Generation of squeezed states of an electromagnetic field by a fluctuating pump

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It is shown that fluctuations of the pump field when squeezed states are generated limits the degree of maximum squeezing and leads to the existence of an optimal length of the nonlinear medium. The role of group synchronism of the interacting wave is explained.

I. INTRODUCTION

The greater part of the experiments[1–4] devoted to generation of squeezed states of an electromagnetic field is based on parametric photon decay (PPD) in a nonlinear medium[5] or on four-wave mixing.[6,7] The low degree of squeezing attained in these experiments poses the problem of a consistent analysis of the causes that limit the degree of squeezing. The influence exerted on the degree of squeezing by losses in the radiation propagating in the medium and by spontaneous emission of particles of the medium in four-wave mixing was analyzed in Ref. 8. In the present paper we analyze the influence exerted on the squeezing by the fluctuations of the amplitude and phase of the electromagnetic wave used as the pump in PDP, or the influence of the reference waves in four-wave mixing.

II. DEGENERATE PARAMETRIC PHOTON DECAY

1. Basic equations

We assume that a pump field of frequency 2ω with fluctuating amplitude and phase

$$\mathscr{E}_{\text{н}}(z=0,\, t)=E_{\text{н}}(t)e^{-2i\omega t}=A(t)e^{i\varphi(t)}e^{-2i\omega t}$$

is applied at the entrance ($z=0$) of a nonlinear nonresonant medium. A photon of frequency 2ω decays in this medium into two equal photons of frequency ω (secondary field). The equations describing the propagation of complex amplitudes of the pump fields E_p and of the secondary wave $a(t)$ take, when the phase-synchronism condition is met, the form[9]

$$\frac{\partial E_p}{\partial z}+\frac{1}{u_p}\frac{\partial E_p}{\partial t}=-\varkappa a^2, \tag{1a}$$

$$\frac{\partial a}{\partial z}+\frac{1}{u}\frac{\partial a}{\partial t}=\varkappa E_p a^*, \tag{1b}$$

where u_p and u are the group velocities of the corresponding waves in the nonlinear medium, $\varkappa=[2\pi\omega/cn(\omega)]\chi^{(2)}$; $\chi^{(2)}$ is the material nonlinear-susceptibility coefficient, $n(\omega)$ is the linear part of the refractive index of the medium, and c is the speed of light in vacuum. Note that the plane-wave approximation is used here and hereafter.

To solve the problem we must derive equations for the secondary-wave photon creation and annihilation operators. Assume that the secondary-wave intensity is low compared with the pump intensity during the entire squeezing process. We can then neglect the right-hand side of Eq. (1a). Its solution satisfying the boundary condition is

$$E_p(\theta)=A(\theta)e^{i\varphi(\theta)}, \tag{2}$$

where $\theta=t-z/u_p$ and $A(t)e^{i\varphi(t)}$ are the values of the pump-wave parameters at $z=0$.

The equations for the operators $\hat a$ and $\hat a^+$ are then

$$\frac{\partial \hat a}{\partial z}+\frac{1}{u}\frac{\partial \hat a}{\partial t}=\varkappa A(\theta)e^{i\varphi(\theta)}\hat a^+, \tag{3a}$$

$$\frac{\partial \hat a^+}{\partial z}+\frac{1}{u}\frac{\partial \hat a^+}{\partial t}=\varkappa A(\theta)e^{-i\varphi(\theta)}\hat a. \tag{3b}$$

The pump field is assumed here to be a classical parameter.

Fluctuations of one of the quadrature components of the field

$$\hat X_s=\frac{\hat a-\hat a^+}{2i} \quad \text{or} \quad \hat X_c=\frac{\hat a+\hat a^+}{2} \tag{4}$$

are then suppressed in the squeezed state of the electromagnetic field. It is therefore expedient to continue the investigation by writing down equations for the operators $\hat X_s$ and $\hat X_c$. Since the pump field is a function of the argument θ, it follows that $\hat X_c=\hat X_c(\theta)$ and $\hat X_s=\hat X_s(\theta)$, and we obtain from (3)

$$\frac{dX_c}{d\theta}=\frac{\varkappa}{v}A(\theta)[\hat X_c\cos\varphi(\theta)+\hat X_s\sin\varphi(\theta)], \tag{5a}$$

$$\frac{d\hat X_s}{d\theta}=-\frac{\varkappa}{v}A(\theta)[\hat X_s\cos\varphi(\theta)-\hat X_c\sin\varphi(\theta)]. \tag{5b}$$

