Nonlinear magnetoelastic waves in easy-plane magnets

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We have investigated the possible types of nonlinear magnetoelastic waves in easy-plane magnetic crystals with relatively small anisotropy in the basal plane, i.e., magnets which are close to an orientational phase transition. We show that in such magnets it is possible to have spiral and periodic nonlinear magnetoelastic waves in addition to solitary waves. We investigate the effects of dissipation in the magnetic subsystem. Apparently the experimentally observed lower bound on the decrease of the sound velocity in ferro- and antiferromagnets as the orientational phase transition point is approached is due to nonlinear effects.

1. INTRODUCTION

As a magnet approaches an orientational phase transition (OPT) point, the "rigidity" of the magnetic subsystem decreases.[1] As a result, weak external excitation leads to large deviations in the magnetization vector **M** from its equilibrium position, and the vibrations in **M** become nonlinear. In addition, there is in this case an effective growth in the magnetoelastic (ME) coupling.[2,3] Therefore, nonlinear ME excitations become important near an OPT, and can give a significant contribution to the thermodynamic and kinetic properties of magnets. When the energy of the external excitation is comparable to the anisotropy energy, solitary, spiral and periodic types of plane nonlinear waves (NLW) can propagate in these magnets. All of these waves reduce to plane waves as the excitation energy decreases. Nonlinear ME phenomena were studied in Refs. 4, 5 by means of an anharmonic expansion. Solitary ME waves were discussed in Refs. 6–11. In this article we will investigate the possible types of nonlinear ME waves in easy-plane ferromagnets with weak anisotropy in the basal plane (e.g., $Mn_{2-x}Cr_xSb$). In these magnets an OPT occurs when the planar anisotropy constant changes sign. The results obtained here are also valid for low-frequency excitations in tetragonal easy-plane antiferromagnets (e.g., NiF_2 and MnO_2) and are easily generalized to rhombohedral and hexagonal magnets.

Experimental investigations of the decrease in the sound velocity v as a magnet approaches an OPT point were carried out in Refs. 12–15. The measurements showed that the velocity decreased by at most a factor of two. This does not agree with the theoretical results, according to which v should decrease to zero.[2,3] One of the reasons for this lower bound on the decrease of v could be the circumstance that near the OPT the intrinsic ME waves are nonlinear. It follows from the estimates presented in the present paper that the magnitude of the field at which ME vibrations in hematite become nonlinear is a few tens of oersteds. These fields are also the fields at which the lower bound on the decrease of the sound velocity is observed. The study of nonlinear ME waves is also of interest in light of the possibility of intentionally exciting nonlinear vibrations of the magnetic subsystem with AC elastic stresses.

2. BASIC EQUATIONS

We will investigate plane ME waves propagating along the symmetry axis of the crystal (the z-axis). In this case the free energy density of a tetragonal ferromagnet can be cast in the form

$$F=M_0^2\Big[\frac{\alpha}{2}(\theta'^2+\varphi'^2\sin^2\theta)+\frac{\beta_1}{2}\cos^2\theta+\frac{\beta_2}{2}\cos^4\theta+\frac{\beta}{4}\sin^4\theta\sin^2 2\varphi+(b_{33}-b_{31})u_z'+\frac{b_{44}}{2}\sin 2\theta(u_x'\cos\varphi+u_y'\sin\varphi)\Big]+\frac{c_{33}}{2}u_z'^2+\frac{c_{44}}{2}(u_x'^2+u_y'^2). \quad (1)$$

