# Electrodynamics of a Josephson medium in a high-temperature superconductor. Impedance in the mixed state

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The microwave response of a high-temperature superconductor with multiple weak links (Josephson medium) is considered. The analysis is based on averaged electrodynamic equations for a Josephson medium in the mixed state that can be produced in such a medium in a very weak lower critical magnetic field $\lesssim 10^{-1}$ Oe. In this case the magnetic flux penetrates the medium in the form of hypervortices, i.e., quantized vortices that are unique to the superconducting medium, have multiple weak links, and exceed in size the London penetration depth and the dimensions of the homogeneous grains separated by weak links. The dependence of the microwave losses on the weak magnetic field, observed in a number of experiments, is incontrovertibly attributed to viscous losses incurred in the oscillatory motion of the hypervortices.

## 1. INTRODUCTION

The presently available aggregate of experimental data on high-temperature superconductors (HTSC) points to a substantial role of inhomogeneities in these materials. In the analysis of HTSC properties it is therefore necessary to distinguish between properties due to the microstructure on an atomic level and the properties connected with large-scale inhomogeneities, i.e., with the macrostructure (twinning domains and twinning boundaries in single crystals, grain boundaries in ceramics). The present article is devoted to the electrodynamics of HTSC, i.e., to a theoretical analysis of the microwave response due to the macrostructure of the material. We consider the theory developed in the present paper to serve two purposes: 1) to present a theoretical qualitative interpretation of the electrodynamic experiments in the widely used inhomogeneous HTSC; 2) to be able to deduce from the large experimental material accumulated by now on the magnetic and electrodynamic properties those quantities that actually pertain to the fundamental bulk mechanism of the superconductivity of HTSC.

Several models of the inhomogeneous macrostructure in HTSC are under discussion at present.[1)] When it comes to single crystals all are connected in one way or another with the role of the twin boundaries. This role is found to be quite significant compared with that of low-temperature superconductors, owing to the exceptionally short coherence length in HTSC (close to the interatomic distance). It is suggested in Ref. 1 that these boundaries stimulate the superconductivity, which sets in on a boundary earlier than in the bulk, as was previously observed in experiment by Khaĭkin and Khlyustikov[2] for low-temperature superconductors. Andreev[3] also considered a case in which the superconductivity sets in on twin boundaries earlier than in the bulk, but suggested an ordering of different symmetry, in which the order parameter at the center of the boundary vanishes. Finally, many authors, for example Deutscher and Müller,[4] assume that twin boundaries are weak links, just as Josephson junctions. Much experimental evidence favors this assumption, particularly experiments to observe magnetic-flux distribution in HTSC by the decoration methods[5]; they indicate that the magnetic fields tends to be concentrated near twin boundaries. We shall adhere to this version of HTSC macrostructure inhomogeneity in the present paper.

A superconductor model with multiple weak links was investigated theoretically already in conjunction with the study of granulated superconductors.[6–8] In further development of this research,[9] an averaged description was considered of an inhomogeneous superconducting medium with weak links (a Josephson medium) for HTSC. It was shown that in such a medium, starting with very weak magnetic fields of the order of the earth's and less there can occur a unique mixed state in which the magnetic flux penetrates the bulk in the form of fluxons that differ in structure from Abrikosov and Josephson vortices. Owing to their large dimension (larger than that of the grains into which the weak links subdivide the superconductor volume) they were named hypervortices in Ref. 9. Our present calculation of the microwave response of a Josephson HTSC medium in a mixed state on hypervortices explains the experimentally frequently observed magnetic-field-dependent microwave absorption in HTSC, which sets in practically at zero values of the magnetic field (see, e.g., Refs. 10–13). This absorption does not accord with the notion of a homogeneous type-II superconductor with a lower critical field of order ten to several hundred Oe, since no such absorption should take place in the Meissner state. This microwave absorption was therefore attributed from the very outset to inhomogenity effects—weak Josephson links and possible glassy behavior of HTSC was suggested. It seems to us that the first attempt to describe this absorption is suggested in Ref. 13 and in the present paper. The authors of Ref. 13 relate the microwave absorption with motion of Abrikosov vortices at the London penetration depth of the electromagnetic wave into the sample. In our paper, on the other hand the absorption is attributed to motion of hypervortices in a Josephson medium at a penetration depth greatly exceeding the London depth for the grain material. We hope to show that it is just this approach to this problem that is internally incontrovertible and self-consistent.

