# Effect of a strong electric field on resonance interaction between particles and hf waves

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(Submitted 23 August 1988)
Zh. Eksp. Teor. Fiz. **95**, 537–546 (February 1989)

It is shown that allowance for a strong low-frequency or constant field can alter substantially the character of wave–particle resonance interaction in an external magnetic field. In the case of a transverse wave, such as a whistler, propagating quasilongitudinally relative to the field, a Čerenkov wave-energy absorption mechanism has been observed, whose contribution to the wave absorption is comparable with that of cyclotron resonance. It is found, in principle, that other types of resonance interaction are possible if electric drift and relativistic effects are neglected. These include cyclotron resonance in a longitudinal wave and resonance at twice the gyrofrequency.

## 1. INTRODUCTION

A consistent theory of resonance interaction between charged particles and high-frequency (hf) waves is based, subject to rather general assumptions, on the drift–eikonal approximation in the case of a weak electric field.[1,2] It is assumed then that the particle electric-drift velocity in a quasistationary field is much lower than its characteristic velocity. Drift theory includes also a strong-electric-field variant corresponding to the magnetohydrodynamic (MHD) approximation,[3,4] in which the electric-drift velocity is comparable with the particle velocity. The MHD approximation accords best with the conditions in space plasma, particularly of the earth's magnetosphere, where strong electric fields are frequently observed.[5,6] Strong electric fields are produced also under laboratory conditions, for example, in a tokamak plasma following particle injection or hf heating.[7]

Various problems of resonant-particle interaction with hf waves are treated in a tremendous number of papers, such as Refs. 8–11. No account has been taken, however, of the influence of a strong quasistationary field. Yet this influence can alter substantially the character of the resonance wave–particle interaction and the mechanism of energy exchange between them

We report here an investigation of the influence of a quasistationary (low-frequency) electric field on the charged-particle dynamics in an hf field under Čerenkov and cyclotron resonance conditions. Weak relativistic effects are taken into account. The hf field is represented as a superposition of rather arbitrary wave packets within the framework of the assumptions of the drift-eikonal theory.

Averaged equations are obtained for weakly relativistic particle motion in the Čerenkov- and cyclotron-resonance regions. It is shown that when a transverse wave propagates quasilongitudinally in an external magnetic field a strong low-frequency electric field strongly influences the wave–particle interaction in the case of Čerenkov resonance but is generally hardly noticeable under cyclotron-resonance conditions.

We consider in detail the case of a whistler propagating along a constant magnetic field. We show that the presence of a strong electric field $a$ in the whistler makes it possible for a particle to experience Čerenkov resonance which is in principle not realizable in a weak electric field. The physical mechanism that produces the Čerenkov resonance in the transverse electromagnetic wave stems from the Lorentz force exerted by the magnetic component of the hf wave on the electrically drifting particle. The nonlinear damping rate of the whistler is calculated for Čerenkov resonance and it is shown that it can be comparable with the damping rate in cyclotron-resonance interaction between a wave and a particle.

The influence of a strong electric field on resonances in a longitudinal wave propagating along a magnetic field is also considered. It is shown that in this case a quasistationary field appears only when the relativistic motion of the particle is taken into account. The character of the particle interaction with a longitudinal wave in the Čerenkov-resonance region is practically unchanged by the presence of a quasistationary electric field. At the same time, cyclotron-resonance interaction can take place between the particle and the longitudinal field, something impossible in principal in a weak electric field. Allowance for weak relativistic effects leads also to the possibility of a specific resonance at twice the gyrofrequency in a strong electric field.

## 2. BASIC EQUATIONS

Neglecting quantum and radiation effects, the motion of a charged particle is described in the weakly relativistic approximation by the equations

$$\frac{d\mathbf{v}}{dt}=\alpha\frac{e}{m}\left\{\mathbf{E}+\frac{1}{c}[\mathbf{vB}]\right\}-\frac{e}{mc^2}\mathbf{v}(\mathbf{vE}). \qquad (2.1)$$

Here $m$ is the particle rest mass and $\alpha = 1 - v^2/2c^2$.

