Generation of Josephson vortices by a current pulse

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The problem of Josephson vortex generation by an applied current pulse injected through the edge of a long junction is solved. The spatial distribution of the phase difference between the junction borders after pulse action is found for a low pulse intensity, and the corresponding direct scattering problem associated with the effective equation of motion–the sine-Gordon equation—is solved. The inverse scattering technique makes it possible to determine the parameters of the generated solitons (free vortices and vortices bound to the edge of the junction) and the continuous spectrum excitations (the plasma waves). Perturbation theory is used to account for the influence of auxiliary factors such as dissipation and the d.c. component of the injected current on vortex dynamics. The condition that facilitates actual penetration of the vortices into the long junction is found.

1. INTRODUCTION

A significant number of experimental and theoretical studies (see, for example, Refs. 1–3) have been devoted to investigations of the dynamics of Josephson vortices (magnetic flux quanta or fluxons) in long Josephson junctions (LJJ). Several methods can be used to experimentally excite vortices in a long junction. One such method involves injecting an external current through the junction edge; as a rule the injection current contains d.c. and pulsed components.[4–6] The resulting excitation of Josephson vortices exhibits a threshold. The pulse area must exceed a certain critical value in order to generate vortices. This dynamical process was investigated numerically by Sakai and Samuelsen[7] within the framework of the familiar simiinfinite Josephson junction model based on the sine-Gordon equation (with a dissipative term) for the nondimensionalized magnetic flux $\varphi(x, t)$.

$$\varphi_{tt}-\varphi_{xx}+\sin\varphi+\gamma\varphi_t=0, \quad x>0, \tag{1}$$

which is supplemented by the boundary condition

$$\varphi_x(0, t)=-{}^1/_2 h(t). \tag{2}$$

The coordinate x, directed parallel to the junction, and the time t are measured in equation (1) in units of the Josephson depth of penetration λ_J and the inverse Josephson frequency ω_J. The parameter λ phenomenologically accounts for dissipative processes caused by tunnelling of normal quasiparticles through the junction, $-\frac{1}{2}h(t)$ is the extraneous current injected through the junction edge ($x=0$). The same model describes the action of an external oscillatory magnetic field on the junction.

Sakai and Samuelsen[7] carried out a numerical analysis of the case

$$h(t)=h_0+h_1(t), \tag{3}$$

where h_0 is the d.c. component of the current, while $h_1(t)$ is the pulsed component with a triangular waveform, which is similar to the experimental conditions of Refs. 4–6. This study also developed a semianalytic approach for determining the threshold conditions for vortex generation in LJJ. However this is not a valid approach, a point already made in Ref. 7 itself. We will provide a consistent theoretical solution to the problem of pulsed excitation of Josephson vortices in the present study. Our approach assumes a short duration T of the pulsed injection current component $h_1(t)$ as well as a small d.c. current component h_0 (in this case the amplitude of the pulsed injection current may reach substantial levels of the order of T^{-1}).

The present treatment and the attendant results have, in our view, a much broader applicability than to nonlinear wave generation in long junctions. Indeed an analysis of any physical system that allows nonlinear soliton excitations to exist must deal with the problem of their generation. The situation where an intense and generally localized pulsed action drives a nonlinear system from equilibrium corresponds to the experimental conditions. The action of such an external force will generate a wave field distribution from which solitons and other system excitation result. If the system is described (after the end of external pulse action) by an exactly integrable equation, such as the sine-Gordon equation, the inverse scattering technique (IST) makes it possible, in principle, to calculate exactly the spectrum of the resulting excitations based on the wave field configuration established by the end of pulse action. In this connection it should be noted that a detailed analysis of some of the simplest initial sine-Gordon wave field configurations was carried out from the viewpoint of IST in Ref. 8, 9.

This clearly indicates that the parameters of the excited solitons are, in the final analysis, determined by the characteristics of the pulsed force, i.e., its intensity and duration. To the best of our knowledge no consistent analytic investigation of such pjroblems has yet been carried out.

