Effects of electric and scalar fields on timelike singularities

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We investigate the effects of electric, scalar, and vector fields on the properties of timelike singularities—which are sources of these fields—and on the structure of their space-time environment, all within the context of the general theory of relativity.

1. INTRODUCTION

According to general relativity, space-time singularities (besides normal matter and physical fields) can serve as sources of a gravitational field. If these singularities are separated from a distant observer by an event horizon, they are black holes. If on the other hand a timelike singularity is not surrounded by a horizon, it is known as a naked singularity. The extreme curvature near one of these sources severely complicates any investigation of its physical meaning and structure, something that is necessary if one is to study the multiplicity of solutions of the Einstein equations that such singularities possess. Without studies of naked singularities, it is impossible to ascertain whether they are produced by gravitational collapse—that is, to resolve the problem of whether cosmic censorship, which is an important factor in global theorems in gravitation, actually forbids such a process from taking place.

The author has developed a method[1] that enables one to determine the type of a timelike singularity, i.e., to find out whether it is a point source, line source, or perhaps takes some other form. It has been shown that apart from the more familiar types of singularities, there is a new kind which cannot exist in a space of finite curvature. In Ref. 1, we did not attempt to name this singularity; here, because of its highly unusual features, we propose to call it *paradoxical*. Point, line, and paradoxical singularities are described by a single family of solutions, the type of source being determined by the functions that enter into it.

The simplest solution describing such sources is the γ-metric, which is obtained from Eq. (1) by setting $\alpha = 0$. The most complicated may contain up to three arbitrary physical functions of three variables.[1] Other possible singularities of no particular type are possible, such as the general solution of the Einstein equations near timelike singularities.[2] Line and paradoxical singularities cannot be produced in a collapse event.

What must change if a singularity is also to be the source of an electric or scalar field? For infinitely long singularities, exact solutions have been obtained for scalar, electric,[3] and magnetic fields.[3,4] In particular, it turns out that the electrostatic field of an infinite string with constant linear mass and charge density at first falls off with increasing distance, but then, due to self-gravitation, it increases, producing a nonremovable singularity at a finite distance.

In the present paper, we find and study solutions describing space-time that contains the simplest form of finite-size timelike singularity, which is also the source of an electric or massless scalar field. The conclusions that we draw from our analysis of these solutions remain valid for any type of point, line, or paradoxical singularity.

An electric field will not affect line or paradoxical singularities, but when a charge exceeds a certain nonzero minimum value, self-gravitation induces a singularity at a finite distance from the source. For a smaller charge, the effect disappears. The structure of space-time near charged point singularities is significantly altered, and the latter ought to have a negative "bare" mass to cancel their infinite field energy.

A point singularity, the source of a scalar field, may have not just a negative but a positive mass as well. When the scalar charge on a singularity is large enough, the latter cannot be a line source. Singularities will not occur at a finite distance from sources of any type; this is also true of a massive scalar field, since the effect also fails to appear for infinitely long singularities.[3]

2. GENERALIZATION OF THE γ-METRIC

We consider space-time with the metric

$$ds^2=\text{th}^{2\mu}(v/2)\exp[2\alpha(v)]\,dt^2 - {}^1\!/_4 L^2\,\text{sh}^2 v\,\text{th}^{-2\mu}(v/2)\exp[-2\alpha(v)] \times[(1+\cos^2 u/\text{sh}^2 v)^{1-\mu^2}(du^2+dv^2)+\cos^2 u\,d\varphi^2]. \qquad (1)$$

For $\alpha = 0$, this becomes the usual γ-metric. A previous analysis[1] has shown that the singularity at $v = 0$ is a point singularity with negative mass when $\mu < 0$, and for $\mu = -1$ it is equivalent to the Schwarzschild metric with negative mass. For $\mu = 0$ the space-time is flat, and when $0 < \mu < 1$, the singularity is a line. For $\mu = 1$ it becomes a virtual singularity. Analytic continuation of this solution yields the Schwarzschild metric with positive mass, in which $v = 0$ corresponds to the horizon of a black hole. For $\mu > 1$, the singularity becomes paradoxical, and is impossible in spaces of finite curvature. For $\mu \geqslant 2$, two directional singularities appear at the points $v = 0$, $u = \pm\pi/2$, which correspond to two infinitely distant points joined by a padadoxical singularity. In the latter case, space-time contains three different spatial infinities.