We assume that the electromagnetic field $\mathbf{E}(\mathbf{r},t)$, $\mathbf{B}(\mathbf{r},t)$ can be resolved into slowly varying (quasistationary) $\mathbf{E}_0(\mathbf{r},t)$, $\mathbf{B}_0(\mathbf{r},t)$ and rapidly varying (high-frequency) $(\mathbf{E}_\sim,\mathbf{B}_\sim)$ components. The hf field is represented as a superposition of noninteracting wave packets

$$\mathbf{E}_\sim=\sum_{1\leqslant s\leqslant N}(\mathbf{E}_s e^{i\theta_s}+\text{c.c.}),$$

$$\mathbf{B}_\sim=\sum_{1\leqslant s\leqslant N}(\mathbf{B}_s e^{i\theta_s}+\text{c.c.}). \qquad (2.2)$$

Here $N$ is the number of packets $\mathbf{E}_s(\mathbf{r},t)$ and $\mathbf{B}_s(\mathbf{r},t)$ are

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slowly varying complex amplitudes, and $\theta_s(\mathbf{r},t)$ is the fast phase (eikonal) of the $s$th packet.

The representation of the hf field in the form (2.2) is in full agreement with the actual conditions of numerous applied problems. The phases $\theta_s$ are described by the equations

$$\frac{d\theta_s}{dt}=-\omega_s(\mathbf{r},t)+\frac{d\mathbf{r}}{dt}\mathbf{k}_s(\mathbf{r},t), \tag{2.3}$$

where

$$\omega_s(\mathbf{r},t)=-\partial\theta_s/dt, \quad \mathbf{k}_s(\mathbf{r},t)=\nabla\theta_s \tag{2.4}$$

are respectively the local frequency and the wave vector of the $s$th quasimonochromatic packet.

The character of the separation of the particle cyclotron rotation in the field $\mathbf{B}_0$ depends on the relation between the electron-drift velocity in the fields $\mathbf{E}_0$ and $\mathbf{B}_0$, on the one hand, and the characteristic particle velocity, on the other.[1-4] In the case of a strong electric field $\mathbf{E}_0$ the cyclotron gyration of the particle is described with the aid of the equation

$$\mathbf{v}=\frac{d\mathbf{r}}{dt}=\mathbf{e}_1v_\parallel+\mathbf{v}_E+v_\perp(\mathbf{e}_2\cos\theta_0+\mathbf{e}_3\sin\theta_0). \tag{2.5}$$

Here $\mathbf{v}_E=c[\mathbf{E}_0\times\mathbf{e}_1]/B_0$ is the electric drift velocity $\mathbf{e}_1=\mathbf{B}_0/|\mathbf{B}_0|$, $\mathbf{e}_2$, $\mathbf{e}_3$ is the basic triad of unit vectors connected with the lines of the magnetic field $\mathbf{B}_0$, $v_\parallel$ and $v_\perp$ are the longitudinal and transverse particle-velocity components, and $\theta_0$ is the gyrophase.

It is expedient to represent the particle equations of motion (21), subject to the substitution (2.5) and with account taken of (2.2)-(2.4), in the form of a multifrequency (or a multiperiod) system:

$$\frac{dx}{dt}=f(t,x,\theta;\varepsilon), \qquad \frac{d\theta}{dt}=\frac{1}{\varepsilon}\omega(t,x)+A(t,x,\theta;\varepsilon). \tag{2.6}$$

The small parameter $\varepsilon$ was introduced in accordance with the approximation of the drift theory of the motion of charged particles and of geometric optics for the waves (2.2); $x=(\mathbf{r},v_\parallel,v_\perp)$ is the vector of the slow variable; $\theta=(\theta_0,\theta_1,\ldots,\theta_N)$ is the vector of the fast variables that contain the gyrophase and the phase of the wave packets (2.2). The frequency vector is $\omega=(\omega_0,\ \nu_1,\ldots,\ \nu_N)$, where $\omega_0=-eB\Gamma/mc\equiv\Omega_0\Gamma$ is the cylotron frequency and $\nu_s=-\omega_s+k_{s\parallel}v_\parallel$.

The right-hand sides of the system (2.6) are the vectors $f=(\mathbf{v},f_\parallel f_\perp)$ and $A=(A_0,A_1,\ldots,A_N)$ where $\mathbf{v}$ is determined by Eq. (2.5) and

$$f_\parallel=a_0+\Big\{a_1\exp(i\theta_0)+a_2\exp(i2\theta_0)$$
$$+\sum_{1\leqslant s\leqslant N}a_{3s}\exp(i\theta_s)+a_{4s}\exp[i(\theta_0+\theta_s)]$$
$$+a_{5s}\exp[i(\theta_0-\theta_s)]+a_{6s}\exp[i(2\theta_0+\theta_s)]$$
$$+a_{7s}\exp[i(2\theta_0-\theta_s)]+\text{c.c.}\Big\}. \tag{2.7}$$

