Spin glass in a two-sublattice model

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A supersymmetric effective Lagrangian is derived which can be used to study the nonergodicity of a system with two nonequivalent sublattices. Equations for the susceptibilities and order parameters of the sublattices on macroscopic and microscopic time scales are derived in the molecular field approximation. A tetracritical point can arise on the de Almeida-Thouless instability lines by virtue of an interaction of two Edwards-Anderson order parameters. The condition for the existence of this point is derived. The behavior of this system in a nonergodic state is analyzed in detail. Frustration of the intrasublattice interaction stimulates a spin-glass phase.

1. INTRODUCTION

The spin-glass problem has attracted much interest because of several unusual properties associated with the nonergodicity of the system.[1] Most of the theoretical work has dealt with the single-sublattice model with an infinite Sherrington-Kirkpatrick interaction range.[2] Some interesting studies on spin glasses with a finite interaction range have also been carried out.[3–6] Experimentally, there have been studies of complex frustrated magnetic materials (e.g., Ref. 7).

The two-sublattice form of spin glass was originally studied theoretically by the method of replicas in Refs. 8 and 9, where some results quite different from those in the single-sublattice case were found. However, the sublattices were assumed to be equivalent in the model used there.

In the present paper we analyze a spin-glass model with two nonequivalent sublattices. On the basis of the dynamic equations and the concept of supersymmetry, we derive an effective Lagrangian for studying the nonergodicity of the system. We use the method of Ref. 10 to construct equations for the susceptibilities, irreversible-response functions, and order parameters of the two spin subsystems. The interaction of the two order parameters of the sublattices is shown to have an interesting result: A tetracritical point may appear on the Almeida-Thouless singularity lines. This effect appears to have an analog in the ordinary theory of phase transitions, where it has been established that the interaction of two order parameters can give rise to a tetracritical point.[11] A condition for the existence of this point is derived in the strong-frustration limit of the intersublattice interaction. The freezing temperature is calculated for various parameters of the theory. As an example, the temperature dependence is constructed for the susceptibilities and Edwards-Anderson parameters in two cases. It is found that the nonequilibrium sublattice susceptibilities have a fairly weak temperature dependence. As in the single-sublattice case, the temperature of the paramagnet-(spin glass) transition in our model decreases with increasing magnetic field in accordance with the familiar two-thirds law.[12] We find those values of the external magnetic field at which the spin-glass phase is completely suppressed. We find that frustration of the intrasublattice interaction stimulates the existence of a spin glass.

2. MODEL AND BASIC EQUATIONS

We adopt a soft Ising model of the spin glass with two nonequivalent subsystems. The Hamiltonian of this system is then given by

$$H=H_0+H_{int},$$
$$H_0=-\sum_{\alpha=1,2}\sum_{i,k}J_{\alpha ik}m_{\alpha i}m_{\alpha k}-\sum_{\alpha,i}m_{\alpha i}h_{\alpha i}+\sum_{\alpha,i}U_\alpha(m_{\alpha i}),$$
$$H_{int}=-\sum_{i,k}J_{ik}m_{1i}m_{2k},\qquad U_\alpha(m_{\alpha i})=m_{\alpha i}^2/2b_\alpha+u_\alpha m_{\alpha i}^4/8, \tag{1}$$

where the index α specifies the sublattice, $m_{\alpha i}$ are the classical fields, $J_{\alpha ik}$ and J_{ik} are the intrasublattice and intersublattice exchange integrals, and $h_{\alpha i}$ are the local magnetic fields. We consider the case of an "ideal" spin glass with random magnetic fields:

$$\langle J_{\alpha ik}\rangle=\langle J_{ik}\rangle=0,\quad \langle h_{\alpha i}\rangle=0,$$
$$\langle J_{\alpha ik}^2\rangle=I_{\alpha ik},\quad \langle J_{ik}^2\rangle=I_{ik},\quad \langle h_{\alpha i}^2\rangle=h_\alpha^2. \tag{2}$$

The quantities $J_{\alpha ik}$, J_{ik} and $h_{\alpha i}$ are assumed to have Gaussian distributions here.

In our case the dynamic equations have the standard form of Langevin equations with random forces $\varepsilon_{\alpha i}(t)$:

$$\frac{1}{\Gamma_\alpha T}\frac{\partial m_{\alpha i}}{\partial t}=-\frac{1}{T}\frac{\partial H}{\partial m_{\alpha i}}+\varepsilon_{\alpha i}(t),$$
$$\langle\varepsilon_{\alpha i}(t)\varepsilon_{\beta j}(t')\rangle=\frac{2}{\Gamma_\alpha T}\delta_{\alpha\beta}\delta_{ij}\delta(t-t'), \tag{3}$$

where Γ_α^{-1} is the bare relaxation time.

