Bifurcation and chaos in a system of optically coupled CO_2 lasers

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(Submitted 26 August 1988)
Zh. Eksp. Teor. Fiz. **95**, 807–816 (March 1989)

Bifurcation and chaotic lasing regimes have been observed for the first time ever in a system of optically coupled CO_2 lasers. The regions of stable laser mode locking are determined analytically and numerically. The dynamic regimes in the absence of mode locking are calculated. Agreement between experimental and numerical results is obtained.

INTRODUCTION

The properties of nonlinear dynamic systems is a subject of considerable interest in the physics of the last decade.[1–3] Contributing to this interest is the observed possibility of onset of chaos in nonlinear systems. The phase trajectories can become quite entangled, and in many cases statistical methods can be used to describe the properties of such systems.[4] Random motion is realized in a set having a dimensionality lower than that of the phase space, and this set, which attracts trajectories, has been named strange attractor. Progress in mathematical analysis of nonlinear dynamic systems, as well as the present scenario of the onset of a transition to chaos,[5,6] have made it possible to use these results for the study of physical objects in nonlinear optics, plasma physics, solid-state theory, and others.

The most suitable, from the viewpoint of the experimental study of chaos, are laser systems. The possibility of a chaotic time behavior of the radiation intensity in microwave quantum generators was first discussed in Ref. 7. It was demonstrated in Ref. 8 that the system of equations describing lasing is equivalent to the system of Lorenz equations,[3] analysis of which yielded solutions of the strange-attractor type. The conditions for the transition to chaos in injection lasers, Q-switched lasers, and gain-switched lasers were subsequently determined numerically.[9] The large number of theoretical computations notwithstanding, the experimental results are relatively few. Bifurcation and chaos were experimentally observed in Q-switched and gain-switched CO_2 lasers.[10–12]

We report here for the first time ever, on the basis of numerical calculation and experiment, that bifurcation and chaos are possible in two optically coupled CO_2 lasers. This investigation was prompted by the need for solving the important practical problem of increasing the brightness of laser emission. One of the methods of solving this problem is to synchronize the fields of a set of cavities by optical coupling.[13] Since the directivity pattern of a set of lasers is determined by the field phases, random variation of the latter should make the emission brightness dependent on the characteristics of the random motion of the system.

EXPERIMENTAL SETUP AND RESULTS

The experimental setup is shown in Fig. 1. Two quasi-stationary waveguide CO_2 lasers 1 and 2, excited by a 10-kHz discharge current, exchange emission via the coupling mirror 3. The laser-beam polarization vector lies in the plane of the drawing. Matching lens 5 is used to exchange laser emission with minimum loss. The active media of the lasers are 1.5 m long, the distance between the common flat mirrors 4 and 6 is 4 m. The diffractive exchange of emission between the lasers through the totally reflecting mirror is prevented by telescoping the beams in the space between the mirror 6 and the closest ends of the discharge tube. The laser detuning $\Delta\omega_{12}$ is effected by varying the optical lengths of the cavities through rotation of the totally reflecting mirror 6. The value of $\Delta\omega_{12}$ was monitored against the intensity-beat frequency of the laser beams brought together through lens 7 on photoreceiver 8 with the optical coupling blocked by screen 9, and also with the aid of photoreceiver 13. The use of a scanning diaphragm made it possible to obtain the spatial distribution of the radiation of the two coupled lasers in the far zone. The coupling coefficient was varied by introducing into the optical channel a set of calibrated attenuators 11, and could be decreased to $M^2 \sim 10^{-4}$, where M^2 is determined by the intensity fraction of the radiation injected from one laser into the other. Another set of calibrated attenuators 12 was inserted into the laser cavities to vary the intracavity losses. The lasing dynamics of one of the coupled lasers was recorded by photoreceiver 13 and a digital oscilloscope. A microcomputer was used for signal processing.

