Distribution of local currents in disordered conductors

I. V. Lerner

Spectroscopy Institute, USSR Academy of Sciences
(Submitted 15 June 1988)
Zh. Eksp. Teor. Fiz. **95**, 253–260 (January 1989)

The current-density distribution function $\varphi(J)$ in disordered conductors is investigated in the framework of a renormalization-group analysis of the extended nonlinear σ model. It is shown that in the weak-localization region the distribution of typical fluctuations is Gaussian, while the probability of large fluctuations of J decreases in accordance with a logarithmically normal (LN) law. The LN part is much more substantial in the distribution function $\varphi(J)$ than in the conductance distribution function, and increases with increase of the quantum corrections. In the region where the average conductivity is determined essentially by quantum corrections, the distribution function $\varphi(J)$ turns out to be fully LN. It is also shown that whereas typical fluctuation currents are not correlated in direction with the external electric field **E**, large fluctuation currents are mainly coaxial with **E**.

1. The conductance of small disordered conductors fluctuates noticeably at low temperatures[1–5] when the realization of the random potential is changed. These fluctuations are manifested in experiment[1] as reproducible aperiodic oscillations of the sample conductance with change of the external magnetic field, of the Fermi energy, etc. The amplitude of the conductance fluctuations turns out[3–4] to be a universal quantity $\sim e^2/\hbar$ for mesoscopic samples (with dimensions L such that $l \ll L \lesssim L_T$, where l is the mean free path of the electrons for scattering by impurities, $L_T = (D\hbar/T)^{1/2}$, and D is the diffusion coefficient). The relative magnitude of the fluctuations of G for samples with good conductivity is, however, low.[3,4]

A quantity whose relative flucutations are not small turns out to be current denstiy $J(\mathbf{r})$ at a given point, for which[5–8] $\langle J^2(\mathbf{r})\rangle \gg \langle J(\mathbf{r})\rangle^2$ as $T\to 0$ (angle brackets denote averaging over realizations of the random potential). Fluctuations of just this quantity should be manifested[8] in fluctuations of the measurable potential difference $\delta V_{a,b}$ between close contacts a and b:

$$\delta V_{a,b} \propto \int_a^b \delta J(\mathbf{r})\, dr$$

in experiment with multicontact mesoscopic samples (see, e.g., Ref. 9). In experiments of this kind the sample dimensions play no role: the potential-difference flucutations turn out to be large provided that the distance between contacts is less than L_T.

In Refs. 6–8 was considered only the variance of the $J(\mathbf{r})$ fluctuations. The fact that the variance is large compared with the mean value arouses interest to the behavior of higher fluctuation moments of the current density. In the present paper are calculated the higher moments $\langle J^m(\mathbf{r})\rangle$ and the distribution function $\varphi(\mathbf{J})$ of the current density is reconstructed on their basis. It turns out that the distribution of the $\mathbf{J}(\mathbf{r})$ fluctuations differs substantially from the fluctuations of the conductance G.

The distribution function $f(G)$ of the mesoscopic fluctuations of the conductance[10] is characterized, even in the region of good metallic conductivity ($g \gg 1$, where $g = 2\pi^2\hbar\langle G\rangle/e^2$ is the dimensionless conductance), by a logarithmically normal (LN) asymptote. This asymptote is a manfestation of the non-universality of the fluctuations, i.e., of the dependence of their magnitude on the disorder (say, on the mean free path l), and not only on the renormalized (physically) conductance $\langle G\rangle$, as would be the case if the flucutations were described in the framework of one-parameter scaling.[11] The LN asymptote of the distribution $f(G)$ is, however long-range, difficult to observe.

It is natural to expect the non-universality to become more pronounced in the distribution $\varphi(\mathbf{J})$ of strongly fluctuating local currents. It will indeed be shown that even in the region of weak localization (where the quantum corrections to the classical Drude conductivity are small), a noticeable LN asymptote appears for the distribution $\varphi(\mathbf{J})$. With increase of the quantum corrections, i.e., with increase of the disorder or with decrease of the temperature, the region of the LN asymptote expands, so that the probability of fluctuations that exceed the variance has not a Gaussian but a logarithmically normal decrease. Finally, in the region where the quantum contribution decreases the conductivity noticeably, the current-density fluctuations become fully non-universal LN fluctuations. It must be emphasized that only the region of metallic conduction is considered, in which the deminsionless conductance $g \gg 1$, although it is small compared with the bare (Drude) value $g_0 \sim (p_F l/\hbar)^{d-1} \gg 1$.