Here θ and φ are the polar and azimuthal angles of the magnetic moment **M**, M_0 is the saturation magnetization, α is the inhomogeneous exchange constant, β_1, β_2, and β are the uniaxial anisotropy and basal-plane anisotropy constants, u_i is the elastic displacement, and b and c are the ME and elastic constants. The equations which describe the ME waves under discussion here have the form

$$\omega_0^{-1}\sin\theta(\dot\theta+r\dot\varphi\sin\theta)=\alpha(\varphi'\sin^2\theta)'-{}^1/_2\beta\sin^4\theta\sin 4\varphi+{}^1/_2 b_{44}\sin 2\theta(u_x'\sin\varphi-u_y'\cos\varphi), \quad (2a)$$

$$\omega_0^{-1}(\dot\varphi\sin\theta-r\dot\theta)=\alpha(\theta''-{}^1/_2\varphi'^2\sin 2\theta)-{}^1/_2\sin 2\theta(\beta_1+2\beta_2\cos^2\theta-\beta\sin^2\theta\sin^2 2\varphi)-(b_{33}-b_{31})u_z'\sin 2\theta+b_{44}\cos 2\theta(u_x'\cos\varphi+u_y'\sin\varphi); \quad (2b)$$

$$\rho\ddot u_x=c_{44}u_x''+{}^1/_2 b_{44}M_0^2(\sin 2\theta\cos\varphi)',$$
$$\rho\ddot u_y=c_{44}u_y''+{}^1/_2 b_{44}M_0^2(\sin 2\theta\sin\varphi)',$$
$$\rho\ddot u_z=c_{33}u_z''+(b_{33}-b_{31})M_0^2(\cos^2\theta)',$$

where $\omega_0=gM_0$, g is the gyromagnetic ratio, r is a dimensionless damping rate, and ρ is the density.

We will investigate solutions of the form $\theta,\varphi,u=f(\zeta)$, where $\zeta=z-vt$, and v is the ME wave velocity. In this case, we obtain from the elasticity equations (2b)

$$u_x'={}^1/_2\varkappa_4\sin 2\theta\cos\varphi,\quad u_y'={}^1/_2\varkappa_4\sin 2\theta\sin\varphi,\quad u_z'=\varkappa_3\cos^2\theta, \quad (3)$$

where

$$\varkappa_3=[(b_{33}-b_{31})M_0^2/c_{33}](v^2/v_l^2-1)^{-1},$$
$$\varkappa_4=(b_{44}M_0^2/c_{44})(v^2/v_t^2-1)^{-1};$$

here and henceforth the dash will denote differentiation with respect to ζ. The equations for the magnetization (2a) can be transformed to the form

$$v\omega_0^{-1}\sin\theta(\theta'+r\varphi'\sin\theta)=-\alpha(\varphi'\sin^2\theta)'+\frac{\beta}{2}\sin^4\theta\sin 4\varphi, \quad (4)$$

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where $\tilde{\beta}_1 = \beta_1 + b_{44}\varkappa_4$, $\tilde{\beta}_2 = \beta_2 + (b_{33} - b_{31})\varkappa_3 - b_{44}\varkappa_4$. Thus, the effect of the elastic subsystem on the magnetic sybsystem is to renormalize the anisotropy constants β_1 and β_2, where this renormalization depends on the sound velocity v. The renormalization shows that the effect of the ME interaction is most important for v close to v_t and v_l. From the second equation (4) for large anisotropy and small damping we obtain

$$\cos\theta = v(\tilde{\beta}_1\omega_0)^{-1}\varphi'. \qquad (5)$$

Including the fact that in this case the excursion of **M** from the basal plane is small ($|\pi/2 - \theta| \ll 1$), and using (5), the first equation (4) can be reduced to an equation of sine-Gordon form with damping

$$\alpha\left(1 - \frac{v^2}{s^2}\right)\varphi'' + \frac{rv}{\omega_0}\varphi' - \frac{\beta}{2}\sin 4\varphi = 0, \qquad (6)$$

where $s^2 = \alpha\tilde{\beta}_1\omega_0^2$. Integrating this equation gives

$$\frac{\alpha}{2}\left(\frac{v^2}{s^2} - 1\right)\varphi'^2 + \frac{\beta}{4}\sin^2 2\varphi = C + \frac{rv}{\omega_0}\int_{\zeta_0}^{\zeta}\varphi'^2\, d\zeta. \qquad (7)$$

The meaning of the constant C can be clarified in the following way. Let us rewrite (6) in the absence of damping in the form $m^*x'' = -\partial U/\partial x$, where $m^* = \alpha(v^2/s^2 - 1)$, $x = \varphi$, $U = \frac{1}{4}\beta\sin^2 2x$. Then the quantity $C = m^*x'^2/2 + U$ is the energy of an effective particle of mass m^* moving the potential U.