Introduction of the factor $\exp[\pm 2\alpha(v)]$ enables us to generalize the γ-metric to the case in which the source carries either an electric or massless scalar charge. The dependence of these factors solely on the coordinate v is suggested by the form of the solution with an infinitely long string [2], and ensures that all nondiagonal components of the Ricci tensor vanish. With $x^0 = t$, $x^1 = v$, $x^2 = u$, and $x^3 = \varphi$, we have

$$R_0{}^0=-R_2{}^2=-R_3{}^3=-g_{11}{}^{-1}(\alpha''+\alpha'\,\text{cth}\,v), \qquad (2)$$

$$R_1{}^1=g_{11}{}^{-1}[\alpha''-2\alpha'^2+\alpha'(\text{cth}\,v-4\mu\,\text{sh}^{-1}v)], \qquad (3)$$

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where a prime denotes a derivative with respect to v.

For completeness, we also construct a solution for a singularity with a source in the form of a semi-infinite string in coordinate space, carrying an electric or scalar charge. The corresponding metric

$$ds^2 = v^{2\mu} \exp[2\alpha(v)]dt^2 - v^{2-2\mu} \exp[-2\alpha(v)] \times [(1+u^2/v^2)^{1-\mu^2}(du^2+dv^2)+u^2 d\varphi^2] \quad (4)$$

is transformed into the exact vacuum solution at $\alpha = 0$.[1] The nondiagonal components of the Ricci tensor are zero, and the diagonal components are given by

$$R_0^0 = -R_2^2 = -R_3^3 = -g_{11}^{-1}(\alpha''+\alpha' v^{-1}),$$
$$R_1^1 = g_{11}^{-1}[\alpha'' - 2\alpha'^2 + (1-4\mu)\alpha' v^{-1}]. \quad (5)$$

3. ELECTRICALLY CHARGED SINGULARITY

Consider Maxwell's equations in the space (1), assuming that the field points in the v-direction and that F_{01} depends only on v. The equation

$$F_{ik,l} + F_{li,k} + F_{kl,i} = 0 \quad (6)$$

is satisfied identically, and from

$$F_{;k}^{ik} = (-g)^{-1/2} \frac{\partial}{\partial x^k}[(-g)^{1/2}F^{ik}] = -4\pi j \quad (7)$$

we obtain

$$F^{01} = -\lambda\, \mathrm{th}^{2\mu}(v/2)\, \mathrm{sh}^{-3} v(1+\cos^2 u/\mathrm{sh}^2 v)^{\mu^2-1} \exp(2\alpha),$$
$$F_{01} = {}^1/_4 \lambda L^2\, \mathrm{th}^{2\mu}(v/2)\, \mathrm{sh}^{-1} v \exp(2\alpha). \quad (8)$$

Next, calculating the components of the traceless energy-momentum tensor we obtain (with $c = G = 1$)

$$R_0^0 = R_1^1 = -R_2^2 = -R_3^3 = -F_{01}F^{01}$$
$$= {}^1/_4 \lambda^2 L^2\, \mathrm{th}^{4\mu}(v/2)\, \mathrm{sh}^{-4} v e^{4\alpha}(1+\cos^2 u/\mathrm{sh}^2 v)^{\mu^2-1}. \quad (9)$$

The relation between R_0^0, R_2^2, and R_3^3 is already satisfied by virtue of (2). Equating (2) and (3) and solving the resulting equation, we find

$$\exp\alpha = [C_1 + C_2\, \mathrm{th}^{2\mu}(v/2)]^{-1}, \quad C_1, C_2 = \mathrm{const}. \quad (10)$$