In view of the fact that this formulation of the soliton generation problem most adequately corresponds to an experimental situation independent of the specific form of the nonlinear, nearly-integrable system, we will briefly outline the general solution scheme. The problem of the linear response of a system to an external pulsed action is examined in the first stage, asssuming that the system was in equilibrium ($\varphi=\varphi_t=0$) prior to the action of the force ($t=0$), and the spatial wave field distribution at time $t=T$ is also calculated. It is obvious that such a calculation will be valid only if the force acts for a time T short enough so that the excited pulse cannot "creep away" due to dispersion. It is important that the analysis of the response to the external

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action can be carried out in the linear approximation. Then using the derived field configuration as the initial condition for the Cauchy problem for $t \geqslant T$ we solve the direct scattering problem within the framework of IST. This makes it possible to obtain a set of so-called scattering data of the discrete and continuous spectra; these describe the further evolution of the pulse. Specifically, if we know the discrete spectrum it is possible to determine completely the parameters of the excited solitons. Since the IST is strictly valid for exactly integrable systems only, the auxiliary factors (responsible for the loss of exact integrability) can be investigated within the framework of perturbation theory[10] by taking the solution of the Cauchy problem as a zeroth approximation. This represents the third stage in solving the soliton generation problem.

The specific results obtained in the present study relating to Josephson vortex (fluxon) generation by a current pulse injected through the edge of a semiinfinite junction can be applied directly to a description of soliton generation in other nonlinear systems described by a perturbed sine-Gordon equation (for example, easy-plane magnetic and nematic liquid crystals), when the spatial range of external pulsed action is significantly less than $V_g T$, where V_g is the highest group velocity of the linear excitations.

2. Initial pulse waveform and solution of the inverse scattering problem

Bearing in mind the use of the inverse scattering technique we will continue Eq. (1) onto the semiaxis $x<0$ by $\varphi(-x,t)=\varphi(x,t)$. Then the following equation will correspond to Eq. (1) with boundary condition (2)

$$\varphi_{tt}-\varphi_{xx}+\sin\varphi+\gamma\varphi_t=h(t)\delta(x). \tag{4}$$

For definiteness we will represent the pulsed injected current component $h(t)$ as a square-wave pulse:

$$h_1(t)=\begin{cases}0, & t<0,\ t>T,\\ a, & 0<t<T.\end{cases} \tag{5}$$

Taking the parameter h_0 in (3) to be small we will first consider the case $h_0=0$.

The term $\sin\varphi$ in Eq. (4) remains of order unity over the time T during which the local pulsed force (5) acts, while under the conditions

$$T\ll 1,\quad |a|T\sim 1 \tag{6}$$

the terms φ_{tt}, φ_{xx} in equation (4) are large (we note that the characteristic spatial dimensions of the excited field region φ are of order $V_g T\sim T\ll 1$). Consequently during the action of the pulsed force Eq. (4) subject to (6) can be replaced by

$$\varphi_{tt}-\varphi_{xx}+\gamma\varphi_t=h(t)\delta(x). \tag{7}$$

The linear wave equation (7) defined on the entire x axis is easily solved and at the time T when the pulse ends the configuration of wave fields φ and φ_t takes the form (see Fig. 1)

$$\begin{aligned}\varphi(x,T)={}&{}^1/_2a\{-x\,\mathrm{sgn}\,x+{}^1/_2(x+T)\,[1+{}^1/_4\gamma(x-T)]\,\mathrm{sgn}(x+T)\\&+{}^1/_2(x-T)\,[1-{}^1/_4\gamma(x+T)]\,\mathrm{sgn}(x-T)\},\end{aligned} \tag{8}$$

$$\varphi_t(x,T)={}^1/_4a(1-{}^1/_2\gamma T)\,[\mathrm{sgn}(x+T)-\mathrm{sgn}(x-T)]. \tag{9}$$

In order to describe the later evolution of the pulse we will examine the functions (8), (9) as initial conditions for the complete equation (4) and will take the parameter γ to be small. Then Eq. (4) is close to an exactly integrable nonlinear sine-Gordon (SG) equation for $t>T$ and the IST can be used to solve the Cauchy poroblem.[11] In the inverse scattering technique the sine-Gordon equation is related to the linear scattering problem

$$\hat{L}\{\varphi,\varphi_t;\lambda\}\Psi(x,t;\lambda)=0$$

for the auxiliary two-component function

$$\Psi(x,t;\lambda)=\begin{pmatrix}\Psi_1(x,t;\lambda)\\ \Psi_2(x,t;\lambda)\end{pmatrix},$$

which λ is the spectral parameter, which takes real positive values. The operator $\hat{L}$ takes the form