## 2. ELECTRODYNAMICS OF MIXED STATE

In the London electrodynamics, the energy and the superconducting current of a superconductor with multiple weak links takes, after averaging over scales exceeding the

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grain size $d$, the form[9]

$$\mathcal{F}=\int dV\left[\frac{g}{2}\left(\nabla\varphi-\frac{2\pi\mathbf{A}}{\Phi_0}\right)^2+\frac{1}{8\pi\mu}(\mathrm{rot}\,\mathbf{A})^2\right], \quad (1)$$

$$\mathbf{j}_s=\frac{2e}{\hbar}\frac{\delta\mathcal{F}}{\delta(\nabla\varphi)}=\frac{2e}{\hbar}g\left(\nabla\varphi-\frac{2\pi\mathbf{A}}{\Phi_0}\right). \quad (2)$$

The total current $\mathbf{j}$ includes also the normal component:

$$\mathbf{j}=\mathbf{j}_s+\mathbf{j}_n, \quad \mathbf{j}_n=\sigma_n\mathbf{E}. \quad (3)$$

Here $\mathbf{E}$ is the electric field, $\mathbf{B}=\mathrm{curl}\,\mathbf{A}$ the magnetic field, $\Phi_0=hc/2e$ the magnetic-flux quantum, $\varphi$ the averaged phase of the order parameter, $g$ the rigidity, and $\mu$ the effective magnetic permeability. A homogeneous superconductor has $\mu=1$ and $g=\hbar^2n_s/4m$ ($n_s$ is the density of the superconducting component of the electron fluid). The values of $\mu$, $g$, and the normal conductivity $\sigma_n$ for a Josephson medium are discussed in Sec. 3.

The superconducting current $\mathbf{j}_s$ is determined from nondissipative equation of motion:

$$\frac{\partial\mathbf{j}_s}{\partial t}=\frac{4\pi^2c^2g}{\Phi_0^2}\mathbf{E}. \quad (4)$$

The total complex conductivity $\sigma(\omega)=j/E$ and the inductance $\mathcal{L}_s$ are equal to

$$\sigma(\omega)=-\frac{c^2}{i\omega\mathcal{L}_s}+\sigma_n, \quad (5)$$

$$\mathcal{L}_s=\Phi_0^2/4\pi^2g. \quad (6)$$

The electromagnetic field penetrates into the superconductor to a depth $1/k$,

$$k^2=4\pi\mu\sigma(\omega)i\omega/c^2=-1/\lambda^2+4\pi\mu\sigma_n i\omega/c^2. \quad (7)$$

Here $\lambda$ is the effective penetration depth for a constant magnetic field ($\omega\rightarrow0$):

$$\lambda^2=\Phi_0^2/16\pi^3g\mu=\mathcal{L}_s/4\pi\mu. \quad (8)$$

For the surface impedance $\zeta=E_t/H_t$ ($E_t$ and $H_t$ are the tangential components of the electric and magnetic fields at the interface with the vacuum) and for the active surface resistance

$$\zeta=\mu\omega/ck\rightarrow-i\mu\omega\lambda/c \text{ for } \omega\rightarrow0, \quad \rho_\square=4\pi c^{-1}\,\mathrm{Re}\,\zeta. \quad (9)$$

In the Meissner state, a nonzero surface resistance is produced only through $\sigma_n$. A noticeable dependence of $\sigma_n$ on the magnetic field (magnetoresistance) is possible here only for sufficiently strong magnetic fields quite remote from the weak field at which the **H**-dependent absorption was observed in HTSC. We turn now to the electrodynamics of the mixed state, in which the magnetic flux penetrates into the superconductor in the form of quantized vortices.