Substituting (10) into (2) and (3) demonstrates that Eq. (9) holds, and from $\lambda^2 L^4 = -64 C_1 C_2 \mu^2$, we have that $C_1 C_2 < 0$. For $C_1 = 0$ or $C_2 = 0$, we obtain a γ-metric with no electric field. In the present case of interest, with $C_1 C_2 \neq 0$, we put $C_1 = A^{-1}$ and $C_2 = -C^2 A^{-1}$ and reduce the solution to the form

$$ds^2 = \frac{A^2\, \mathrm{th}^{2\mu}(v/2)\, dt^2}{[1-C^2\, \mathrm{th}^{2\mu}(v/2)]^2} - \frac{L^2\, \mathrm{sh}^2 v}{4A^2\, \mathrm{th}^{2\mu}(v/2)}\left(1-C^2\, \mathrm{th}^{2\mu}\frac{v}{2}\right)^2$$
$$\times\left[\left(1+\frac{\cos^2 u}{\mathrm{sh}^2 v}\right)^{1-\mu^2}(du^2+dv^2) + \cos^2 u\, d\varphi^2\right],$$
$$\mu, L, A, C = \mathrm{const}, \quad (11)$$

$$F_{01} = 2A\mu C\, \mathrm{th}^{2\mu}(v/2)\, \mathrm{sh}^{-1} v[1-C^2\, \mathrm{th}^{2\mu}(v/2)]^{-2} \quad (12)$$

The factor A in g_{00} can be changed by scaling the time, and in the other components by renormalizing the parameter L. We thus set $A = |1-C^2|$ for $C^2 \neq 1$, ensuring Galilean invariance at asymptotically large distances. For $C^2 = 1$, we set $A = 1$.

The metric (11) contains a singularity at $v = 0$, and that singularity is nonremovable for $\mu \neq 0$ and $\mu \neq 1$. For $C^2 > 1$ one more singularity appears at $v = v_0$, where $\mathrm{th}^{2\mu}(v_0/2) = C^{-2}$. As one approaches these singularities, the field (12) tends to infinity. For $C^2 \neq 1$, space-time becomes asymptotically flat as $v \to \infty$. We can determine the mass of the source from the form of g_{00} at large distances:

$$M = \mu L(1+C^2)/2(1-C^2). \quad (13)$$

Its charge can be obtained from Gauss's law:

$$Q = \int j^0 (-g)^{1/2} dV = -\frac{1}{4\pi}\oint (-g)^{1/2} F^{01}\, d\varphi\, du = \mu C L A^{-1}. \quad (14)$$

This expression is consistent with the form of the field at large distances, $E = (-F^{01}F_{01})^{1/2}$, when $C^2 \neq 1$.

Consider the structure of space-time (11) for different values of μ and C^2. For $\mu = 0$, we have a flat space with no electric field. For $\mu > 0$, the form of the metric near the singularity $v = 0$ is practically the same as (11) with $C = 0$, $L = L/|1-C^2|$. Therefore, for $0 < \mu < 1$ we have a line source, and for $\mu > 1$ a paradoxical source, while for $\mu \geqslant 2$ there are two directional singularities located at the points $v = 0$, $u = \pm\pi/2$, corresponding to infinitely distant points. In the vicinity of these points the electric field strength also has a directional singularitity, and tends either to zero or infinity, depending on how we approach it.

For $C^2 < 1$, space-time is regular for $v > 0$. The mass of the singularity (13) is positive, and exceeds its charge. It also exceeds the mass of an electrically neutral singularity, due to the contribution of the electric field energy to the overall mass. As $C^2 \to 1$ for a singularity with finite mass and charge, the parameter L must decrease as $L = \tilde{L}(1-C^2)$.

For $C^2 > 1$, space-time has singularities at $v = 0$ and $v = v_0$. If we consider the singularity at $v = 0$ to be a line or paradoxical source with a larger charge than for $C^2 < 1$, we see that due to self-gravitation, the electric field strength then begins to increase with increasing v, and tends to infinity as one approaches v_0.

This effect, then, which makes its appearance in an infinitely long singularity no matter what its charge,[3] occurs in finite-size sources only for $Q > Q_{\min} > 0$. Since according to (14) the parameters μ, L, *and* C do not, in and of themselves, determine any quantities that are amenable to measurement, and (13) does not hold because of the lack of flat asymptotic behavior, this limiting value is not manifested in terms of any measurable quantities.

For $\varphi, \mu = \mathrm{const}$, the distance from the source $v = 0$ to the singularity $v = v_0$ depends on u. If $\mu < 1$, it is at a maximum at $u = 0$ and a minimum at $u = \pm\pi/2$. On the other hand, for $\mu > 1$, it increases with decreasing $\cos u$. For $\mu \geqslant 2$, this distance diverges at $u = \pm\pi/2$, due to the appearance of the aforementioned directional singularities.