$$\begin{aligned}\hat{L}={}&\hat{I}\frac{\partial}{\partial x}-\frac{i}{2}\Big[\Big(\lambda-\frac{1}{4\lambda}\cos\varphi\Big)\hat{\sigma}_3\\&-\frac{\hat{\sigma}_2}{4\lambda}\sin\varphi+\frac{\hat{\sigma}_1}{2}(\varphi_x-\varphi_t)\Big],\end{aligned} \tag{10}$$

where $\hat{\sigma}_\alpha$ ($\alpha=1,2,3$) are the Pauli matrices, while $\hat{I}$ is the unit matrix. When conditions (6) hold the direct scattering problem for the initial conditions (8), (9) can be solved approximately. For this process we write $\hat{L}\Psi=0$ for (10) as

$$(\Psi_1)_x=\alpha(x,\lambda)\Psi_1+\beta(x,\lambda)\Psi_2, \tag{11}$$

$$(\Psi_2)_x=-\alpha(x,\lambda)\Psi_2-\beta^*(x,\lambda)\Psi_1, \tag{12}$$

where

$$\alpha(x,\lambda)=\frac{i}{2}\Big(\lambda-\frac{1}{4\lambda}\cos\varphi\Big), \tag{13}$$

$$\beta(x,\lambda)=\frac{i}{2}(\varphi_x-\varphi_t)-\frac{1}{8\lambda}\sin\varphi, \tag{14}$$

where the asterisk denotes complex conjugation. In order to find the scattering data we will specify the asymptotic values of the function Ψ for $x=\pm\infty$, i.e., we will construct the so-called Jost function:

$$\Psi(x,\lambda)\to\begin{pmatrix}0\\ e^{-ik(\lambda)x/2}\end{pmatrix},\quad x\to-\infty, \tag{15}$$

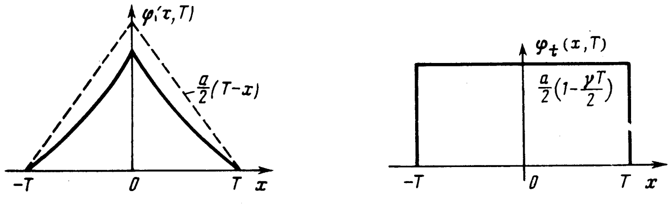

FIG. 1. Distribution of wave fields φ and φ_t at the time $t=T$ when the pulsed force stops.

$$\Psi(x,\lambda)\to\begin{pmatrix} b(\lambda)e^{ik(\lambda)x/2} \\ a(\lambda)e^{-ik(\lambda)x/2}\end{pmatrix}, \quad x\to+\infty, \tag{16}$$

where $k(\lambda)=\lambda-1/4\lambda$ represents the wave number in the sine-Gordon equation. The coefficients $b(\lambda)$ and $a(\lambda)$ are called the backward and forward scattering amplitudes, respectively. After this normalization has been chosen the solution of Eqs. (11), (12) for $x<-T$ obviously has the form of (15).

As the subsequent analysis will demonstrate it is sufficient to know the behavior of the amplitude $a(\lambda)$ for $k\sim 1$, i.e., near the point $\lambda=\pm 1/2$, in order to investigate the vortex generation process, i.e., in order to determine the threshold conditions behind discrete spectrum development. For $k\ll T^{-1}$ the direct scattering problem can be solved approximately in each of the following intervals specified by initial conditions (8), (9).

Interval $-T<x<0$. Introducing the notation $y=B(x+T)$, where $B=a(1-\frac{1}{2}\gamma T)$, we will represent the initial conditions (8), (9) as

$$\varphi=y+\frac{\gamma}{4B}y(y-2BT),\qquad \varphi_x-\varphi_t=\frac{\gamma y}{2}, \tag{17}$$

where $y\sim 1$. Taking into account that $B\sim T^{-1}\gg 1$ virtue of (6) we will find the solution of Eqs. (11), (12) with the initial conditions (17) as the first terms of a series in powers of B^{-1}:

$$\Psi_1=\frac{i\gamma}{16B}Ay^2+\frac{A}{8\lambda B}(\cos y-1),$$

$$\Psi_2=A-\frac{iA}{2B}\left(\lambda y-\frac{1}{4\lambda}\sin y\right). \tag{18}$$