The stationary points of Eq. (6) are the points $\varphi = n\pi/4$, $\varphi' = 0$ ($n = 0, \pm 1, \ldots$). If we neglect damping, the phase portrait in the region $0 < C < \beta/4$ ($\beta > 0$) has foci. They are found at the points $\varphi = n\pi/2$ for $v > s$ and $\varphi = \pi/4 + n\pi/2$ for $v < s$. In the regions $C > \beta/4$ and $C < 0$ the phase trajectories have saddle points: $n\pi/2$ for $v > s$ and $\pi/4 + n\pi/2$ for $v < s$. The trajectories for $C = \beta/4$ ($v > s$) and $C = 0$ ($v < s$) are the corresponding separatrices. We note that if φ is a solution to Eq. (6), then $\varphi + n\pi/2$ is also a solution. Constant solutions to Eq. (6) exist in the region $0 \leqslant C \leqslant \beta/4$, and have the form

$$\varphi = {}^1/_2\arcsin(4C/\beta)^{1/2} + n(\pi/2). \qquad (8)$$

3. TYPES OF NONLINEAR MAGNETOELASTIC WAVES

We will carry out an analysis of the nonconstant solutions of Eq. (6) in the absence of attenuation for various relations between v and s, and also between the integration constant C and β (Fig. 1).

A. $v > s$. In this case solutions to Eq. (6) exist with $C > 0$.

1. In the region $C > \beta/4$, we obtain from (6), (5) and (3) (for the sake of simplicity we present only the $n = 0$ solutions)

$$\begin{aligned}
\varphi &= \frac{\eta}{2}\arcsin\left[\mathrm{sn}\left(\frac{\zeta-\zeta_0}{k\Delta}, k\right)\right],\\
\theta &= \arccos\left[\frac{\eta}{2}\frac{v\delta}{sk\Delta}\mathrm{dn}\left(\frac{\zeta-\zeta_0}{k\Delta}, k\right)\right],\\
u'_{x,y} &= \frac{\eta v\delta\varkappa_4}{2\sqrt{2}\,sk\Delta}\mathrm{dn}\left(\frac{\zeta-\zeta_0}{k\Delta}, k\right)\left[1 \pm \mathrm{cn}\left(\frac{\zeta-\zeta_0}{k\Delta}, k\right)\right]^{1/2},\\
u_z' &= \left(\frac{v\delta}{2sk\Delta}\right)^2\varkappa_3\,\mathrm{dn}^2\left(\frac{\zeta-\zeta_0}{k\Delta}, k\right).
\end{aligned} \qquad (9)$$

Here $\Delta = \{(\alpha/2\beta)|1 - v^2/s^2|\}^{1/2}$ and $\delta = (\alpha/\tilde{\beta}_1)^{1/2}$ are the characteristic length scales, and $\eta = \pm 1$. The Jacobi elliptic functions $\mathrm{sn}(u,k)$, $\mathrm{cn}(u,k)$ and $\mathrm{dn}(u,k)$ have periods in u which are $\Lambda_s = \Lambda_c = 4K$ and $\Lambda_d = 2K$ respectively, where

$$K = \int_0^{2\pi}(1 - k^2\sin^2\alpha)^{-1/2}\, d\alpha$$

is the complete elliptic integral and k is the modulus of the elliptic functions ($0 < k < 1$). In the case under study $k = (\beta/4C)^{1/2}$, while the periods with respect to ζ are $\Lambda_{s,c} = 4Kk\Delta$, $\Lambda_d = 2Kk\Delta$. Based on the expressions for θ in (9), the condition that the deviation of θ from $\pi/2$ be small takes the form

$$2k^2\left|\left(1 - \frac{v^2}{s^2}\right)\frac{\tilde{\beta}_1}{\beta}\right| \gg 1. \qquad (10)$$