Equations (5) have for a constant phase the solution

$$\hat X_s(\theta,\eta)={}^1/_2[\hat X_{s0}(\eta)(1-\cos\varphi)-\hat X_{c0}(\eta)\sin\varphi]e^{\lambda}+{}^1/_2[\hat X_{s0}(\eta)(1+\cos\varphi)+\hat X_{c0}(\eta)\sin\varphi]e^{-\lambda}, \tag{6a}$$

$$\hat X_c(\theta,\eta)={}^1/_2[\hat X_{c0}(\eta)(1+\cos\varphi)-\hat X_{s0}(\eta)\sin\varphi]e^{\lambda}+{}^1/_2[\hat X_{c0}(\eta)(1-\cos\varphi)+X_{s0}(\eta)\sin\varphi]e^{-\lambda}, \tag{6b}$$

where $\hat X_{c0}$ and $\hat X_{s0}$ are the values of the quadrature-component operators at $z=0$, and

$$\eta=t-\frac{z}{u},\quad \lambda=\frac{\varkappa}{v}\int_\eta^\theta A(\theta')d\theta'.$$

We introduce the definitions

$$\Delta X_s=[\langle\hat X_s^2\rangle-\langle\hat X_s\rangle^2]^{1/2},\quad \Delta\hat X_c\neq\langle\hat X_c^2\rangle-\langle\hat X_c\rangle^2]^{1/2}. \tag{7}$$

We assume the field to be normalized in such a way that the uncertainty relation takes the form

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$$\Delta X_s \Delta X_c \geq 1/4. \quad (8)$$

If $\varphi = 0$, it follows from (6) that

$$\Delta X_c = \Delta X_{c0} e^{\lambda}, \quad \Delta X_s = \Delta X_{s0} e^{-\lambda}. \quad (9)$$

Assume that the secondary radiation evolves from vacuum fluctuations, or that a wave of frequency ω in a coherent state is applied at the entrance into a nonlinear system. We have then $\Delta X_{s0} = \Delta X_{c0} = 1/2$ and as the process evolves in the nonlinear medium the fluctuations of one of the quadrature components (ΔX_s in our case) becomes smaller than the vacuum fluctuations, and a squeezed state of the electromagnetic field sets in. The degree Sq of the squeezing is given by the simple equation

$$\mathrm{Sq} = \Delta X_{s0} / \Delta X_s = 1/2 e^{\lambda}. \quad (10)$$

The fluctuations of the second component grow in this case, so that the condition

$$\Delta X_c \Delta X_s \geq 1/4$$

remains satisfied.

If $\varphi \neq 0$, the fluctuations ultimately increase exponentially in both components [see (6)]. This does not mean, however, that no squeezing takes place. The coordinate frame in which the squeezing takes place is simply rotated through an angle $\psi = \varphi/2$ relative to the initial one. In fact, at $\varphi =$ const the transformation

$$\hat{X}_c = \hat{X}_c' \cos(\varphi/2) - \hat{X}_s' \sin(\varphi/2),$$
$$\hat{X}_s = \hat{X}_s' \cos(\varphi/2) + \hat{X}_c' \sin(\varphi/2) \quad (11)$$

reduces Eqs. (5) to

$$\frac{d\hat{X}_c'}{d\theta} = \frac{\varkappa}{v} A(\theta) \hat{X}_c', \quad \frac{d\hat{X}_s'}{d\theta} = -\frac{\varkappa}{v} A(\theta) X_s'. \quad (12)$$

These equations coincide with (5) if $\varphi = 0$. The constant phase at the entrance into the nonlinear medium determines only the orientation of the axes of the squeezed state, but does not influence the squeezing process. The situation changes radically if φ is a time dependent random quantity.

2. Phase fluctuation. Role of group synchronism

We assume that the pump fluctuations are set only by the boundary conditions—there are no fluctuation sources in the medium itself. In this case the phase at each point of the pump-wave profile is constant during the wave propagation in the nonlinear medium. In the language of mathematics, the phase is constant on the characteristic of the pump wave $\theta =$ const. Therefore, if the group velocities of the waves are equal, the interaction of the secondary wave with the pump wave at each point of the latter takes place at a constant phase, even though the phase varies randomly from point to point. As shown above, a constant phase does not limit the degree of squeezing, and determines only the orientation of the axis along which the squeezing takes place. The phase fluctuations at the entrance into the nonlinear medium cause fluctuations of the squeezing direction, but should not influence the degree of squeezing under group synchronism conditions.