For $C^2 > 1$, we may also consider the singularity at $v = v_0$ as a source of electric and gravitational fields; space-time is then described by Eq. (11) for $v_0 < v < \infty$. It displays flat asymptotic behavior as $v \to \infty$, and Eq. (13) is valid, giving $M < 0$, and in conjunction with (14), $M < -|Q|$. Near the singularity $v = v_0$, the space-time (11) acquires the form

$$ds^2 \xrightarrow[v\to v_0+0]{} x^{-1}\,d\tau^2 - dx^2 - x\,dy^2 - x\,dz^2,$$

$$x = \text{const}\,(v-v_0)^2, \tag{15}$$

$$\tau=\text{const}\cdot t,\quad y=\text{const}\cdot u,\quad z=\text{const}\cdot\varphi.$$

If we construct a diagram that enables us to determine the type of singularity,[1] we find that the singularity $v = v_0$ corresponds to a point source. Evaluating the integral

$$\int R_0{}^0(-g)^{1/2}\,dV = \oint (-g)^{1/2} g^{00}\Gamma_{00}{}^1\,du\,d\varphi \tag{16}$$

as a function of the coordinate v of the surface of integration, we find that the source has an infinite negative bare mass. The total mass of the source and its surrounding field within the volume bounded by the surface v = const increases with increasing v, approaching the negative value of M in (13) as $v\to\infty$. We encounter a similar situation in classical electrodynamics, with an infinite negative mass of a point source being cancelled by the infinite energy of its surrounding electric field. We thus see that this problem persists within the framework of general relativity as well.

The interpretation of the metric (11) is somewhat altered for $\mu = 1$, due to the fact that the singularity $v = 0$ becomes virtual, and as we have seen, corresponds to the horizon of a black hole. Following the transformation

$$r=L\,\mathrm{ch}^2(v/2)+C^2L(1-C^2)^{-1}$$

the solution of (11) with $\mu = 1$ and $C^2\neq 1$ reduces to the Reissner-Nordstrøm metric[5]

$$ds^2=\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)dt^2-dr^2\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1}$$

$$-r^2(du^2+\cos^2 u\,d\varphi^2) \tag{17}$$

with M and Q taken from (13) and (14). There are two horizons, located at $r = L/(1-C^2)$ and $r = C^2L/(1-C^2)$. The first corresponds to $v = 0$, and the second is located in the region into which we analytically continued (11). The metric (17) has a true singularitity at $r = 0$, corresponding to $v = v_0$. At small r, its asymptotic behavior is described by (15). The source also has an infinite negative bare mass and infinite electric field energy. For $C^2 < 1$, both horizons lie at $r > 0$, and we are dealing with a charged black hole. In that event, the overall mass M of the singularity plus the field is positive. For $C^2 > 1$, both horizons lie at $r < 0$; that is, they are completely absent, and we have a naked Reissner-Nordstrøm singularity with $M < 0$. It is not possible to obtain a naked singularity from (11) with $0 < M < Q$.

For $C^2 = \mu = 1$, the metric (11) reduces to another well-known solution, the electromagnetic Bertotti-Robinson universe,[5] and there is no singularity. For $C^2 = 1, \mu\neq 1$, we obtain a space-time with a line $(0<\mu<1)$ or paradoxical $(\mu > 1)$ source which tends toward the Bertotti-Robinson solution with increasing distance from the source.

For $\mu < 0$, we obtain no new solutions, as the metric (11) is symmetric under the interchange $\mu\to-\mu, C\to C^{-1}$.

In similar fashion, we may use the solution (4) to construct a metric with a semi-infinite charged source,

$$ds^2=\frac{v^{2\mu}\,dt^2}{(1-C^2v^{2\mu})^2}-v^{2-2\mu}(1-C^2v^{2\mu})^2$$

$$\times\left[\left(1+\frac{u^2}{v^2}\right)^{1-\mu^2}(du^2+dv^2)+u^2\,d\varphi^2\right], \tag{18}$$

$$F_{01}=2\mu Cv^{2\mu-1}/(1-C^2v^{2\mu})^2.$$

The physical meaning of this metric is disclosed in a manner similar to (11). The main difference is that there will be a singularity at $v = v_0 = |C|^{1/\mu}$ no matter how small its charge.

