Cherenkov interaction of spin waves with charge carriers in the ferromagnetic semiconductor $HgCr_2Se_4$

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We have investigated the effect of an electric field E on the decay ΔH_k of spin waves with frequency 4.7 GHz in the ferromagnetic semiconductor $HgCr_2Se_4$ at temperatures $T = 4.2$–77 K. The spin waves were excited by the method of longitudinal pumping; the character of the observed variation of ΔH_k (E) is similar to the analogous behavior of ultrasonic attenuation in semiconductors. The results are explained by the magnetoelectric effect, i.e., the excitation of an electric field by a spin wave propagating through the magnetic semiconductor. The model explains the peculiarities in the behavior of ΔH_k as a function of various factors, and estimates of ΔH_k are in reasonable agreement with experiment.

The coexistence of interacting subsystems of charge carriers and spin waves (SW) in magnetic semiconductors leads to a number of new physical phenomena arising from the possibility of transferring the kinetic energy of the charge carriers to the spin waves when they are subjected to an electric field. Among these phenomena we should mention heating of the magnon system by hot carriers[1] and amplification of SW by drifting charge carriers.[2]

In models in which the magnon system is heated by charge carriers, it is assumed that the charge carriers (band electrons), which are heated by the electric field, transfer energy to magnons, which then are also heated. In magnetic semiconductors such as europium chalcogenides or chromium chalcogenide spinels, the magnons are weakly coupled to the lattice (the Debye temperature for these compounds is $T_D \approx 300$ K, while the Curie temperature is $T_c \approx 100$ K). A consequence of this is that the lattice and magnon temperatures can be different. Results of experimental investigations of the magnetization, microwave, optical, and electrical properties of magnetic semiconductors in strong electric fields[3] are found to be in reasonable agreement with estimates carried out based on this model.

By "amplification" of SW by charge carriers we mean a decrease of the attenuation of the SW (or even a reversal of its sign, i.e., generation) due to variation of the electronic contribution to the attenuation of SW under the action of an electric field. It has been predicted that SW can be amplified in a ferromagnet either by a beam of charged particles,[2] or because of the drift of intrinsic carriers.[4] In order to have coherent SW amplification, it is necessary to fulfil the condition for Cherenkov radiation, i.e., the drift velocity of charged carriers (e.g., the beam) must be larger than the SW phase velocity $V_\phi = \omega/k$ (ω is the spin wave frequency, k is its wave vector). A large quantity of experimental and theoretical work has been devoted to the study of amplification of magnetostatic waves in layered ferrite-semiconductor structures (see the review of Ref. 5). These structures have the advantage of combining in one system the most useful properties of magnets and semiconductors. For example, yttrium iron garnet, whose SW attenuation parameter $2\Delta H_k \leqslant 0.5$ Oe, has been used in such structures with n-type InSb crystals with carrier mobility $\mu = 8 \cdot 10^5$ cm^2/V·sec (Ref. 5). However, despite these advantages, no experimental scheme for amplification of magneto-static waves has been able to provide the required slowing of the wave (the condition $V_{dr} > V_\phi$); this because real attainable carrier velocities in solid state plasmas are in the range 10^6–10^7 cm/sec, while the phase velocity of SW excited in such structures is almost an order of magnitude higher.

The charge carrier mobility of most magnetic semiconductors is typically too small to amplify magnetostatic waves, although within the last year some progress has been made in the technology of these materials: values of $\mu \approx 10^3$ cm^2/V·sec and $2\Delta H_k = 1$ Oe have been obtained.[7] It is noteworthy that attempts have been made to explain some of the results of investigations of the properties of magnetic semiconductors (current-voltage characteristics,[8] magnetoresistance,[9] and transmission of microwave signals[10]) on the basis of this SW amplification model. However, these conclusions are ambiguous, because the material parameters are such that the model results are used at the limit of their applicability. For example, the results of Ref. 9 can also be satisfactorily explained on the basis of a model in which the magnon system is heated by charge carriers.

Thus, in order to slow coherent waves with frequency $\sim 10^{10}$ Hz enough to satisfy the Cherenkov condition ($V_{dr} > V_\phi$), the experimental scheme must involve SW with $k = 10^5$–10^6 cm^{-1}. In our previous paper[11] we studied the effect of an electric field on the attenuation of SW with $k \approx 10^6$ cm^{-1}, excited by the method of transverse pumping in $HgCr_2Se_4$. One significant result reported in Ref. 11 was the detection of a nonmonotonic variation of the threshold for SW excitation with the intensity of an electric field applied to the sample in the regime where the Cherenkov condition is fulfilled. The basis for discussing these results was the SW amplification model. However, the transverse-pumping method does not allow excitation of waves with various phase velocities; furthermore, the precision with which the value of ΔH_k can be determined is not high in this case. The simplest and, comparatively speaking, the most widespread method of exciting and measuring the attenuation of coherent SW with $k = 10^4$–10^6 cm^{-1} is the method of longitudinal pumping.[12]

In this paper we report the results of our investigations of the effect of an electric field on the attenuation of SW parametrically excited in the ferromagnetic semiconductor

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$HgCr_2Se_4$ by the method of longitudinal pumping.