The function $\varphi(\zeta)$ in (9) describes a nonuniform rotation of **M** around the symmetry axis (i.e., a spiral wave) either clockwise or counterclockwise for $\eta = \pm 1$, respectively. The angle θ and the component of the distortion tensor u_i' ($i = x,y,z$) varies periodically with a period Λ determined by the period of the elliptic functions. The rotation velocity of the magnetization φ' also varies periodically. For φ' and $u_{x,y'}$ the period is $\Lambda = \Lambda_s$, while for θ and u_z' the period is $\Lambda = \Lambda_d$. The period and amplitude of these variables clearly depend on k. For $k \to 0$ ($C \to \infty$) we have $K \to \pi/2$ and $\Lambda \to 0$. The vibration amplitudes φ', θ and $u_{x,y'}$ decrease in this case, while the amplitude $u_{z'}$ increases. The values of the quantities φ', θ and u_i' averaged over a period grow; however, this growth is limited by condition (10). Thus, the rotation **M** becomes more uniform as k decreases. As $k \to 1$ ($C \to \beta/4$) we have $K \to \infty$ and $\Lambda \to \infty$. The scale of the nonuniformity of φ', θ and u_i', which is determined by the quantity $k\Delta$, increases to the value Δ. In this limit, the amplitude variation of these quantities increases while their average values decrease. The solution (9) is transformed into a sequence of domain boundaries for φ and a sequence of solitons with the same sign for θ and u_i'.

2. In the interval $0 < C < \beta/4$ we have

$$\begin{aligned}
\varphi &= \frac{\eta}{2}\arcsin\left[k\,\mathrm{sn}\left(\frac{\zeta-\zeta_0}{\Delta}, k\right)\right],\\
\theta &= \arccos\left[\frac{\eta v k\delta}{2s\Delta}\mathrm{cn}\left(\frac{\zeta-\zeta_0}{\Delta}, k\right)\right],\\
u'_{x,y} &= \frac{\eta v k\delta\varkappa_4}{2\sqrt{2}\,s\Delta}\mathrm{cn}\left(\frac{\zeta-\zeta_0}{\Delta}, k\right)\left[1 \pm \mathrm{dn}\left(\frac{\zeta-\zeta_0}{\Delta}, k\right)\right]^{1/2},\\
u_z' &= \left(\frac{vk\delta}{2s\Delta}\right)^2\varkappa_3\,\mathrm{cn}^2\left(\frac{\zeta-\zeta_0}{\Delta}, k\right),
\end{aligned} \qquad (11)$$

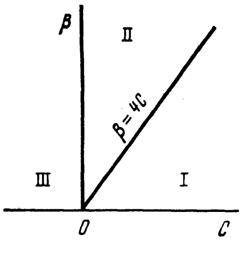

FIG. 1. Regions where various types of nonlinear waves exist: I ($C > \beta/4$)—spiral nonlinear waves for $v > s$; II ($0 < C < \beta/4$)—periodic nonlinear waves for $v \gtrless s$; III ($C < 0$)—spiral nonlinera waves for $v < s$; the solitary nonlinear waves are at the boundary lines $C = \beta/4$ ($v > s$) and $C = 0$ ($v < s$).