The quantities $f_\perp$ and $A_0$ are similar, with the coefficients $a_i$ replaced by $b_i$ and $c_i$, respectively. Let us write down these coefficients:

$$a_0=\mathbf{v}_E\mathbf{e}_1'+\frac{v_\perp^2}{2}\operatorname{div}\mathbf{e}_1+\left(\Gamma-\frac{v_\parallel^2}{c^2}\right)F_{0\parallel},$$
$$a_1=\frac{v_\perp}{2}[\mathbf{e}_-\mathbf{e}_1'+\mathbf{v}_E(\mathbf{e}_1\nabla)\mathbf{e}_1]-\frac{v_\perp}{2c^2}(i\omega_0v_\parallel+F_{0\parallel})\mathbf{v}_E\mathbf{e}_-,$$
$$a_2=\frac{v_\perp^2}{4}\mathbf{e}_-(\mathbf{e}_-\nabla)\mathbf{e}_1,\qquad a_{3s}=\Gamma\tau_1+\frac{1}{c^2}\left(\frac{v_\perp^2}{2}\mathbf{e}_1[\mathbf{v}_E\Omega_s]\right.$$
$$\left.-v_\parallel\mathbf{u}_0\mathbf{F}_s\right), \tag{2.7a}$$

$$a_{4s}=\alpha_{1+}\mathbf{e}_-,\qquad a_{5s}=\alpha_{1-}\mathbf{e}_+,$$
$$a_{6s}=\beta_{1+}^{\alpha\beta}e_-^{\beta}e_-^{\alpha},\qquad a_{7s}=\beta_{1-}^{\alpha\beta}e_+^{\beta}e_+^{\alpha};$$
$$b_0=-\frac{v_\perp}{2}[\operatorname{div}\mathbf{u}_0-\mathbf{e}_1(\mathbf{e}_1\nabla)\mathbf{u}_0]+\frac{v_\perp v_\parallel}{c^2}F_{0\parallel},$$
$$b_1=-\frac{1}{2}\mathbf{e}_-\mathbf{u}_0'-\frac{1}{2c^2}(i\omega_0R+v_\parallel F_{0\parallel})\mathbf{v}_E\mathbf{e}_-,$$
$$b_2=-\frac{v_\perp}{4}\left[\mathbf{e}_-(\mathbf{e}_-\nabla)\mathbf{u}_0+\frac{i\omega_0}{c^2}(\mathbf{v}_E\mathbf{e}_-)^2\right],$$
$$b_{3s}=-\frac{v_\perp}{c^2}\left\{\left(\mathbf{F}_{s\perp}+\frac{1}{2\omega_s}[[\mathbf{k}_s\mathbf{F}_s]\mathbf{u}_0]\right)\mathbf{v}_E+\mathbf{F}_s\mathbf{u}_0\right\},$$
$$b_{4s}=\alpha_{2+}\mathbf{e}_-,\qquad b_{5s}=\alpha_{2-}\mathbf{e}_+,\qquad b_{6s}=\tau_{4+}\mathbf{e}_-,\qquad b_{7s}=\tau_{4-}\mathbf{e}_+; \tag{2.7b}$$

$$c_0=-\frac{1}{2}\mathbf{e}_1\operatorname{rot}\mathbf{u}_0-\frac{i}{2}\mathbf{e}_-\mathbf{e}_+'-\frac{\Omega_0v_E^2}{2c^2},$$
$$c_1=\frac{ib_1}{v_\perp}-\frac{iv_\perp}{4}\mathbf{e}_-(\mathbf{e}_-\nabla)\mathbf{e}_+,$$
$$c_2=\frac{ib_2}{v_\perp},\qquad c_{3s}=\left(\Gamma-\frac{v_E^2}{2c^2}-\frac{iv_\parallel F_{0\parallel}}{2c^2\omega_0}\right)\Omega_{s\parallel}-\frac{iv_\parallel}{2c^2\omega_0}\mathbf{F}_0\Omega_s,$$
$$c_{4s}=\frac{ib_{4s}}{v_\perp}+\alpha_{3+}\mathbf{e}_-,\qquad c_{5s}=-\frac{ib_{5s}}{v_\perp}+\alpha_{3-}\mathbf{e}_+,$$
$$c_{6s}=\frac{ib_{6s}}{v_\perp},\qquad c_{7s}=-\frac{ib_{7s}}{v_\perp}. \tag{2.7c}$$