The non-Gaussian character of $\varphi(\mathbf{J})$ is due to the rapid increase of all the higher ($m \geqslant 3$) irreducible moments (cumulants) $\langle J^m(\mathbf{r})\rangle_c$ with increase of L (or of L_T if $T\neq 0$). (Only cumulants with numbers $m \gg 1$ increase rapidly for the fluctuations of G.) These cumulants cannot be described (if $m \geqslant 3$) in the framework of the usual nonlinear σ model,[12,13] and hence in the framework of one-parameter scaling.[11] To calculate them we must see the results of a renormalization-group (RG) analysis of the expanded[10,14] nonlinear σ model.

2. We begin with the determination of the principal contribution to the cumulants $\langle J(\mathbf{r}_1)...J(\mathbf{r}_m)\rangle_c$ in the first nonvanishing order of perturbation theory. These cumulants are connected with the local conductivities $\sigma_{\alpha\beta}(\mathbf{r},\mathbf{r}')$ by the relation

$$\left\langle \prod_{i=1}^m J_{\alpha_i}(\mathbf{r}_i)\right\rangle_c = \int \left\langle \prod_{i=1}^m \sigma_{\alpha_i\beta_i}(\mathbf{r}_i,\mathbf{r}_i')\right\rangle_c \times \prod_{i=1}^m E_{\beta_i}(\mathbf{r}_i')\, d^d\mathbf{r}_i', \tag{1}$$

 0038-5646/89/010143-05$04.00 © 1989 American Institute of Physics

where $\mathbf{E}(\mathbf{r}')$ is the effective value of the electric field which, as shown in Ref. 8, must be taken equal to the homogeneous external field when the cumulants are calculated.

In Fig. 1, drawn by the technique of Ref. 15, with separated diffusion (cooperon) propagators $D(\mathbf{r})$ denoted by wavy lines, are shown several types of diagrams for the cumulants (1). The solid lines denote the electron Green's functions $\mathcal{G}(\mathbf{r}_i,\mathbf{r}_j)$, the light circles correspond to the current coordinates $\mathbf{r}_i$, and the dark ones to the field coordinates $\mathbf{r}_i'$, over which the integration is carried out in (1). The Green's functions $\mathcal{G}(\mathbf{r}_i-\mathbf{r}_j)$ decrease exponentially if $|\mathbf{r}_i-\mathbf{r}_j|\gtrsim l$.[16] The coordinates of all the sources located on one electron loop are therefore equal accurate to l. The diffusion propagators decrease slowly with r (as a power law if $d=3$ or logarithmically if $d=1$). Consequently diagrams in which currents $\mathbf{J}(\mathbf{r}_i)$ (light circles) are contained in several electron loops (for example, diagrams of type d in Fig. 1) make a nonlocal contribution to (1).

The local contribution, of greatest interest to us, to the current-density cumulants is made by diagrams in which all the light circles are on a single loop (a–c,e). The most significant of these diagrams are those having the highest degree of infrared divergence—the diagrams with the largest number of loops with dark circles. For even moments (with numbers $2n$) these are the diagrams of type a, which contain n "external" loops with two dark circles and $n-1$ "internal" diffusions or cooperons (diagrams of this type, containing a smaller number of internal diffusions, are topologically impossible—they do not break up into $2n$ current loops of type a'). The diagrams for the odd (with numbers $2n+1$) moments are those of type e, which contain n external loops and n internal diffusions. In these diagrams are carried out the largest number of independent integrations, each of which yields a factor L^{4-d}.