If, however, the group velocities differ, the secondary wave either lags or leads the pump wave. In either case, the secondary wave goes over from one characteristic of the pump wave to another, and the wave interaction takes place under conditions in which the phase fluctuates at each point of the secondary wave. This prevents establishment of even a local squeezing direction, thereby limiting in final analysis the degree of squeezing. The calculation below confirms these qualitative considerations.

We assume in the calculation that the amplitude is constant: $A(\theta) = A_0$. With an aim of obtaining substantial degrees of squeezing, we assume a small change $\Delta\varphi$ of the pump-wave phase during the time of passage of the wave through the nonlinear medium, $\Delta\varphi \ll 1$. Equations (5) can then be solved by perturbation theory. The orientation of the coordinate frame in $\hat{X}_c$ and $\hat{X}_s$ are defined is chosen such that at the entrance into the nonlinear medium

$$\varphi(t, z=0) = 0. \quad (13)$$

We have then a first-order approximation

$$\hat{X}_c \approx \hat{X}_{c0}(\eta) \exp\left[\frac{\varkappa A_0}{v}(\theta - \eta)\right],$$
$$\frac{d\hat{X}_s}{d\theta} \approx -\frac{\varkappa A_0}{v}\left[\hat{X}_s - \Delta\varphi(\theta, \eta) \exp\left[\frac{\varkappa A_0}{v}(\theta - \eta)\right] \hat{X}_{c0}(\eta)\right], \quad (14)$$

where $\Delta\varphi(\theta,\eta) = \varphi(\theta) - \varphi(\eta)$, and

$$\hat{X}_s(\theta, \eta) = \hat{X}_{s0}(\eta) \exp\left[-\frac{\varkappa A_0}{v}(\theta - \eta)\right] + \hat{X}_{c0}(\eta) \frac{\varkappa A_0}{v} \exp\left[-\frac{\varkappa A_0}{v}(\theta - \eta)\right] \times \int_\eta^\theta \Delta\varphi(\theta, \eta) \exp\left(2\frac{\varkappa A_0}{v}\theta'\right) d\theta'. \quad (15)$$

3. Case of diffusing phase

Further analysis requires assumptions concerning the character of the phase fluctuations. Assume that $\varphi(t)$ is a Wiener diffusion process, so that

$$\overline{[\varphi(t') - \varphi(t'')]^2} = \Delta\Omega |t' - t''|.$$

Then, taking (13) into account,

$$\overline{\Delta\varphi(\theta', \eta) \Delta\varphi(\theta'', \eta)} = \Delta\Omega \min\{|\theta' - \eta|, |\theta'' - \eta|\}. \quad (16)$$

It is known[9] that in the absence of the amplitude fluctuations the diffusion coefficient $\Delta\Omega$ of the phase of an electromagnetic wave is equal to the width of the intensity spectrum of this wave.

Taking (16) into account, we obtain from (15)

$$\Delta X_s \approx \Delta X [e^{-\zeta} + (2\zeta - 3) e^{\zeta}/8q]^{1/2}, \quad (17)$$

where

$$\Delta X_{s0} = \Delta X_{c0} = \Delta X, \quad \zeta = 2\varkappa A_0(\theta - \eta)/v$$
$$= 2\varkappa A_0 z, \quad q = 2\varkappa A_0 / v\Delta\Omega.$$

In the calculation of (17) it was taken into account that $\langle \hat{X}_c \hat{X}_s + \hat{X}_s \hat{X}_c \rangle = 0$, and terms of order unity compared with e^{ζ} were neglected.

It follows from (17) that the degree of squeezing reaches a maximum at a point z_0 defined by the relation

$$(2\zeta_0 - 1) e^{2\zeta_0}/8q = 1. \quad (18)$$

The maximum squeezing Sq_m turns out to be connected with the parameters of the nonlinear medium and of the pump:

$$\frac{\zeta_0-1}{\zeta_0-0.5} Sq_m^4 \frac{1}{q} = 1. \qquad (19)$$

According to (19), large degrees of squeezing require large values of q. Thus, to reach a squeezing of the order of 10 we need $q \approx 10^4$. Numerical analysis shows that at large q the solution of (18) can be approximated by the relation

$$\zeta_0 \approx \lg q + 0.7. \qquad (20)$$

Equation (19) takes then the more elegant form

$$Sq_m^4 q^{-1} \lg q \approx 1, \qquad (19')$$

and the accuracy of this approximation for $q \geqslant 10^4$ is better than 10%.