4. SINGULAR SOURCE OF A MASSLESS SCALAR FIELD

We now consider a massless scalar field with energy-momentum tensor

$$T_{ik}=\psi_{,i}\psi_{,k}^*+\psi_{,i}^*\psi_{,k}-g_{ik}\psi_{,l}\psi^{*,l} \tag{19}$$

in the space-time (1). If ψ depends only on v, the field equation

$$\frac{\partial}{\partial x^i}\left[(-g)^{1/2}g^{ik}\frac{\partial\psi}{\partial x^k}\right]=0 \tag{20}$$

yields

$$\psi=\frac{\eta}{2(2\pi)^{1/2}}\ln\mathrm{th}\frac{v}{2}+\text{const},\qquad \eta=\text{const},$$

$$R_0{}^0=R_2{}^2=R_3{}^3=0,\ R_1{}^1=g_{11}{}^{-1}\cdot 2|\eta|^2\,\mathrm{sh}^{-2}v. \tag{21}$$

Substituting this result into (2) and (3), we obtain the metric

$$ds^2=\mathrm{th}^{2\nu}\frac{v}{2}dt^2$$

$$-\frac{L^2}{4}\mathrm{th}^{-2\nu}\frac{v}{2}\mathrm{sh}^2v\left[\left(1+\frac{\cos^2u}{\mathrm{sh}^2v}\right)^{1-\mu^2}(du^2+dv^2)\right.$$

$$\left.+\cos^2u\,d\varphi^2\right],\qquad \nu^2=\mu^2-|\eta|^2. \tag{22}$$

We obtain the mass of the singularity from the form of g_{00} in this asymptotically flat solution:

$$M=L\nu/2. \tag{23}$$

Let us analyze the space-time represented by (22). For $\nu<0$ we obtain a point source with negative mass. For $\nu>0$ and a weak scalar charge $|\eta|^2 < 1/2$, three types of sources are possible. For $|\eta|^2 <\mu^2 < 1/2 + (1/4-|\eta|^2)^{1/2}$ we have a line source, $1/2 + (1/4-|\eta|^2)^{1/2} <\mu^2 < 1 + |\eta|^2$ gives a point source with positive mass, and $\mu^2 > 1 + |\eta|^2$ gives a paradoxical source of mass $M > L/2$. As the scalar charge is raised to $|\eta|^2 > 1/2$, line sources are no longer possible, and for $|\eta|^2 <\mu^2 < 1 + |\eta|^2$, i.e., $0 < M < L/2$, we have a point source with positive mass; For $\mu^2 > 1 + |\eta|^2$, we have a paradoxical singularity. Note that for small $|\eta|$, point sources of positive mass are separated from point sources of negative mass by a zone of line sources. As $|\eta|$ approaches zero, a zone of positive-mass point sources is compressed down to the point $\mu = 1$, or in other words, to a black hole.

For a semi-infinite filamentary source of a massless scalar field, we obtain the metric

$$ds^2=v^{2\nu}dt^2-v^{2-2\nu}\left[\left(1+\frac{\cos^2u}{\mathrm{sh}^2v}\right)^{1-\mu^2}(du^2+dv^2)+u^2\,d\varphi^2\right],$$

$$\nu^2=\mu^2-|\eta|^2,\quad \psi=\frac{\eta}{2(2\pi)^{1/2}}\ln v+\text{const}. \tag{24}$$

Both in this instance and that of an infinite string,[3] there are no singularities at a finite distance from the source.

