Features of resonance absorption, due to spin-orbit interactions, by a system of two-dimensional electrons

A. E. Koshelev and V. Ya. Kravchenko

Solid State Physics Institute, Academy of Sciences of the USSR
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We derive the kinetic equations for the density-matrix components and the quasiclassical transport equation for densities and fluxes for a two-dimensional electron system with spin-orbit interactions. Using the transport equations we evaluate the main characteristics of combined resonance—the amplitude, line shape, linewidth, and saturation. We also consider nonlinear resonance effects—second-harmonic generation and rectifying effect—and spin resonance when alternating electric and magnetic fields act simultaneously.

1. INTRODUCTION

The two-dimensional electron gas in heterojunction MIS structures or on the surfaces of semiconductors is the result of size quantization in an axisymmetric potential well, so that the symmetry admits the presence, in the Hamiltonian of these electrons, of a spin-orbit term which is linear in the momentum[1-3]:

$$\hat{\mathscr{H}}_{so} = -{}^1/_2[\hat{\boldsymbol{\sigma}}\mathbf{V}]\hat{\mathbf{p}}. \tag{1}$$

The vector $\mathbf{V}$ which has the dimensions of a velocity, is here oriented at right angles to the plane of the two-dimensional states. The operator $\hat{\mathscr{H}}_{so}$ modifies the normal spectrum of the electrons in a magnetic field which connects the Landau quantization with the spin.[1] Resonance absorption of an alternating electric field occurs at frequencies which depend on $\mathbf{V}$ and which can be realized either as cyclotron resonance (CY) corresponding to transitions without spin changes, or as combined resonance (CR)—transitions between the spin branches without a change in the number of the Landau level.[1]

An analysis of the experimental data[4,5] about resonance absorption in the GaAs–$Al_xGa_{1-x}As$ heterojunctions was given in Refs. 2 and 3, where the manifestation of the spin-orbit interaction (1) was revealed and an estimate was given of the spin-orbit constant $\alpha = \hbar V/2$. Dorozhkin and Ol'shanskiĭ,[6] using their own measurements and also those of Ref. 7, estimated the spin-orbit coupling constant for the hole channel on the (110) surface of silicon using the discontinutiy in the Shubnikov oscillations of the conductivity. According to the estimates in those papers the spin-orbit constant is $\alpha \sim (1\text{–}6)\times 10^{-10}$ eV·cm. We use in the present paper the transport equations for the densities and the fluxes to obtain the basic characteristics of the resonance absorption of two-dimensional electrons in an arbitrarily oriented magnetic field $\mathbf{B}$ and alternating electric and magnetic fields. We consider nonlinear effects—saturation of the resonance, resonance excitation of the second harmonic, and rectification effect.

To describe the two-dimensional system we use the Hamiltonian

$$\hat{\mathscr{H}} = \hat{\mathbf{p}}^2/2m + \hat{\mathscr{H}}_{so} - {}^1/_2\boldsymbol{\Omega}_s\hat{\boldsymbol{\sigma}}, \tag{2}$$

where $\hat{\mathbf{p}} = -i\nabla - e\mathbf{A}/c$, $\mathbf{A}$ is the vector potential, and $\boldsymbol{\Omega}_s = g\mu_B\mathbf{B}$. It is well known that in semiconductors the band mass m is smaller than the "spin" mass m_0 which determines the spin magnetic moment of the electron. In a paper by the present authors and Khmel'nitskiĭ[8] the manifestation of CR was studied for an electron system coupled to a dislocation and described by a Hamiltonian similar to (2). It was shown that the shape of the CR line observed experimentally[9] enables one to explain a number of specific properties of the electron system. In the two-dimensional case the CR picture turns out to be even richer and can offer new possibilities of studying the properties of electron systems.

2. KINETIC EQUATIONS AND TRANSPORT EQUATIONS

We shall start from the Liouville equation for the Wigner distribution function:

$$\hat{f}(\mathbf{p},\mathbf{r}) = \int d\mathbf{u}\exp\left(i\mathbf{p}\mathbf{u} + i\frac{e}{c}\int\limits_{\mathbf{r}-\mathbf{u}/2}^{\mathbf{r}+\mathbf{u}/2}\mathbf{A}\,d\mathbf{l}\right)\hat{f}\left(\mathbf{r}-\frac{\mathbf{u}}{2},\mathbf{r}+\frac{\mathbf{u}}{2}\right),$$

where $f(\mathbf{r}_1,\mathbf{r}_2)$ is the density matrix—a 2×2 matrix in the spin indexes—which has the form

$$\frac{\partial\hat{f}}{\partial t} + i[\hat{\mathscr{H}},\hat{f}]_- + \frac{1}{2}\left[\frac{\partial\hat{\mathscr{H}}}{\partial p_j},\frac{\partial\hat{f}}{\partial r_j}\right]_+ + \frac{1}{2}\left[eE_j + \frac{e}{c}\left[\frac{\partial\hat{\mathscr{H}}}{\partial\mathbf{p}}\mathbf{B}\right]_j\right. \\ \left. + \frac{1}{2}\frac{\partial(\boldsymbol{\Omega}_s\hat{\sigma})}{\partial r_j}, \frac{\partial\hat{f}}{\partial p_j}\right]_+ = \hat{\mathrm{St}}(\hat{f}). \tag{3}$$