In a nonlinear medium, the path length $l_{opt} = \zeta_0/2 \varkappa A_0$ defined by Eqs. (18) and (20) is optimal, since further development of the process leads to degradation of the squeezed state. For example, doubling the path length in a nonlinear medium compared with the optimal at $q \gg 1$ increases the fluctuations in the squeezed component back to the initial values ΔX_0. At half l_{opt} we have $Sq(l_{opt}/2) \approx Sq_m^{1/2}$.

It is more convenient in experiment to choose not the optimal path length in the nonlinear medium, but the optimal pump-source intensity. Equation (17) can be tested for maximum squeezing as a function of the pump intensity at a given path length l in a nonlinear medium. The result is

$$(A_0^2)_{opt} \approx (\ln \gamma)^2/4\varkappa^2 l^2, \qquad (21)$$

$$Sq_m^4 \gamma^{-1} \ln \gamma \approx 1, \qquad (19'')$$

where $\gamma = 4/v\Delta\Omega l$. It should be noted that when l is increased the maximum degree of squeezing decreases. Therefore, if the pump supply has power to spare, a short path in a nonlinear medium is preferable.

Since the degree of squeezing enters in (19') raised to the fourth power, a relatively small increase of the squeezing must be "paid for" by an appreciable increase of q. Evidently, in view of the parameter q, the degree of squeezing is strongly influenced by the difference between the group velocities of the pump and of the secondary wave, this difference being smaller the larger the degree of squeezing for the same pump-wave intensity and spectrum width. This conclusion agrees fully with the qualitative reasoning above.

For a specific nonlinear material, one can seek a polarization direction and a wavelength for which the phase and group velocities of the pump and secondary waves are simultaneously equal.

The possibility of simultaneously meeting the phase and group synchronism conditions for PPD was investigated in Ref. 10. It was found, in particular, that at a pump wavelength $\approx 0.53\ \mu m$ there exists in the nonlinear crystal $LiIO_3$ a direction in which the conditions of group and phase synchronism are met simultaneously. However, an analysis of the data of Ref. 11 for nonlinear optical materials shows that the possibilities of ensuring the double synchronism are quite limited. The most direct way of obtaining a high degree of squeezing is therefore the use of pump with highly monochromatic radiation. The required degree of monochromaticity can be estimated for individual nonlinear media. Examples of such estimates are given below.

4. Stationary random phase

Assume now that the pump-radiation phase is determined by a stationary random process with a Lorentz spectrum of half-width $\Delta\Omega_\varphi$. According to the boundary condition (13), the phase at the entrance into the nonlinear medium is certainly zero. This means that in our chosen coordinate frame, in accord with the assumptions made, $\Delta\varphi(\theta,\eta)$ in each cross section z is a stationary process, but with $\overline{\Delta\varphi^2}$ dependent on z. The dependence should be such that $\overline{\Delta\varphi^2(\theta,\eta)} = 0$ for $z = 0$ and $\overline{\Delta\varphi^2(\theta,\eta)} \to \overline{\varphi^2}$ for large z. This condition can be met by choosing for the correlation function of $\Delta\varphi$ the expression:

$$\overline{\Delta\varphi(\theta', \eta)\Delta\varphi(\theta'', \eta)} = \overline{\Delta\varphi^2(\theta-\eta)} \exp(-\Delta\Omega_\varphi|\theta'-\theta''|), \qquad (22)$$

where

$$\overline{\Delta\varphi^2(\theta-\eta)} = \overline{\varphi^2}\{1-\exp[-\Delta\Omega_\varphi(-\theta-\eta)]\}. \qquad (23)$$

Substituting (22) in (15) we obtain by obvious transformations

$$Sq^{-2} = e^{-\zeta} + \frac{\overline{\varphi^2}}{4} \frac{q}{q+1} (1-e^{-\zeta/q}) e^{\zeta},$$

where $q = 2\varkappa A_0/v\Delta\Omega_\varphi$. With this equation we easily calculate the optimal length of the nonlinear medium and the maximum attainable degree of squeezing. We have

$$l_{\text{опт}} \approx \frac{1}{\varkappa A_0} \lg \frac{q}{\overline{\varphi^2}}, \qquad (24)$$

$$Sq_m^4 \left(\frac{q}{\overline{\varphi^2}}\right)^{-1} \lg \frac{q}{\overline{\varphi^2}}. \qquad (25)$$