We obtain the principal contribution to the even moments in the lowest possible order of perturbation theory (diagams a):

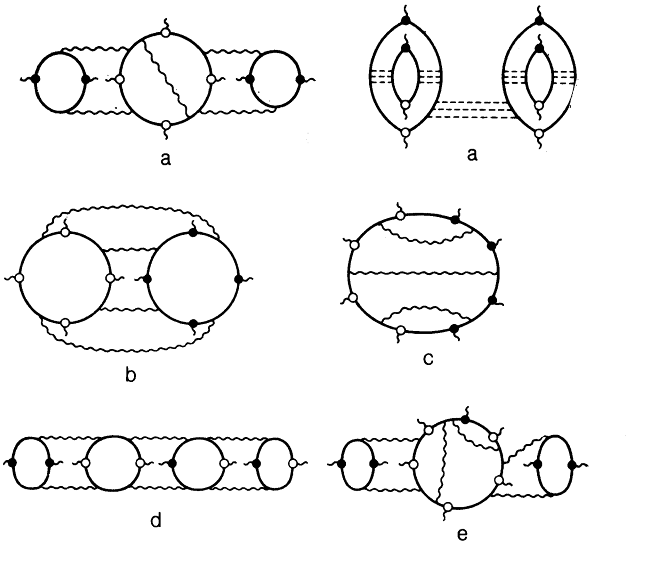

FIG. 1. Diagrams for mth-order cumulants ($m=4$ for diagrams a–d and $m=5$ for diagram e): the main contribution is made by diagrams of type a (of type e for odd cumulants). For clarity, one of the diagrams of this type is drawn (a') by the usual impurity graphic technique,[16] where a closed loop corresponds to each of the conductivities $\sigma_{\alpha\beta}(\mathbf{r},\mathbf{r}')$ and a triad of dashed lines to the diffusions.

$$\left\langle \prod_{i=1}^{2n} J_{\alpha_i}(r_i) \right\rangle_c \sim s_{\alpha_1\ldots\alpha_{2n}} \left(\frac{e^4}{\hbar^2}\mathbf{E}^2\right)^n \gamma^n \xi^{n-1} g l^{4n-2} \prod_{i=2}^{2n} \delta(r_1-r_i). \tag{2}$$

The δ functions here are the result of averaging over the fast electronic degrees of freedom. This expression is therefore valid only if $|\mathbf{r}_i-\mathbf{r}_j|>l$. For current-density cumulants in one point (or at distances $|\mathbf{r}_i-\mathbf{r}_j|<l$) the same diagrams yield

$$\left\langle \prod_{i=1}^{2n} J_{\alpha_i}(\mathbf{r}) \right\rangle_c \sim s_{\alpha_1\ldots\alpha_{2n}} (\mathbf{J}_0^2)^n \gamma^n \xi^{n-1}, \tag{3}$$

where $\mathbf{J}_0 \equiv \langle \mathbf{J}(\mathbf{r})\rangle = \sigma\mathbf{E}$ is the mean value of the current density ($\sigma \neq GL^{2-d}$ is the conductivity, $s_{\alpha_1\ldots\alpha_{2n}}$ is an absolutely symmetric tensor of order $2n$ (proportional to the sum of all possible products of the Kronecker symbols). The parameters γ and ξ in (2) and (3), with respect to which the actual selection of the diagrams is made, take for $d=2$ the form

$$\gamma = g^{-1}(L/l)^2 \gg 1, \quad \xi = g^{-1}\ln(L/l). \tag{4}$$

For $T\neq 0$ it is necessary here to replace L by L_T. The selection remains the same also for $d=3$ when $\xi\sim(\hbar/p_F l)^2$, and $\gamma\sim\xi L/l\gg 1$. Note that ξ is the usual perturbative parameter of weak-localization theory, whereas the parameter γ appears only when current-density fluctuations are considered. It is relative to this parameter that the variance of the fluctuations $\langle J^2(\mathbf{r})\rangle_c$ is large[6] compared with the mean value J_0 (this can be seen from (3) for $n=1$).

We obtain in the same manner the main contribution (proportional to the highest power of γ) to the odd-order cumulants (diagram e):

$$\left\langle \prod_{i=1}^{2n+1} J_{\alpha_i}(\mathbf{r}) \right\rangle_c \sim s_{\alpha_1\ldots\alpha_{2n+1}\beta} J_{0\beta} (\mathbf{J}_0^2)^n \gamma^n \xi^n. \tag{3'}$$

Let us clarify the difference between the selection of diagrams for the current-density fluctuations and for the fluctuations of the conductance G. The cumulants of G are connected with the cumulants of the local conductivity by the obvious equation

$$\langle G^{2n}\rangle_c = L^{-4n} \left\langle \left[\int \sigma(\mathbf{r},\mathbf{r}')\,d\mathbf{r}\,d\mathbf{r}' \right]^{2n} \right\rangle_c.$$

Additional integration [compared with Eq. (1)] eliminates the δ function in (2), and the contribution of diagrams a (or d) turns out to be proportional to the small factor $(l/L)^{2n}$. Of fundamental importance here (for not too large n) diagams d (Ref. 10), which describe power-law correlations in the cumulants $\langle \mathbf{J}_{\alpha 1}(\mathbf{r}_1)\ldots\mathbf{J}_{\alpha m}(\mathbf{r}_m)\rangle_c$. On the other hand, the contribution of the same diagrams to the cumulants $\langle J^m(\mathbf{r})\rangle_c$ of the local current density fluctuations is small compared with (3) in terms of the same parameter γ^{-1}.