SAMPLES AND METHOD OF INVESTIGATION

The compound $HgCr_2Se_4$ (Ref. 13) is a ferromagnetic semiconductor with a Curie temperature of 106 K. Crystals of $HgCr_2Se_4$ have the normal spinel structure, i.e., the Cr^{+3} ions occupy octahedral sites, while the Hg^{+2} ions occupy tetrahedral sites. The chalcogen anions form a close-packed face-centered cubic lattice with a unit-cell parameter of 1.074 nm. The magnetic properties of $HgCr_2Se_4$ are caused by the Cr^{+3} ions. The magnetization, when calculated in formal units equals $6\mu_B$, which corresponds to a saturation magnetization of $4\pi M_0 = 4400$ G at $T = 0$. The real values of the magnetization are smaller ($\approx 5.8\,\mu_B$), which apparently is due to lack of stoichimetry in the anion and cation sublattices which occurs during crystal growth. In the most perfect nonconducting crystals of $HgCr_2Se_4$, the SW attenuation parameter satisfies $2\Delta H_k = 1$–2 Oe.[7,14]

Compared to other magnetic semiconductors, $HgCr_2Se_4$ is the most convenient compound for investigating amplification effects, at least from a semiconductor standpoint. In $HgCr_2Se_4$ the electron effective mass is $m_e = 0.2m_0$ (Ref. 15), while the carrier mobility at 4.2 K can attain the value $\mu = 1800$ cm^2/V·sec (Ref. 6). Within comparatively wide limits we can also vary the carrier concentration by doping the crystals with impurities or annealing them in a Hg or Se atmosphere.[6,16]

In this work we used crystals prepared using a technology described in Ref. 17. According to Ref. 16, single crystals are strongly compensated while being grown; at high temperatures they have the characteristics of semiconductors with p-type conductivity. As the temperature decreases below T_k, the character of the conductivity changes to quasimetallic n-type. By annealing the crystal in mercury or selenium vapor, we can change the ratio of the carrier concentrations for the two carrier types until the crystal enters a state with a fixed conductivity at all temperatures. Crystals with p-type carriers have the lower mobility $\mu \approx 5$ cm^2/V·sec.

We carried out measurements on a number of samples which had both n- and p-type conductivities in the ferromagnetic region. In this paper we are essentially reporting results for a single sample, which was made weakly conducting ($\sigma \approx 10^{-4}\ \Omega^{-1}\,\mathrm{cm}^{-1}$ at 77 K) by annealing it in mercury vapor. The temperature dependence of the electrical conductivity of the sample under study ($\sigma_{300\,\mathrm{K}} = 1\ \Omega^{-1}\,\mathrm{cm}^{-1}$) has a minimum for $T = T_k$ ($\sigma = 6\cdot 10^{-3}\ \Omega^{-1}\,\mathrm{cm}^{-1}$). Its electrical conductivity at 77 K and 4.2 K equals $7\cdot 10^{-3}\ \Omega^{-1}\,\mathrm{cm}^{-1}$ and $2\cdot 10^{-2}\ \Omega^{-1}\,\mathrm{cm}^{-1}$, respectively. The mobility of the charge carriers, as determined by Hall effect measurements, equalled 7, 35, and 50 cm^2/V·sec for 300, 77, and 4.2 K, respectively.

In the method of longitudinal pumping, the SW become unstable under the action of an AC magnetic field parallel to the external constant magnetic field. A pump photon decays into two magnons with momenta $\mathbf{k}_1 = -\mathbf{k}_2$ and with frequencies $\omega_{k_1} = \omega_{k_2} = \omega_p/2$, where ω_p is the pump frequency. For the case $\Delta H_{k_1} \neq \Delta H_{k_2}$ the threshold magnetic field h_{th} for parametric excitation of spin waves equals[12,13]

$$h_{\mathrm{th}} = \frac{2\Delta H_k \omega_p}{\omega_M \sin^2\theta_k}, \qquad \Delta H_k = (\Delta H_{k_1} \Delta H_{k_2})^{1/2}, \tag{1}$$