To find the expectation value in (3) we could use, for example, the diagram technique of Ref. 13. We find it more convenient to work with an effective Lagrangian which can be constructed on the basis of the idea of supersymmetry. We introduce the superfields[10]

$$\Phi_{\alpha i}(t)=m_{\alpha i}(t)+[\theta_\alpha^*\eta_{\alpha i}(t)+\eta_{\alpha i}^*(t)\theta_\alpha]-\theta_\alpha^*\theta_\alpha\varphi_{\alpha i}(t), \tag{4}$$

where θ_α and θ_α^* are supersymmetric coordinates, and $\eta_{\alpha i}$, $\eta_{\alpha i}^*$, and $\varphi_{\alpha i}$ are anticommuting and commuting variables, respectively. Using (1)–(4), we find the following effective Lagrangian in the mean field approximation:

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$$L_{eff}=L_1+L_2,$$

$$L_1=\iint dt\,dt'\iint d\theta_1^{\cdot}\,d\theta_1\,d\xi_1^{\cdot}\,d\xi_1\Big\{-\frac{1}{2\Gamma_1 T}\sum_i P(\theta_1,\xi_1)$$

$$\times\Big[\Phi_{1i}(\theta_1,t)\frac{\partial\Phi_{1i}(\xi_1,t)}{\partial t}+2\Phi_{1i}(\theta_1,t)\Phi_{1i}(\xi_1,t')\Big]\delta(t-t')$$

$$-\frac{1}{2T^2}[h_1^2+4I_1G_1(\theta_1,\xi_1,t-t')+4I_0G_2(\theta_1,\xi_1,t-t')]$$

$$\times\sum_i\Phi_{1i}(\theta_1,t)\Phi_{1i}(\xi_1,t')$$

$$+\frac{1}{T}\sum_i U_1[\Phi_{1i}(t)]\delta(t-t')\delta(\theta_1-\xi_1)\Big\},$$

$$P(\theta,\xi)=-(\theta^{\cdot}-\xi^{\cdot})(\theta+\xi),\quad \delta(\theta-\xi)=-(\theta^{\cdot}-\xi^{\cdot})(\theta-\xi),$$

$$I_\alpha=\sum_k I_{\alpha ik},\quad I_0=\sum_k I_{ik}. \tag{5}$$

The second term, L_2, is found from L_1 through the interchange of indices $1\rightleftarrows 2$, so all the equations written below are symmetric under this interchange. Since the Lagrangian $L_{\rm eff}$ is a single-site Lagrangian in the approximation, it is sufficient to consider only one sublattice; the effect of the second is incorporated in the term containing I_0 in (5). The correlation function G_α which figures in the effective Lagrangian is given by

$$G_\alpha(\theta,\xi,t-t')=\langle\Phi_{\alpha i}(\theta,t)\Phi_{\alpha i}(\xi,t')\rangle, \tag{6}$$

where the expectation value is carried out with Lagrangian (5). We see from (1) and (5) that it is possible to construct a standard perturbation theory in the term Φ (Ref. 4).

Following Ref. 10, we find a system of equations for the spin correlation function D_α and the retarded and advanced Green's functions $G_\alpha^\pm$:

$$(G_1^\pm)^{-1}-S_1^\pm+\frac{4I_1}{T^2}G_1^\pm+\frac{4I_0}{T^2}G_2^\pm=0,$$

$$(G_2^\pm)^{-1}-S_2^\pm+\frac{4I_2}{T^2}G_2^\pm+\frac{4I_0}{T^2}G_1^\pm=0,$$

$$D_1=-\Big[\Big(1-\frac{4I_1}{T^2}G_1^-G_1^+\Big)\Big(1-\frac{4I_2}{T^2}G_2^-G_2^+\Big)$$

$$-\Big(\frac{4I_0}{T^2}\Big)^2G_1^-G_1^+G_2^-G_2^+\Big]^{-1}$$

$$\times\Big[B_1G_1^-G_1^++\frac{4}{T^2}(I_0B_2-I_2B_1)G_1^-G_1^+G_2^-G_2^+\Big],$$

$$D_2=-\Big[\Big(1-\frac{4I_1}{T^2}G_2^-G_1^+\Big)\Big(1-\frac{4I_2}{T^2}G_2^-G_2^+\Big)$$

$$-\Big(\frac{4I_0}{T^2}\Big)^2G_1^-G_1^+G_2^-G_2^+\Big]^{-1}$$

$$\times\Big[B_2G_2^-G_2^++\frac{4}{T^2}(I_0B_1-I_1B_2)G_2^-G_2^+G_1^-G_1^+\Big], \tag{7}$$

where

$$S_\alpha^+=\frac{1}{T}\Big(\frac{1}{b_\alpha}\pm\frac{i\omega}{\Gamma_\alpha}\Big)-\Sigma_\alpha^\pm, \tag{8}$$

$$B_\alpha=-\frac{2}{\Gamma_\alpha T}-\sigma_\alpha. \tag{9}$$

No de Almeida-Thouless singularity arises in first-order perturbation theory, so we need to go to second order in the anharmonicity constants u_1 and u_2. The eigenenergy parts of $\Sigma_\alpha^\pm$ and σ_α then take the form

$$\Sigma_\alpha^\pm=-\frac{3u_\alpha}{2}G^\pm(\omega=0)$$

$$+\frac{9u_\alpha^2}{2T^2}\iint\frac{d\omega_1\,d\omega_2}{(2\pi)^2}G_\alpha^\pm(\omega-\omega_1-\omega_2)D_\alpha(\omega_1)D_\alpha(\omega_2),$$

$$\sigma_\alpha(\omega)=\frac{3u_\alpha^2}{2T^2}\iint\frac{d\omega_1\,d\omega_2}{(2\pi)^2}D_\alpha(\omega-\omega_1-\omega_2)D_\alpha(\omega_1)D_\alpha(\omega_2). \tag{10}$$

Equations (7)–(10) are the basic equations of our theory.