Here $[,]_\mp$ indicates, respectively, a commutator or an anticommutator, $\hat{\mathrm{S}}\mathrm{t}(\hat{f})$ is the collision operator, and $\hat{\mathscr{H}}$ is the same as (2) with the quasimomentum $\mathbf{p}$ replacing the operator $\hat{\mathbf{p}}$. Equation (2) is valid for electric and magnetic fields which vary slowly in space. If we write $\hat{f}(\mathbf{p},\mathbf{r})$ in the form

$$\hat{f} = {}^1/_2(F^0 + \mathbf{F}\hat{\boldsymbol{\sigma}}), \tag{4}$$

we get from (2) and (3) a set of equations for the scalar F^0 and the vector $\mathbf{F}$ parts of the distribution function:

$$\frac{\partial F^0}{\partial t} + \left(e\mathbf{E} + \frac{e}{c}\left[\frac{\mathbf{p}}{m}\mathbf{B}\right]\right)_j\frac{\partial F^0}{\partial p_j} - \frac{e}{2c}\left[\mathbf{V}\left[\mathbf{B}\frac{\partial}{\partial\mathbf{p}}\right]\right]\mathbf{F} \\ + \frac{1}{2}\frac{\partial\boldsymbol{\Omega}_s}{\partial r_j}\frac{\partial\mathbf{F}}{\partial p_j} + \frac{p_j}{m}\frac{\partial F^0}{\partial r_j} - \frac{1}{2}\mathbf{V}\left[\frac{\partial}{\partial\mathbf{r}}\mathbf{F}\right] = \mathrm{Sp}\,(\hat{\mathrm{St}}\,(F^0,\mathbf{F})), \tag{5a}$$

$$\frac{\partial\mathbf{F}}{\partial t} + [\boldsymbol{\Omega}_s + [\mathbf{V}\mathbf{p}],\mathbf{F}] + \left(e\mathbf{E} + \frac{e}{c}\left[\frac{\mathbf{p}}{m}\mathbf{B}\right]\right)_j\frac{\partial\mathbf{F}}{\partial p_j} \\ - \frac{1}{2c}\left[\mathbf{V}\left[\mathbf{B}\frac{\partial}{\partial\mathbf{p}}\right]\right]F^0 \\ + \frac{1}{2}\frac{\partial\boldsymbol{\Omega}_s}{\partial r_j}\frac{\partial F^0}{\partial p_j} + \frac{p_j}{m}\frac{\partial\mathbf{F}}{\partial r_j} - \frac{1}{2}\left[\mathbf{V}\frac{\partial}{\partial\mathbf{r}}\right]F^0 = \mathrm{Sp}\,(\hat{\sigma}\,\hat{\mathrm{St}}\,(F^0,\mathbf{F})). \tag{5b}$$

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It is necessary to note that if the external fields are constant in space, the kinetic equations (3) and (5a), (5b) are exact—they do not contain the semiclassical approximation. In particular, when $\mathbf{E}=0$ and the magnetic field is at right angles to the plane, the equilibrium functions F_0^0 and $\mathbf{F}_0$ making the left hand sides of Eqs. (5a) and (5b) equal to zero can be obtained from the exact wave functions and eigenvalues of the energy.[2]

In what follows we conduct our analysis in the semiclassical approximation $\varepsilon_F, T \gg \Omega_c$. In that limit we can use the classical expression for the collision integral.[8] In the case when the mean free path is considerably shorter than the other spatial scales of the problem, such as the size of the specimen in the $z=0$ plane and the electromagnetic wavelength, and the spin-orbit interaction is weak,

$$V(\overline{p^2})^{1/2} \ll \Omega_s, \quad 1/\tau, \tag{6}$$

we can obtain from the kinetic equations (5a) and (5b) transport equations for the densities and fluxes:

$$\begin{aligned} \rho &= \int F^0(\mathbf{p})\,(d\mathbf{p}), \quad \boldsymbol{\mu} = \int \mathbf{F}(\mathbf{p})\,(d\mathbf{p}), \\ I_j^0 &= \int \left[\frac{p_j}{m}F^0 + \frac{1}{2}[\mathbf{VF}]_j\right](dp), \\ I_j^k &= \int \left[\frac{p_j}{m}F^k + \frac{1}{2}[\mathbf{n}_k\mathbf{V}]_j F^0\right](d\mathbf{p}). \end{aligned} \tag{7}$$

Here ρ and $\boldsymbol{\mu}$ are the particle and spin polarization densities per unit area (the magnetic-moment density is $\mathbf{M}=\mu_B g\boldsymbol{\mu}/2$), I_j^0 and I_j^k are the components of the particle and spin polarization flux densities (here and henceforth the upper index in I_j^k refers to the spin and the lower one to the coordinate). One can show in the same way as in Ref. 8 that in the transport equations one gets from the collision operator, in the leading order in V, relaxation terms that are diagonal in the flux components and have the same relaxation time τ for I_j^0 and I_j^k. The set of transport equations is the following:

$$\frac{\partial\rho}{\partial t} + \frac{\partial I_j^0}{\partial r_j} = 0, \tag{8a}$$

$$\frac{\partial\mu^k}{\partial t} + \frac{\mu^k - \mu_0^k}{T_2} + \left(\frac{1}{T_1} - \frac{1}{T_2}\right)(\boldsymbol{\mu}-\boldsymbol{\mu}_0, \boldsymbol{\zeta})\zeta_k + \frac{\partial I_j^k}{\partial r_j} + [\boldsymbol{\Omega}_s\boldsymbol{\mu}]_k + mV(I_k^z - \delta_{kz}I_l^l) = 0, \tag{8b}$$