where θ_k is the angle between the vector $\mathbf{k}$ and the magnetization direction in the sample, and $\omega_M = 4\pi\gamma M$. It is clear that the first spin wave to become unstable is the one with $\theta_k = \pi/2$, if ΔH_k does not depend on θ_k. For these magnons the wave numbers are given by the expression

$$k = [(H_c - H)/D]^{1/2}, \tag{2}$$

$$H_c = 1/2\lambda[(\omega_p^2 + \omega_M^2)^{1/2} - \omega_M] + N_z M, \tag{3}$$

where H is the intensity of the external magnetic field, N_z is the demagnetization factor of an ellipsoidal sample, and D is the inhomogeneous exchange constant. For $HgCr_2Se_4$ the value $D = 2.2\cdot 10^{-10}$ Oe·cm^2 is determined from the temperature dependence of the thermoelectric power.[19]

In Fig. 1 we show a block diagram of the setup for investigating the effect of an electric field on the attenuation of spin waves. We note several of its features: the microwave source is a pulsed magnetron with a frequency of 9.4 GHz and a maximum power of ≈ 5 kW. The sample, in the shape of a rectangular parallelepiped with dimensions $2.5\times 1.8\times 0.3$ mm^3, was positioned in an antinode of the magnetic field of a rectangular cavity such that $\mathbf{H}\|\mathbf{h}$. The signals, which are proportional to the microwave powers incident on (P_{inc}) and reflected from (P_{ref}) the resonator, are recorded simultaneously with a stroboscopic power meter and are fed to the input of a two-coordinate plotter. An additional absorption takes place when spin waves are excited, which is manifested in a deviation of the dependence $P_{\mathrm{ref}} = f(P_{\mathrm{inc}})$ from linear (Fig. 1). The value of the AC magnetic field is determined through the value of P_{inc} and is calibrated using the pump threshold in a ferrite with known ΔH_k. The electric field is applied to the sample through ohmic contacts prepared at the opposite faces of the parallelepiped (along the 1.8 mm dimension). In order to avoid heating, the investigations were carried out in the pulsed regime: the durations of the electrical and microwave pulses were 2 μsec; the pulses were synchronized with each other, and the repetition rate of the pulses was ≈ 16 Hz.

MEASUREMENT RESULTS

The curves of the dependence of the threshold field h_{th} on intensity of the constant magnetic field for a nonconducting sphere have a characteristic "butterfly" shape.[18] The calculated values of H_c obtained from (3) using the results of ferromagnetic resonance investigations on these samples (the magnetic anisotropy field is negligibly small, while the

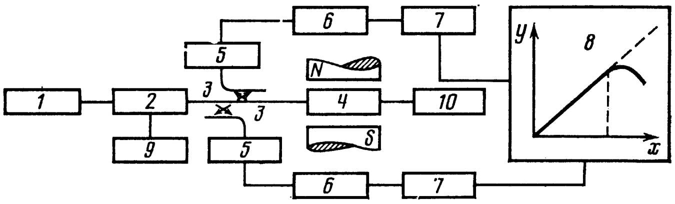

FIG. 1. Block diagram of setup: 1—microwave magnetron; 2—precision attenuator; 3—directional coupler; 4—resonator with sample; 5—attenuator; 6—detector; 7—stroboscopic voltmeter; 8—two-coordinate plotter; 9—automated scanning system for microwave power; 10—electrical pulse generator.

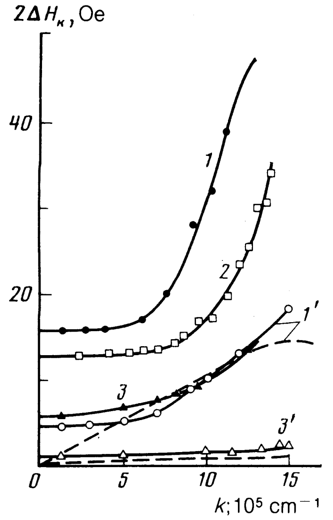

FIG. 2. Dependence of the SW attenuation parameter for conducting (curves 1—3) and weakly conducting (traces 1′, 3′) crystals at 77, 60 and 4.2 K, respectively. The broken traces 1′ and 3′ are the contribution from intrinsic magnetic processes in nonconducting crystals at 77 and 4.2 K, respectively.

g-factor is 1.994 ± 0.005 at 77 K), coincide with experimental values of H_c to good accuracy. The form of the "butterfly" curve measured on conducting samples differs from that of the curve measured on nonconducting crystals: the minimum of the curve h_{th} (H) is mildly sloping, as is usual for magnetic semiconductors[7], which hinders a precise determination of k. However, in contrast to Ref. 7, the experimental values of H_c do not differ strongly (to within ~50–100 Oe) from the calculated values.