In a nonergodic state, a difference arises between the equilibrium (Gibbs-averaged) susceptibility $\chi_{\alpha0}$ and the nonequilibrium (time-averaged) susceptibility χ_α. The difference is determined by an irreversible-response function (or Sommers parameter[14]):

$$\Delta_\alpha=T(\chi_{\alpha0}-\chi_\alpha)=G_\alpha^-(\omega=0)-g_\alpha,\quad g_\alpha=\lim_{\omega\to0}G_\alpha^-(\omega). \tag{11}$$

The function Δ is obviously a measure of the nonergodicity.

In the nondegenerate phase ($\Delta_\alpha=0$) it is easy to derive the system of Sherrington-Kirkpatrick equations[2] for g_1, g_2 and for the Edwards-Anderson parameters q_1, q_2:

$$\Big(1-\frac{4I_1}{T^2}g_1^2\Big)q_1=\Big(\frac{h_1^2}{T^2}+\frac{3u_1^2}{2T^2}q_1^3\Big)g_1^2+\frac{4I_0}{T^2}g_1^2q_2,$$

$$\Big(1-\frac{4I_2}{T^2}g_2^2\Big)q_2=\Big(\frac{h_2^2}{T^2}+\frac{3u_2^2}{2T^2}q_2^3\Big)g_2^2+\frac{4I_0}{T^2}g_2^2q_1,$$

$$\Big(\frac{1}{b_1T}-\beta_1\Big)g_1-\frac{4I_1}{T^2}g_1^2-\frac{4I_0}{T^2}g_1g_2-1=0,$$

$$\Big(\frac{1}{b_2T}-\beta_2\Big)g_2-\frac{4I_2}{T^2}g_2^2-\frac{4I_0}{T^2}g_1g_2-1=0,$$

$$\beta_\alpha=-\frac{3u_\alpha}{2}(g_\alpha+q_\alpha)+\frac{9u_\alpha^2}{2T^2}\Big(q_\alpha^2g_\alpha+q_\alpha g_\alpha^2+\frac{g_\alpha^3}{3}\Big). \tag{12}$$

The phase transition to the spin-glass phase is determined by the pole in $D_\alpha(\omega)$ as $\omega\to0$ (the relaxation time becomes infinite). We thus find the following equation for the Almeida-Thouless singularity line:

$$\Big[1-\Big(\frac{4I_1}{T^2}+\frac{9u_1^2}{2T^2}q_1^2\Big)g_1^2\Big]\Big[1-\Big(\frac{4I_2}{T^2}+\frac{9u_2^2}{2T^2}q_2^2\Big)g_2^2\Big]$$

$$-\Big(\frac{4I_0}{T^2}g_1g_2\Big)^2=0. \tag{13}$$

In deriving equations for the degenerate phase ($\Delta_\alpha\neq0$) we use Sompolinsky's hypothesis,[15] which allows us to write

$$D_\alpha(\omega)=D_{\alpha0}(\omega)+D_{\alpha s}(\omega),\quad G_\alpha^-(\omega)=G_{\alpha0}^-(\omega)+G_{\alpha s}^-(\omega),$$

$$D_{\alpha s}(\omega)=\sum_{j=0}^{k}\frac{2q_{\alpha j}'\Gamma_{\alpha j}}{\omega^2+\Gamma_{\alpha j}^2},\quad G_{\alpha s}^-(\omega)=-\sum_{j=0}^{k}\frac{i\Delta_{\alpha j}'\Gamma_{\alpha j}}{\omega+i\Gamma_{\alpha j}},$$

$$\Delta_{\alpha i}=-\sum_{j=i}^{k}\Delta_{\alpha j}',\quad q_{\alpha i}=\sum_{j=0}^{i}q_{\alpha j}',\quad \Gamma_i\to 0,\quad \frac{\Gamma_i}{\Gamma_{i+1}}\to 0, \tag{14}$$

where $D_{\alpha0}(\omega)$ and $G_{\alpha0}^-(\omega)$ satisfy the usual fluctuation-dissipation theorem. After substituting (14) into (17)–(10), we find some equations for $\Delta_{\alpha i}$ and $q_{\alpha i}$. In the limit $k\to\infty$ the quantity i/k becomes a continuous variable x, which varies over the interval [0,1], while $\Delta_{\alpha i}$ and $q_{\alpha i}$ become functions $\Delta_\alpha(x)$ and $q_\alpha(x)$. The function $q(x)$ is called the "Parisi parameter."[16] Omitting the details of the calculations, we write the final result:

$$\left[1-\left(\frac{4I_1}{T^2}+\frac{3u_1^2}{2T^2}q_{10}^2\right)(\Delta_1+q_1)^2\right]q_{10}-\frac{4I_0}{T^2}(\Delta_1+g_1)^2q_{20}=\frac{h_1^2}{T^2}(\Delta_1+g_1)^2,$$

$$\left[1-\left(\frac{4I_2}{T^2}+\frac{3u_2^2}{2T^2}q_{20}^2\right)(\Delta_2+g_2)^2\right]q_{20}-\frac{4I_0}{T^2}(\Delta_2+g_2)^2q_{10}=\frac{h_2^2}{T^2}(\Delta_2+g_2)^2, \tag{15}$$

$$\left\{\left[1-\left(\frac{4I_1}{T^2}+\frac{9u_1^2}{2T^2}q_1^2(x)\right)(\Delta_1(x)+g_1)^2\right]\times\left[1-\left(\frac{4I_2}{T^2}+\frac{9u_2^2}{2T^2}q_2^2(x)\right)\times(\Delta_2(x)+g_2)^2\right]-\left(\frac{4I_0}{T^2}\right)^2(\Delta_1(x)+g_1)^2(\Delta_2(x)+g_2)^2\right\}q_\alpha'(x)=0,$$