$$\frac{\partial I_j^0}{\partial t} + \frac{I_j^0}{\tau} - \frac{1}{2}\left[\mathbf{V}\frac{\partial\boldsymbol{\mu}}{\partial t}\right]_j + [\boldsymbol{\Omega}_c\mathbf{I}^0]_j + \frac{\overline{p^2}}{2m^2}\frac{\partial\rho}{\partial r_j} + \frac{V}{2}\left(\frac{\partial I_j^x}{\partial y} - \frac{\partial I_j^y}{\partial x}\right) - \frac{1}{2m}\mu^l\frac{\partial\Omega_{sl}}{\partial r_j} = \frac{e\rho}{m}E_j, \tag{8c}$$

$$\frac{\partial I_j^k}{\partial t} + \frac{I_j^k}{\tau} + e_{nlk}\Omega_{sn}I_j^l + e_{ijn}\Omega_{cn}I_j^k + V\left[\left(\frac{\overline{p^2}}{2m}\mu^z - \frac{\rho}{2}\Omega_{sz}\right)\delta_{kj} - \left(\frac{\overline{p^2}}{2m}\mu^j - \frac{\rho}{2}\Omega_{sj}\right)\delta_{kz}\right] + \frac{\overline{p^2}}{2m^2}\frac{\partial\mu_k}{\partial r_j} - \frac{V}{2}\frac{\partial}{\partial r_l}[e_{lkz}I_j^0 + e_{jkz}I_l^0] - \frac{\rho}{m}\frac{\partial\Omega_{sk}}{\partial r_j} = \frac{eE_j}{m}\mu^k. \tag{8d}$$

Here $\boldsymbol{\mu}=\chi_0\mathbf{B}$ is the vector of the equilibrium spin polarization (χ_0 is the static spin susceptibility), $\zeta=\mathbf{B}/B$ is a unit vector along the constant magnetic field, e_{ijk} is an antisymmetric unit tensor, $\boldsymbol{\Omega}=e\mathbf{B}/mc$, and

$$\frac{\overline{p^2}}{2m} = \begin{cases}\varepsilon_F/2 \\ T\end{cases}, \quad \chi_0 = \frac{1}{2}g\mu_B\rho_0\begin{cases}2/\varepsilon_F \\ T\end{cases}$$

for the degenerate and nondegenerate cases, respectively. In deriving these equations we have used the approximate relations

$$\int (dp)\,p_ip_jf_k \approx \frac{1}{2}\overline{p^2}\delta_{ij}\mu^k,$$

and we have also introduced the phenomenological spin-spin (T_2) and spin-lattice (T_1) relaxation times. We assume that $T_{1,2} \gg \tau$. Furthermore, we consider the spatially uniform case when we can drop in Eqs. (8a)–(8d) the terms with spatial derivatives.

3. COMBINED RESONANCE

We consider the situation when an alternating electric field $\mathbf{E}(t)=\mathbf{E}\exp(-i\omega t)$ acts upon the two-dimensional electrons, as dues a constant magnetic field $\mathbf{B}$ oriented at an angle ϑ to the normal $\mathbf{n}_z$ to the surface (the x axis lies in the plane of the vectors $\boldsymbol{\zeta}$ and $\mathbf{n}_z$) (Fig. 1). It is convenient to let the upper indexes (of the spin polarization) refer to the $(\xi, \eta = n_y, \zeta)$ coordinate system and the lower indexes of I_j^0 and I_j^k to the (x,y) system in the plane. Moreover, we introduce circular components of the fluxes and the spin densities:

$$I_a = I_x + iaI_y, \quad I^\beta = I^\xi + i\beta I^\eta,$$
$$\mu^\beta = \mu^\xi + i\beta\mu^\eta \quad (a, \beta = \pm 1).$$

To take into account saturation effects it is necessary to write down not only the equations for the alternating components I_a^0, I_a^ζ, I_a^β, and μ^β, but also the equations for the rectified components $\bar{\mu}^\zeta$ and $\bar{I}_a^\beta$ (the components μ^ζ, $\bar{I}_a^\zeta$, and $\bar{\mu}^\beta$ are of higher order in the small V than to μ^β, $\bar{I}_a^\beta$, and $\bar{\mu}^\zeta$, respectively, and we therefore need not consider them for CR)

$$[1+i\tau(a\zeta_z\Omega_c-\omega)]I_a^0 - {}^1/_4\omega\tau V\sum_\beta \beta(1+a\beta\zeta_z)\mu^\beta = \nu\rho E_\rho, \tag{9a}$$

$$[1/T_2 + i(\beta\Omega_s-\omega)]\mu^\beta + {}^1/_2 mV\sum_a[(1+a\beta\zeta_z)I_a^\zeta - a\beta\zeta_x I_a^\beta] = 0, \tag{9b}$$

$$[1+i\tau(a\zeta_z\Omega_c-\omega)]I_a^\zeta - {}^1/_2 mVD\sum_\beta(1+a\beta\zeta_z)\mu^\beta = \nu\bar{\mu}^\zeta E_a, \tag{9c}$$

$$[1+i\tau(a\zeta_z\Omega_c+\beta\Omega_s-\omega)]I_a^\beta + mVDa\beta\zeta_x\mu^\beta = 0, \tag{9d}$$

$$\bar{\mu}^\zeta/T_1 - {}^1/_4 mV\sum_{a,\beta}(1-a\beta\zeta_z)\bar{I}_a^\beta = 0, \tag{9e}$$

$$[1+i\tau(a\zeta_z\Omega_c+\beta\Omega_s)]\bar{I}_a^\beta + mVD(1-a\beta\zeta_z)(\bar{\mu}^\zeta-\mu_0) = \frac{\nu}{2}E_{-a}^*\mu^\beta. \tag{9f}$$