The choice of the coordinate frame in which $\overline{\Delta\varphi^2} = 0$ at the entry into the nonliner medium is neither formal nor trivial. Not formal, because it is on the basis of this frame that the experimenter detects the squeezing. Not trivial, because the frame must be oriented in step with the variation of the pump phase $\varphi(t)$. Squeezed states are usually revealed by homodyning the detected signal,[1,2] i.e., by mixing it nonlinearly with a reference oscillation of the same frequency to separate the slowly varying signal amplitude. In the homodyne system the coordinate frame is thus tied to the phase of the reference oscillation. This means that the reference-oscillation phase must be a random process strictly identical to the process $\varphi(t)/2$. This can be done if the fluctuations of the pump-wave parameters are strictly correlated with the fluctuations of the parameters of the reference oscillations. In Refs. 1 and 2, for example, the pump and reference oscillations were produced by the same generator. If, on the other hand, the pump phase fluctuations are not correlated with those of the reference phase, it is impossible to meet condition (13) reliably in any point of space. Equation (23) must then be replaced by the stationary correlation function

$$\overline{\Delta\varphi(\theta')\Delta\varphi(\theta'')} = \overline{\varphi^2} \exp(-\Delta\Omega_\varphi|\theta'-\theta''|).$$

This replacement yields for the maximum degree of squeezing

$$Sq_m^4 \overline{\varphi^2} q/(q+1) \approx 1. \qquad (26)$$

The most remarkable feature of this result is that the squeezing has a finite limit even as $q \to \infty$. Since $q \sim \nu^{-1}$, the squeezing remains finite also when the group velocities are matched.

A wave with a random but small stationary phase can be represented by a sum of a monochromatic wave of amplitude A_0 and a random wave with zero mean amplitude and uniform phase distribution in the interval $[-\pi,\pi]$.[9] If the random-wave field has a Gaussian distribution with variance σ, the variance of the combined-wave phase is given by

$$\overline{\varphi^2}=\sigma/A_0^2, \tag{27}$$

and Eq. (26) takes the form

$$\mathrm{Sq}_m^4 \frac{\sigma}{A_0^2}\frac{q}{q+1} \approx 1. \tag{26'}$$

5. Amplitude fluctuations

We consider now the influence of amplitude fluctuations, assuming the phase to be constant. In this case the squeezing is described by Eqs. (12). Assume that $A(\theta) = A_0 + \alpha(\theta)$, where A_0 is the average pump-wave amplitude, and $\alpha(\theta)$ is its fluctuating part and has a Gaussian probability distribution. By simple calculations we can obtain then from (12)

$$\Delta X_s=\Delta X_{s0} \exp(-\varkappa A_0 z)\exp\left[\overline{(\Delta\lambda)^2}/2\right], \tag{28}$$

$$\overline{(\Delta\lambda)^2}=\frac{\varkappa^2}{\nu^2}\int_\eta^\theta d\theta' \int_\eta^\theta d\theta''\, \overline{\alpha(\theta')\alpha(\theta'')}. \tag{29}$$

The succeeding calculation depends on the assumed spectral characteristics of the random quantity $\alpha(\theta)$. Let $\alpha(\theta)$ have a Lorentz spectrum of width $\Delta\Omega_A$. Then

$$\overline{\alpha(\theta')\alpha(\theta'')}=\overline{\alpha^2} \exp(-\Delta\Omega_A|\theta'-\theta''|), \tag{30}$$

$$\frac{1}{2}\overline{(\Delta\lambda)^2}=\frac{\varkappa^2\overline{\alpha^2}}{\nu\,\Delta\Omega_A}z-\frac{\varkappa^2\overline{\alpha^2}}{\nu^2\,\Delta\Omega_A^2}\left[1-\exp(-\Delta\Omega_A \nu z)\right]. \tag{31}$$