3. The perturbation-theory corrections of higher order in g^{-1} cannot increase the degree of the parameter γ in (2) and (3). However, just as in the case of the conductance fluctuations, it is necessary to sum the corrections in all or-

ders over ξ. Confining ourselves here to the case $d=2$, we represent the cumulants in the form

$$\left\langle \prod_{i=1}^{m} \sigma_{\alpha_i\beta_i}(\mathbf{r}_i,\mathbf{r}_i') \right\rangle = \left(\frac{-e^2}{16\pi\hbar N^2}\right)^m \times \prod_{i=1}^{m} \operatorname{tr}\left\{ \frac{\delta^2}{\delta A_{\alpha_i}(\mathbf{r}_i)\,\delta A_{\beta_i}(\mathbf{r}_i')} \int \frac{\mathscr{D}Q}{Z} e^{-F[\mathbf{A}]} \right\}_{\substack{A=0\\ N=0}}, \qquad (5)$$

where

$$Z = \int \mathscr{D}Q \exp\{-F[0]\}.$$

Expression (5) is obtained from the corresponding expression derived in Refs. 10 and 14 for the conductance fluctuations, by replacing the partial derivatives with variational ones. Here $F[A]$ is the generating functional of the expanded nonlinear σ model[14]

$$F[A] = \tilde{g} \int \operatorname{tr}(\nabla Q)^2\,dr + \sum_{n\geqslant 2} z_n \int \operatorname{tr}(\nabla Q)^{2n}\,dr. \qquad (6)$$

The symbol ∇_α denotes the covariant derivative: $\Delta_\alpha Q \equiv \partial_\alpha Q - i[A_\alpha, Q]$, Q is a matrix field of definite symmetry,[10,14] $Q^2 = 1 (Q \in Sp(2nN)/Sp(nN) \otimes Sp(nN), Sp$ is a symplectic group, and in the final results the number of replicas is $N=0$), the vector indices for ∇_α in (6) have been left out for simplicity, $\tilde{g} \equiv \pi\hbar G/16e^2$, the nonrenormalized value of the charge is $z_n(0) \sim l^{2n-2} g_0$, and $g_0 \sim p_F l/\hbar \gg 1$.

The schematic rule of setting the diagrams in correspondence with the vertices of the functional (6) is the following. A closed electron loop corresponds to the tr symbol, a circle to the field $A(r)$ and a wavy line to interaction of the diffusion modes.[13] Clearly, the diagrams obtained within the framework of the usual σ model[12] [to which the first term of (6) corresponds] contain only loops that have no more than two free ends (circles). These are the diagrams of Fig. d, the contributions of which to the cumulants $\langle J^n(\mathbf{r})\rangle_c$ we shall neglect. Diagrams a, which make the main contribution to the cumulants (2), are obtained by perturbation theory from (5) by taking into account in the functional (6) the vertex containing $\operatorname{tr}(\nabla Q)^{2n}$.

Summation of all the principal logarithmic corrections in (2) reduces to renormalization of the functional (6) and to calculation of the contribution of diagram a with renormalized values of the charges z_n. The functional (6) has been renormalized in Ref. 14. The procedure of calculating the cumulants (1) using (5) and (6) is similar to the procedure of calculating $\langle G^n\rangle_c$, described in Ref. 14. As a result we obtain for cumulants of even order

$$\left\langle \prod_{i=1}^{2n} J_{\alpha_i}(\mathbf{r}_i) \right\rangle_c \sim s_{\alpha_1\ldots\alpha_{2n}} \left(\frac{e^4}{\hbar^2}\mathbf{E}^2\right)^n \gamma^n \exp(C_n^0 u)\, g l^{4n-2} \prod_{i=2}^{2n} \delta(\mathbf{r}_1 - \mathbf{r}_i). \qquad (7)$$