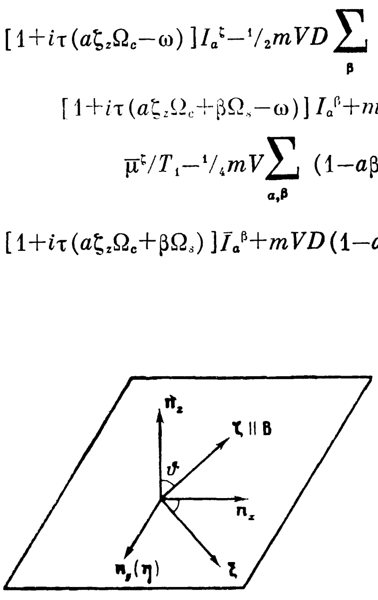

FIG. 1. Oreintation of the coordinate systems.

We introduced here the static mobility $\nu = e\tau/m$ and the diffusion coefficient $D = \overline{p^2}\tau/2m^2$. Eliminating the spin fluxes I_a^ζ, I_a^β, and $\overline{I}_a^\beta$, we get equations for μ^ζ and μ^β which are the same as the Bloch equations. It follows from the set (9) that the spin density component μ^1, which contributes to the flux I_a^0, experiences resonance at the frequency $\omega = \Omega_c$ [see Eq. (9b)]. The expression for the resonant spin density μ^1 can be written in the form

$$\mu^1 = -\frac{mVB}{2\omega}\sum_a (1+a\zeta_z)\nu_a(\omega)E_a\chi(\omega), \tag{10}$$

where the $\nu_a(\omega) = \nu/(1 + i\tau(a\zeta_z\Omega_c - \omega))$ are the circular mobility components, made up of the transverse ($\nu_{xx} = \nu_{yy}$) and Hall ($\nu_{xy} = -\nu_{yx}$) components,

$$\chi(\omega) = \frac{\omega\tilde{T}_2(1-i(\Omega_s-\omega-\Delta\omega)\tilde{T}_2)\chi_0}{1+\tilde{T}_2^2(\Omega_s-\omega-\Delta\omega)^2+\tilde{T}_1\tilde{T}_2(mV/4)^2\left|\sum_a (1+a\zeta_z)\nu_a E_a\right|^2} \tag{11}$$

is the resonance susceptibility,

$$1/\tilde{T}_2 = 1/T_2 + {}^1/_4 D(mV)^2 \operatorname{Re} S(\omega),$$

$$\frac{1}{\tilde{T}_1} = \frac{1}{T_1} + \frac{1}{2}D(mV)^2\sum_a \frac{(1+a\zeta_z)^2}{1+\tau^2(\Omega_s+a\zeta_z\Omega_c)^2},$$

T_2 and T_1 are, respectively, the spin-spin and spin-lattice relaxation times,

$$\Delta\omega = {}^1/_4 D(mV)^2 \operatorname{Im} S(\omega)$$

is the shift in the resonance frequency, and

$$S(\omega) = \sum_a \left[\frac{(1+a\zeta_z)^2}{1+i\tau(a\zeta_z\Omega_c-\omega)} + \frac{2\zeta_x^2}{1+i\tau(a\zeta_z\Omega_c+\Omega_s-\omega)}\right].$$

The corrections to the spin-spin and spin-lattice relaxation times arise due to the random changes in the spin precession axis in scattering by impurities and phonons (Ref. 1).[1)] Using Eq. 9(a) we write the conductivity flux density in the following form:

$$eI_a^0 = \sum_{a_1} (\sigma_a^{(0)}(\omega)\delta_{aa_1} + \sigma_{aa_1}^{CR}(\omega))E_{a_1}, \tag{12}$$

where $\sigma_a^{(0)} = e\rho_0\nu_a(\omega)$ and the part of the effective conductivity which describes CR has the form

$$\sigma_{aa_1}^{CR} = -{}^1/_8(mV)^2(1+a\zeta_z)(1+a_1\zeta_z)\nu_a(\omega)\nu_{a_1}(\omega)B\chi(\omega). \tag{13}$$

The intensity of the absorbed energy per unit area of the conducting layer is

$$Q = \frac{1}{2}\operatorname{Re} eE^*I^0 = \frac{1}{4}\operatorname{Re}\sum_{a,a_1}\sigma_{aa_1}E_a^*E_{a_1}. \tag{14}$$

The resonance absorption of a wave which is linearly polarized at an angle φ to the x axis ($E_a = E_0 \exp ia\varphi$) is given by the following expression:

$$Q_{CR} = {}^1/_2 E_0^2(\cos^2\varphi \operatorname{Re}\sigma_{xx}^{CR} + \sin^2\varphi \operatorname{Re}\sigma_{yy}^{CR}), \tag{15}$$

where

$$\sigma_{xx}^{CR} = -{}^1/_4(mV)^2B\chi(\omega)(\nu_{xx}+i\zeta_z\nu_{yx})^2, \tag{15a}$$

$$\sigma_{yy}^{CR} = -{}^1/_4(mV)^2B\chi(\omega)(\zeta_z\nu_{xx}+i\nu_{yx})^2, \tag{15b}$$

If the magnetic field is not at right angles to the conducting layer ($\zeta_z \neq 1$) the absorption is anisotropic and depends on the direction of the polarization:

$$\left|\frac{\sigma_{xx}^{CR}}{\sigma_{yy}^{CR}}\right| = \frac{1+\tau^2(\omega+\zeta_z^2\Omega_c)^2}{\zeta_z^2(1+\tau^2(\omega+\Omega_c)^2)}. \tag{16}$$

In particular, if the magnetic field is parallel to the surface ($\zeta_z = 0$) there is no resonance absorption for waves which are polarized at right angles to the field.