$$\times\left\{\left[1-\left(\frac{4I_1}{T^2}+\frac{9u_1^2}{2T^2}q_1^2(x)\right)(\Delta_1(x)+g_1)^2\right]\times\left[1-\left(\frac{4I_2}{T^2}+\frac{9u_2^2}{2T^2}q_2^2(x)\right)\times(\Delta_2(x)+g_2)^2\right]-\left(\frac{4I_0}{T^2}\right)^2(\Delta_1(x)+g_1)^2(\Delta_2(x)+g_2)^2\right\}\Delta_\alpha'(x)=0, \tag{16}$$

where

$$q_{\alpha0}=q_\alpha(x=0),\quad q_\alpha=q_\alpha(x=1),$$
$$\Delta_\alpha(x=1)=0,\quad \Delta_\alpha=\Delta_\alpha(x=0). \tag{17}$$

It is not difficult to see that q_α and $q_{\alpha0}$ are the Edwards-Anderson parameters at microscopic and macroscopic frequencies. In the approach which we are taking here, we find not the functions $q_\alpha(x)$ and $\Delta_\alpha(x)$, themselves but only their boundary values.[10] In the vicinity of the Almeida-Thouless singularity, where $q_\alpha'(x)$ and $\Delta_\alpha'(x)$ are nonvanishing, the two equations in (16) are the same. In this case the expression in braces vanishes. At $x = 1$, this gives us condition (13), while at $x = 0$ we have

$$\left[1-\left(\frac{4I_1}{T^2}+\frac{9u_1^2}{2T^2}q_{10}^2\right)(\Delta_1+g_1)^2\right]\times\left[1-\left(\frac{4I_2}{T^2}+\frac{9u_2^2}{9T^2}q_{20}^2\right)(\Delta_2+g_2)^2\right]-\left[\frac{4I_0}{T^2}(\Delta_1+g_1)(\Delta_2+g_2)\right]^2=0, \tag{18}$$

and it follows from (13) and (18) that on the Almeida-Thouless singularity line we have $q_\alpha = q_{\alpha0}$ and $\Delta_\alpha = 0$.

Interestingly, the fact that the parameters Δ_α vanish on the de Almeida-Thouless singularity line can be proved through direct derivation of the Sommers equations for our model. In second-order perturbation theory these equations take the form

$$\frac{g_1}{\Delta_1+g_1}-\left(\frac{1}{b_1T}-\beta_1\right)g_1+\frac{4I_1}{T^2}g_1^2+\frac{4I_0}{T^2}g_1(g_2+\Delta_2)+\left(\frac{4I_1}{T^2}+\frac{9u_1^2}{2T^2}q_1^2\right)\Delta_1g_1=0,$$

$$\frac{g_2}{\Delta_2+g_2}-\left(\frac{1}{b_2T}-\beta_2\right)g_2+\frac{4I_2g_2^2}{T^2}+\frac{4I_0}{T^2}g_2(g_1+\Delta_1)+\left(\frac{4I_2}{T^2}+\frac{9u_2^2}{2T^2}q_2^2\right)\Delta_2g_2=0, \tag{19}$$

where β_α are defined in (12). Using (12) and (19), we find

$$\left[\frac{g_1}{g_1+\Delta_1}-\left(\frac{4I_1}{T^2}+\frac{9u_1^2}{2T^2}q_1^2\right)g_1^2\right]\times\left[\frac{g_2}{g_2+\Delta_2}-\left(\frac{4I_2}{T^2}+\frac{9u_2^2}{2T^2}q_2^2\right)g_2^2\right]-\left(\frac{4I_0}{T^2}g_1g_2\right)^2=0 \tag{20}$$

it follows that under condition (13) the quantities Δ_1 and Δ_2 are zero.

3. POSSIBLE EXISTENCE OF A TETRACRITICAL POINT

Let us analyze Eqs. (12), (13), (15), and (18). Using the first two of Eqs. (12), we rewrite condition (13) as

$$(h_1^2-3u_1^2q_1^3)(h_2^2-3u_2^2q_2^3)+4I_0q_1(h_1^2-3u_1^2q_1^3)+4I_0q_2(h_2^2-3u_2^2q_2^3)=0. \tag{21}$$

We assume that the Edwards-Anderson order parameter q_α for each sublattice is determined by exclusively its own local magnetic field h_α. It is then a simple matter to verify that condition (21) holds if

$$h_1^2=3u_1^2q_1^3,\quad h_2^2=3u_2^2q_2^3. \tag{22}$$

It can be seen from (12), (15), (22), and the definition of the susceptibilities in (11) that the quantities $q_{\alpha0}$ and $\chi_{\alpha0}$ are independent of the temperature, while q_α and χ_α are independent of h_α. We can thus write

$$q_\alpha(T,h)=q_{\alpha c}(T),\quad g_\alpha(T,h)=g_{\alpha c}(T), \tag{23}$$

where $q_{\alpha c}$ and $g_{\alpha c}$ are the values of q_α and g_α on the $h_{\alpha c}(T)$

phase curves. The six functions q_{ac}, g_{ac}, and h_{ac} are defined by the simultaneous solution of the six equations (12) and (22). We will not determine $h_{ac}(T)$, however; we will instead assume that they are known functions of the temperature.[10] From (12), (15), and (22) we then find

$$q_{\alpha 0}^3=h_\alpha^2/3u_\alpha^2,\quad \chi_{10}^2=q_{10}(4I_1q_{10}+4I_0q_{20}+9u_1^2q_{10}^3/2)^{-1},$$
$$\chi_{20}^2=q_{20}(4I_2q_{20}+4I_0q_{10}+9u_2^2q_{20}^2/2)^{-1},$$

$$q_\alpha^3=h_{\alpha c}^2/3u_\alpha^2,\quad \chi_1^2=q_1(4I_1q_1+4I_0q_2+9u_1^2q_1^3/2)^{-1},$$
$$\chi_2^2=q_2(4I_2q_2+4I_0q_1+9u_2^2q_2^3/2)^{-1}. \tag{24}$$