The constant A is determined from the condition for matching the functions (18) with the functions (15) at $x=-T$ (here $y=0$):

$$A=\exp\left(\frac{i}{2}k(\lambda)T\right). \tag{19}$$

The interval $0<x<T$. We introduce the notation $z=B(x-T)$ and represent the initial conditions (8), (9) as

$$\varphi=-z+\frac{\gamma z}{4B}(z+2BT),\qquad \varphi_t-\varphi_x=2B-\frac{\gamma}{2}(z+2BT). \tag{20}$$

We note that by virtue of condition (6b) $z\sim 1$ in (10), since $B\gg 1$, $BT\sim 1$, as before. An approximate solution of Eqs. (11)–(14) in this interval takes the form

$$\Psi_1=(C_1+A_1(z))e^{iz/2}+(C_2+A_2(z))e^{-iz/2}, \tag{21}$$

$$\Psi_2=-(C_1+A_1(z))e^{iz/2}+(C_2+A_2(z))e^{-iz/2}, \tag{22}$$

where

$$A_1(z)=-\frac{C_2}{2B}\left(\lambda e^{-iz}+\frac{iz}{4\lambda}\right)-\frac{i\gamma C_1}{8B}\left(\frac{z^2}{2}+2BTz\right)+D_1, \tag{23}$$

$$A_2(z)=\frac{C_1}{2B}\left(\lambda e^{iz}-\frac{iz}{4\lambda}\right)+\frac{i\gamma C_2}{8B}\left(\frac{z^2}{2}+2BTz\right)+D_2, \tag{24}$$

while the constants C_1, C_2, D_1 and D_2 must be determined from the matching conditions of the Jost functions in each order in the small parameter B^{-1}. In lowest order we have

$$C_1=-\frac{A}{2}e^{iBT/2},\qquad C_2=\frac{A}{2}e^{-iBT/2}. \tag{25}$$

In order to simplify the process of obtaining final results we will express the Jost coefficient $a(\lambda)$ directly through the constant D_1 and D_2 determined in (23) and (24). We find from the matching condition of the functions (21), (22) and (16) for ($x=T$) (i.e, for $z=0$)

$$a(\lambda)e^{-ik(\lambda)T/2}=\Psi_2|_{z=0}=A\cos\frac{BT}{2}-\frac{i\lambda A}{2B}\sin\frac{BT}{2}+(D_2-D_1). \tag{26}$$

We have used (25) in writing (26). By matching the functions (21), (22) with the functions (18) at $x=0$ it is possible to find D_2-D_1 to first order in B^{-1}, which finally yields

$$a(k)=e^{ikT}\left\{\left(1-\frac{ikT}{2}\right)\cos\frac{BT}{2}+\frac{\gamma BT^2}{4}\sin\frac{BT}{2}-\frac{ik}{B}\sin\frac{BT}{2}\right\}, \tag{27}$$

$$k=k(\lambda)=\lambda-1/4\lambda.$$

It is possible to find the amplitude $b(\lambda)$ analogously:

$$b(\lambda)=e^{-ikT/2}\left\{-iA\sin\frac{BT}{2}-\frac{\lambda A}{2B}\cos\frac{BT}{2}+D_2+D_1\right\}$$

$$=-ie^{-ikT}\left[\left(1-\frac{ikT}{2}\right)\sin\frac{BT}{2}+\frac{i\gamma BT^2}{4}\cos\frac{BT}{2}\right]. \tag{28}$$

We again point out that the results (27), (28) are valid under conditions (6) and also for $k\ll T^{-1}$.

3. ANALYSIS OF SCATTERING DATA: FREE VORTICES AND VORTICES BOUND TO THE EDGE OF THE JUNCTION

We can easily determine that in the lowest approximation in T (or B^{-1}) and for $k\ll B^{-1}$, T the scattering data (27), (28) formally correspond to the initial condition (9) together with $\varphi(x,T)=0$, which were calculated in Ref. 8. However, as will become clear from the analysis below, terms of order T, B^{-1} which differentiate our result from the result of Ref. 8, have a substantial influence on the free vortex excitation threshold.