where $k=(4C/\beta)^{1/2}$. The period of the elliptic functions $\Lambda_5=4K\Delta$ and $\Lambda_d=2K\Delta$ depend on k only through $K(k)$. All variables, including φ (in contrast to the previous case), are periodic functions. Their amplitudes grow as k increases; however, the change in $|\varphi|$ is less than $\pi/4$, while θ and u_i' are limited by the condition $2k^{-2}|(1-v^2/s^2)\tilde{\beta}_1/\beta|\gg 1$

$$\omega_{1,2}^2=\frac{1}{2(2K\Delta_0/\pi)^2}([s_0^2+(s_0^2+v_t^2)(\tilde{q}\Delta_0)^2]$$
$$\pm\{[s_0^2+(s_0^2-v_t^2)(\tilde{q}\Delta_0)^2]^2+4ps_0^2v_t^2(\tilde{q}\Delta_0)^2[1+(\tilde{q}\Delta_0)^2]\}^{1/2}), \quad (12)$$

where

$$\tilde{q}=2Kq/\pi,\ \Delta_0^2=\alpha/2\beta,\ s_0^2=\alpha\beta_1\omega_0^2,\ p=H_{\rm ME}/H_A,$$
$$H_{\rm ME}=b_{44}^2M_0^3/C_{44},\ H_A=\beta_1M_0,$$

ω and q are the frequency and wave number of the periodic nonlinear wave.

As $k\to 0$ ($C\to 0$) these waves reduce to linear (harmonic) ME waves:

$$\varphi\to\frac{\eta}{2}k\sin\left[\frac{\zeta-\zeta_0}{\Delta}\right],$$
$$\theta\to\frac{\pi}{2}-\left[kv\cos\left(\frac{\zeta-\zeta_0}{\Delta}\right)\Big/\ (2\beta_1\omega_0\Delta)\right]^2.$$

The magnetization vibrates sinusoidally relative to the equilibrium position $\varphi_0=n\pi/2$, remaining practically in the basal plane (the deviation of θ from $\pi/2$ is quadratic in k). The dispersion relation for these waves is determined by Eq. (12), in which $\tilde{q}=q$, since as $k\to 0$ we have $K\to\pi/2$. From this it is clear that in the limit of weak external excitation the periodic nonlinear waves transform into the usual linear ME waves whose renormalized velocities for small wave numbers q have the well-known forms of quasimagnon and quasiphonon branches:

$$\tilde{s}_0=s_0(1+pv_t^2/s_0^2)^{1/2},\ \tilde{v}_t=v_t(1-p)^{1/2}.$$

Let us recall that in the case under discussion the system is far from the static OPT point, at which there is a static excursion of **M** from the basal plane ($H_A\gg H_{\rm ME}$).

As $k\to 1$ ($C\to\beta/4$) the period $\Lambda\to\infty$ and $\omega_{1,2}\to 0$. The functions $\varphi(\zeta)$ and $\theta(\zeta)$, $u_i'(\zeta)$ transform into a periodic domain structure and a sequence of solitons of alternating sign.

3. In the case $C=\beta/4$ the solution to the problem consists of a moving solitary wave:

$$\varphi=\operatorname{arctg}\left\{\exp\left[\frac{\eta(\zeta-\zeta_0)}{\Delta}\right]\right\}-\frac{\pi}{4},$$
$$\theta=\arccos\left\{\frac{\eta v\delta}{s\Delta}\frac{\exp[\eta(\zeta-\zeta_0)/\Delta]}{1+\exp[2\eta(\zeta-\zeta_0)/\Delta]}\right\},$$
$$u_{x,y}'=\frac{\eta v\delta\varkappa_4}{\sqrt{2}s\Delta}\frac{\exp[\eta(\zeta-\zeta_0)/\Delta]\{1\pm\exp[\eta(\zeta-\zeta_0)/\Delta]\}}{\{1+\exp[2\eta(\zeta-\zeta_0)/\Delta]\}^{3/2}}, \quad (13)$$
$$u_z'=\left(\frac{v\delta}{s\Delta}\right)^2\varkappa_3\frac{\exp[2\eta(\zeta-\zeta_0)/\Delta]}{\{1+\exp[2\eta(\zeta-\zeta_0)/\Delta]\}^2}.$$

This solution is intermediate with respect to φ between the spiral wave (9) for $C>\beta/4$ and the periodic nonlinear wave (11) for $0<C<\beta/4$.