When lasers 1 and 2 operated independently in the absence of optical coupling, their outputs were modulated at double the pump-current frequency 20 kHz. The depth of this modulation did not exceed 10%. Under emission-exchange conditions but in the absence of phase locking, at a coupling coefficient M^2 smaller than its threshold value M_t^2 (Ref. 14), the output radiation was modulated at a frequency $\Delta\omega_{12}$ that was varied in the experiments in the range 10^5–10^7 Hz. The modulation depth in this regime (beat regime) reached 50–100%. At $M^2 > M^2_{\rm thr}$ lasers 1 and 2 were synchronized and an interference pattern with almost 100% contrast was observed in the far zone. Since the value of the coupling-coefficient phase, i.e., the phase shift in the coupling channel, was not monitored in the experiment, all the measurements were performed within times 10^{-4}–10^{-3} s in which the phase remained unchanged. The operating regimes of the coupled lasers (locking, beats, etc.) were additionally monitored in the course of the measurements with the aid of the devices 8 and 10 (Fig. 1). In the absence of laser synchronization and under conditions when the detuning frequency $\Delta\omega_{12}$ approached the natural frequency

$$\Omega = \left[\frac{cg_t}{\tau}\left(\frac{g_0}{g_{\rm II}} - 1\right)\right]^{1/2}$$

of the radiation-field oscillations in the laser cavity,[10] the intensity oscillations of the coupled lasers become more

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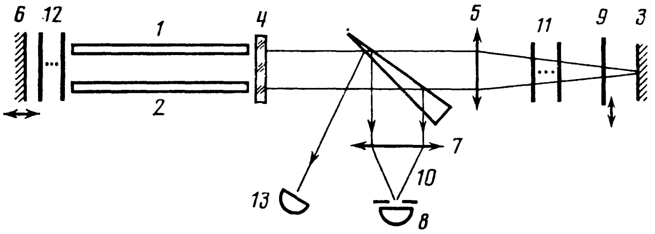

FIG. 1. Experimental setup: 1, 2—CO_2-laser discharge tubes; 4, 6—cavity mirrors; 3—coupling mirror; 5—matching lens; 7—focusing lens; 9—movable screen; 10—scanning diaphragm; 11, 12—calibrated attenuators; 8, 13—photoreceivers.

complex and the modulation depth was almost 100%. In the expression above, τ is the collisional-relaxation time of the upper laser level, while g_0 and g_t are respectively the weak-signal gain and the threshold gain.

The natural frequency Ω was determined in experiment in the following manner. Calibrated attenuators 12 were introduced into the laser cavities and the threshold gain g and the ratio g_0/g_t were determined from the lasing cutoff. The relaxation time was estimated from the known g_0 and g_t and from the measured delay of the lasing pulse relative to the pump-current pulse. The obtained values of g_0, g_t, and τ were substituted in the expression for Ω. The frequency Ω was varied by varying the discharge current (actually the parameters g_0 and τ) and a second lasing threshold was fixed. The lasing properties of the coupled lasers were experimentally investigated for two values of the natural frequency Ω, corresponds to $\Omega/2\pi = 100$ kHz and 150 kHz. For both natural frequencies, the radiation-intensity spectrum exhibited at $\Delta\omega_{12} \approx \Omega$ frequencies corresponding to doubling and quadrupling of the period $2\pi/\Delta\omega_{12}$. Typical oscillograms of the intensity for doubling and quadrupling of the period are shown in Figs. 2 and 3. The frequencies 10 and 20 kHz in these figures are connected with the electric circuit used to excite the lasers. In individual cases, a more complicated intensity spectrum was observed when M^2 and $\Delta\omega_{12}$ was varied (Fig. 4).