Here

$$R=3v_\perp^2+v_\parallel^2+v_E^2,\qquad \tau_1=F_{s\parallel}+\mathbf{e}_1[\Omega_s\mathbf{v}_E],$$
$$\alpha_{1\pm}=\mp\frac{i}{2}\Gamma v_\perp\Omega_s-\frac{v_\perp}{2c^2}\tau_1\mathbf{v}_E-\frac{v_\perp v_\parallel}{2c^2}\mathbf{F}_s,\ \beta_{1\pm}^{\alpha\beta}=\pm\frac{iv_\perp}{4c^2}v_E^{\alpha}\Omega_s^{\beta},$$
$$\alpha_{2\pm}=\frac{1}{2}\Gamma(\mathbf{F}_s+\tau_{3\pm})-\frac{1}{2c^2}[v_\perp^2\mathbf{F}_s+(\mathbf{F}_s\mathbf{u}_0)\mathbf{v}_E],$$
$$\alpha_{3\pm}=\pm\frac{iv_\perp}{2c^2}\left(-\mathbf{F}_s\mathbf{u}_0\pm\frac{i\mathbf{F}_s[\mathbf{e}_1\mathbf{k}_s]}{\omega_s}\right)\mathbf{v}_E,$$
$$\tau_{3\pm}=\pm\frac{i}{\omega_s}(v_\parallel[\mathbf{k}_s\mathbf{F}_s]\pm i[\mathbf{v}_E[\mathbf{k}_s\mathbf{F}_s]]),$$
$$\tau_{4\pm}=-\frac{v_\perp}{4c^2}(2\mathbf{F}_s+\tau_{3\pm})(\mathbf{v}_E\mathbf{e}_\mp).$$

The quantities $A_s$ are defined by the equation

$$A_s=\mathbf{k}_s\mathbf{v}_E+{}^1/{}_2v_\perp\mathbf{k}_s[\mathbf{e}_-\exp(i\theta_0)+\mathbf{e}_+\exp(-i\theta_0)]. \tag{2.8}$$

We use in (2.7) and (2.8) the notation

$$\mathbf{e}_\pm=\mathbf{e}_2\pm i\mathbf{e}_3,\qquad (\ldots)'=(\partial/\partial t+v_\parallel\mathbf{e}_1\nabla+\mathbf{v}_E\nabla)(\ldots),$$
$$\mathbf{u}_0=\mathbf{v}_E+\mathbf{e}_1v_\parallel,\qquad k_{s\parallel}=\mathbf{k}_s\mathbf{e}_1,\qquad \mathbf{F}_{s,0}=(e/m)\mathbf{E}_{s,0},$$
$$\Gamma=1-(v_E^2+v_\parallel^2+v_\perp^2)/2c^2,\qquad \Omega_s=-e\mathbf{B}_s/mc.$$

A particular case of the system (2.6) is one with a rapidly rotating phase, first considered by Bogolyubov and Zubarev.[12]

The system (2.6) with coefficients (2.7) and (2.8) describes only the case of quasilongitudinal propagation of wave packets relative to the field $\mathbf{B}_0$. If their propagation direction is arbitrary, the right-hand sides of the system (2.6) become much more complicated.[1,2] Below consider quasilongitudinal wave propagation.

## 3. AVERAGED EQUATIONS OF MOTION OF A RESONANT PARTICLE IN THE MHD APPROXIMATION

Averaging the multifrequency system (2.6) means smoothing (integration) over all the fast phases contained in this system. This operation simplifies substantially the description of the particle motion. When there are resonance relations between the frequencies, the descritption of the motion is made complicated by the fact that a phase combination corresponding to resonant combination of the frequencies cannot be regarded as a fast variable. The space of the slow variables is thereby broadened, and it is necessary to add to the averaged equations an equation, smoothed over the remaining fast phases, for the resonant dephasing. The general scheme for averaging the equations of motion of resonance particles was considered in Refs. 1 and 13. It follows from (2.6) and (2.7) that in the zeroth order of the expansions in the parameter $\varepsilon$ there can exist Čerenkov and cyclotron resonances defined respectively by the following relations between the frequencies[1)]

$$\nu=-\omega+kv_{\parallel}\approx 0, \qquad \omega_0\pm\nu\approx 0. \tag{3.1}$$

A resonance is also possible with a frequency ratio

$$2\omega_0\pm\nu\approx 0. \tag{3.1a}$$

It can be seen from the general equations (2.6) and (2.7) that the resonances (3.1a) are of purely relativistic origin.