The basic characteristics of the resonance—amplitude, shape, width—depend significantly on the relations between the reciprocal relaxation time $1/\tau$ and the characteristic frequencies Ω_s and Ω_c.

1. The simplest case is when $\Omega_s \lesssim \Omega_c \ll 1/\tau$. In this case, irrespective of the dependence on the orientation of the magnetic field and the polarization of the wave, the absorption curve has an antiresonance shape with a dip in the region of the spin frequency. The resonance width equals.

$$1/\tilde{T}_2 = 1/T_2 + D(mV)^2(1+\zeta_x^2/2), \tag{17}$$

and the resonance frequency shift $\Delta\omega$ is much smaller than its width. For a wave polarized along the x axis the amplitude is independent of the direction of the magnetic field and for a wave polarized along y the amplitude is proportional to ζ_x^2.

2. In the opposite limiting case $\Omega_c \gtrsim \Omega_s \gg 1/\tau$ there occurs not only spin resonance, but also the broader and stronger cyclotron resonance the frequency of which depends on the inclination of the field ($\zeta_z\Omega_c$). When the resonance frequencies are the same ($\zeta_z = \Omega_s/\Omega_c$) the characteristics of the spin resonance are changed appreciably. The spin-resonance frequency shift

$$\Delta\omega = \frac{\overline{p^2}V^2}{4}\frac{(\Omega_s^2-\zeta_z^2\Omega_c)(\Omega_s(1+\zeta_z^2)+2\zeta_z^2\Omega_c)}{(\Omega_s^2-\zeta_z^2\Omega_c^2)^2+4(\Omega_s/\tau)^2} \tag{18a}$$

and its width

$$\frac{1}{\tilde{T}_2} = \frac{1}{T_2} + \frac{\overline{p^2}V^2}{4\tau}\left(\frac{2\zeta_x^2}{\zeta_z^2\Omega_c^2+1/\tau^2} + \frac{(1+\zeta_x^2)(\zeta_z^2\Omega_c^2+\Omega_s)+4\zeta_z^2\Omega_c\Omega_s}{(\zeta_z^2\Omega_c^2-\Omega_s^2)^2+4(\Omega_s/\tau)^2}\right) \tag{18b}$$

depend resonantly on the inclination of the field and the frequency shift changes sign for $\zeta_z \approx \Omega_s/\Omega_c$. The shape of the CR curve has for any polarization of the wave the shape of ordinary Lorentz resonance for practically the whole ζ_z range and is distorted only in a small neighborhood of a "resonance" inclination of the field $\zeta_z \approx \Omega_s/\Omega_c$. When

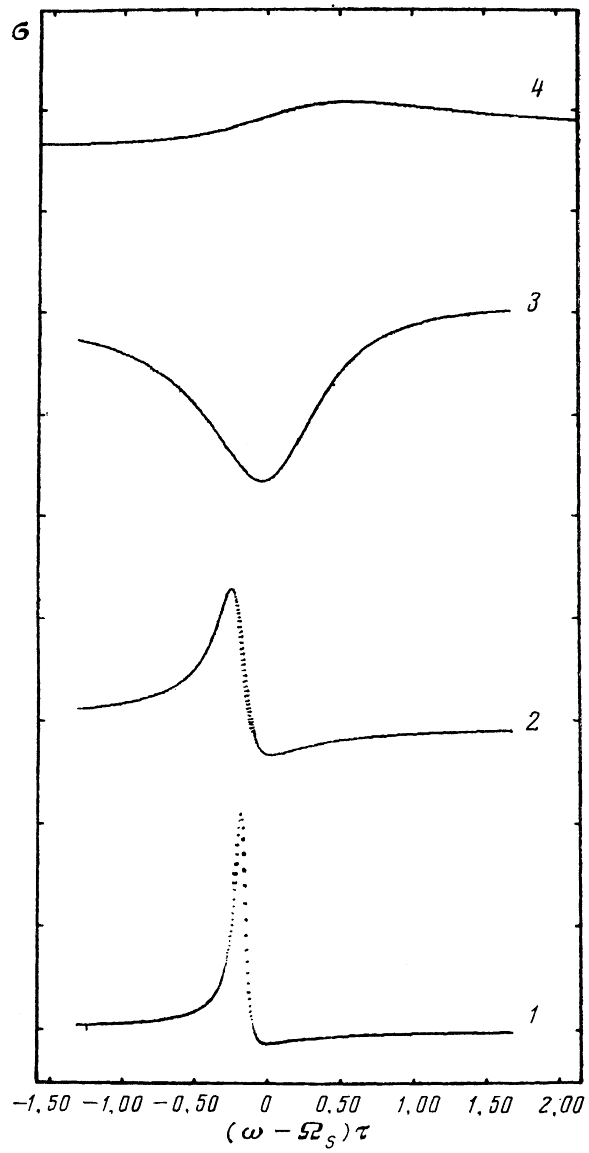

FIG. 2. Curves of combined resonance in a strong magnetic field for different orientations of the field relative to the normal to the surface: 1—$\vartheta = 0°$, 2—$\vartheta = 22.5°$, 3—$\vartheta = 35.3°$, 4—$\vartheta = 90°$ ($\Omega_s = \Omega_c \cos\vartheta$). We used the following parameter values: $\varepsilon_F\tau = 15$, $\Omega_c\tau = 6$, $\Omega_s\tau = 2$, $mV^2\tau/2 = 10^{-3}$.