The presence of vortices in a superconductor is taken into account by replacing $\mathbf{E}$ by $\mathbf{E}+c^{-1}[\mathbf{v}_L\times\mathbf{B}]$ in Eqs. (3) and (4) that relate the currents $\mathbf{j}_s$ and $\mathbf{j}_n$ with the electric field $\mathbf{E}$; here $\mathbf{v}_L$ is vortex velocity and $\mathbf{B}$ is the average induction inside the superconductor and is equal to $\Phi_0 n_v$ ($n_v$ is the two-dimensional density of the vortices). To obtain a closed system of electrodynamic equations we need an equation of motion for the vortices. It takes in the general case the form of some tensor linear relation connecting the current with the displacement of the vortices, and also their time derivative (velocity). We, however, wish to confine ourselves to a simplest vortex-motion model, which has been successfully used for mixed states on Abrikosov vortices. The vortex equation of motion is then

$$c^{-1}[\mathbf{\Phi}_0\mathbf{j}]=-k\mathbf{r}_L-\eta\mathbf{v}_L, \quad (10)$$

where $\Phi_0$ is the modulus of a vector directed along the vortex, $\mathbf{r}_L$ is the vortex displacement, and $\mathbf{v}_L=\partial\mathbf{r}_L/\partial t$. Thus, according to (10), the Lorentz force [the left-hand side of (10)] is balanced by the restoring pinning force $-k\mathbf{r}_L$ and by the viscous force $-\eta\mathbf{v}_L$. Eliminating the vortex displacements from the equations, we find that the complex conductivity in the mixed state $\sigma_B(\omega)$ in a plane perpendicular to the magnetic field differs from the conductivity $\sigma(\omega)$ in the Meissner state (5) by a normalization factor that depends on the induction $B$:

$$\sigma_B(\omega)=\left\{1-\sigma(\omega)\frac{B\Phi_0}{c^2}\frac{i\omega}{k-i\omega\eta}\right\}^{-1}\sigma(\omega)$$

$$\approx-\frac{c^2}{4\pi\lambda^2\mu i\omega}\left(1+\frac{B\Phi_0}{4\pi\lambda^2\mu}\frac{1}{k-i\omega\eta}\right)^{-1}. \quad (11)$$

This expression agrees with the analogous expressions for the conductivity in the mixed state, obtained by others in the same model but with $\mu=1$ (see, e.g., Refs. 13 and 16). They, however, used this model as applied to Abrikosov vortices and took the penetration depth $\lambda$ to the London depth $\lambda_L$ into the superconductor bulk. We, however, shall use hereafter expression (11) for $\sigma_B$ for the mixed state of a Josephson medium with hypervortices, in which case the effective depth will be substantially larger than the London depth $\lambda_L$.

From (11) in the limit $\omega\rightarrow0$, neglecting pinning ($k=0$), we obtain the resistance to the viscous flow

$$\rho_f=1/\sigma_B(0)=B\Phi_0/c^2\eta, \quad (12)$$

which is known to be determined from the slope of the IVC in a magnetic field at currents exceeding the critical value and corresponding to detachment of the vortices from the pinning centers.[15] In this case the suppression of the pinning is due to nonlinearity. According to (11), however, the effect of pinning can be suppressed also by the frequency $\omega$ if $\omega\gg\omega_\eta=k/\eta$, and in this case one can speak of a viscous vortex-motion regime. If the term $\sim B$ in the denominator of (11) is small compared with unity, substitution of (11) in (9) leads to frequency-independent active surface resistance in the mixed state:

$$\rho_\square=B\Phi_0/2\lambda c^2\eta=\rho_f/2\lambda. \quad (13)$$

According to (12) and (13) $\rho_f$, just as the surface resistivity $\rho_\square$ is proportional to $B$. It must be remembered, however, that we are considering a case when there are many vortices at the effective penetration depth, i.e., the external magnetic field noticeably exceeds the lower critical field of the transition to the mixed state, while $B/\mu$ is approximately equal to the external magnetic field $H$. In the case of a homogeneous superconductor $\mu=1$, while for a Josephson medium $\mu$ can be much less than unity owing to the diamagnetic response of the grains.[9] It must also be added that the linear dependence of $\rho_f$ of $\rho_\square$ on $B$ presupposes that the hypervortex density is not too high, i.e., $B\ll\Phi_0/d^2$.

We examine now the self consistency in the region where the foregoing theory is applicable. The theory is valid so long as enough vortices are located at the penetration depth $\lambda$, and their effect can be described in terms of their density, i.e., the distances between the vortices are much smaller than the penetration depth $\lambda$. This takes place only far enough from the transition into the mixed state, when $B \gg B_{c1} = \mu H_{c1} \sim \Phi_0/\lambda^2$. At the same time, the authors of Ref. 13 propose that they are dealing with a captured flux corresponding to a field weaker than the critical field for the transition into a truly mixed state, while microwave absorption by ordinary Abrikosov vortices takes place at the London penetration depth $\lambda_L$ which they estimate to be $\sim 200$ Å. But if this is so, the microwave-response theory developed here and in Ref. 13 cannot be used (the penetration depth is much smaller than the distances between vortices) and it is difficult to expect a loss growth nonlinear in $B$, even if it is assumed that some number of vortices has landed, via static fluctuations, in a narrow layer of thickness $\lambda_L$. We assume therefore that the use of the here-discussed microwave-response theory is self-consistent and is internally noncontradictory for weak fields only if it is applied to a mixed state of a Josephson medium with hypervortices.