According to the IST results the zeros of the function $a(\lambda)$ correspond to soliton excitations; these lie in the upper half-plane of the complex spectral parameter λ. A stationary breather—the fluxon and antifluxon bound state oscillating at the frequency $\cos\mu$ and having an amplitude $0<\mu<\pi/2$—corresponds to the complex conjugate pair of roots

$$\lambda_{1,2}=\pm{}^1/_2\exp(\pm i\mu).$$

The only zero of the function $a(\lambda)$ lying on the imaginary axis at the point

$$\lambda_1={}^1/_2 i[(1+v)/(1-v)]^{1/2},$$

corresponds to a free fluxon travelling with speed v ($v^2<1$):

$$\varphi_{fl}(x,t)=4\sigma\,\mathrm{arctg}\exp\left(-\frac{x-\xi}{(1-v^2)^{1/2}}\right), \tag{29}$$

where $\sigma=\pm 1$ and $\xi=vt+\xi(0)$ are the polarity and coordinate of the center of the vortex, respectively. In this problem by virtue of the condition

$$\varphi(-x, t)=\varphi(x, t)$$

the free fluxons will appear only in pairs with polarities ± 1 and velocities $\pm v$. It is in fact obvious that only a fluxon travelling at a positive velocity (having the polarity $\sigma = \mathrm{sgn}\, a$) is physically meaningful (its imaginary partner travelling with negative velocity is its "mirror image" with respect to the junction edge). Similarly, for $x > 0$ the breather solution in fact describes a single fluxon oscillating in the effective potential well near the junction edge.

Substituting expression (27) into $a(k) = 0$ yields

$$\left(1-\frac{ikT}{2}\right)\operatorname{ctg}\frac{BT}{2}+\frac{\gamma BT^2}{4}=\frac{ik}{B}. \tag{30}$$

The solution of Eq. (30) with $\operatorname{Im}\lambda > 0$ yields the characteristics of the excited solitons. A simple analysis shows that the breather (a fluxon oscillating near the junction edge) is generated by the injected current pulse if the pulse area exceeds the threshold

$$(|a|T)_{thr}=2\pi(1+\gamma T). \tag{31a}$$

Similarly, from (30) we obtain the threshold condition for generation of bound vortices:

$$|a|T\geqslant a_N T=2\pi(2N-1)(1+\gamma T). \tag{31b}$$

It is clear from (31a) and (31b) that the presence of dissipation in the system requires an increase in the injected current power in order to generate the same number of fluxons. The evolution of an initial pulse in a dissipative sine-Gordon system was studied numerically in Ref. 12, which also noted this same trend.

The search for the threshold conditions for the creation of free fluxons (29) and for determination of their initial velocities is of fundamental interest. Analysis of Eq. (30) indicates that the corresponding threshold condition takes the form $a(k = i) = 0$, i.e., free fluxons are generated when

$$|a|T\geqslant 2\pi(1+\gamma T)+4T/\pi. \tag{32a}$$

The boundary conditions for generation of N free fluxons can be found analogously:

$$|a|T\geqslant \tilde{a}_N T=2\pi(2N-1)(1+\gamma T)+4T/\pi(2N-1). \tag{32b}$$

The polarities of all generated fluxons match the sign of the effective amplitude a.

In addition to vortices the applied current pulse also generates non-soliton excitations (plamsa waves) described by the continuous spectrum in terms of the inverse scattering technique. The primary characteristic of the continuous spectrum is the scattering amplitude (28). Specifically, at the generation threshold for a single fluxon, i.e., when the equality in relation (32a) holds, we have ($k \ll T^{-1}$)

$$b(\lambda)\equiv b(k(\lambda))=-i(1-ikT/2)\cos(T/\pi+\pi\gamma T/4). \tag{33}$$

The fundamental physical characteristic of these waves is the spectral density $\mathscr{E}(k) = dE_{\mathrm{em}}/dK$ of their energy E_{em}. According to Ref. 11

$$\mathscr{E}(k)=\pi^{-1}\ln(1-|b(k)|^2)^{-1}. \tag{34}$$

Using relation (34) it is possible to estimate the energy contained in the nonsoliton part of the pulse-generated wave field as

$$E_{em}=\frac{4}{\pi}\int_0^\infty \mathscr{E}(k)\,dk\approx\frac{1}{T}\int_0^1 \mathscr{E}\left(\frac{x}{T}\right)dx\sim\frac{\mathrm{const}}{T}. \tag{35}$$

Comparing (35) to the energy E_{fl} of the created fluxon (in our notation $E_{\mathrm{fl}} = 8$) we find that only a small fraction ($\sim T$) of the pulse energy is expended in the creation of the fluxon, and all remaining energy is expended in generating the relatively "useless" nonsoliton wave field. We note in this connection that the semianalytic technique used in Ref. 7 did not take account of the nonsoliton part of the excited wave field.