$\zeta_z = \Omega_s/\Omega_c$ the CR curve takes the antiresonance shape and the "dip" occurs in the center of the broader cyclotron-resonance curve. The CR amplitude also depends resonantly on the inclination of the field and for a wave polarized along the x axis

$$|\sigma_{xx}^{\mathrm{CR}}| \sim \frac{(\Omega_s+\zeta_z^2\Omega_c)^2}{(\zeta_z^2\Omega_c^2-\Omega_s^2)^2+(2\Omega_s/\tau)^2}, \tag{19a}$$

and for a wave polarized along y

$$|\sigma_{yy}^{\mathrm{CR}}| \sim \frac{\zeta_z^2(\Omega_c+\Omega_s)^2}{(\zeta_z^2\Omega_c^2-\Omega_s^2)^2+(2\Omega_s/\tau)^2}. \tag{19b}$$

We show the CR features mentioned here in Fig. 2.

4. RESONANCE SECOND-HARMONIC GENERATION[2)]

As the electron system described by the Hamiltonian (2) has a center of inversion, here is a second-harmonic current response which is proportional to the square of the electric field:

$$eI_{a\mathrm{II}}^0 = \sum_{a_1,a_2} s_{aa_1a_2}E_{a_1}E_{a_2}. \tag{20}$$

We note that there is no rectified conduction current (although there are spin responses $\bar{\mu}^\beta$ and $\bar{I}_a^\beta$ at the zeroth harmonic). Formally this is connected with the fact that in the homogeneous case, under electric excitation, the connection between I^0 and the spin polarization in (8c) is realized only through $\partial\mu/\partial t$.

To evaluate $I_{a\,\mathrm{II}}^0$ we need add to the set (9a)–(9f) equations for the second harmonics $\mu_{\mathrm{II}}^{\zeta,\beta}$ and $I_{a,\mathrm{II}}^{a,\beta,\zeta}$:

$$eI_{a\mathrm{II}}^0 = \frac{1}{2} mV\omega\nu_a(2\omega)\left[\sum_\beta \beta(1+a\beta\zeta_z)\mu_{\mathrm{II}}^\beta + 2a\zeta_x\mu_{\mathrm{II}}^\zeta\right], \tag{21a}$$

$$\left(\frac{1}{T_2}+i(\beta\Omega_s-2\omega)\right)\mu_{\mathrm{II}}^\beta + \frac{mV}{2}\sum_a [(1+a\beta\zeta_z)I_{a\mathrm{II}}^\zeta - \zeta_x a\beta I_{a\mathrm{II}}^\beta]=0, \tag{21b}$$

$$\left(\frac{1}{T_1}-2i\omega\right)\mu_{\mathrm{II}}^\zeta - \frac{mV}{4}\sum_{a,\beta}(1-a\beta\zeta_z)I_{a\mathrm{II}}^\beta=0, \tag{21c}$$

$$(1+i\tau(a\zeta_z\Omega_c+\beta\Omega_s-2\omega))I_{a\,\mathrm{II}}^\beta + mVD(a\beta\zeta_x\mu_{\mathrm{II}}^\beta + (1-a\beta\zeta_z)\mu_{\mathrm{II}}^\zeta) = {}^1/_2\nu E_a\mu^\beta, \tag{21d}$$

$$(1+i\tau(a\zeta_z\Omega_c-2\omega))I_{a\mathrm{II}}^\zeta - {}^1/_2 mVD\sum_\beta (1+a\beta\zeta_z)\mu_{\mathrm{II}}^\beta=0. \tag{21e}$$

On the right-hand side of Eq. (21d) one must substitute the first harmonic of the spin polarization μ^β obtained by solving the set (9b)–(9f). The second harmonics of the spin polarizations μ_{II}^ζ and μ_{II}^β, which determine the response of the conduction current (21a), have resonances both near the spin resonance [when $\omega \approx \Omega_s$ the right hand side of (21d) with μ^1 is at resonance] and near half the spin frequency [when $\omega \approx \Omega_c/2$ the resonance is caused by the first term of (21b)]. In the region of the resonances the coefficients $s_{aa_1a_2}$ of the (20) have the form

1. When $\omega \approx \Omega_s$

$$s_{aa_1a_2} \approx -\frac{i(mV)^3}{32\Omega_s}\zeta_x(a+2a_1+aa_1\zeta_z)(1+a_2\zeta_z) \times \nu_a(2\omega)\nu_{a_1}(\omega)\nu_{a_2}(\omega)B\chi(\omega). \tag{22}$$

2. When $\omega \approx \Omega_s/2$

$$s_{aa_1a_2} = \frac{i(mV)^3}{16\Omega_s}\zeta_x(1+a\zeta_z)(1+a_2\zeta_z) \times a_1\nu_a(2\omega)\nu_{a_1}(0)\nu_{a_2}(\omega)B\chi(\omega). \tag{23}$$

We note that the second harmonic is generated provided the external magnetic field is not at right angles to the conducting layer. The ratio of the resonance responses at the first and the second harmonics is, to order to magnitude, equal to

$$\frac{I_{\mathrm{II}}^0}{I_{\mathrm{I}}^0} \sim \frac{mV}{\hbar\Omega_s}\zeta_x\nu E. \tag{24}$$

For parameter values $V \sim 10^5$ cm/s, $B \sim 0.1$ T, and $\nu \sim 10^3$ cm^2/V·s the characteristic value of the field strength at which the responses at the two harmonics are comparable is $\sim 10^2$ V/cm. Near half the spin frequency ($\omega \approx \Omega_0/2$), a spike appears also on the first harmonics. The amplitude of this second harmonic in the current is

$$I_1{}^0 \sim I_1{}^0 (mV\zeta_x \nu E/\hbar\Omega_s)^2.$$

5. SIMULTANEOUS ACTION OF ALTERNATING ELECTRIC AND MAGNETIC FIELDS

One often measures in experiment resonance reflection or transmission of an electromagnetic wave (e.g., Ref. 4). In that case the spin resonance can be excited both by the electric and by the magnetic component of the wave. Moreover, the presence of an alternating magnetic field leads to a new effect—the resonant generation of direct current (rectifying effect).