To analyze, on the basis of the foregoing theory, microwave absorption in HTSC in weak magnetic fields, we need estimates of the parameters of the medium. These are the subject of the sections that follow.

### 3. EFFECTIVE PARAMETERS OF A JOSEPHSON MEDIUM

The effective rigidity $g$ is determined from an analysis of the phase distribution along the superconducting current passing through a grain and a weak coupling. The superconducting current is connected with the phase differences $\delta_g \varphi$ and $\delta_j \varphi$ on the grain and on the junction by the relations

$$j_s = \frac{en_s\hbar}{2m}\frac{\delta_\Gamma\varphi}{d}, \quad j_s = j_c \sin \delta_j\varphi \sim j_c\delta_j\varphi, \tag{14}$$

where $j_c = 2eE_J/\hbar$ is the critical current through the weak coupling, and $E_J$ is the energy of the weak coupling per unit area. This yields an expression for the current $j_s$ through the average phase gradient along the current line, $\nabla\varphi = (\delta_g \varphi + \delta_j \varphi)/d$ (the junction thickness is small compared with the grain size $d$):

$$j = 2eg\nabla\varphi/\hbar, \quad 1/g = 4m/\hbar^2 n_s + 1/E_J d. \tag{15}$$

Thus, $g = \hbar^2 n_s/4m$ if $E_J d \gg \hbar^2 n_s/4m$, i.e., the coupling is actually "not weak" and does not influence noticeably the properties of the superconductor. The weak coupling only renormalizes the parameters of the medium in the opposite limit $E_J d \ll \hbar^2 n_s/4m$. In this case $g = E_J d$. Note that $E_J$ coincides here with the energy of an isolated weak coupling only far from the phase transition into the coherent state. Owing to the grain phase fluctuations, this transition takes place at a temperature lower than the superconducting transition temperature for the grain material.[6–8]

The effective magnetic permeability $\mu$ is determined from the condition that the magnetic energy in (1), expressed in terms of the average field $B$, is equal to the energy of the nonaveraged inhomogeneously distributed (over the grain volume) magnetic field that decreases with increase of the weak coupling in the scale of the London penetration depth $\lambda_L = (\Phi_0/h)(m/\pi n_s)^{1/2}$. Obviously, for $d$ much less than $\lambda_L$ the permeability $\mu$ is close to unity, $1 - \mu \approx d^2/\lambda_L^2$. In the opposite limit $d \gg \lambda_L$ we have $\mu \sim \lambda_L/d$, i.e., it is small because of the weak penetration of the magnetic field into the bulk of the grain.

We consider now the effective complex conductivity of a Josephson medium, or more accurately speaking, the first two terms of its Laurent expansion in the frequency, viz., the term $\sim 1/\omega$ connected with the inductance, and the frequency-independent normal conductivity [see (5)]. The inductance is directly connected with the rigidity $g$ by Eq. (6), so that it has already been determined by us; expression (15) for $g$ shows that the inductance of the medium, which is inversely proportional to $g$, is the direct sum of the grain-bulk and the weak-coupling (junction) inductances. In the derivation of (15), however, we considered only the superconducting channel for the electric current $j_s$, and not the more rigorous condition that the total current $\mathbf{j} = \mathbf{j}_s + \mathbf{j}_n$ be constant. We shall see presently, however, that this approximation is fully justified at low frequencies $\omega$.