In analyzing the dynamics of the intial pulse for times $t > T$ we have so far neglected the influence of dissipation on the nature of fluxon motion (dissipation entered only into the threshold characteristics). In fact oscillations of the fluxon bound to the junction edge and described by the "half-breather" will experience damping slowly due to dissipation, and this bound state vanishes as $t\to\infty$, i.e., the fluxon is anniliated with its image. Regarding the free fluxon generated by the pulse when condition (32) holds, the motion of this fluxon is decelerated by dissipation and by a certain time t_0 it ends up at a point located at a distance of

$$\xi_0=v/\gamma, \tag{36}$$

from the junction edge, where v is the velocity at which the vortex is created.[2)] If the weak attractive force F_{attr} of the fluxon travelling away from the junction edge towards its mirror image is ignored, $t_0 = \infty$. In fact t_0 is finite and under the action of force F_{attr} the fluxon will eventually travel backward, enter a bound state and experience damping due to dissipation. The expression for the force F_{attr} for $\xi\gg 1$ (ξ is the effective vortex coordinate measured from the junction edge) is well known[13]:

$$F_{attr}(\xi)\approx -e^{-2\xi}. \tag{37}$$

If, in addition to dissipation, we account for the small d.c. component h_0 of the applied current injected through the junction edge, the resulting fluxon may go to infinity. At the same time it is easily demonstrated that when $h_0\neq 0$ an auxiliary force

$$F_{rep}\approx 4\sigma h_0 e^{-\xi} \tag{38}$$

will act on vortex (29) in addition to the attractive force F_{attr} (37). The force F_{rep} is repulsive when $\sigma h_0 > 0$, i.e., $ah_0 > 0$ as clearly indicated from (38). It follows from (37) and (38) that in this condition the total force $F_{\mathrm{attr}} + F_{\mathrm{rep}}$ acting on the fluxon consists of the force of attraction to the junction edge in the range $\xi < \xi_m$ and the force of repulsion in the range $\xi > \xi_m$, where

$$\xi_m\approx\ln(1/|h_0|)$$

(we have assumed that $\ln|h_0|^{-1}\gg 1$). It is clear that the resulting fluxon will go to infinity if ξ_0, defined in (36), exceed ξ_m. We can therefore conclude that when dissipation is present a fluxon will escape from the junction edge if its initial velocity v exceeds the critical value

$$v_{cr}=\gamma\ln|h_0|^{-1}. \tag{39}$$

According to (30) and the results of the IST, the initial velocity of the generated vortex can be represented as

$$v^2=(k_0^2-1)/k_0^2, \tag{40}$$

where $k_0(k_0^2>1)$ is the root of the equation $a(ik_0)=0$. As follows from (27),

$$k_0=-B\frac{\cos(BT/2)+(\gamma BT^2/4)\sin(BT/2)}{\sin(BT/2)+(BT/2)\cos(BT/2)}.$$

The condition $v^2>v_{cr}{}^2$ suggests that in addition to the threshold values $a_1, a_2,\ldots$ (see (31), 32)) there is a certain auxiliary threshold value $a_*>a_1$ for the fluxon going to infinity. If $\gamma\ \ln|h_0|^{-1}<1$, then a_* lies in the range $a_1<a_*<a_2$. In principle when $T\sim1$ we could have a_* greater than a_2. If $a_1<a<a_*$, i.e., $v^2<v_{cr}{}^2$ (specifically, when $\gamma\ln|h_0|^{-1}<1$), the fluxon will return to the junction edge and will be anniliated there due to dissipative losses.

In actual experiments a long Josephson junction will have a long, yet finite length L. Hence all preceding results will be valid when $L\gg\xi_0$. If $L<\xi_0$, which could occur with very low dissipation, the condition for a vortex to arrive at the other junction edge appears as $|h_0|>e^{-L}$ and is independent of the initial vortex velocity.

The condition for each subsequent fluxon going to infinity can be obtained analogously.

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[2)]This fact was excluded from the analysis of numerical results in Ref. (12), thereby giving rise to inaccuracies in their treatment.

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