These relations, along with (12) and (15), in principle constitute a complete solution of the problem. However, these equations cannot be solved analytically for the general case. We will accordingly restrict the discussion to particular cases.

If the intrasublattice interaction of the spins is far stronger than the intersublattice interaction ($I_1,I_2 \gg I_0$), we can set $I_0 = 0$. In this case, all of the equations derived above break up into two independent subsystems. For each sublattice we find the result of the one-sublattice model,[10] as is easily shown. Experimentally, one should observe two points of transition to a spin glass in such a situation (the susceptibility curve will have two slope changes).

We are interested primarily in the opposite case, in which frustration of the intersublattice interaction dominates ($I_0 \gg I_1,I_2$). In this case both sublattices can undergo a transition to the spin-glass phase simultaneously at some temperature T_f. This point is obviously the point at which the two curves $h_{1c}(T_f) = 0$ and $h_{2c}(T_f) = 0$ intersect. In other words, on de Almeida-Thouless singularity lines a tetracritical point arises; below this point, the order parameters of the two subsystems are nonzero. The situation is shown qualitatively in Fig. 1.

To determine T_f we first set $I_1 = I_2 = 0$. In this approximation we find from (12) and (13)

$$T_f={}^4/_3x_1x_2(I_0/u_1u_2)^{1/2}, \tag{25}$$

where x_1 and x_2 are the solutions of the system

$$x_1^4-3x_1^2+12-a_1(x_1/x_2)^{1/2}=0,$$
$$x_2^4-3x_2^2+12-a_2(x_2/x_1)^{1/2}=0. \tag{26}$$

Here

$$x_\alpha=(3u_\alpha/T_f)^{1/2}g_{\alpha f},\quad a=3b_1^{-1}(u_2/u_1I_0^2)^{1/4},$$

$$a_2=3b_2^{-1}(u_1/u_2I_0^2)^{1/4},\quad g_{\alpha f}=g_\alpha(T=T_f). \tag{27}$$

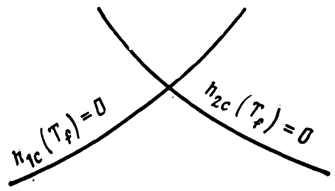

FIG. 1. Appearance of a tetracritical point on the de Almeida-Thouless singularity lines by virtue of the interaction between the two order parameters of the sublattices.

Equations (26) do not have exact solutions for arbitrary α_1 and α_2. It is possible, however, to find the condition under which it has positive solutions, i.e., the condition for the existence of a tetracritical point. In the Appendix we prove that a tetracritical point arises only if

$$39/4\leqslant a_1\leqslant 12,\quad 39/4\leqslant a_2\leqslant 12. \tag{28}$$

Figure 2 shows the freezing temperature as a function of the parameters α_1 and α_2. As these parameters increase, T_f decreases, as is easily seen. The reason is that for given anharmonicity constants an increase in α_1 and α_2 is equivalent to a decrease in the degree of frustration [see (27)]. The maximum value, reached at $\alpha_1 = \alpha_2 = 39/4$, is

$$T_{f\,max}=2(I_0/u_1u_2)^{1/2}. \tag{29}$$

As we will see below, expression (29) corresponds to the point of transition to a metastable spin-glass phase.

In the case $\alpha_1 = \alpha_2 = \alpha$, Eqs. (26) can be solved exactly ($x_1 = x_2$); we find

$$g_{1f}=\frac{I_0^{1/2}}{2u_1}\left(\frac{u_1}{u_2}\right)^{1/4}\varphi(a),\quad g_{2f}=\frac{I_0^{1/2}}{2u_2}\left(\frac{u_2}{u_1}\right)^{1/4}\varphi(a),$$
$$T_f=\frac{2I_0}{3(u_1u_2)^{1/2}}\varphi(a),$$
$$\varphi(a)=3-(4a-39)^{1/2},\quad 0\leqslant\varphi(a)\leqslant 3. \tag{30}$$

Let us consider the case in which α_1 and α_2 differ only slightly. Setting $a_2 = a_1 + \varepsilon = a + \varepsilon$ where $\varepsilon \ll 1$, we find, in first order in ε,

$$\Delta T_f/T_f(\varepsilon=0)=-\varepsilon/\varphi(a)\,[3-\varphi(a)], \tag{31}$$

where ΔT_f is the shift of the transition temperature from $T_f(\varepsilon = 0)$, which is determined by (30). The minus sign in (31) means that T_f decreases with increasing a_α, in agreement with the general conclusion derived above.