Here $\exp[C_n^0 u] \equiv \exp[u(n^2-n)]$, C_n^0 is the largest of the eigenvalues arising on renormalization of z_n (Ref. 14), and $u = \ln(\sigma_0/\sigma)$ is a non-universal parameter that depends both on the nonrenormalized (σ_0) and the renormalized (σ) values of the conductivity. It is known[11-13] that $\sigma/\sigma_0 = 1 - \xi$ for $d=2$. A similar expression is obtained for the odd-order cumulants. To transform to the one-point correlator it is necessary to analyze the corresponding transition from (2) to (3). It can be seen that it reduces to regularization of the δ function at zero in accordance with the rule $\delta(0) \sim p_F(\hbar l)^{-1} \sim g l^{-2}$. For a one-point correlator of any order we obtain thus

$$\left\langle \prod_{i=1}^{m} J_{\alpha_i}(\mathbf{r}) \right\rangle_c \sim \begin{cases} s_{\alpha_1\ldots\alpha_{2n}} (\mathbf{J}_0^2)^n \gamma^n e^{u(n^2-n)}, & m = 2n \\ s_{\alpha_1\ldots\alpha_{2n-1}\beta} J_{0\beta} (\mathbf{J}_0^2)^{n-1} \gamma^{n-1} e^{u(n^2-n)}, & m = 2n-1. \end{cases} \qquad (8)$$

Note that to go over to the perturbative expressions (2) and (3) it would be necessary to retain in (7) and (8) all the eigenvalues C_n^i appearing upon renormalization of z_n. The resultant expression would contain a sum of the form

$$\sum_i \exp(C_n^i u),$$

and the transition to the perturbative expressions in the limit $un^2 \ll 1$ would be obtained by expanding these factors in terms of u and expanding u in terms of ξ. Expressions (7) and (8) are valid for moments with numbers $n \gtrsim u^{-1}$, for which the contribution of all eigenvalues but the largest is negligible. Obviously, for moments with such numbers the perturbative expressions (2) and (3) cannot be used even in the region of low localization ($\xi \ll 1$). Note that the variance, as can be seen from (8), is not renormalized, so that the perturbative expression (2) remains valid for it.

A special examination is needed for the behavior of the cumulants (7). As shown in (6), the fluctuating currents contained in the variance are not correlated in direction with the external field $\mathbf{E}$. It is seen from (8) that, as before, there are no such correlations for higher even cumulants. It will be made clear below, however, at the largest m the higher fluctuations moments are determined by the contribution of diagrams b, which is of the form

$$\left\langle \prod_{i=1}^{m} J_{\alpha_i}(\mathbf{r}) \right\rangle_c \sim e^{u(m^2-m)} \prod_{i=1}^{m} J_{0\alpha_i}. \qquad (9)$$

The fluctuation currents are here fully correlated in direction with the external field $\mathbf{E}$. This will be shown below to determine in essence the pattern of the distribution of large fluctuating currents in disordered conductors.

4. The procedure of reconstructing the distribution function over the cumulants is well known, and is described in Ref. 10 for the case of conductances. The distribution is close to Gaussian if $K_m \ll \Delta^{m/2}$, and differs greatly from Gaussian if

$$K_m > \Delta^{m/2}, \qquad (10)$$

where K_m is an mth order cumulant and $\Delta \equiv K_2$ is the variance. In this case, when the inequality (10) is satisfied only starting with a certain number m_0, the distribution asymptote that turns out to be non-Gaussian is the one farther the larger m_0. The distribution here is symmetric if $K_{2n+1} - 0$

and is almost symmetric if the inequality (10) holds only for even moments.