For variable electric and magnetic fields acting simultaneously, the resonance part of the spin polarization can be obtained from Eqs. (10) and (11) through the substitution

$$\frac{mV}{2}\sum_a (1+a\zeta_z)\nu_a(\omega)E_a \to \frac{mV}{2}\sum_a (1+a\zeta_z)\nu_a(\omega)E_a - ig\mu_B\tilde{B}_1,$$

where $\widetilde{B}_1 = \widetilde{B}_\xi + i\widetilde{B}_\eta$ is the circular component of the alternating magnetic field. When energy is absorbed there occurs, beside the terms which describe "pure" CR and ESR, an "interference" term proportional to the product of the electric and mangetic fields:

$$\dot{Q} = \dot{Q}_E + \dot{Q}_B + \dot{Q}_{EB}. \qquad (25)$$

The first term describes here the electric dipole contribution to the absorption, which is given by Eq. (14), the second term describes the usual ESR:

$$\dot{Q}_B = \frac{B}{8}|g\mu_B\tilde{B}_1|^2 \operatorname{Re}\chi(\omega), \qquad (25a)$$

and the third term

$$\dot{Q}_{EB} = -\frac{mV}{8} g\mu_B B \sum_a \operatorname{Im}((1+a\zeta_z)\nu_a(\omega)\chi(\omega))\operatorname{Re}(\tilde{B}_1 E_a{}^*) \qquad (25b)$$

gives the interference contribution to the absorption, caused by the superposition of the fields causing the spin transitions (the magnetic field and the effective magnetic field from the spin-orbit interaction).

The presence of an alternating contribution to the cyclotron frequency leads to resonance generation of a direct current:

$$e\overline{I_a{}^0} = \sum_{a_1} \tilde{B}_{1z}{}^* (b_{aa_1}E_{a_1} + c_a\tilde{B}_1), \qquad (26)$$

where the coefficient b_{aa_1} and c_a are given by the equations

$$b_{aa_1} = -\frac{ia}{2c}\nu_a(0)\sigma_{aa_1}^{\rm CR}, \qquad (27a)$$

$$c_a = \frac{a}{8c}\nu_a(0)(1+a\zeta_z)\nu_a(\omega)mVB\chi(\omega). \qquad (27b)$$

6. CONCLUSION

The analysis of the transport equations shows that the amplitude of the CR is in reality determined by the responses of the conduction current I_a^0 and the spin flux I_a^ξ to the variable electric field. One should therefore expect that the expression obtained for this amplitude [Eq. (13)] remains valid in leading order in V also in more complicated situations (e.g., when $1/\tau$, $T \lesssim \Omega_c\zeta_z$ and the Shubnikov oscillations of the conductivity become important), provided we replace $\nu_a(\omega)$ by the real mobility value that determines the response of the current to the alternating field. This statement can immediately be checked for free electrons in any Landau level and also for states which are localized at an impurity (in that case one can in the approximation which is linear in V evaluate the dipole moment matrix element between states with different spin directions). In both cases Eq. (13) gives the correct value of the CR amplitude (see Appendix).

The CR line shape is determined by the relation between the different components of the mobility (the real and imaginary parts of the diagonal and the Hall components). In particular, when Ω_c, $\omega \ll 1/\tau$ and the real part of the diagonal component of the mobility is larger than the other components, the line has an antiresonance shape.

The transport equations (8) do not give a spin-orbit splitting of the cyclotron resonance (this splitting was evaluated in Refs. 2 and 3 for $V\sqrt{p^2} \gg \Omega_c$ under conditions which were te opposite of the ones used by us when we wrote down the transport equations). One can obtain the splitting by expanding the scheme used: the equations for the fluxes contain is a link to higher moments $\langle (p_ip_j - \langle p_ip_j\rangle) f_\beta\rangle$, which we neglected. If we take this link into account and write down equations for these moments we can obtain a splitting of the cyclotron resonance frequency $\Delta\Omega_c \sim mV^2$. We assume, however, that the inequality $\Delta\Omega_c \ll 1/\tau$ holds and we neglect this splitting.

The ratio of the CR and ESR intensities is of the order of

$$\dot{Q}_{\rm CR}/\dot{Q}_{\rm ESR} \sim (mV\nu E/g\mu_B\tilde{B}). \qquad (28)$$

If we take typical values for the hole channel on the (110) surface of Si[6]: $m \approx 0.35m_0$, $V \approx 10^5$ cm/s, and $\nu \approx 3\times10^3$ cm^3/V·s, we get an estimate $\dot{Q}_{\rm CR}/\dot{Q}_{\rm ESR} \sim 10^6$, if the electric and magnetic field strength have the same values. However, the mobility changes within wide limits depending on the frequency, the magnetic field, and the impurity density, and, moreover, due to the large value of the permittivity in semiconductors, the electric field strength in an electomagnetic wave is smaller than the magnetic induction and therefore it may turn out that real situations where the CR and the ESR are of the same order of magnitude can in actual fact be realized. It was shown in Sec. 5 that the absorbed power contains not only terms describing "pure" CR and ESR but also an interference term proportional to the product $\widetilde{B}E$. This contribution, in particular, changes sign when one varies the direction of the wave propagation and it can be shown experimentally.