Near T_f we can derive exact general expressions for all quantities in terms of x_1 and x_2. These expressions are too lengthy to reproduce here, however; we will write the results only for the case $a_1 = a_2$:

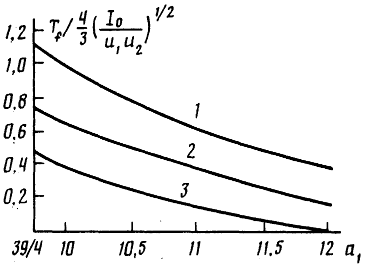

FIG. 2. Freezing temperature versus the parameters a_1 and a_2. 1—$a_2 = 10$; 2—$a_2 = 11$; 3—$a_2 = 12$.

$$q_{\alpha 0}=(h_\alpha^2/3u_\alpha^2)^{1/3}, \quad h_{\alpha c}=3^{1/2}u_\alpha(g_{\alpha f}|\tau|)^{3/2},$$
$$q_\alpha=q_{\alpha c}=(h_{\alpha c}^2/3u_\alpha^2)^{1/3}=g_{\alpha f}|\tau|, \quad |\tau|=|(T-T_f)/T_f|\ll 1,$$
$$\chi_{10}=\frac{1}{2(I_0q_{20}/q_{10})^{1/2}}\left[1-\frac{9u_1^2q_{10}^3}{16I_0q_{20}}\right],$$
$$\chi_{20}=\frac{1}{2(I_0q_{10}/q_{20})^{1/2}}\left[1-\frac{9u_2^2q_{20}^3}{16I_0q_{10}}\right],$$
$$\frac{\Delta_1}{T}=\frac{9u_1^2q_{1c}^2}{32(I_0u_1/u_2)^{3/2}}\left[1-\left(\frac{h_1}{h_{1c}}\right)^{4/3}\right],$$
$$\chi_1=\frac{1}{2(I_0q_2/q_1)^{1/2}}\left[1-\frac{9u_1^2q_1^3}{16I_0q_2}\right];$$
$$\chi_2=\frac{1}{2(I_0q_1/q_2)^{1/2}}\left[1-\frac{9u_2^2q_2^3}{16I_0q_1}\right]$$
$$\frac{\Delta_2}{T}=\frac{9u_2^2q_{2c}^2}{32(I_0u_2/u_1)^{3/2}}\left[1-\left(\frac{h_2}{h_{2c}}\right)^{4/3}\right]. \qquad (32)$$

From the latter equations we see that we have $\chi \propto h^{4/3}$ and $\Delta(h=0)\propto|\tau|^2$. The same field dependence and temperature dependence have been derived for the one-sublattice model[10] (the only difference is in the numerical coefficients). This result is apparently a consequence of the use of the molecular field approximation.

As an example, we write the results of the numerical calculations for the temperature dependence of the Edwards-Anderson parameters and for the susceptibilities in the two particular cases

$$a_1=a_2=11, \quad u_1=0{,}722, \quad u_2=u_1/2,$$

$$a_1=11{,}5, \quad a_2=11, \quad u_1=0{,}528, \quad u_2=u_1/2.$$

Here we have used $I_0^{1/2}$ as a unit of energy, so we have $u_\alpha, h_\alpha, T\propto I_0^{1/2}$; $\chi_\alpha, b_\alpha\propto I_0^{1/2}$; while the other quantities, g_α, q_α, and Δ_α, are dimensionless. In the numerical calculation, $I_0^{1/2}$ drops out. In each case, the parameter values are chosen to satisfy $T_f=I_0^{1/2}$. Note also that in the limit $T\to 0$ we have $q_\alpha=$ const, while $g_\alpha\propto T$. Curves 1 and 2 in Fig. 3 show the temperature dependence of the parameters q_1 and q_2 in cases a) and b), respectively. We see that the Edwards-Anderson parameters decrease with increasing a_α. The reason is that an increase in a_α prevents the existence of a spin glass, as we noted earlier. The horizontal lines in Figs. 4 and 5 correspond to temperature-independent equilibrium sublattice susceptibilities. As can be seen from Figs. 4 and 5, the nonequilibrium susceptibilities depend rather weakly on the

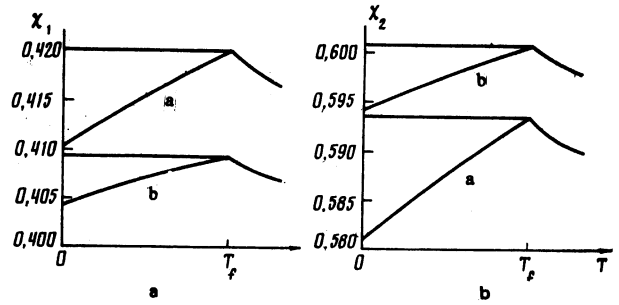

FIG. 4. Susceptibilities of (a) the first and (b) the second sublattice for cases a) and b). The horizontal lines show the equilibrium values.

temperature in our model. We believe this weak dependence is a general feature of a "soft" spin-glass model. Since the irreversible-response functions Δ_α are related to $\chi_{\alpha 0}$ and χ_α by (11), we will not reproduce the corresponding curves for them here.