It turns out that the distribution function $\varphi(\mathbf{J})$ has substantially different forms in the weak-localization region $\xi \ll 1$, where $u \approx \xi$, and in the region in which $u = \ln(\sigma_0/\sigma) \gtrsim 1$. Obviously, if $g_0 > g \gg 1$ this region does not lie outside the limits of applicability of the RG analysis carried out in the one-loop approximation. From Eq. (8), which is valid at $u \gtrsim 1$ for all cumulants, we obtain in this region

$$\frac{K_m}{\Delta^{m/2}} = \begin{cases} e^{u(n^2-n)}, & m=2n \\ \gamma^{-1/2} e^{u(n^2-n)}, & m=2n-1 \end{cases}. \tag{11}$$

The angle factors have not been taken into account here. Thus, even cumulants of all orders are large compared with the variance in the considered region $u \gtrsim 1$. Odd cumulants are immaterial for numbers $n < n_0^{1/2}$, where

$$n_0 \sim u^{-1} \ln \gamma, \tag{12}$$

i.e., the distribution is almost symmetric. Since the ratio (11) is large, all the moments are determined by their irreducible parts, i.e., by the cumulants. The variation, proportional to $\exp(un^2)$, of the moments denotes here that, just as a number of distribution functions of one-dimensional systems,[18–20] the distribution function $\varphi(\mathbf{J})$ becomes logarithmically normal. Calculations similar to those in Ref. 10 yield

$$\varphi(\mathbf{J}) = \frac{1}{2(\pi u)^{1/2}|\mathbf{J}-\mathbf{J}_0|} \exp\left[-\frac{1}{u}\ln^2\left(\frac{|\mathbf{J}-\mathbf{J}_0|}{D}\right)\right], \quad D = e^{-u}\Delta^{1/2}. \tag{13}$$

We emphasize once more that the distribution (13) is exact in the region $u \gtrsim 1$. The function $\varphi(\mathbf{J})$ differs significantly in form from the conductance distribution function $f(G)$,[10] which is, in the same region $u \gtrsim 1$, mainly Gaussian with an LN asymptote that characterizes only fluctuations δG that exceed the variance substantially. Note that the typical currents $J \sim \Delta$ in the distribution (13) exceed considerably the average current J_0 and, moreover, owing to the slow decrease of $\varphi(\mathbf{J})$ compared with Gaussian (and exponential), the probability of currents noticeably larger than typical turns out to be small.

In the region of weak localization, for moments with numbers $n < \xi^{-1/2}$, the cumulants are small: $K_m/\Delta^{m/2} \sim \xi^{m/2} \ll 1$. The region of moments with numbers $\xi^{-1/2} < n < \xi^{-1}$ is intermediate, and for $n \gtrsim \xi^{-1}$ the moments are characterized by the relation (8), which leads to an LN asymptote of type (13) in this region. For small ξ, this asymptote is long-range, but with increase of the quantum correction the non-Gaussian region of φ increases, so that at $u = \ln(1-\xi)^{-1} \sim 1$ the entire distribution function becomes the nonGaussian LN function (13).

We have analyzed up to now a purely two-dimensional case. The perturbative analysis (Sec. 2) shows that the employed diagram selection is valid also for $d = 3$. One can hope that, just as in Refs. 10 and 17, at least a qualitatively $2+\varepsilon$ expansion is permissible (a quantitative ε expansion meets with quite serious difficulties[14]). The parameter $u = \ln(\sigma_0/\sigma) \approx \ln(1-g_c/g)^{-1}$ increases then abruptly[17] as the Anderson transition is approached ($g_c = \varepsilon^{-1}$ is the critical value of the conductance), and at small ε this increase is within the limits of applicability of the performed RG analysis. Consequently, all the moments are characterized near the transition by the $\exp(un^2)$ dependence (8), so that the distribution function $\varphi(\mathbf{J})$ becomes LN, just as at $d = 2$ in the region $u \sim 1$.

We note that if $n \sim n_0$ [Eq. (12)] the contributions of diagrams a (7) and b (9) turn out to be of the same order, while at $n > n_0$ the contribution (9) predominates. This changes the asymptote of $\varphi(\mathbf{J})$ both in the region $u \gtrsim 1$ and in the weak-localization region, viz., the asymptotic remains LN, but the factor preceding the logarithm in the exponential is decreased by a factor of four. (It is interesting that it is just the diagrams b which determine the LN asymptote of the conductance distribution function.[10]) In Eq. (9), however, the current densities $\mathbf{J}$ are fully correlated in direction with the field $\mathbf{E}$, so that this asymptote is valid only for currents that are coaxial with the field. High moments (with numbers $n > n_0$) for currents that are not correlated with the field can be shown to be determined by diagrams of type b. Their contribution leads to an LN asymptote with the logarithm preceded by a factor half as large as in (13). The probability of large currents fluctuations in a direction perpendicular to $\mathbf{E}$ is therefore small compared with like fluctuations but coaxial with $\mathbf{E}$. Thus, in contrast to typical fluctuating currents that are not correlated in direction with $\mathbf{E}$, large fluctuating currents are in the main coaxial with the field, i.e., they form closed current loops stretched along the field. This seems quite natural: large fluctuations are determined by random-potential realization that have relatively low probability and form a region with altered conductivity σ. The presence of these regions is manifested by the large fluctuating currents that are coaxial with the field.