The function $\varphi(\zeta)$ in (13) describes a domain boundary involving left-handed and right-handed rotations of **M** ($\eta=\pm1$) between states with $\varphi=\pm\pi/4$, i.e., between states of **M** lying along the difficult axes. The functions $\theta(\zeta)$ and $u_i'(\zeta)$ describe a soliton and antisoliton respectively for $\eta=\pm1$. As the velocity v decreases, the width of the domain wall and soliton Δ decrease, while the amplitude of the solitons grows to a value limited by inequality (10) for $k=1$.

The results obtained so far show that the changes in the type of nonlinear wave with decreasing C from spiral to solitary to periodic wave for $C=\beta/4$, and also from periodic nonlinear wave to the constant phase with $C=0$, occur in a way similar to phase transitions. As we pass from one type of nonlinear wave to another, the symmetry which characterizes the dynamics of the magnetic state changes. Thus, whereas the constant phase is characterized by continuous translational symmetry, in the region of nonlinear waves the translation symmetries have a definite period in ζ. As we approach the region where the solitary wave can exist (i.e., the point $C=\beta/4$) this period satisfies $\Lambda\to\infty$ ($q\to 0$). For the spiral nonlinear wave, however, the translations are characterized by a new finite period. As an order parameter for the various phase transitions we can take, e.g., the value of the squared angle $\theta-\pi/2$ averaged over a period. This parameter is different from zero in the regions where the spiral and periodic nonlinear waves exist, and reduces to zero for the solitary wave and constant phases.

B. $v<s$. In this case a nonconstant solution to Eq. (6) exists for $C<\beta/4$.

1. For $C<0$ the solution $\varphi(\zeta)$ of Eq. (6) differs from the one presented in (9) by a shift of $\pi/4$. The other variables are determined from (9) when we replace the function cn in $u_{x,y'}$ by sn. The modulus of the elliptic functions $k=(1-4C/\beta)^{-1/2}$.

2. In the interval $0<C<\beta/4$ the solution $\varphi(\zeta)$ to Eq. (6) differs from (11), again by a shift of $\pi/4$, while the dependences of θ and u_i' on ζ are determined using (11). In this case the modulus satisfies $k=(1-4C/\beta)^{1/2}$. The relation between ω and q is determined by Eq. (12), once we replace β with $-\beta$. In this case, the quasiacoustic branch is stable ($\omega^2>0$) for all $\tilde{q}$, while the quasimagnon branch is stable for $\tilde{q}>\Delta_0^{-1}$. As $\tilde{q}$ decreases to the value $\tilde{q}=\Delta_0^{-1}$, the nonlinear waves corresponding to vibrations of **M** relative to the difficult axis are transformed into a solitary wave of the domain-boundary type separating constant states with $\varphi=0$ and $\varphi=\pi/2$.

3. For $C=0$ the variables are determined by Eq. (13), with φ replaced by $\varphi-\pi/4$ and u_x' and u_y' exchanged in this expression.

In case B we obtain the same types of solutions as in case A. The only difference is that the easy and difficult axes have changed places: in the spiral nonlinear wave the rotation of **M** slows down along the easy, not the difficult axis; the nonlinear vibration takes place relative to the difficult axis, and the solitary wave constitutes a domain boundary between the state $\varphi=0$ and $\varphi=\pi/2$ (and not between the states $\varphi=\pm\pi/4$). An analogous result is obtained when the sign of β is reversed without changing the relation between v and s. Consequently, we can assume that when the sign of the factor $(1-v^2/s^2)$ changes a dynamic OPT takes place, analogous to the static OPT which occurs when the sign of β changes.[1]