The frequency combinations (3.1) and (3.1a) correspond respectively to the phase combinations $\psi\equiv\theta$, $\psi=\theta_0\pm\theta$ and $\psi=2\theta_0\pm\theta$, where $\theta$ is the phase of one of the wave packets (2.2). It is assumed that the resonances due to particle interactions with individual waves (2.2) do not overlap.

In the Čerenkov-resonance region ($\nu\approx 0$), the desired averaged equations of particle motions are

$$\frac{dv_{\parallel}}{dt}=a_0+(a_3e^{i\psi}+\text{c.c.}), \qquad \frac{dv_{\perp}}{dt}=b_0+(b_3e^{i\psi}+\text{c.c.}),$$

$$\frac{d\mathbf{r}}{dt}=\mathbf{u}_0, \qquad \frac{d\psi}{dt}=\nu+\mathbf{k}\mathbf{v}_E. \tag{3.2}$$

In the cyclotron-resonance region ($\psi=\theta_0+\theta$) the averaged motion of a particle is described by the equations

$$\frac{dv_{\parallel}}{dt}=a_0+(a_4e^{i\psi}+\text{c.c.}), \qquad \frac{dv_{\perp}}{dt}=b_0+(b_4e^{i\psi}+\text{c.c.}),$$

$$\frac{d\mathbf{r}}{dt}=\mathbf{u}_0, \qquad \frac{d\psi}{dt}=\omega_0+\nu+c_0+\mathbf{k}\mathbf{v}_E+(c_4e^{i\psi}+\text{c.c.}). \tag{3.3}$$

In the resonant region at double the gyrofrequency ($\psi=2\theta_0+\theta$) the averaged motion of the particles is described by the equations

$$\frac{dv_{\parallel}}{dt}=a_0+(a_6e^{i\psi}+\text{c.c.}), \qquad \frac{dv_{\perp}}{dt}=b_0+(b_6e^{i\psi}+\text{c.c.}),$$

$$\frac{d\mathbf{r}}{dt}=\mathbf{u}_0, \qquad \frac{d\psi}{dt}=2\omega_0+\nu+2c_0+\mathbf{k}\mathbf{v}_E+(2c_6e^{i\psi}+\text{c.c.}). \tag{3.4}$$

The coefficients of Eqs. (3.2)–(3.4) are described by Eqs. (2.7) and (2.8) with the subscript $s$ omitted.

We emphasize that the equations derived are valid for quasilongitudinal wave propagation. Equations (3.2)–(3.4) are still quite complicated. To show the effects of a strong electric field in pure form, we shall simplify to the utmost the conditions of the problem. We assume the magnetic field $\mathbf{B}_0$ to be stationary and homogeneous and the electric field $E_{0\parallel}$ to have no longitudinal component, and we neglect relativistic effects.

## 4. INFLUENCE OF STRONG ELECTRIC FIELD ON PARTICLE MOTION IN THE CYCLOTRON-RESONANCE REGION

Consider a linearly polarized wave $\mathbf{E}\sim=(\frac{1}{2}\mathscr{E}e^{i\varphi},0,0)$, propagating at a small angle to a magnetic field $\mathbf{B}_0=(0,0,B_0)$ in the $yz$ plane: $\mathbf{k}=(0,k\sin\alpha, k\cos\alpha)$, $\tan\alpha\ll 1$. We assume $\mathbf{E}_0=(E_0,0,0)$, and then $\mathbf{v}_E=(0,-v_E,0)$. The system (3) takes in this case the simpler form

$$\frac{dv_{\parallel}}{dt}=\frac{v_{\perp}}{2}[\mathbf{e}_1\boldsymbol{\Omega}]\mathbf{e}_-e^{i\psi}+\text{c.c.},$$

$$\frac{dv_{\perp}}{dt}=\frac{1}{2}(\mathbf{F}+[\boldsymbol{\Omega}\mathbf{u}_0])\mathbf{e}_-e^{i\psi}+\text{c.c.},$$

$$\frac{d\mathbf{r}}{dt}=\mathbf{u}_0, \qquad \frac{d\psi}{dt}=\omega_0+\nu+\frac{i}{v_{\perp}}\frac{dv_{\perp}}{dt}. \tag{4.1}$$

It is evident hence that in longitudinal wave propagation ($\alpha=0$) a strong electric field $\mathbf{E}_0$ does not influence the cyclotron resonance at all. For a small angle $\alpha$ (the only case when the system (3.3) is valid) the electric drift causes a shift of the resonance frequency. The law governing the transverse energy of the particle is determined, according to (4.1), by the approximate equation

$$v_{\perp}^2=v_{0\perp}^2+(e\mathscr{E}/4m\omega)^2t^2[\omega-k(v_{0\parallel}\cos\alpha-v_E\sin\alpha)]^2.$$

We see hence that a strong electric field hardly alters the mechanism of cyclotron-resonance interaction of a particle with a transverse wave.