We have considered up to now local current-density fluctuations. It is they which determine[8] the contribution made to measurable voltage fluctuations between close contacts in multicontact systems. Of course, it is possible to have other experimental geometries in which spatial correlations in the current density are important. The diagrams that make the main contribution to such correlations are obtained from diagrams a′ by drawing intraloop diffusions that enclose all the free ends. It is most convenient to calculate contributions of this kind in the Langevin scheme.[7] The cumulants calculated here must then be regarded in such a scheme as cumulants of extraneous currents. Note that as a result we arrive at an LN asymptote and at spatially separated current densities in the distribution function.

The author is deeply grateful to B. L. Al'tshuler, A. G. Aronov, V. E. Kravtsov, B. Z. Spivak, and V. I. Yudson for valuable discussions.

[1] R. A. Webb, S. Washburn, C. P. Umbach, and R. S. Laibowitz, Phys. Rev. Lett. **54**, 485 (1985).
[2] A. D. Stone, *ibid.* **54**, 2692 (1985).
[3] B. L. Al'tshuler, Pis'ma Zh. Eskp. Teor. Fiz. **41**, 530 (1985) [JETP Lett. **41**, 648 (1985)].
[4] P. A. Lee and A. D. Stone, Phys. Rev. Lett. **55**, 1622 (1985).
[5] B. L. Al'tshuler and D. E. Khmel'nitskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 291 (1985) [JETP Lett. **42**, 359 (1985)].
[6] A. G. Aronov, A. I. Zyuzin, and B. Z. Spivak, *ibid.* **43**, 431 (1986) [**43**, 555 (1986)].
[7] A. I. Zyuzin and B. Z. Spivak, Zh. Eksp. Teor. Fiz. **93**, 994 (1987) [Sov. Phys. JETP **66**, 560 (1987)].
[8] C. L. Kane, P. A. Lee, and D. DiVincenzo, MIT Preprint, 1987.
[9] W. J. Skocpol, P. M. Mankiewich, R. E. Howard, *et al.* Phys. Rev. Lett. **58**, 2347 (1987).

[10]B. L. Al'tshuler V. E. Kravtsov, and I. V. Lerner, Zh. Eksp. Teor. Fiz. **91**, 2276 (1986) [Sov. Phys. JETP **64**, 1352 (1986)].
[11]E. Abrahams, P. V. Anderson, D. C. Licciardello, and T. V. Ramkrishnan, Phys. Rev. Lett. **43**, 718 (1979).
[12]F. Wegner, Z. Phys. B**38**, 113 (1980).
[13]K. B. Efetov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Zh. Eksp. Teor. Fiz. **79**, 1120 (1980) [Sov. Phys. JETP **52**, 568 (1980)].
[14]V. E. Kravtsov, I. V. Lerner, and V. I. Yudson, *ibid.* **94**, No. 7, 255 (1988) [Sov. Phys. JETP. **67**, 1441 (1988)].
[15]L. P. Gor'kov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 248 (1979) [JETP Lett. **30**, 248 (1979)].
[16]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, *Quantum Field-Theoretical Methods in Statistical Physics*, Pergamon, 1965, §39.
[17]B. L. Al'tshuler, V. E. Kravtsov, and I. V. Lerner, Zh. Eksp. Teor. Fiz. **94**, No. 4, 258 (1988) [Sov. Phys. JETP **67**, 795 (1988)].
[18]A. A. Abrikosov, Sol. St. Comm. **37**, 997 (1981).
[19]V. I. Mel'nikov, Fiz. Tverd. Tela (Leningrad) **23**, 782 (1981) [Sov. Phys. Solid State **23**, 444 (1981)].
[20]B. L. Al'tshuler and V. N. Prigodin, Zh. Eksp. Teor. Fiz. **95**, 348 (1989) [Sov. Phys. JETP **68**, (1989), this issue].

Translated by J. G. Adashko