Let us examine the effect of an external magnetic field on the tetracritical point. In weak fields ($h\ll I_0^{1/2}$) we find from (12) and (13)

$$T_f(h)=T_f(h=0)(1-\gamma h^{2/3}),$$
$$\gamma=3^{7/6}\{T_f^{1/2}(h=0)[4x_1^4-6x_1^2-{}^1\!/_2a_1(x_1/x_2)^{1/2}]$$
$$\times[4x_2^4-6x_2^2-{}^1\!/_2a_2(x_2/x_1)^{1/2}]-a_1a_2/4\}^{-1}$$
$$\times\{x_2(1-x_2^2)u_2^{-1/6}[a_1(x_1/x_2)^{1/2}+6x_1^2-4x_1^4]$$
$$+x_1(1-x_1^2)u_1^{-1/6}[a_2(x_2/x_1)^{1/2}+6x_2^2-4x_2^4]\}. \qquad (33)$$

It is easy to see that we have $\gamma>0$ at $x_1, x_2<1$. According to the numerical calculations described in the Appendix, solutions $x_1, x_2<1$ of Eqs. (26) are possible only in the interval $10<a_1, a_2<12$. Accordingly, for this region of the values of the parameters a_α the external magnetic field suppresses the spin-glass phase in accordance with the two-thirds law.[12] Elsewhere in the interval $39/4<a_1, a_2<10$ the coefficient γ is negative. This result means that the temperature of the paramagnetic-(spin glass) transition increases rather than decreases with increasing field. We thus assume that the spin-glass phases which correspond to the interval $39/4<a_1$, $a_2<10$ are metastable.

In the case $a_1=a_2[x_1=x_2=(\varphi(a)/\lrcorner\,)^{1/2}]$, expression (33) simplifies substantially, becoming

$$\gamma=\frac{3^{5/3}(u_1u_2)^{1/4}}{2I_0^{1/2}}(u_1^{-1/6}+u_2^{-1/6})\frac{[1-\varphi(a)/2]}{\varphi(a)[3-\varphi(a)]}. \qquad (34)$$

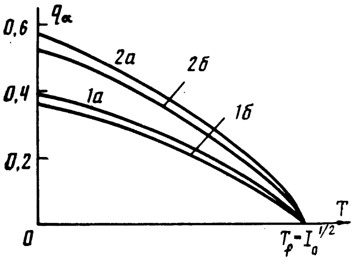

FIG. 3. Temperature dependence of the Edwards-Anderson parameters q_1 (1*a*, 1*b*) and q_2 (2*a*,2*b*) for cases a) and b).

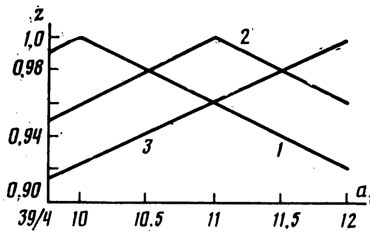

FIG. 5. Dependence of the solution of Eq. (A4) on a_1 and a_2. 1—$a_2=10$; 2—$a_2=11$; 3—$a_2=12$.

It is not difficult to see that the coefficient γ is positive for $10 < a < 12$ $(0 < \varphi(a) < 2)$ and negative for $39/4 < a < 10$ $(2 < \varphi(a) < 3)$. For cases a) and b) introduced above, Eqs. (34) and (33) $(x_1 = 0.5168, x_2 = 0.5345)$ yield $\gamma = 1.8$ and $\gamma = 1.3$, respectively.

Let us find the critical external magnetic field, at which the spin-glass phase is totally suppressed $(T_f = 0)$. From (2) and (22) we find

$$h_k = 3^{1/2} u_\alpha q_\alpha^{3/2}(T=0), \tag{35}$$

where $q_\alpha(T=0)$ is the value of q_α at absolute zero, which can be found only by numerical methods. For particular cases a) and b), the values of h_k are approximately $0.25 I_0^{1/2}$ and $0.18 I_0^{1/2}$, respectively.

We now incorporate the intrasublattice interaction. As before, we assume that the intersublattice frustration is dominant, i.e., $I_0 \gg I_1, I_2$. Expanding in the small parameters I_1/I_0 and I_2/I_0, we then find the shift of the freezing temperature:

$$\Delta T_f/T_f(I_\alpha=0) = 3(x_1^4 + x_2^4 + \tilde{B}_1 x_2^6 + \tilde{B}_2 x_1^6 + 12\tilde{B}_1\tilde{B}_2 - 12)^{-1}$$
$$\times\left[\frac{x_1}{x_2}\left(\frac{u_2}{u_1}\right)^{1/2}(1+2\tilde{B}_2+\tilde{B}_1\tilde{B}_2)\frac{I_1}{I_0} + \frac{x_2}{x_1}\left(\frac{u_1}{u_2}\right)^{1/2}(1+2\tilde{B}_1+\tilde{B}_1\tilde{B}_2)\frac{I_2}{I_0}\right],$$
$$\tilde{B}_\alpha = 1 + x_\alpha^2/2 - x_\alpha^4/2. \tag{36}$$

It can be verified that $\Delta T_f/T_f$ in (36) is positive for all a_1 and a_2 which satisfy (28), i.e., that T_f increases with increasing values of the parameters I_1 and I_2. In other words, the frustration of the intrasublattice interaction stimulates the existence of a spin glass. This is a natural result, since T_f must be proportional to the degree of frustration of the system as a whole.

In the case $a_1 = a_2$, Eq. (36) can be rewritten in the simpler form

$$\frac{\Delta T_f}{T_f(I_\alpha=0)} = 3\left[\left(\frac{u_2}{u_1}\right)^{1/2}\frac{I_1}{I_0} + \left(\frac{u_1}{u_2}\right)^{1/2}\frac{I_2}{I_0}\right]\frac{16+2\varphi(a)-\varphi^2(a)}{\varphi(a)[3-\varphi(a)]}. \tag{37}$$

We see that ΔT_f is positive for all a. We will not reproduce here the lengthy expressions for the other quantities.