In Fig. 2 we show the dependence of the SW attenuation parameter on wave number for weakly-conducting ($\sigma = 10^{-4}$–10^5 Ω^{-1} cm^{-1}) and conducting crystals of $HgCr_2Se_4$ crystals in the absence of electric fields. The dashed curves show the contributions of intrinsic (magnetic) processes at 77 K and 4.2 K. It is clear that in the nonconducting crystals the SW attenuation can essentially be accounted for by the intrinsic processes; of these, the fundamental contributions at 77 K and 4.2 K are given by three-magnon dipole processes in which parametric magnons combine with thermal magnons, and four-magnon exchange scattering processes (see Ref. 7). In the conducting crystals the SW attenuation is three to six times larger than in the nonconducting ones.

In Fig. 3 we show the dependence on electric field intensity E of the quantity

$$\delta_k(E) = \Delta H_k - \Delta H_k^0 \tag{4}$$

for various values of k at 77 K and 4.2 K in the $\mathbf{E} \perp \mathbf{H}$ configuration (i.e., it is assumed that the electric field is directed along the SW propagation direction). Here ΔH_k, ΔH_k^0 are the SW attenuation parameters with and without the electric field, respectively, as determined from Eq. (1), assuming that $\theta_k = \pi/2$. Note that a number of important patterns follow from these dependences.

1. The variation of the SW attenuation with electric field intensity is nonmonotonic, and has an N-shaped character: ΔH_k is a maximum for E_{cr1} and a minimum for E_{cr2}.

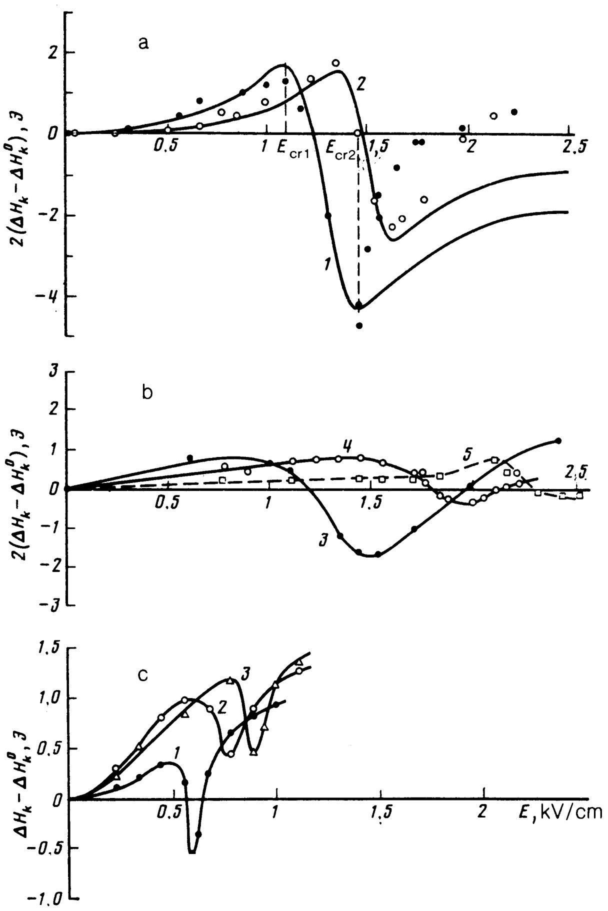

FIG. 3. Relative variation of the SW attenuation $\Delta H_k - \Delta H_k^0$ with the intensity of the applied electric field: (a) $T = 77$ K, $\sigma = 7 \cdot 10^{-3}$ Ω^{-1} cm^{-1}. Curves 1—$k = 9.2 \cdot 10^5$ cm^{-1} ($H = 750$ Oe), 2—$k = 6.3 \cdot 10^5$ cm^{-1} ($H = 849$ Oe). The points are experimental, the solid traces are calculations using (10) and (11); (b) $T = 77$ K, $\sigma = 7 \cdot 10^{-3}$ Ω^{-1} cm^{-1}. Traces 3—$k = 13 \cdot 10^5$ cm^{-1} ($H = 630$ Oe), 4—$k = 4 \cdot 10^5$ cm^{-1} ($H = 920$ Oe), 5—$k = (2–3) \cdot 10^5$ cm^{-1} ($H = 931$ Oe); (c) $T = 4.2$ K, $\sigma = 2.2 \cdot 10^{-2}$ Ω^{-1} cm^{-1}. Traces 1—$k = 8.10^5$ cm^{-1} ($H = 651$ Oe), 2—$k = 6 \cdot 10^5$ cm^{-1} ($H = 707$ Oe), 3—$k = 4 \cdot 10^5$ cm^{-1} ($H = 747$ Oe).