All the results presented above were obtained for defi-

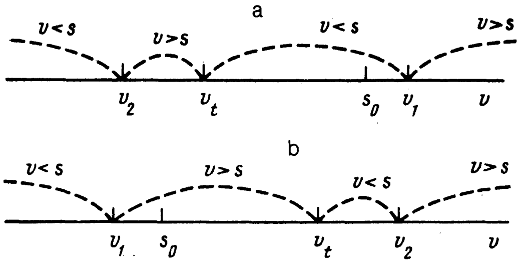

FIG. 2. Regions of nonlinear wave velocities corresponding to the conditions $v \gtrless s$ for (a) $v_t < s_0$, (b) $v_t > s_0$.

nite relations between v and s ($v \gtrless s$). However, the value of s itself depends on v. Solving the relation between v and s, we find that the condition $v > s$ is satisfied in the regions $v > \max\{v_1, v_2\}$ and $\min\{v_1, v_2\} < v < v_t$, while the condition $v < s$ is satisfied for $v_t < v < \max\{v_1, v_2\}$ and $v < \min\{v_1, v_2\}$ (Fig. 2). Here,

$$v_{1,2}^2 = {}^1\!/_2\{s_0^2 + v_t^2 \pm [(s_0^2 - v_t^2)^2 + 4ps_0^2 v_t^2]^{1/2}\},$$
$$\max(\min)\{v_1, v_2\} = \begin{cases} v_1(v_2) & \text{for } v_t < s_0 \\ v_2(v_1) & \text{for } v_t > s_0 \end{cases}. \quad (14)$$

In the absence of the ME interaction $s = s_0$, and consequently for the magnetic nonlinear waves, the conditions listed above have the form $v \gtrless s_0$. Inclusion of the interaction leads to the appearance of a small interval $|v_t - v_2|$ around v_t in which a relation between v and s is established which is opposite to the one which holds without the ME interaction. This also causes a shift in the boundary $v = s$ from s_0 to v_1.

4. EFFECT OF DISSIPATION

Dissipation leads to a decrease in the wave energy. Thus, if the excitation energy of the wave is initially larger than the anisotropy energy, then as it propagates a wave which starts out as a spiral wave changes first to a periodic nonlinear wave and then to a linear wave.

For weak damping ($r \ll 1$) we can find the following relaxation correction to φ:

$$\varphi_r = \frac{r}{\delta^2}\cos\theta(\zeta)\int_{\zeta_c}^{\zeta}\frac{d\zeta'}{\cos^2\theta(\zeta')}\int_{\zeta_0}^{\zeta'}\cos^2\theta(\zeta'')\,d\zeta''. \quad (15)$$

In the limit $k \to 1$ we have from (9), (11), (15)

$$\varphi_r = \frac{\eta r v k \Delta}{4\alpha\omega_0}\left(\mathrm{ch}\,\frac{\zeta-\zeta_0}{\nu\Delta} - \mathrm{ch}^{-1}\frac{\zeta-\zeta_0}{\nu\Delta}\right), \quad (16)$$

where

$$\nu = \begin{cases} k & \text{for } C > \beta/4 \text{ и } C < 0 \\ 1 & \text{for } 0 < C < \beta/4 \end{cases}.$$

For $k = 0$, in the case $v > s$ the full expression for the azimuthal angle has the form

$$\varphi + \varphi_r = \frac{\eta}{2}\left[1 - \frac{rv(\zeta-\zeta_0)}{2\alpha\omega_0}\right]\begin{cases} \dfrac{\zeta-\zeta_0}{k\Delta}, & C > \beta/4 \\ k\sin\dfrac{\zeta-\zeta_0}{\Lambda}, & 0 < C < \beta/4 \end{cases}. \quad (17)$$

From (17) the scale of the damping $\zeta - \zeta_0 \sim 2\alpha\omega_0/rv$. For $\zeta - \zeta_0 \ll \nu\Delta$, i.e., in the initial period of variation of **M**, for any value of k in the interval $v > s$.