Passage of current through a Josephson medium at low frequency can be described by the equivalent electric circuit shown in Fig. 1, which includes the inductances

$$L_g = \frac{d}{\pi^2}\frac{\Phi_0^2 m}{\hbar^2 n_s}, \quad L_j = \frac{\Phi_0^2}{4\pi^2 E_J} \tag{16}$$

of the grain and the junction, as well as their active resistances $R_j$ and $R_g = d/\sigma_{ng}$, where $\sigma_{ng}$ is the normal conductivity of the material inside the grain. All the quantities are defined per unit junction area and per grain–junction layer. For such a scheme, the total impedance is

$$Z(\omega) = (-c^2/i\omega L_g + 1/R_g)^{-1} + (-c^2/i\omega L_j + 1/R_j)^{-1}. \tag{17}$$

Expanding the admittance of the medium $\sigma(\omega) = d/Z(\omega)$ in powers of the frequency and retaining the first two terms, we obtain Eq. (5) in which $\mathscr{L}_s = (L_g + L_j)/d$, which agrees fully, according to (6) and (16), with the corresponding expression for $g$ in (15), and the normal conductivity is

$$\sigma_n = dL_j^2/R_j(L_g + L_j)^2 + \sigma_{ng}L_g^2/(L_g + L_j)^2. \tag{18}$$

We see thus that the distribution of the active resistance between the grain and the junction has no effect whatever, at low frequency $\omega$, on the total inductance of the medium. On the other hand, however, the inductance distribution among the grain and the junction influences substantially the normal active conductivity. If the coupling is very weak, then $L_j \gg L_g$ and the active resistance of the grain is completely shunted by its low inductance and, according to (18), the effective normal conductivity depends only on the junction

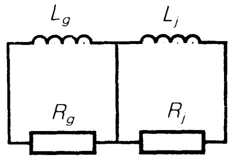


FIG. 1. Equivalent circuit of a Josephson medium. $R_g$ and $L_g$—active resistance and inductance of grain, $R_j$ and $L_j$— the same for weak coupling (Josephson junction).

resistance:

$$\sigma_n \approx d/R_j + \sigma_g (4E_J\, dm/\hbar^2 n_s)^2 \underset{E_J\to 0}{\longrightarrow} d/R_j. \tag{19}$$

For lack of direct data on the conductivity $\sigma_n$ we shall estimate it below (Sec. 5), by comparison with experiment, using the experimental data on the normal-state conductivity $\sigma_{T>T_c}$. We compare therefore these conductivities close to both sides of the superconducting transition. Above the transition

$$\sigma_{T>T_c} = (R_j/d + 1/\sigma_r)^{-1}, \tag{20}$$

where $\sigma_g$ is the conductivity in the grain and $R_j$ is the resistance in the intergrain boundary of thickness $b$ ($b \ll d$). Assuming that the grain and intergrain-boundary resistances are continuous in a superconducting transition, it can be assumed that $\sigma_g \approx \sigma_{ng}$ and that $R_j$ is the same in expressions (18) and (20) for $\sigma_n$ and $\sigma_{T>T_c}$. It is evident then from the same expressions that $\sigma_n$ and $\sigma_{T>T_c}$ do not become matched at the transition temperature, since $\sigma_n$ depends on the ratio of the inductance $L_g$ and $L_j$. If the coupling is weak, i.e., $L_j \gg L_g$ and the conductivity $\sigma_n$ is determined by the active resistance of the weak coupling according to (19), then $\sigma_n \approx \sigma_{T>T_c}$ (i.e., the active resistance of the medium is continuous in the transition) only if at $T > T_c$, too, all the resistance is in the intergrain boundary (twin boundary of the single crystal) and $\sigma_g \gg d/R_j$. If, however, the integrain-boundary conductivity is the same as in the grain, i.e., $R_j = b/\sigma_g$ ($b$ is the width of the boundary), then $\sigma_{T>T_c} = \sigma_g$ and, according to (19),

$$\sigma_n = \sigma_{T>T_c} d/b. \tag{21}$$

The normal conductivity below $T_c$ can thus exceed appreciably the conductivity in the normal state above $T_c$. However, when considering such a "jump" of the effective conductivity in a superconducting transition, it must be borne in mind that it takes place in a finite temperature interval whose width depends on frequency. This dependence is explained by the fact that expression (18) for $\sigma_n$ is based on an expansion in terms of frequency, whose validity range becomes narrower as $T_c$ is approached, for in this limit $L_g$, $L_j \to \infty$.