## 5. POSSIBILITY OF ČERENKOV RESONANCE IN A TRANSVERSE WAVE WITH ALLOWANCE FOR ELECTRIC DRIFT

We know that cyclotron resonance of a charged particle take place in a transverse wave propagating along a magnetic field, but no Čerenkov resonance is possible in principle. The situation is radically altered if the quasistationary electric field is strong. Let us consider, to be specific, a circularly polarized wave, such as a whistler mode.

Interest in whistlers is due both to their major role in a number of magnetosphere-plasma phenomena[5] and in connection with active experiments being carried out in outer space.[9] The choice of longitudinal wave propagation is justified because under magnetosphere conditions the averaged energy flux of whistler signals cannot deviate from the magnetic field lines by more than 19°29′ (Ref. 6).

Putting

$$\mathbf{B}_0=(0,\ 0,\ B_0),\quad \mathbf{E}_0=(E_0,\ 0,\ 0),\quad \mathbf{B}_\sim=(B\sin\varphi,\ B\cos\varphi,\ 0),$$
$$\mathbf{E}_\sim=(\mathscr{E}\cos\varphi,\ \mathscr{E}\sin\varphi,\ 0),\quad \mathbf{k}=(0,\ 0,\ k),$$

we obtain from (3.2) under the conditions indicated above

$$\frac{dv_\parallel}{dt}=-\frac{e\mathscr{E}kv_E}{m\omega}\cos\psi,\quad \frac{dv_\perp}{dt}=0,$$
$$\frac{d\psi}{dt}=-\omega+kv_\parallel. \tag{5.1}$$

Hence if follows that Čerenkov resonance is possible in such a wave. There is no such resonance without allowance for electric drift. Equations (5.1) are entirely similar to the thoroughly investigated equations of motion of a resonant particle in the field of a finite-amplitude electrostatic wave.[8,9]

Putting $\xi=(1/2)(\psi-\pi/2)$ we obtain from (5.1) the energy integral

$$\dot{\xi}^2=\frac{1}{\tau^2\varkappa^2}(1-\varkappa^2\sin^2\xi), \tag{5.2}$$

where

$$\varkappa=\left(\beta+\frac{2\beta}{H}\right)^{1/2}\operatorname{sign}\dot{\xi},\quad \beta=\frac{e\mathscr{E}^2k^2v_E}{m\omega},\quad 2\beta=\frac{1}{\tau^2}.$$

The constant $H$ is the Hamiltonian of Eqs. (5.1). The solutions of (5.2) are

$$F(\xi,\varkappa)-F(\xi_0,\varkappa)=t/\varkappa\tau,\ |\varkappa|<1,$$
$$F(\xi,1/\varkappa)-F(\xi_0,1/\varkappa)=t/\tau,\ |\varkappa|>1.$$

Here $\xi_0=\xi|_{t=0}$ and $F(\xi,\varkappa)$ is an elliptic integral of the first kind:

$$F(\xi,\varkappa)=\int_0^\xi\frac{d\varphi}{(1-\varkappa^2\sin^2\varphi)^{1/2}}.$$

The values of the parameter $\varkappa$ determine the trapped ($\varkappa>1$) and the untrapped ($\varkappa<1$) particles. The quantity $\tau$ characterizes the trapping region and determines the period of the nonlinear oscillations of the particle in the wave field. The only difference from the case of an electrostatic wave[8] is that different symbols are used for the parameters $\varkappa$ and $\tau$, which depend in our case on the electric-drift velocity.