Nonlinear resonance effects—second harmonic generation [Eq. (17)–(19)] and the rectifying effect [(21), (22a), (22b)]—may be of interest for experimental observations.

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APPENDIX

Let us assume that in the description of the electron quantum states we can take the spin-orbit operator (1) to be a perturbation. We show that the CR amplitude, i.e., the absorption of energy from a variable electric field $\mathbf{E}(t)$ in transitions between spin sublevels of any Landau level [or of a state localized at an impurity] is determined by the response of the velocity to the field $\mathbf{E}(t)$, i.e., by the quantum-mechanical value of the mobility, similar to Eq. (13).

The quantum mechanical expression for the absorption Q in a transition from a state $|\uparrow\lambda\rangle$ to a state $|\downarrow\lambda\rangle$, where λ is the number of the Landau level, under the action of the perturbation $-e\mathbf{E}(t)\mathbf{r}$ has the form

$$\dot{Q}=\frac{\pi}{2}e^2\Omega_s\sum_{a,a_1}E_a^*E_{a_1}\langle\uparrow\lambda|r_{-a_1}|\downarrow\lambda\rangle\langle\downarrow\lambda|r_{-a}|\uparrow\lambda\rangle\mu_\lambda^\zeta\delta(\Omega_s-\omega). \tag{A1}$$

Here $\mu_\lambda^\zeta=f_{\downarrow\lambda}-f_{\uparrow\lambda}$ is the spin polarization of the level λ, i.e., the difference between the probabilities of occupying the sublevels with down and up spin, respectively, $r_a=x+iay$ $(a=\pm1)$. The wave functions with first-order corrections in $\hat{\mathscr{H}}_{so}$ of (1) are the following:

$$\Psi_{\downarrow\lambda}=\Psi_{\downarrow\lambda}{}^0-\frac{mV}{2}\sum_{\lambda'}\Big[\zeta_x\frac{\langle\lambda'|\hat{v}_y|\lambda\rangle_0}{E_\lambda-E_{\lambda'}}\Psi_{\downarrow\lambda'}+\frac{1}{2}\frac{\langle\lambda'|\zeta_z\hat{v}_y-i\hat{v}_x|\lambda\rangle_0}{E_\lambda-E_{\lambda'}-\Omega_s}\Psi_{\uparrow\lambda'}\Big],$$
$$\Psi_{\uparrow\lambda}=\Psi_{\uparrow\lambda}{}^0-\frac{mV}{2}\sum_{\lambda'}\Big[-\zeta_x\frac{\langle\lambda'|\hat{v}_y|\lambda\rangle_0}{E_\lambda-E_{\lambda'}}\Psi^0_{\uparrow\lambda'}+\frac{1}{2}\frac{\langle\lambda'|\zeta_z\hat{v}_y+i\hat{v}_x|\lambda\rangle_0}{E_\lambda-E_{\lambda'}+\Omega_s}\Psi^0_{\downarrow\lambda'}\Big], \tag{A2}$$

where $\Psi^0_{\uparrow,\downarrow\lambda}$ and $E_\lambda\pm\Omega_s/2$ are the zeroth-approximation functions and energies, and $\hat{\mathbf{v}}=m^{-1}(\mathbf{p}-e\mathbf{A}/c)$. Using (A2) we can write the coordinate matrix element in the following form:

$$\langle\uparrow\lambda|r_{-a}|\downarrow\lambda\rangle=\frac{imV}{4e}(1+a\zeta_z)v_a{}^\lambda(\Omega_s),$$
$$v_a{}^\lambda(\omega)=\frac{e}{2}\sum_{\lambda'}\Big[\frac{\langle\lambda|r_a|\lambda'\rangle_0\langle\lambda'|\hat{v}_{-a}|\lambda\rangle_0}{E_{\lambda'}-E_\lambda-\omega}+\frac{\langle\lambda'|r_a|\lambda\rangle_0\langle\lambda|v_{-a}|\lambda'\rangle_0}{E_{\lambda'}-E_\lambda+\omega}\Big]. \tag{A3}$$

Here the $v_a^\lambda(\omega)$ are the circular components of the mobility, determining the response of the velocity of the variable field $\mathbf{E}$. Indeed, one can easily show that $\langle\lambda|\hat{v}_a|\lambda\rangle=v_a^\lambda(\omega)E_a$ for the perturbation $-e\mathbf{E}\mathbf{r}$. Using (A3) in (A1) we get for the loss an expression of the type of the (14) with an appropriate $\sigma_{aa_1}^{\rm CR}$ which is the same as (13) when in the latter we change from a Lorentz resonance to a δ function.

1)In atomic semiconductors the times T_1 and T_2 are determined by the spin-orbit interaction, modulated by the lattice vibrations; according to Ref. 11, $T_{1,2}^{-1}\propto(\Delta g)^2/\tau$, where Δg is the shift in the g factor upon deformation. Since $\Delta g\propto V$, both terms in $\widetilde{T}_{1,2}^{-1}$ are of the same order of smallness in the spin-orbit interaction ($\propto V^2$).

2)The nonresonance response to the second harmonic at high frequencies ($\omega\gg\Omega_s$, $1/\tau$) is considered in Ref. 12.

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