As the frequency increases and approaches the plasma frequency, the capacitive contribution to the impedance becomes important. For a more complicated equivalent circuit than in Fig. 1, with the capacitances of the junction and of the grain added, the expression for the admittance becomes quite unwieldy. We confine ourselves therefore to a discussion of the plasma frequency of a Josephson medium for the case of greatest interest in which, owing to the shunting of the grain conductances by the small inductance, the properties of the medium are determined only by weak couplings. A noticeably electric field is produced for the plasma oscillations only inside a weak coupling. We can then introduce, besides the magnetic permeability $\mu$ that reflects the inhomogeneity of the magnetic-field distribution, the effective dielectric constant $\varepsilon$ of the medium, which takes into account the inhomogeneity of the electric field $\mathbf{E}$ concentrated in the weak coupling. From the condition that the electric-field energy must be equal to $\varepsilon E^2/8\pi$, where $\mathbf{E}$ is the average electric field, we obtain $\varepsilon \sim d/b$. After substituting $\varepsilon = d/b$ and $g = E_J d$, the plasma frequency of the Josephson medium

$$\omega_p^2 = 4\pi g c^2/\Phi_0^2 \varepsilon \tag{22}$$

coincides with the plasma requency of an isolated Josephson junction[17] and is independent of the grain size $d$. This natural result is evidence that the concept of a Josephson medium is not contradictory. One can consider also the propagation, in such a medium, of transverse waves with spectrum

$$\omega^2 = \omega_p^2 + c^2 k^2, \tag{23}$$

where the velocity $c = c/(\varepsilon\mu)^{1/2}$ is much lower than the speed of light, in view of the large values of $\varepsilon\mu$. It is also important that the plasma-oscillation frequency $\omega_p$ in a Josephson medium is considerably lower than in a homogeneous superconductor and is $\gtrsim 10^{10}$ s$^{-1}$. It is legitimate therefore to treat these oscillations phenomenologically, without resorting to the more complicated microscopic theory. Experimental observation of plasma complicated microscopic theory. Experimental observation of plasma oscillations near $T_c$ is impossible in view of the large damping ($\mathrm{Im}\,\omega \sim \sigma_n$), but as the temperature is lowered, and $\sigma_n$ correspondingly decreased, this problem may apparently be experimentally solvable. For a more profound analysis of this possibility, as well as of the possibility of observing waves with spectrum (23), it is necessary to take seriously into account the static random fluctuations of the Josephson medium, i.e., consider the effects of glassy behavior.

## 4. DYNAMIC PARAMETERS OF VORTICES

To obtain from the equations of Sec. 2 theoretical estimates of the microwave absorption it is necessary to determine the dynamic parameters of the vortices, viz., the pinning-force parameter $k$ and the viscous-friction coefficient $\eta$. The friction coefficient $\eta$ for Abrikosov vortices in the mixed state of homogeneous type-II superconductors was intensively investigated experimentally and theoretically in Refs. 15 and 18 in connection with a determination of the flow resistance $\rho_f$ (12). A rigorous theoretical determination requires a theory of the processes that take place in the core of the moving vortex. The time-dependent Ginzburg–Landau theory was used for this purpose in the case of a dirty homogeneous type-II superconductor.[18] But if numerical coefficients of order unity can be neglected, the results of this theory are given by the simpler and cruder Bardeen–Stephen model.[19] Viscous losses are defined in this model as ohmic losses in a core having the size of the correlation length $\xi$, assumed to go over completely into a normal state with conductivity $\sigma_n$. Then

$$\eta = \frac{\Phi_0^2 \sigma_n}{c^2 \xi^2} = \frac{\Phi_0 \sigma_n}{c^2} H_{c2}, \tag{24}$$

where $H_{c2} = \Phi_0/\xi^2$ is the upper critical magnetic field.

Let us attempt to use this estimate for a hypervortex in a Josephson medium. In the general case it is natural to define the core dimension as the one of that core-center region whose description requires allowance for nonlinear effects. The dimension of a hypervortex core is therefore determined by the dimension $d$ of the grain. Indeed, at a distance $\sim d$ from the hypervortex center the number of weak couplings crossed by a current line around the center is of the order of unity. When the weak coupling condition $E_J d \ll n_s \hbar^2/m$ is met, the phase jump at the coupling is also of the order of unity, i.e., the current through the coupling is comparable with the critical and a nonlinear expression must be used for it. In the upshot, the friction coefficient for hypervortices is defined as

$$\eta \sim \Phi_0^2 \sigma_n / c^2 d^2 \sim \Phi_0 \sigma_n B_d / c^2, \qquad (25)$$

where $B_d = \Phi_0/d^2$—the analog of the field $H_{c2}$ in expression (24) for the Abrikosov vortex— is the average magnetic field (magnetic induction) at which the theory of an effective Josephson medium ceases to be valid.[9]