Starting from the kinetic equation for the resonance particles:

$$\frac{\partial f}{\partial t}+v_\parallel\frac{\partial f}{\partial z}-\frac{e\mathscr{E}kv_E}{m\omega}\cos\psi\frac{\partial f}{\partial v_\parallel}=0, \tag{5.3}$$

one can obtain by a standard procedure[8-11] the particle distribution function in the resonance region

$$f(\xi,\dot{\xi},\varkappa,v_{0\perp},t)$$
$$=f_0(v_{0\perp})+\frac{1}{k\varkappa\tau}\operatorname{dn}\left[F(\xi,\varkappa)-\frac{1}{\tau\varkappa};\varkappa\right]\left(\frac{\partial f}{\partial\dot{\xi}}\right)_{v_\parallel=\omega/k}$$

Here $\operatorname{dn}(\xi,\varkappa)$ is a Jacobi elliptic function with modulus $\varkappa$.

The wave damping rate $\gamma(t)$ can be calculated by starting from the energy conservation law

$$dW/dt=-\mathbf{j}\mathbf{E}_\sim=\gamma W, \tag{5.4}$$

where $\mathbf{j}=-e\int\mathbf{v}f dv$ is the density of the wave-induced current. The whistler energy density is

$$W=\frac{B^2}{8\pi}\frac{1}{(1-\omega/\omega_c)},\quad \omega_c=\frac{eB_0}{m_0c}. \tag{5.4a}$$

Performing the calculations, we get

$$\gamma(t)=\gamma_L\sum\frac{64}{\pi}\int_0^1 d\varkappa\left\{\frac{2\pi n\sin(\pi nt/K\varkappa\tau)}{\varkappa^5K^2(1+q^{2n})(1+q^{-2n})}\right.$$
$$\left.+\frac{(2n+1)\pi^2\sin[(2n+1)\pi t/K\varkappa\tau]}{K^2(1+q^{2n+1})(1+q^{-2n-1})}\right\}. \tag{5.5}$$

Here $q=\exp(\pi K'/K)$, $K'=K(\varkappa)(1-\varkappa^2)^{1/2}$, $K(\varkappa)=F(\pi/2,\varkappa)$ is a complete elliptic integral of the first kind, and $\gamma_L$ is the linear absorption coefficient.

Passage of a whistler through a plasma is accompanied by absorption of the latter, both in cyclotron resonance and in Čerenkov resonance, if the electric field is strong. The whistler absorption coefficient for cyclotron resonance is given by[9]

$$\gamma(t)=\frac{8\omega_{pe}^2\omega}{k^2c^2}\left(1-\frac{\omega}{\omega_c}\right)\int_0^\infty dw_0\,w_0^3\left(\frac{\partial f}{\partial v_\parallel}+\frac{\omega}{kw}\frac{\partial f}{\partial w}\right)_{v_\parallel=\frac{\omega-\omega_c}{k}}$$
$$\times\int_s\int d\varkappa\,dF\frac{\sin(2amF)}{\varkappa^3}\operatorname{dn}\left(F-\frac{1}{\varkappa\tau}\varkappa\right), \tag{5.6}$$

where $\omega_{pe}^2=4\pi e^2n/m$.

We consider now the relative contributions of the two absorption mechanisms, assuming a Maxwellian particle distribution function. It follows then from (5.5) and (5.6) that the ratio of the wave damping rates in the cyclotron and Čerenkov resonances is

$$\frac{\gamma_L^{cy}}{\gamma_L^{ch}}=\pi\left(\frac{v_{th}}{v_E}\right)^2\exp\left[\frac{\omega_c(2\omega-\omega_c)}{2v_{th}^2k^2}\right], \tag{5.7}$$

where $v_{th}$ is the thermal velocity of the particles. This shows that the two whistler-absorption mechanisms make comparable contributions at the frequency $\omega=\omega_c/2$ corresponding to the maximum phase velocities of these waves.[6] For $\omega<\omega_c/2$ Čerenkov resonance is the more effective mechanism for $\omega>\omega_c/2$ cyclotron absorption.

## 6. INFLUENCE OF STRONG ELECTRIC FIELD ON RESONANCES IN A LONGITUDINAL WAVE

Under the simplifying assumptions considered in Secs. 4 and 5, the effects of a strong electric field in resonance interaction of particles with a longitudinal wave traveling along a magnetic field $\mathbf{B}_0$ are quite weak. They are due entirely to relativistic effects.