4. CONCLUSION

We have shown that in the case of pronounced frustration of the intersublattice interaction a tetracritical point can exist. When such a point does exist, the susceptibility of the system should have only a single slope change. This is apparently the situation in, for example, the frustrated system of the garnet MnFeG (Ref. 7).

Note that if we assume the sublattices to be equivalent, as in Refs. 8 and 9, a tetracritical point no longer arises, since the point $h_{1c}(T) = h_{2c}(T)$ and the two lines $h_{1c}(T_f) = 0$ and $h_{2c}(T_f) = 0$ simply coincide, rather than intersecting.

It would be interesting to examine the intermediate case in which the degrees of frustration of the intersublattice and intrasublattice interactions are approximately the same $(I_1, I_2 \sim I_0)$. This question can be treated using the equations derived in this paper.

The model which we have used in this paper can be generalized to study a state in which ferrimagnetism coexist with a spin glass (it is necessary to assume that the first moments of the distribution of the random exchange interactions are nonzero).

I am deeply indebted to Prof. Wu Dien Ky for a useful discussion of this study.

APPENDIX

The condition for the existence of a tetracritical point is equivalent to the condition for the existence of nonnegative solutions of Eqs. (26) in the text proper. From (26) we have

$$a_1(x_1/x_2)^{1/2} = (x_1^2 - 3/2)^2 + 39/4 \geqslant 39/4,$$
$$a_2(x_2/x_1)^{1/2} \geqslant 39/4, \quad a_1 a_2 \geqslant (39/4)^2. \tag{A1}$$

Assuming $a_1 > a_2$ and introducing the auxiliary variable $z = (x_1/x_2)^{1/2}$, we find

$$x_1^2 = 1/2[3 - (4a_1 z - 39)^{1/2}], \quad x_2^2 = 1/2[3 - (4a_2/z - 39)^{1/2}]. \tag{A2}$$

From the condition that the solutions x_1 and x_2 be nonnegative we find

$$a_1 a_2 \leqslant 12^2. \tag{A3}$$

Inequalities (A1) and (A3) constitute a necessary condition. The sufficient condition is the condition for the existence of a positive solution of the equation

$$z^4 - [3 - (4a_1 z - 39)^{1/2}]/[3 - (4a^2/z - 39)^{1/2}] = 0 \tag{A4}$$

in the interval $[39/4a_1, 4a_2/39]$. The sufficiency is easily checked.

Furthermore, the symmetry of Eqs. (26) under the interchange of the sublattice indices means that we can always choose $z \geqslant 1$ [it is not difficult to show that in the case $a_1 > a_2$ we choose $z = (x_1/x_2)^{1/2}$, while in the case $a_1 < a_2$ we choose $z = (x_2/x_1)^{1/2}$]. This result, along with (A1)–(A3), gives us condition (28) of the text proper. Figure 5 shows the result of a solution of (A4) for various values of the parameters a_1 and a_2. From these results and (A2) we find the x_1 and x_2 used in the text proper.

[1] K. H. Fischer, Phys. Status Solidi **b116**, 357 (1983); **130**, 13 (1985).
[2] D. Sherrington and S. Kirkpatrick, Phys. Rev. Lett. **35**, 1792 (1975).
[3] H. J. Bouchiat, J. Phys. **C16**, L145 (1983); **17**, 3963 (1984).
[4] B. Barbara, A. P. Malozemoff, and S. E. Barnes, J. Appl. Phys. **55**, 1655 (1984).
[5] A. P. Malozemoff, S. E. Barnes, and B. Barbara, Phys. Rev. Lett. **51**, 1704 (1983).
[6] L. B. Ioffe and M. V. Feĭgel'man, Zh. Eksp. Teor. Fiz. **89**, 654 (1985) [Sov. Phys. JETP **62**, 376 (1985)].
[7] V. I. Sokolov, Izv. Vyssh. Uchebn, Zaved., Fiz. **10**, 91 (1984).
[8] I. Ya. Korenblit and E. F. Shender, Zh. Eksp. Teor. Fiz. **89**, 1785 (1985) [Sov. Phys. JETP **62**, 1030 (1985)]; Pis'ma Zh. Eksp. Teor. Fiz. **42**, 370 (1985) [JETP Lett. **42**, 456 (1985)].
[9] I. Ya. Korenblit, Ya. V. Fedorov, and E. F. Shender, Zh. Eksp. Teor. Fiz. **92**, 710 (1987) [Sov. Phys. JETP **65**, 400 (1987)].

[10]S. L. Ginzburg, Zh. Eksp. Teor. Fiz. **85**, 2171 (1983) [Sov. Phys. JETP **58**, 1260 (1983)].
[11]Yu. A. Izyumov and V. N. Syromyatnikov, *Fazovye perekhody i simmetriya kristallov* (*Phase Transitions and Symmetry of Crystals*), Nauka, Moscow, 1984.
[12]J. R. L. De Almeida and D. J. Thouless, J. Phys. **A11**, 983 (1978).
[13]S. Ma, *Critical Phenomena*, Benjamin, New York, 1976 (Russ. Transl. Mir. Moscow, 1984).
[14]H. J. Sommers, Z. Phys. **B31**, 301 (1978).
[15]H. Sompolinsky, Phys. Rev. Lett. **47**, 935 (1981).
[16]G. Parisi, Phys. Rev. Lett. **43**, 1574 (1979).

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