ANALYTIC AND NUMERICAL INVESTIGATION

To describe the lasing dynamics of two coupled lasers we start with the point model for the fields[14] and the gains of the active media:

$$\dot{A}_1 = {}^1/_2(g_1 - g_t)A_1 + MA_2\cos(\psi - \varphi),$$
$$\dot{A}_2 = {}^1/_2(g_2 - g_t)A_2 + MA_1\cos(\psi + \varphi),$$
$$\dot{\varphi} = -\Delta + M\left[\sin(\psi - \varphi)\frac{A_2}{A_1} - \sin(\psi + \varphi)\frac{A_1}{A_2}\right], \qquad (1)$$
$$\dot{g}_{1,2} = \frac{g_0 - g_{1,2}}{\tau} - A_{1,2}g_{1,2}.$$

Here $A_{1,2}$ are the field amplitudes in the lasers, $\varphi = \varphi_1 - \varphi_2$ is the phase difference of the fields, g_0 is the cavity unsaturated gain per pass, $g_{1,2}$ and g_t are respectively the signal gains and loss in the first and second lasers, respectively. The coefficient of optical coupling between the lasers is determined by the amplitude M and phase ψ of the coupling. All the quantities with dimension of time, viz., τ (relaxation time), Δ^{-1} (Δ is the laser-frequency detuning), and A^{-2} (the stimulated-transition time) are made nondimensional by division by the time of light-beam passage through the cavity.

The implementation of the various dynamic regimes depends on many parameters of the problem: M, ψ, Δ, τ, g_0, and g_t. The stationary-lasing conditions can be analytically investigated most completely at the coupling-phase values $\psi = 0, \pi$, and $\pm\pi/2$. The laser energy needed for coupling is usually low ($M \ll g_t$), and the system (1) can be analyzed assuming M to be small.

If the coupling coefficient is real (i.e., for $\psi = 0$ and π) stationary solutions of (1) exist for $\Delta \leqslant 2M$. If $\psi = 0$, the field phase difference in the stationary solutions is given by

$$\varphi = -\arcsin(\Delta/2M), \quad \varphi = -\pi + \arcsin(\Delta/2M).$$

The first of the solutions corresponds to in-phase lasing, and the second to antiphase lasing. An investigation of the stability of these two solutions shows that at $\psi = 0$ the in-phase solution is stable and the second solution is unstable.

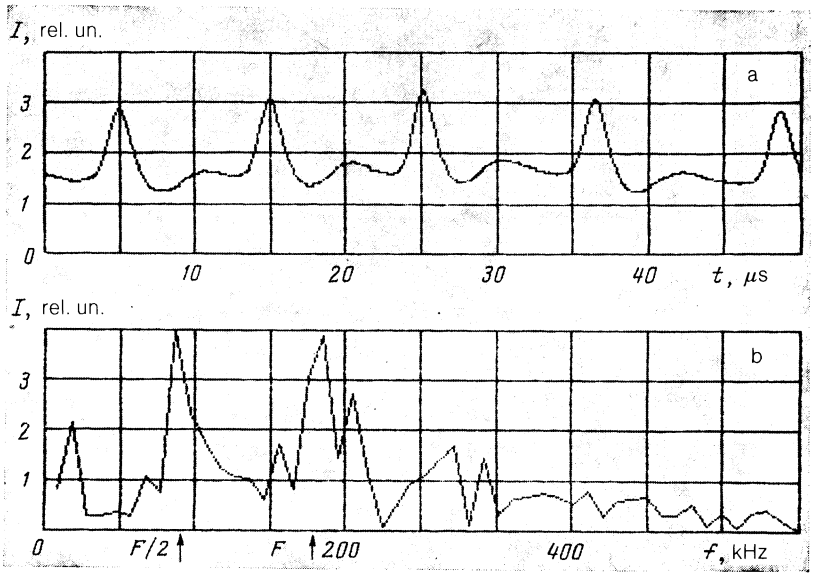

FIG. 2. Oscillogram (a) and spectrum (b) of radiation intensity I in the period-doubling regime. $\Omega/2\pi = 150$ kHz, $\Delta\omega_{12}/2\pi = F = 180$ kHz.