It must be emphasized that expression (25) contains just the normal conductivity $\sigma_n$ of the medium below $T_c$, and not the conductivity in the normal state above $T_c$, as assumed in the original Bardeen–Stephen model.[15,19] The difference between $\sigma_n$ and $\sigma_{T>T_c}$ is immaterial for Abrikosov vortices in a homogeneous superconductor, but it can be appreciable, as shown in Sec. 3, for a Josephson medium. To show that (25) contains just $\sigma_n$, we estimate the Ohmic losses in a hypervortex core of size $d$. For sufficiently weak couplings these losses occur only in weak couplings, and the rate of energy dissipation in the core per unit hypervortex length is $\dot{Q} \sim dV^2/R_j$, where $V$ is the voltage drop across the weak bond and is equal to

$$V = \frac{\hbar}{2e} \frac{\partial(\delta\varphi_g)}{\partial t} \sim \frac{\hbar}{2e} \frac{v_L}{d} \sim \frac{\Phi_0}{d} \frac{v_L}{c}. \qquad (26)$$

Here $\delta\varphi_g$ is the phase difference on the weak coupling, the order of magnitude of the spatial gradient along which being of the order of $\sim 1/d$. Then

$$\dot{Q} \sim \Phi_0^2 v_L^2 / c^2 R_j d \sim \Phi_0^2 \sigma_n v_L^2 / c^2 d^2. \qquad (27)$$

On the other hand, the dissipation rate $\dot{Q}$ is connected with $\eta$ by the relation $\dot{Q} = \eta v_L^2$, from which follows expression (25) for $\eta$. An estimate of the Ohmic losses outside the core makes a contribution of the same order to $\eta$.

We substitute now the estimate (25) for $\eta$ in the expressions (12) and (13) for the flow resistivity $\rho_f$ and for the surface resistivity $\rho_\square$:

$$\rho_f \sim \rho_n \frac{Bd^2}{\Phi_0} = \rho_n \frac{B}{B_d}, \qquad \rho_\square \sim \frac{\rho_n}{2\lambda} \frac{Bd^2}{\Phi_0} = \frac{\rho_n}{2\lambda} \frac{B}{B_d}. \qquad (28)$$

These expressions go over into analogous expressions for Abrikosov vortices upon the substitutions $d \to \xi$ and $B_d \to H_{c2}$.

The foregoing estimates of the microwave losses were obtained, just as in Ref. 13, for the viscous vortex-flow regime, when the pinning force $\sim k$ can be disregarded. Hypervortex pinning in a Josephson medium is the inevitable consequence of the medium's inhomogeneity. Indeed, displacement of a hypervortex by one grain, i.e., by a distance of order $d$, is accompanied by surmounting a potential barrier of magnitude $\sim E_J d$ per unit length of the supervortex (see Ref. 20). From this we get an estimate from the pinning-force parameter:

$$k \sim E_J / d. \qquad (29)$$

The pinning force influences substantially the microwave response at low frequencies that are lower than and of the order of

$$\omega_\eta = \frac{k}{\eta} \sim \rho_n \frac{E_J d c^2}{\Phi_0^2} \sim \rho_n \frac{c^2}{\lambda^2 \mu} \sim \rho_\square \frac{c^2}{\lambda \mu} \frac{B_d}{B}. \qquad (30)$$

Pinning is closely related to the irreversibility and glassy behavior of superconductors. These questions were recently considered, in particular, in Ref. 21 for an exactly solvable model. It was established that substantial glassy effects can be expected for magnetic fields that are strong enough compared with the screening-current fields. We consider here the inverse limiting case of a weak magnetic field, when the magnetic flux in the bulk consists of individual weakly interacting fluxons (the condition $H < \Phi_0/d^2$). The randomness and disorder effects might influence the results in the frequency region $\omega < \omega_\eta$. For the viscous regime $\omega > \omega_\eta$ considered mainly in the present paper, the results cannot be noticeably sensitive to disorder effects.