It follows from (28) and (31) that for $\beta \equiv \nu A_0 \Delta\Omega_A / \varkappa \overline{\alpha^2} > 1$ the squeezing is unlimited if the length of the nonlinear medium is unbounded. If, however, $\beta < 1$, the degree of squeezing is limited to

$$\mathrm{Sq}_m = \exp\left\{\frac{\varkappa A_0}{\nu\,\Delta\Omega_A}-\frac{\varkappa^2\overline{\alpha^2}}{\nu^2\,\Delta\Omega_A^2}(1-\beta)\ln\frac{1}{1-\beta}\right\}, \tag{32}$$

which is reached at a length

$$l_{\rm opt}=\frac{1}{\nu\Delta\Omega_A}\ln\frac{1}{1-\beta}. \tag{33}$$

The length determined by Eq. (33) is the optimum, and when increased above $l_{\rm opt}$ the degree of squeezing at the exit is smaller than Sq_m.

In group synchronism ($\nu \to 0$) the amplitude fluctuations do not limit the degree of squeezing, for $\overline{(\Delta\lambda)^2} \to 0$ in this case. In general, amplitude fluctuations are not as dangerous as phase fluctuations. If they are small enough, their presence only slows the squeezing down somewhat. In the presence of pump amplitude fluctuations, however, notwithstanding the squeezing of one of the quadrature components, the product $\Delta X_s \Delta X_c$ increases:

$$\Delta X_s \Delta X_c={}^1/_4 \exp\left[\overline{(\Delta\lambda)^2}\right]. \tag{34}$$

6. Numerical estimates

We present now concrete numerical examples. We consider PDP in a nonlinear medium of the "banana" type; the pump and parametric-signal wavelengths are 530 nm and 1.06 μm, respectively; $\chi^{(2)} = 1.32\times10^{-11}$ m/V $= 3\times10^{-8}$ CGS units. The difference of the group velocities for these two wavelengths can be calculated by using the dependence of the "banana" nonlinear-susceptibility tensor on the wavelength.[11] It turns out that $c\nu = 0.16$ in the case of interest to us. Therefore, according to (19'), to obtain a tenfold squeezing with the aid of a "banana," the connection between the pump intensity I and the width of the spectrum should be $\Delta\Omega \approx 10^3 I^{1/2}$ ($\Delta\Omega$ in Hz and I in W/cm^2). A pump of intensity $I \approx 10^3$ W/cm^2 must be monochromatic, $\Delta\Omega/\Omega \approx 10^{-11}$. An estimate based on (20) of the optimal path length in the nonlinear medium with the same parameters leads to a value $l_{\rm opt} \approx 10^2$ cm. Clearly, to set up a real experiment we must either have a pump intensity substantially higher than 10^3 W/cm^2 or employ a cavity in which the photon mean free path in the nonlinear medium is much longer than the medium.

III. FEATURES OF SQUEEZING IN DEGENERATE FOUR-WAVE MIXING

Two possible experimental setups are discussed in experiment: forward scattering[6] and backward scattering.[7] What is squeezed in both cases is a superposition of waves scattered in four-wave mixing. The forward-scattering scheme is considered preferable from the standpoint of the effect of the losses.[7] A wave in a squeezed state is obtained by merging the scattered waves.[6,7]

By using an anisotropic nonliner medium, however, it is possible to generate scattered waves that are parallel to one another (see Fig. 1), by automatically effecting their merging.[12] We shall use this scheme as the basis of the analysis that follows.

The equations for the complex amplitudes E_j in four-wave forward mixing take under phase-synchronism conditions the form[9]

$$\frac{1}{u}\frac{\partial E_3}{\partial t}+\frac{\partial E_3}{\partial z}=-i\beta E_1E_2E_4^* \tag{35a}$$

$$\frac{1}{u}\frac{\partial E_4}{\partial t}+\frac{\partial E_4}{\partial z}=-i\beta E_1^*E_2^*E_3 \tag{35b}$$

$$\frac{1}{u_0}\frac{\partial E_{1,2}}{\partial t}+\frac{\partial E_{1,2}}{\partial z_{1,2}}=0. \tag{35c}$$

In these equations $\beta = 2\pi\omega\chi^{(3)}/cn(\omega)$, $\chi^{(3)}$ is the third-order nonlinear susceptibility coefficient of the substance, u and u_0 are the group velocities of the scattered and primary waves,

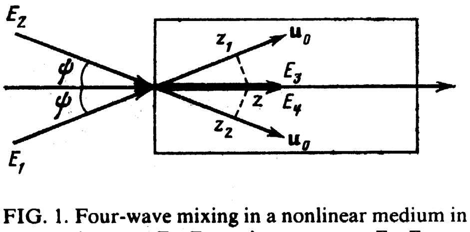

FIG. 1. Four-wave mixing in a nonlinear medium in the case of collinear scattered waves, E_1, E_2—primary waves. E_3, E_4—scattered waves.

and $z_{1,2}$ are the coordinates in the propagation direction of the corresponding waves—the "intrinsic" coordinates. As before, attenuation of the primary waves in the course of scattering is neglected, as indicated by the zero right-hand side of Eq. (35c).