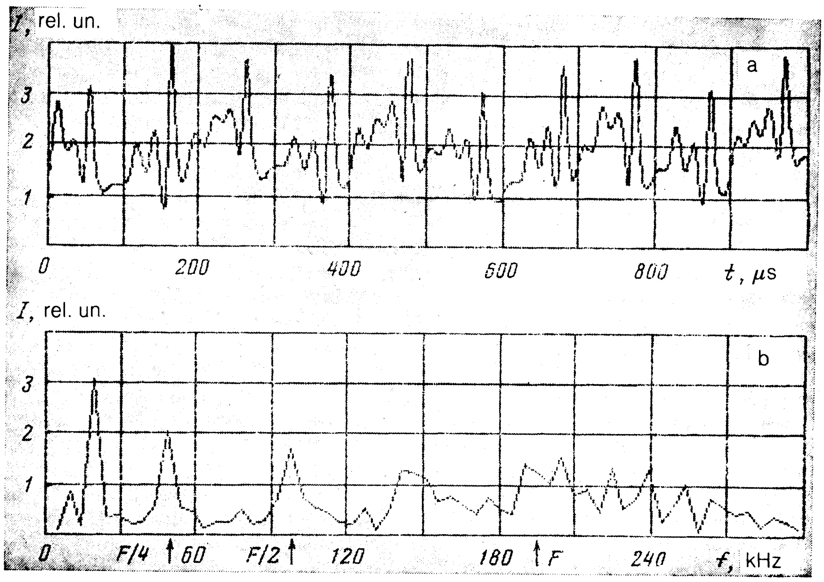

FIG. 3. Oscillogram (a) and spectrum (b) of radiation intensity when the period is quadrupled; $\Omega/2\pi = 150$ kHz, $\Delta\omega_{12}/2\pi = F = 192$ kHz.

At $\psi = \pi$ there are also two stationary solutions corresponding to in-phase $\varphi = \arcsin(\Delta/2M)$ and antiphase $\varphi = \pi - \arcsin(\Delta/2M)$ lasing, the latter being stable.

Thus, at real optical-coupling coefficients and for $\Delta \leqslant 2M$ stationary lasing is effected for all the remaining parameters of the problem. If the frequency deviation exceeds the critical value $2M$, the lasing becomes nonstationary. Investigations have shown that the highest-quality (in the sense of maximum conversion of the active-medium energy into radiation) regime is the one in which the time dependences of the field amplitudes $A_{1,2}(t)$ are equal. This follows from the symmetry of the equations at $\psi = 0$ and π. The analysis of the system can be reduced to an investigation of the emission of one periodically Q-switched laser. This problem was investigated experimentally and numerically in Refs. 10 and 12, where a possibility of onset of chaos in the system was observed. In contrast to Refs. 10 and 12, where the Q-switching period was set by external devices, in the present case of coupled lasers the period is determined by the equation for the phase

$$\dot{\varphi} = -\Delta \mp 2M \sin\varphi,$$

where the upper and lower signs correspond to coupling-coefficient phases $\psi = 0$ and $\psi = \pi$, respectively.

In the case of an imaginary coupling coefficient at $\psi = \pm\pi/2$ the condition for the existence of stationary solutions takes the form[15]

$$|\Delta| \leqslant \frac{2M^2}{g_0 - g_t}\frac{g_0}{g_t}. \tag{2}$$

If (2) is satisfied, four stationary solutions are possible with close field amplitudes and with the following phases:

$$\varphi = -\frac{1}{2}\arcsin\left(\frac{\Delta}{2M^2}\frac{g_0 - g_t}{g_0/g_t}\right),$$

$$\varphi = \pi + \frac{1}{2}\arcsin\left(\frac{\Delta}{2M^2}\frac{g_0 - g_t}{g_0/g_t}\right),$$

$$\varphi = \pm\left[\frac{\pi}{2} - \frac{1}{2}\arcsin\left(\frac{\Delta}{2M^2}\frac{g_0 - g_t}{g_0/g_t}\right)\right].$$

The first two solutions correspond to in-phase and antiphase lasing, and the second pair corresponds to lasing with a phase shift close to $\pm\pi/2$. Even if (2) is satisfied, however, the stationary solutions may turn out to be unstable. In accord with the Hurwitz criterion, the system (1) linearized for small perturbations near the stationary solutions leads to

$$\frac{1}{\tau} > \frac{4M^2}{g_0 - g_t}\sin^2\varphi,$$

$$\frac{1}{\tau^2}\left[1 - 4M^2\left(\frac{g_0/g_t}{g_0 - g_t}\right)^2\sin^2\varphi\right]$$

$$> \frac{16M^4}{(g_0 - g_t)^2}\left[\cos^2\varphi - \frac{(g_0/g_t)^2}{\tau(g_0 - g_t)}\sin^2\varphi\right]\sin^2\varphi. \tag{3}$$