$$\varphi + \varphi_r = \frac{\eta}{2}(\delta_{k\nu} + k\delta_{1\nu})\left(\frac{\zeta-\zeta_0}{\nu\Delta}\right)\left[1 - \frac{rv\nu\Delta}{2\alpha\omega_0}\left(\frac{\zeta-\zeta_0}{\nu\Delta} - \frac{1+k^2}{6}\left(\frac{\zeta-\zeta_0}{\nu\Delta}\right)^2\right)\right] \quad (18)$$

where $\delta_{k\nu}$ is the Kronecker symbol.

Relaxation can decrease the solitary wave velocity without changing its form. In this case, based on energy analysis[16] we can obtain the time dependence of the velocity decay:

$$\frac{dv}{dt} = -r\beta_1\omega_0 v\left|1 - \frac{p}{1 - v^2/v_t^2} - \frac{v^2}{s_0^2}\right|. \quad (19)$$

For small velocities the relaxation time for the velocity of a solitary wave satisfies $\tau_v = [r\beta_1\omega_0(1-p)]^{-1}$. Thus, the ME coupling increases τ_v.

The results presented above were obtained using the condition that the velocity v not be close to s. However, for $v \approx s$ and for large damping the first term in (6) is small. Then the solution of this equation has the form

$$\varphi = \frac{1}{2}\,\mathrm{arctg}\left[\exp\left(-\frac{\zeta-\zeta_0}{\Delta_r}\right)\right] \quad (20)$$

and describes the motion of a relaxing solitary wave of the 45° domain wall type between the states $\varphi = \pi/4$ and $\varphi = 0$ of width $\Delta_r = rv/(2\beta\omega_0)$.

5. CONCLUDING REMARKS

The results we have obtained are also valid for easy-plane antiferromagnets such as, e.g., tetragonal NiF_2, rhombohedral α-Fe_2O_3 and $FeBO_3$, in the presence of interaction between elastic waves and low-frequency spin waves.

We have already noted that as we approach the OPT point, e.g., when the constant β decreases with temperature,[17-19] the ME vibrations become nonlinear. The system can also be made to approach an OPT through elastic stress or magnetic fields. In this case, the appearance of a nonlinear wave apparently could be the cause of the experimentally observed lower bound on the decrease of the sound velocity in magnets as the OPT is approached, in particular when this approach is mediated by a magnetic field in easy-plane antiferromagnets.[12-15] In this case the value of the field H_c below which the ME vibrations become significantly nonlinear can be estimated from a comparison of the Zeeman and ME energy: $MH_\Pi \approx bL^2u^{(0)}$. Here $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$, $\mathbf{L} = \mathbf{M}_1 - \mathbf{M}_2$, where $\mathbf{M}_1$ and $\mathbf{M}_2$ are the magnetic moments of the sublattices, and $u^{(0)}$ is the amplitude of the sound wave. Setting $bL^2 \approx 10^7$ erg/cm^3, $u^{(0)} \approx 10^{-6}$, $M \approx 1$ Oe, we obtain $H_c \approx 10$ Oe, which is in full agreement with the results of these experiments.

The expressions obtained here for the deformations show how nonlinear ME waves can be efficiently excited by external AC elastic stresses. For example, setting the values of v and k for a specific material, we can determine the amplitude of the elastic deformations u_i' using (11), and also the spatial and temporal periods Λ and T. If we select an external excitation with such a u_i' and T (or Λ), then nonlinear waves described by Eq. (11) should be excited with the given v and k. The nonlinear ME wave can also be excited by a variable magnetic field, and also by varying the magnitude and sign of the anisotropy constant β, e.g., by varying the temperature.

We can observe ME nonlinear waves, e.g., by using magnetooptic effects such as rotation of the plane of polarization or Bragg diffraction.

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