The difference $\delta(E_{cr1}) - \delta(E_{cr2})$ at 4.2 K is considerably smaller than at 77 K. At 60 K this difference is close to twice as small as the result for 77K, while the dependence $E_{cr2} = f(V_\phi)$ is in this case practically the same for both (Fig. 4).

2. The N-shaped variation in $\delta_k(E)$ which is observed against the background of monotonic increase in ΔH_k with E is especially noticeable at 4.2 K (Fig. 3c).

3. As k decreases, the values of E_{cr1} and E_{cr2} increase: at 4.2 K the value of E_{cr} is approximately half that at 77 K. In Fig. 4 we show the dependence of the values of E_{cr2} on the SW phase velocity for three different temperatures. The dashed curves denote the functions $V_{dr} = V_\phi$ at 77 K and 4.2 K. It is clear that the resonance variation of the SW attenuation is observed only when the Cherenkov radiation condition is fulfilled.

4. To the limits of our measurement accuracy, the experimental results do not change when the direction of E is reversed.

5. No variation of the SW attenuation is observed if the electric field direction is perpendicular to the SW propaga-

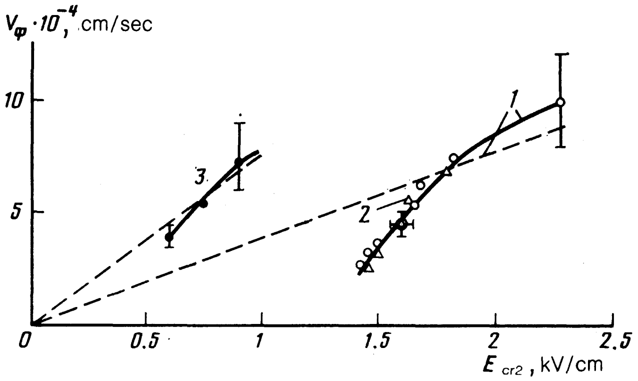

FIG. 4. Dependence of the electric field intensity E_{cr1} on the phase velocity of spin waves at 77, 60, and 4.2 K (curves 1–3). The broken traces 1 and 3 are the functions $V_{dr} = V_\phi$ at 77 K and 4.2 K; Δ—curve 2 at $T = 60$ K.

tion direction (E||H), or if the microwave and electric field are shifted in time by a value of $\sim 1\,\mu$sec or more. This quantity ($1\,\mu$sec) is apparently determined by the lengths of the microwave fronts and electric pulses; its size indicates that the variation in ΔH_k in an electric field corresponds to rapidly acting processes.

6. An electric field is not observed to have any influence on the threshold for SW excitation in nonconducting crystals ($\sigma = 10^{-4}$–$10^{-5}\,\Omega^{-1}\,\text{cm}^{-1}$) and in p-type crystals with carrier mobilities $\mu \approx 5$ cm^2/V·sec and conductivity $\sigma = 10^{-1}\,\Omega^{-1}\,\text{cm}^{-1}$) at 77 K.

DISCUSSION

The Joule heating of the sample in an electric field intensity of 3 kV/cm can attain a value of 1–1.5 and 6–7 K for 77 K and 4.2 K, respectively. As the temperature increases, the spin wave attenuation grows (see Fig. 2). In addition, it is clear from Eqs. (2) and (3) that, because of the decrease in magnetization under the conditions of our experiment (in which the external magnetic field is maintained constant), waves will be excited in the electric field with higher k and correspondingly higher attenuation (Fig. 2). Hence, the monotonic growth of ΔH_k with E (for example, what is shown in Fig. 3c) can be explained by heating. As an example, part of this heating can be associated with independent heating of the magnon system by charge carriers heated in the electric field.

The nonmonotonic variation of ΔH_k (E) is apparently caused by Cherenkov interaction of the parametrically excited spin waves with the drifting carriers. The following facts argue in favor of this:

1. The effect is seen only in the region where $V_{dr} > V_\phi$ (Fig. 4). For example, at 77 K for $k = 9\cdot 10^5$ cm^{-1} we have $V_\phi = 3\cdot 10^4$ cm/sec, $E_{cr_2} = 1.45$ kV/cm, $V_{dr} = 5\cdot 10^4$ cm/sec.

2. The effect is sensitive to the mutual orientation of the electric and magnetic fields; it is absent if the electric field is perpendicular to the direction of propagation of the spin waves.

3. The peculiarity pointed out in Paragraph 4 of the previous section is explained by the fact that spin waves are excited by longitudinal pumping with $\mathbf{k}_1 = -\mathbf{k}_2$; therefore the contribution to ΔH_k from the Cherenkov interaction should not depend on the direction of **E**.