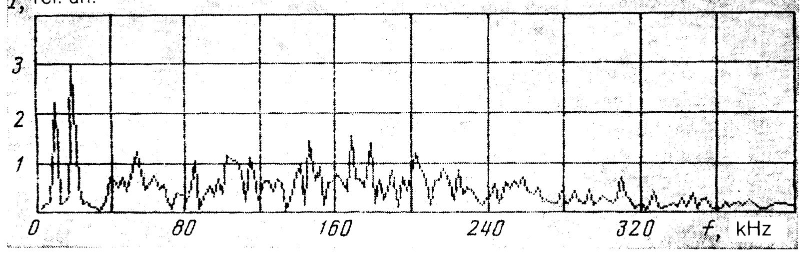

FIG. 4. Intensity spectrum of chaotic signal.

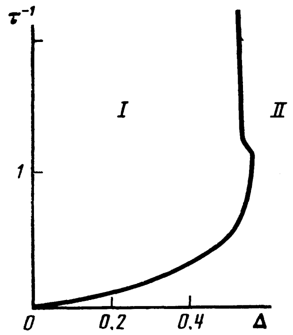

FIG. 5. Stability pattern of in-phase lasing at $g_0 = 1.4$, $g_t = 1.4$, $M = 0.3$, and $\psi = \pi/2$: 1—stable lasing; 2—unstable.

Figure 5 shows the stability region of the stationary solutions for $\psi = \pm\pi/2$, in terms of $1/\tau$ and Δ.

Various physical factors govern the stability of stationary lasing regimes at real and imaginary values of the laser optical-coupling coefficient. If the coupling coefficient is real, the lasing is stable because the field-energy losses in in-phase (or antiphase) lasing are minimal. If the field mismatch time of the two lasers ($\sim 1/\Delta$) exceeds that evolution time of the in-phase (or antiphase) field structure ($\sim M^{-1}$) the stationary regime is stable. This condition is equivalent to $M > \Delta$ and agrees with the rigorous mathematical analysis.

In the case of an imaginary coupling coefficient the regimes are stable because the relaxation time of the medium is finite. The point is that a single laser with an active medium is equivalent to a damped oscillating system. The deviation of the intensity or of the gain from stationary values leads to damped oscillations with a frequency

$$\Omega' = \left[\frac{1}{\tau}(g_0 - g_t)\right]^{1/2}$$

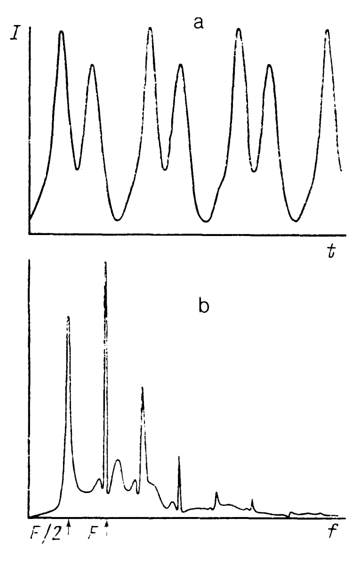

FIG. 6. Calculated variations of the intensity (a) and spectrum (b) for the doubled period.