The solutions of Eqs. (35c) can be written in the general form

$$E_{1,2}(t, z_{1,2})=E_{1,2}(t-z_{1,2}/u_0).$$

At the point z, the waves $E_{3,4}$ interact with primary waves for which $z_{1,2} = z\cos\psi$. Therefore

$$E_{1,2}(t-z_{1,2}/u_0)\equiv E_{1,2}(t-z\cos\psi/u_0). \qquad (36)$$

As already noted, what is squeezed in four-wave mixing is not each scattered wave but their superposition[6,7]:

$$\hat{b}=\hat{a}_3\pm i\hat{a}_4. \qquad (37)$$

It is therefore more convenient to analyze the equations for the operators $\hat{b}$ and b^+

$$\frac{\partial\hat{b}}{\partial z}+\frac{1}{u}\frac{\partial\hat{b}}{\partial t}=\beta A_1(\theta)A_2(\theta)e^{i\varphi(\theta)}\hat{b}^+, \qquad (38a)$$

$$\frac{\partial\hat{b}^+}{\partial z}+\frac{1}{u}\frac{\partial\hat{b}^+}{\partial t}=\beta A_1(\theta)A_2(\theta)e^{-i\varphi(\theta)}\hat{b}, \qquad (38b)$$

in which A_1 and A_2 are the amplitudes of the primary waves, φ_1 and φ_2 are their phases,

$$\theta=t-z\cos\psi/u_0, \quad \varphi(\theta)=\varphi_1(\theta)+\varphi_2(\theta).$$

This equation is mathematically equivalent to Eq. (3). The results of Sec. II can therefore be transformed into the results from Eq. (38) by making the substitutions

$$\varkappa A_0\to\beta A_{10}A_{20}, \quad \nu\to 1/u-\cos\psi/u_0,$$

and by taking $\Delta\Omega$, $\Delta\Omega_\varphi$, and $\overline{\varphi^2}$ to mean the characteristics of the combined random process $\varphi(t)=\varphi_1(t)+\varphi_2(t)$.

One of the features of four-wave mixing is the possibility of transforming amplitude fluctuations into phase fluctuations. The point is that in a cubic nonlinear medium the wave vector of the primary waves depends on their amplitudes[13]

$$k_1=k+\beta(A_1^2+2A_2^2), \quad k_2=k+\beta(A_2^2+2A_1^2), \qquad (39)$$

where k is the wave vector in the limit of infinitely small reference-wave intensities. [We have neglected in (39) the influence of the angle between the wave vectors of the primary waves, assuming them to be small.] For fluctuating amplitudes, the phase-synchronism condition

$$\mathbf{k}_1+\mathbf{k}_2=\mathbf{k}_3+\mathbf{k}_4 \qquad (40)$$

can be met only in the mean, so that the equations describing the compression process contain a fluctuating pump phase

$$\varphi(\theta,\eta)=1.5\beta\int_\eta^\theta[\Delta I_1(\theta')+\Delta I_2(\theta')]d\theta', \qquad (41)$$

ΔI_1 and ΔI_2 are the fluctuating intensity terms of the primary waves ($\overline{\Delta I_1}=\overline{\Delta I_2}=0$).

If $\Delta I_{1,2}$ is regarded as a stationary δ-correlated process, then $\varphi(\theta,\eta)$ is a diffusion process with a diffusion coefficient $\Delta\Omega=4.5\pi\beta^2(G_1+G_2)$, where $G_{1,2}$ are the spectral densities of the amplitude fluctuations.[9] In this case the computational part of the problem is similar to that considered in Sec. II.3.

Consideration of four-wave mixing in resonant isotropic media requires allowance for the inevitable losses to resonance absorption, and also for the influence of spontaneous emission of the nonlinear medium. In isotropic media it is impossible to satisfy the phase synchronism condition at a zero angle between the scattered waves. All this calls for a special analysis.

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