Many authors[2,20] have investigated various mechanisms (inductive, s-d exchange, spin-orbit[21]) for the Cherenkov interaction of spin waves with charge carriers both in the hydrodynamic ($kl < 1$) and collisionless ($kl > 1$) regimes (where l is the mean free path of an electron). Under the conditions of our experiment, the inequality $kl < 1$ is always fulfilled. In this case our estimates suggest that out of all these mechanisms the dominant one which results in amplification is induction. For this mechanism

$$\Delta H_{k,\text{ind}} = h_{\text{ind}}(1+\cos^2\theta_k)(1-\mathbf{n}\mathbf{V}_{dr}/V_\phi),$$
$$h_{\text{ind}} = (k_0/k)^2 2\pi M/\omega\tau_M, \quad \mathbf{n}=\mathbf{k}/k, \quad k_0=\omega\varepsilon^{1/2}/c, \tag{5}$$

where ε is the dielectric permittivity of the crystal, and $\tau_M = \varepsilon/\sigma$ is the Maxwell relaxation time. Substituting (5) in (1), we obtain for the quantity[4]

$$\delta_k(E) = -\frac{h^2_{\text{ind}}}{2\Delta H_k^0}\left(\frac{V_{dr}}{V_\Phi}\right)^2. \tag{6}$$

Estimates of the SW amplification based on (6) give a value for $\delta_{k,\text{ind}}(E)$ which is many orders of magnitude smaller than that observed in experiment. In addition, the mechanisms discussed in Refs, 2, 20 ought to lead to a monotonic dependence of ΔH_k on E.

The character of the nonmonotonic variation of ΔH_k on E (Fig. 3) is in many ways similar to analogous behavior of the electronic attenuation of sound in piezoelectrics (the acoustoelectric effect[22]). Therefore, it is natural to assume that in magnetic semiconductors there exists a magnetoelectric effect, through which an electric field of noninducitve nature is generated when SW propagate in a ferromagnetic semiconductor. This field bunches the charge carriers, and the subsequent relaxation of the inhomogeneous distribution of bulk charge leads to an additional SW attenuation which is sensitive to the carrier drift. Without loss of generality, we can write for the potential of this anomalous electric field

$$\varphi_{\text{anom}}(\mathbf{k},\omega) = -(4\pi\mu_B/e)\beta_z(\mathbf{k},\omega)\mathbf{m}(\mathbf{k},\omega), \tag{7}$$

where $\mathbf{m}(\mathbf{k},\omega)$ is the variable component of the SW magnetization, and $\beta_z(\mathbf{k},\omega) = (\beta_{zx},\beta_{zy},0)$ is a certain vector whose explicit form is determined by the crystal symmetry. In order to find the electronic attenuation it is necessary to calculate the additional power absorbed as the SW propagates:

$$P(\mathbf{k},\omega) = {}^1\!/_2\,\mathrm{Re}\{\mathbf{j}(\mathbf{k},\omega)[\mathbf{e}^*(\mathbf{k},\omega)+\mathbf{e}^*_{\text{anom}}(\mathbf{k},\omega)]\}$$
$$= \frac{\omega\varepsilon}{8\pi}|\varphi_{\text{anom}}(\mathbf{k},\omega)|^2\frac{\tilde{\omega}\tau_M}{(1+k^2L_D^2)^2+(\tilde{\omega}\tau_M)^2},$$

where $\mathbf{j}(\mathbf{k},\omega)$, $\mathbf{e}(\mathbf{k},\omega)$ are the Fourier components of the current and field, $\mathbf{e}_{\text{anom}}(\mathbf{k},\omega) = i\mathbf{k}\varphi_{\text{anom}}(\mathbf{k},\omega)$ is the intensity of the anomalous electric field, $\omega = \omega(1-\mathbf{n}\mathbf{V}_{dr}/V_\phi)$, and L_0 is the Debye screening length. From the expression for the energy density of a ferromagnet (Ref. 2, chapter 1) we can obtain an expression for the spin wave energy $W(\mathbf{k},\omega)$. Using the dispersion relation for spin waves (Ref. 2, chapter 2), we obtain the magnitude of the additional attenuation in the form

$$\Delta H_k = \frac{P(\mathbf{k},\omega)}{2\gamma W(\mathbf{k},\omega)} = h_k\frac{\tilde{\omega}\tau_M}{(1+k^2L_D^2)^2+(\tilde{\omega}\tau_M)^2}, \tag{8}$$

$$h_k=2\pi M\left(\frac{\mu_B}{e}\right)^2 k^2\varepsilon\left\{\left[1+\left(\frac{\omega_M}{\omega_p}\right)^2\sin^4\theta_k\right]^{1/2}[\beta_{zx}{}^2+\beta_{zy}{}^2]\right.$$
$$\left.-\frac{\omega_M}{\omega_p}\sin^2\theta_k[(\beta_{zx}{}^2-\beta_{zy}{}^2)\cos 2\varphi_k+2\beta_{zx}\beta_{zy}\sin 2\varphi_k]\right\}. \quad (9)$$