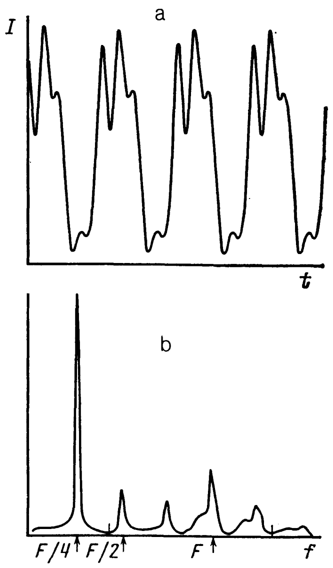

FIG. 7. Calculated dependences of the intensity (a) and spectrum (b) for the quadrupled period.

and a decrement $(1/2)\ (g_0/g_t\tau)$ (it is assumed that $\Omega' \gg \frac{1}{2}(g_0/g_t\tau)$. Detuning leads to power oscillations in the lasers, and the relaxation of the medium tends to damp these oscillations. Clearly, stationary lasing is possible at a sufficiently large optical-coupling coefficient and at a sufficient damping decrement. Note that slightly above the lasing threshold

$$M > \frac{g_0 - g_t}{g_0/g_t}$$

the stationary regime is stable in the limit as $\tau \to 0$ if the inequality $\Delta < 2M$ is satisfied, in analogy with the case of a real coefficient.

In the parameter region where stationary solutions do not exist or are unstable, the dynamics of generation by the coupled lasers was investigated by numerically integrating the system (1). Phenomena typical of nonlinear dynamics systems[9] were observed. By way of example, Figs. 6 and 7 show the temporal and spectral dependencies of the emission intensity of the lasers, corresponding to the period doubling and quadrupling regimes. The fundamental period is equal to the time of energy transfer from one laser to the other. Thus, for an imaginary coupling coefficient, the fundamental period is determined by the detuning and by the amplitude of the coupling coefficient

$$T = 2\pi/(\Delta^2 + 4M^2)^{1/2}.$$

The corresponding phase portraits for the period doubling and quadrupling regimes are shown in Fig. 8. In these regimes the frequencies of energy exchange between the lasers were chosen to be close to the frequencies of the relaxation oscillations in the lasers. Besides the frequency components F, $F/2$, and $F/4$ corresponding to the fundamental, doubled, and quadrupled bifurcation periods, the emission spectrum showed also components resulting from frequency combinations in the nonlinear systems: $3/2F$, $2F$, $5/2F$ (Fig. 3) and $3/4F$, $5/4F$ (Fig. 7). Upon variation of the parameters the calculations revealed also regimes corresponding to tripling of the period, and more complicated ones corresponding to

