. JETP **62**, 1298 (1985)].
[6]L. G. Aslamazov and M. V. Fistul', *ibid.* **81**, 382 (1981) [**54**, 206 (1981)].
[7]L. G. Aslamazov and M. V. Fistul', *ibid.* **86**, 1516 (1984) [59, 887 (1984)].
[8]L. V. Keldysh, *ibid.* **47**, 1515 (1964) [**20**, 1018 (1965)].
[9]A. I. Larkin and Yu. N. Ovchinnikov, *ibid.* **68**, 1915 (1975) [**41**, 960 (1975)].
[10]A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect*, Wiley, 1982.
[11]N. B. Delone and V. P. Kraĭnov, Atom in a Strong Electric Field [in Russian], Energoatimizdat, 1984.
[12]R. A. Vardanyan and B. I. Ivlev, Zh. Eksp. Teor. Fiz. **65**, 2315 (1973) [Sov. Phys. JETP **38**, 1156 (1974)].
[13]D. G. McDonald, F. R. Peterson, J. D. Cupp, *et al.*, Appl. Phys. Lett. **24**, 395 (1974).
[14]S. I. Vedeneev, M. A. Gubin, and V. A. Stepanov, Zh. Eksp. Teor. Fiz. **88**, 1395 (1985) [Sov. Phys. JETP **61**, 832 (1985)].

Translated by J. G. Adashko