Calculating the quantity $\Delta H_k = (\Delta H_{k_1}\Delta H_{k_2})^{1/2}$ and assuming that $\Delta H_k^0 > \Delta H_k(E)$, we find that

$$\delta_k(E)=2h_k\frac{\omega\tau_M}{(1+k^2L_D^2)^2+\omega^2\tau_M^2}f(\eta,E), \quad (10)$$

$$f(\eta,E)=\frac{1-\alpha^2}{(1+\alpha^2)^2-4\eta\alpha^2}-1, \quad (11)$$

where

$$\alpha=\frac{E}{E_0},\quad E_0=\frac{\omega}{k\mu}\frac{1}{\eta^{1/2}}\frac{1}{\cos\psi_k},$$
$$\eta=\frac{\omega^2\tau_M^2}{(1+k^2L_D^2)^2+\omega^2\tau_M^2},\quad \psi_k=\varphi_k-\varphi_E,$$

φ_k, φ_E are the azimuthal angles for the vectors **k** and **E**.

The solid trace in Fig. 3 shows a calculation of the variation of the SW attenuation in the electric field $\delta_k(E)$ from Eqs. (10) and (11) using the values $\omega\tau_M = 8$, $L_0 = 8\cdot 10^7$ cm, $\mu = 35$ cm^2/V·sec, obtained on the basis of constant-current measurements. We took $h_k = 5.7$ and 3.4 Oe for $k = 9.2\cdot 10^5$ and $6.3\cdot 10^5$ cm^{-1}, respectively, while $\theta_k = 90°$, $\cos\varphi_k = 0.73$, i.e., it was assumed that in the azimuthal plane there is a certain specific direction with which the electric field makes an angle $\approx 45°$. It is clear that Eqs. (10) and (11) explain the basic features of the behavior of $\delta_k(E)$, i.e., the presence of two extrema and the growth of $\delta_k(E)$ with increasing k. A certain difference between the experimental and theoretical values of $\delta_k(E)$ is apparently connected with the fact that ΔH_k depends on the angle θ_k. The values of k and ΔH_k must be determined from minimization of Eq. (1). The problem in this case is greatly complicated, because the values of k and θ_k will depend on E. We can expect (when the inequality $\Delta H_k \gg h_k$ is taken into account) that the character of $\delta_k(E)$ remains the same as in Fig. 3.

Let us investigate possible causes of the magnetoelectric effect. One of the mechanisms for the appearance of an anomalous electric field consists of a modulation of the conduction band edge $\varepsilon_c = \varepsilon_c(\mathbf{M},\mathbf{H})$ by the variable component of the SW field.[23] If the dependence of ε_c on **M** and **H** is caused by the isotropic *s-d* exchange interaction of the carriers with the magnetic moments of the Cr^{+3} ions, then

$$\beta_z=\beta n_z\mathbf{n},\ \beta=|d\varepsilon_c/d(\mu_B H)|$$

and the quantity (9) takes the form

$$h_k=h_k{}^0P(\theta_k),\ h_k{}^0=2\pi M(\mu_B/e)^2\varepsilon\beta^2k^2,$$
$$P(\theta_k)=\sin^2\theta_k\cos^2\theta_k\{[(1+(\omega_M/\omega_p)^2\sin^4\theta_k]^{1/2}$$
$$-(\omega_{\overline{M}}/\omega_p)\sin^2\theta_k\}. \quad (12)$$

The quantity β is the magnetic analogue of the deformation potential. An estimate of β can be made rigorously only in the spin-wave approximation.[23] Using values of the exchange splitting of the conduction band $\Delta(4.2\text{ K}) = 1.5$ $A_c \approx 1$ eV (A_c is the exchange integral), the effective electron mass $m = 0.2\, m_e$ and $D = 2\cdot 10^{-10}$ Oe·cm^2, we obtain

$$\beta_{SW}=\begin{cases}1.5\cdot 10^3(1-M/M_0), & T<9.7\text{K}\\ 5.2kT/\hbar\omega_M, & T>9.7\text{K}\end{cases} \quad (13)$$