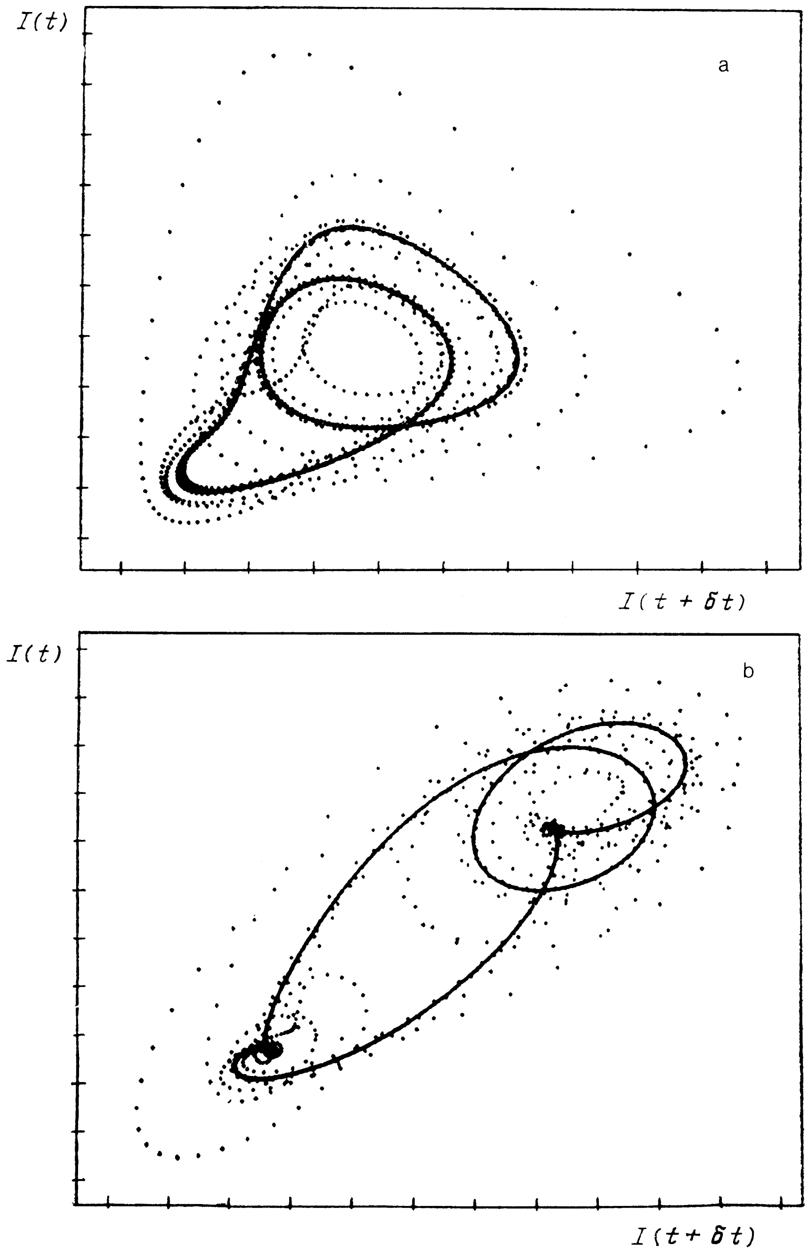

FIG. 8. Phase portrait. Doubling (a) and quadrupling (b) of the period.

onset of chaos in the system. This behavior is illustrated in Fig. 9, which shows the time dependence of the interference term $J = A_1(t)A_2(t)\cos\varphi(t)$. It is seen that the character of the pulsation is similar to the self-modulation regime in the region of the Lorenz attractor.[9]

CONCLUSION

We have thus shown experimentally, for the first time ever, that a complex dynamic behavior of the emission intensity can be observed in a system consisting of two optically coupled CO_2 lasers if the frequency of energy exchange

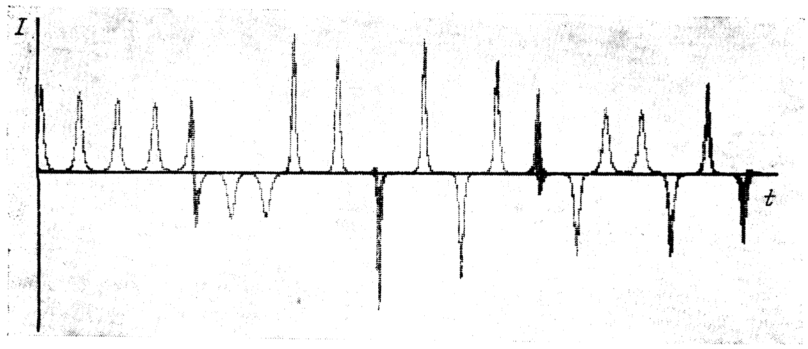

FIG. 9. Time variation of the interference term $J = A_1(t)A_2(t)\cos\varphi(t)$ in the stochastic regime.

between the laser is close to the natural frequency Ω of the relaxation oscillations. The transition to chaos is via bifurcation of the period-doubling type. The conditions for the realization of various regimes are not stringent when the gain exceeds a threshold value on the order of unity, and are achieved by retuning the natural frequency of one of the lasers. By varying the gain, the cavity Q, and the gas-mixture pressure it is possible to shift rather simply the regions where various regimes are realized and vary the spectral composition of the output radiation. The emission brightness of the two lasers depends on the phase shift φ of the fields. In a fully developed chaos there exist field phase differences near which the trajectories exist substantially longer than at the other values of φ. The probability of finding a system at a definite value φ is determined by the correlator of the interference term $J = A_1A_2\cos\varphi(t)$ and is expressed in terms of the correlation time, the correlation being one of the characteristics of the attractor.[4] We note in conclusion that a study of the dynamics of generation of two optically coupled lasers is the first step in the investigation of the possible generation regimes of assemblies of lasers.

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Translated by J. G. Adashko