5. COMBINED EFFECT OF ELECTRIC AND MASSLESS SCALAR FIELDS

If the singularity in (1) is a source of both an electric and massless scalar field, one can readily find possible solutions. One of these is a natural generalization of (1), (12), and (21):

$$ds^2=B(v)\,dt^2-\frac{L^2\,\mathrm{sh}^2 v}{4B(v)}\left[\left(1+\frac{\cos^2 u}{\mathrm{sh}^2 v}\right)^{1-\mu^2}\right.$$

$$\left.\times(du^2+dv^2)+\cos^2 u\,d\varphi^2\right], \qquad (25)$$

$$B(v)=A^2\,\mathrm{th}^{2\nu}\frac{v}{2}\left(1-C^2\,\mathrm{th}^{2\nu}\frac{v}{2}\right)^{-2}, \qquad \nu^2=\mu^2-|\eta|^2,$$

$$\psi=\frac{\eta}{2(2\pi)^{1/2}}\ln\mathrm{th}\frac{v}{2}+\mathrm{const}, \qquad F_{01}=2\nu CA^{-1}B(v)\,\mathrm{sh}^{-1}v.$$

Here, the combined effect of the two fields reduces to the sum of their effects individually.

A second solution arises for $\mu^2<|\eta|^2$:

$$B(v)=A^2\sec^2\left(\nu\ln\mathrm{th}\frac{v}{2}+C\right), \quad \nu^2=|\eta|^2-\mu^2, \quad C=\mathrm{const},$$

$$F_{01}=\nu A^{-1}B(v)\,\mathrm{sh}^{-1}v, \quad \psi=\frac{\eta}{2(2\pi)^{1/2}}\ln\mathrm{th}\frac{v}{2}+\mathrm{const}. \qquad (26)$$

This solution has an infinity of singularities, but the field ψ is regular throughout. When we consider the region between two singularities, we obtain a point source with infinite negative mass, and because of self-gravitation, the field produces a singularity at a finite distance from the source. This is clearly an unphysical solution.

The third metric, which is obtained for $\mu^2=|\eta|^2$, takes the form

$$B(v)=A^2\left(\ln\mathrm{th}\frac{v}{2}+C\right)^{-2}, \quad C=\mathrm{const},$$

$$F_{01}=A^{-1}B(v)\,\mathrm{sh}^{-1}v, \quad \psi=\frac{\eta}{2(2\pi)^{1/2}}\ln\mathrm{th}\frac{v}{2}+\mathrm{const}. \qquad (27)$$

It describes a space-time with no source at $v=0$, which can be obtained from (16). Here the scalar and electric fields are concentrated by the intrinsic gravitational field. Notice that one can draw a parallel between (25)–(27) and the three solutions for an infinite charged filament obtained in Ref. 3.

6. THE EFFECT OF MASSIVE SCALAR AND VECTOR FIELDS ON THE PROPERTIES OF TIMELIKE SINGULARITIES

If we have a massive scalar or vector field whose source is a timelike singularity, the corresponding exact solution cannot be constructed. Even in the much simpler case of an infinite filament, the resulting systems of equations cannot be solved.[3,6] A qualitative examination of the effects of massive fields is nevertheless possible. If we consider the Einstein equations and the equations for the nongravitational fields, it is clear that terms which include the mass of the field quanta turn out to be much smaller than the leading terms, and the asymptotic form of the space-time is describable by solutions for the massless field. Massive fields will therefore affect the type of singularity in just the same way as the massless fields considered above.

The issue is somewhat more complicated when we investigate space-time far from a singularity. For a massive scalar field, there will be no singularities at a finite distance from the source, inasmuch as there are none for an infinitely long filament either.[3] If in fact such a filament were the source of a massive vector field, there would evidently be a singularity engendered by its self-gravitation.[6] For finite-length singularities, one should expect that this effect would be observed if the vector charge were to exceed some threshold, but it is extremely difficult to obtain the corresponding solutions.

7. CONCLUSION

We have thus shown that if a line or paradoxical singularity of any type also possesses an electric or vector charge, it will affect neither the properties of space-time near the source nor the type of singularity. At the same time, far from a source that carries a large charge, self-gravitation may induce a time singularity. A point source of a gravitational, electric, or vector field ought to have negative bare mass, and the space-time outside it should then be regular. No new types of singularity appear in the latter case.

If a timelike singularity carries a scalar charge, a new type of positive-mass point singularity intermediate between line and paradoxical singularities becomes possible. As the scalar charge increases, this type completely expels line sources, engendering a region of negative-mass point sources. It is easily seen that sources of this type cannot be formed in a collapse, since when a naked singularity is initially formed (if in fact it can be), one expects small values of μ and $|\eta|$. Space-time outside any type of singularity with a scalar charge is regular, and singularities cannot result from self-gravitation of the field.

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