We include in this the fact that in the spinel structure there are four magnetic Cr^{+3} ions which fit into the primitive unit cell. Then at 77 K we have $\beta_{sw} = 1.1\cdot 10^3$. However, this estimate of β_{sw} for $HgCr_2Se_4$ may turn out to be too low, because the temperature 77 K is close to $T_k = 106$ K. It is known that in calculating ε_c using the spin-wave approximation a contribution to the true magnetization enters in which is fully compensated. If the calculations are carried out in the molecular-field approximation for the spin correlators, then at high temperatures a contribution to the true magnetization appears. Then

$$\beta=\beta_{SW}+\frac{A_c\Omega}{(2\mu_B)^2}\frac{\partial M}{\partial H}. \quad (14)$$

Here Ω is the volume of a unit cell. Making use of the experimental value of the susceptibility of the true magnetization $\partial M/\partial H \approx 10^{-2}$ at 77 K, we obtain from (14) that $\beta = 2.5\cdot 10^3$. Then for $4\pi M = 3000$ G, $\varepsilon = 16$,

$$h_k{}^0\,[\text{Oe}]=60(10^{-6}k\,[\text{cm}^{-1}])^2. \quad (15)$$

Minimizing the threshold field (1) while taking into account the carrier contribution from Eqs. (10)–(15) shows that for $E = 0$ the minimum threshold is for SW with $\theta_k = \pi/2$, i.e., for the magnetoelectric effect mechanism under discussion the current carriers do not contribute to the value of h_{th}. However, when $h_k^0 \gtrsim 4\Delta H_k^0$ and $E > E_0$ ($V_{dr} > V_\phi$) the SW with minimum threshold are those with $\theta_k \neq \pi/2$. For example, when $E = E_{cr2}$ for $k = 9.2\cdot 10^5$ cm^{-1}, $\Delta H_k^0 = 20$ Oe and $h_k^0 = 60$ Oe, the spin waves with the minimum threshold are those with $\theta_k \approx \pi/3$. In this case, from (10)–(12) we have $\delta_k(E_{cr2}) = -5.3$ Oe. This value of $\delta_k(E)$ numerically coincides with the experimentally-observed value of the decrease of the SW attenuation parameter in an electric field (Fig. 3). According to this model of the magnetoelectric effect, spin wave amplification is possible ($h_{th}\to 0$) if

$$h_k{}^0 \gtrsim (6-8)\Delta H_k{}^0.$$

The character of the variation $\delta_k(E)$ as the temperature decreases coincides qualitatively with the decrease of β as a function of temperature from (13). However, quantitatively the effect at liquid helium temperature should be almost an order of magnitude smaller. It is possible that this is caused by Joule and magnon heating in the electric field: it is necessary to assume that the magnon temperature is ~ 20 K. This is in keeping with the assumption we used above to explain the monotonic growth of ΔH_k with E (Fig. 3).

We remark that as the symmetry of the crystal decreases from T or T_d (loss of a center of symmetry) it becomes possible for terms in β_z to appear which are linear in **n**; in this case, h_k, $\propto \sin^2\theta_k$ and $h_k \neq 0$ for $\theta_k \neq \pi/2$.

There is still another mechanism which could cause an electric field to appear when SW propagate, which is related to impurity centers in conducting crystals of $HgCr_2Se_4$. The fact is that conduction electrons in this material arise because of the appearance of selenium vacancies. Each vacancy, together with its three neighboring Cr^{+3} ions, forms an impurity center which appearently binds two electrons.

One electron from the center is ejected into the conduction band (because the bottom of the latter is strongly decreased). The other electron, which is in the lowest bound state, is smeared out among the three Cr^{+3} ions (one sometimes speaks of this impurity center as a Cr^{+3} ion). Since the crystal is compensated, certain impurity centers will in general be empty. The transfer of an electron from a filled center to an empty one under the action of the AC magnetization component of the SW could be the cause of the broadening of the ferromagnetic resonance line in ferrites.[19]

CONCLUSIONS

1. We have observed a variation of the decay of parametrically-excited SW with electric field intensity in the ferromagentic semiconductor $HgCr_2Se_4$ when the Cherenkov radiation condition is fulfilled.

2. Our results are explained by excitation of an electric field of noninductive nature by a spin wave.[23] In a wide-gap magnetic semiconductor the energy of an electron depends on the magnetization, and the electrons tend to collect in those regions where their energy is lowest. Because of this, the SW gives rise to a spatially inhomogeneous electron distribution.

3. Based on this model, we have calculated a free-carrier contribution to the SW attenuation which depends on carrier drift and which explains the basic features of the variation of attenuation with various parameters. Estimates of the variation of the SW attenuation with the electric field direction and temperature are found to be in reasonable agreement with the experimental results.

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