Indeed, from the general averaged equations (3.2) and (3.3) we have in the Čerenkov-resonance region

$$\frac{dv_\parallel}{dt}=\left(1-\frac{3v_\parallel^2+v_\perp^2+v_E^2}{2c^2}\right)\frac{e}{m}\mathscr{E}\cos\psi,$$
$$\frac{dv_\perp}{dt}=-\frac{ev_\perp v_\parallel}{mc^2}\mathscr{E}\cos\psi,\quad \frac{d\psi}{dt}=-\omega+kv_\parallel, \tag{6.1}$$

and in the cyclotron-resonance region

$$\frac{dv_\parallel}{dt}=\frac{ev_\perp}{2c^2m}\mathscr{E}v_E\sin\psi,\quad \frac{dv_\perp}{dt}=\frac{ev_\parallel}{2c^2m}\mathscr{E}v_E\sin\psi,$$

$$\frac{d\psi}{dt}=\Omega_0\left(\Gamma-\frac{v_E^2}{2c^2}\right)-\omega+kv_\parallel+\frac{v_\parallel v_E e}{2v_\perp c^2m}\mathscr{E}\cos\psi. \tag{6.2}$$

It follows from (6.1) that the electric drift has little effect on Čerenkov resonance in a longitudinal wave. In a sufficiently strong field $E_0$, owing to relativistic effects, particle capture by a longitudinal wave of finite amplitude becomes weaker in the Čerenkov-resonance region. This slows down the establishment of an ergodic state of the resonance particles. The system (6.1) leads to the following energy integral in the dephasing space:

$$\frac{1}{2}\dot{\Phi}^2+\frac{1}{\tau^2}\sin\Phi=\text{const}. \tag{6.3}$$

The depth of the potential well is characterized by the quantity

$$\frac{1}{\tau^2}=k\left(1-\frac{3v_{0\parallel}^2+v_{0\perp}^2+v_E^2}{2c^2}\right)\frac{e}{m}\mathscr{E},$$

which determines the possibility of particle trapping by the wave.

Damping of a longitudinal wave by the usual Čerenkov-resonance mechanism predominates over the cyclotron mechanism if relativistic effects are immaterial. Equation (6.2) shows that, if electric drift is taken into account, cyclotron resonance (with a Doppler frequency shift) becomes possible in a longitudinal wave of sufficient amplitude. This effect is of purely relativistic origin.

In analogy with Sec. 5, we can calculate also the damping rate of a longitudinal wave for cyclotron resonance. The potential-well depth is in this case

$$\frac{1}{\tau^2}=\frac{ekv_{0\perp}v_E}{2c^2m}\mathscr{E}. \tag{6.4}$$

The resonances $2\omega_0\pm\nu\approx0$ are realized only in a transverse wave propagating along the magnetic field. The average motion of a charged particle in the resonance region $2\omega_0+\nu=0$ is defined by the equations

$$\frac{dv_\parallel}{dt}=-\frac{v_\perp^2v_E}{4c^2v_\Phi}\frac{e}{m}\mathscr{E}\sin\psi,$$

$$\frac{dv_\perp}{dt}=-\frac{v_\perp v_E}{4c^2}\left(1-\frac{v_\parallel}{v_\Phi}\right)\frac{e}{m}\mathscr{E}\sin\psi,$$

$$\frac{d\psi}{dt}=2\Omega_0\left(\Gamma-\frac{v_E^2}{2c^2}\right)-\omega+kv_\parallel+\frac{v_E}{4c^2}\left(1-\frac{v_\parallel}{v_\Phi}\right)\frac{e}{m}\mathscr{E}\cos\psi,$$

$$v_\Phi=\omega/k. \tag{6.5}$$

In analogy with the preceding cases, the particle trajectories in a wave of finite amplitude can be determined by a familiar technique.[8-11] The trapping of the resonance particles is determined by the quantity

$$\frac{1}{\tau^2}=\frac{kv_{0\perp}^2v_E}{4c^2v_\Phi}\frac{e}{m}\mathscr{E}. \tag{6.6}$$

## 7. CONCLUSION

The foregoing analysis shows that the character of the resonant interaction between a charged particle and a wave is substantially altered in a strong quasistationary electric field. In particular, Čerenkov resonance becomes possible in a transverse wave propagating along a magnetic field, and cyclotron resonance, which is a purely relativistic effect, is possible in a longitudinal wave. Of relativistic origin likewise is the resonance at twice the gyrofrequency in a transverse wave propagating along a magnetic field.

Comparison of the damping rates of a transverse wave in cyclotron and Čerenkov resonances has shown that the two absorption mechanism are comparable. A similar conclusion holds also for a longitudinal wave in the case of an electric drift of sufficiently high velocity.

[1)]The averaged variables will be denoted below by the same letters as the exact ones.


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Translated by J. G. Adashko