## 5. COMPARISON WITH EXPERIMENT

Let us estimate the surface resistivity $\rho_\square$ for YBaCuO by starting from the fact that the typical resistivity of this material in the normal state is $\rho_{T>T_c} \sim 10^{-3}$ Ω·cm, and the grain dimension is $d \sim 10^{-4}$ cm. In weak fields, expulsion of the 80–90% of the flux is observed (see, e.g. Ref. 22), and assuming that these fields correspond to a mixed state on hypervortices, it can be assumed that $\mu \sim 0.1$, i.e., the hypervortex is "reticular" (see Ref. 9). Using then the estimate $g \sim E_J d \sim T_c/d$ discussed in Ref. 9, and also relation (8), we obtain an effective penetration depth $\lambda \sim 10^{-3}$ cm. The London penetration depth can be estimated from the critical magnetic field $H_i$ at which Abrikosov vortices begin to appear in the grains. This yields $\lambda_L \sim 10^{-5}$ cm (see Refs. 9, 13, and 22), which agrees with the estimate $\mu \sim \lambda_L/d \sim 0.1$. The result of the succeeding estimate of $\rho_\square$ in accordance with (28) depends on whether we make the assumption $\rho_n \sim \rho_{T>T_c}$, i.e., the entire resistance above $T_c$ is determined by the resistance of the integrain boundaries, or the assumption $\rho_n \sim \rho_{T>T_c}$ (see Sec. 3). In the former case we obtain $\rho_\square/B \sim 0.03$ Ω/Oe and in the latter $\rho_\square/B \sim 0.3 \times 10^{-4}$ Ω/Oe. In Ref. 13 is cited a value $\rho_\square/H = 0.02$ Ω/Oe determined for YBaCuO from microwave absorption. This corresponds, assuming $\mu \approx B/H \sim 0.1$, to $\rho_\square/B = 0.02$ Ω/Oe, in the interval between the two foregoing estimates obtained for $\rho_\square/B$ from the normal resistance. This is evidence that the interpretation of the microwave absorption as being absorption by hypervortices does not contradict the experiment. More categorical conclusion might be drawn in the presence of reliable experimental values of $\rho_n$ below the transition, and of the other parameters used in the theory, as well as by verifying the relation (13) between $\rho_\square$ and $\rho_f$. It must be borne in mind, however, that the resistivity $\rho_f$ to the flow pertains to a mixed state of the hypervortices, so that it must be determined from a reduction of the section of the $\rho(H)$ curve at small $H$, for which we have not found reliable experimental data in the literature. The degree of reliability of our comparison of the theory with experiment suffers also from the fact that it was necessary to use in the theoretical relations parameters taken from various experimental studies, i.e., for different samples.

To conclude this section we note that the frequency $\omega_\eta$ above which pinning forces can be disregarded amounts according to (30) to $4\cdot 10^8$ $s^{-1}$ for the parameters assumed above, noticeably lower than the frequency $\sim 10^4$ MHz at which the absorption was measured.[13] This justified the comparison of the experiment with the theoretical equations for purely viscous flow of the vortices.

## 6. CONCLUSION

Our present analysis offers evidence that the experimentally observed appreciable microwave absorption by HTSC in weak fields can be attributed to the presence, for such fields, of a mixed state of hypervortices. This agrees with other indirect confirmation of the existence of a mixed hypervortex state. The most complete experimental verification of the theory, developed in the present paper, of the electrodynamic response of a Josephson medium would be an all-inclusive experimental determination of the various experimental parameters for one and the same samples. It is very important to have the still nonexistent reliable data on the reactive part of the surface impedance of the bulk admittance.

The concept of a Josephson medium is, of course, not universal, but pertains apparently to most presently investigated HTSC. To eliminate effects of multiple weak couplings, single crystals are needed with sufficiently large twin domains. This technological requirement is of fundamental importance for technical applications, for only in this case can one count on fabrication ideal magnetic screens and cavities of very high $Q$ from HTSC materials.

The present paper can be regarded only as the first step in the development of an electrodynamic theory of real HTSC materials. The next step of the theory should be allowance for nonlinear effects and for the random character of the network of weak couplings and of the associated pinning and irreversibility, and also a rigorous derivation of equations of motion of hypervortices, capable of yielding not only the friction coefficient but also the transverse force that is produced when these vortices are displaced and leads to the Hall effect in the mixed state.

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[1)] In view of the tremendous flow of literature of HTSC, with the inevitable duplication of results, the choice of our bibliography is partly random.


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