Mutual influence of resonant spin-flavor precession and resonant neutrino oscillations

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The mutual influence of resonant spin flavor precession and resonant neutrino oscillations in matter is considered for Majorana neutrinos. The case of overlapping resonances is discussed in detail and an approximate analytic solution is obtained for it. Numerical calculations of the oscillation and precession probabilities are carried out for solar neutrinos. It is shown that, as a rule, these two processes suppress one another but, acting together, they convert ν_{eL} into other types of neutrino more effectively than they do separately.

1. INTRODUCTION

Resonant neutrino spin-flavor precession (NSFP) in matter has recently been considered in Refs. 1–3 and, independently, in Ref. 4. NSFP can occur if the neutrinos have flavor off-diagonal magnetic moments. In a transverse magnetic field, left-handed neutrinos of a given type transform into right-handed neutrinos (or antineutrinos) of another type. In the absence of matter, NSFP is suppressed as compared with ordinary (flavor-conserving) precession because neutrinos of different flavor have nondegenerate energies: $\Delta E \approx \Delta m^2/2E$ (for $E \gg m_1, m_2$).[5] However, in matter, the kinetic energy difference can be canceled by the difference between the potential energies of interaction between different types of neutrino and the medium, so that resonant amplification of precession can occur[1–4] in the case of a sufficiently slow (adiabatic) change in the density of matter and field strength. This effect is analogous to the Mikheev-Smirnov-Wolfenstein (MSW) effect in neutrino oscillations,[6,7] but differs from it in exhibiting a different dependence of the adiabatic parameter on neutrino energy.

Possible consequences of resonant NSFP were examined in Refs. 1–3 for solar neutrinos, and the allowed ranges of parameter values were obtained for which this phenomenon can explain the observed deficit of solar neutrinos[8] and the anticorrelation between the neutrino count rate and solar activity.[9,10] Possible methods of experimental detection of resonant NSFP were also discussed. However, no acount was taken of possible neutrino mixing effects. A combined analysis of oscillations and NSFP in matter was performed in Ref. 4 for Dirac neutrinos. Numerical calculations were made for several values of the parameters of the problem, but the mutual influence of oscillations and precession was not fully investigated. In particular, the case of overlapping resonances was not examined, and the calculations were performed on the assumption that the diagonal magnetic moments were zero, which is not a natural assumption for Dirac neutrinos.

In the present paper, we report a combined analysis of oscillations and NSFP in matter for Majorana neutrinos. An analytic solution of the problem is obtained for a uniform magnetic field $B_\perp$ and a medium of constant density $\rho = \text{const}$. This solution is then used to investigate the adiabatic regime in the case of slowly varying $B_\perp(r)$ and $\rho(r)$. The estimated interaction between neutrino oscillations and precession is confirmed by direct numerical calculations.

2. NEUTRINO MAGNETIC MOMENTS

If the neutrinos mix, the flavor states ν_i ($i = e, \mu, \tau, \ldots$) participating in weak interaction are linear combinations of the states ν_a ($a = 1,2,3,\ldots$) with a particular mass. In general, there are no reasons to suppose that the unitary transformation $\nu_i \Rightarrow \nu_a$ that diagonalizes the neutrino mass matrix will also diagonalize the electromagnetic moment $\tilde{\mu}_{ab}$. Hence, in the basis of states with a particular mass, this matrix will not in general be diagonal. Its diagonal elements determine the magnetic and electric dipole moments of the neutrinos, whereas the off-diagonal elements are the transition moments that are responsible for the $\nu_b \to \nu_a\, \gamma$ radiative decays (for $m_b > m_a$). The magnetic moments μ_{ab} are specified by the Hermitian part of the matrix $\tilde{\mu}_{ab}$ and the electric moments ε_{ab} by the anti-Hermitian part. The neutrino spin precession (NSP) can be due to moments of either type. As noted in Refs. 5 and 11, only the combination $(\mu^2 + \varepsilon^2)^{1/2}$ appears in all the formulas for ultrarelativistic neutrinos. For the sake of convenience, we shall use the phrase "magnetic moment" when we refer to this particular combination.

For Dirac neutrinos, both the diagonal and the off-diagonal magnetic moments can lead to transitions between left-handed (active) ν_L neutrinos and inactive right-handed ν_R neutrinos. If the neutrinos are Majorana particles, their diagonal magnetic moments are $\mu_{ii} = 0$ because of CPT invariance; the off-diagonal moments μ_{ij} ($i \neq j$) give rise to transitions in a transverse field between left-handed ν_{iL} neutrinos and right-handed ν_{jR}^c antineutrinos, which are also active.[1)]

The neutrino spin precession (both ordinary and flavor) can play an important part in the dynamics of neutrinos from the Sun and from supernovas, and also in the early stages of the evolution of the Universe. For solar neutrinos, NSP effects can be significant only if the neutrino magnetic moments are large enough, i.e., $\mu \gtrsim 10^{-11}\,\mu_B$, where $\mu_B = e/2m_e$ is the Bohr magneton[1–3,5,11–13] (these restrictions become somewhat less stringent in the case of NSFP). This condition is in agreement with existing experimental limits on μ_{ij} (see the discussion in Ref. 3). We note that much more stringent restrictions have recently been obtained as a result of analysis of neutrino events due to the supernova SN 1987A,[14–16] but these require additional analysis[17,18] and, in any case, they do not refer to the off-diagonal magnetic moments of Majorana neutrinos, which will be of particular interest to us in the present paper. Theoretical diagonal and off-diagonal magnetic moments of the order of $10^{-11}\mu_B$–

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$10^{-10}\mu_B$ can be obtained, for example, from models with a charged $SU(2)_L$-singlet scalar.[19–22] In the case of Majorana neutrinos, this requires the existence of more than one Higgs particle doublet.[19,21]

3. EVOLUTION EQUATION FOR A NEUTRINO SYSTEM

For Dirac neutrinos, the evolution problem involves a large number of parameters, even with only two types of neutrino. These parameters are Δm^2, the mixing angle θ_0, and the magnetic moments μ_{11}, μ_{22}, and μ_{12}. We shall therefore confine our attention to the simplest case of Majorana neutrinos. We shall consider that there are only two new neutrino flavors. To be specific, we shall examine transitions between the different components ν_e and ν_μ and take the flavor basis $(\nu_{eL}, \nu_{eR}^c, \nu_{\mu L}, \nu_{\mu R}^c)$.

In the absence of matter, and in the basis of the eigenstates of the mass matrix $(\nu_{1L}, \nu_{1R}^c, \nu_{2L}, \nu_{2R}^c)$, the effective Hamiltonian describing the evolution of the neutrinos is $H = H_k + H_B$, where H_k determines the kinetic energies of the neutrinos and H_B describes their interactions with the transverse magnetic field $B_\perp$ For ultrarelativistic neutrinos, we have

$$H_k=E\cdot\hat{1}+(2E)^{-1}\,\mathrm{diag}(m_1^2,\ m_1^2,\ m_2^2,\ m_2^2), \tag{1}$$

where $\hat{1}$ is the unit matrix. The nonzero elements of the matrix H_B are

$$(H_B)_{14}=(H_B)_{41}=-(H_B)_{23}=-(H_B)_{32}=\mu_{12}B_\perp. \tag{2}$$

We can transform from the basis of the eigenstates of the mass matrix to the flavor basis, using the unitary matrix

$$U=\begin{pmatrix} c & s \\ -s & c \end{pmatrix}\otimes\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};\quad c=\cos\theta_0,\quad s=\sin\theta_0. \tag{3}$$

We note that the matrix H_B then retains its form.

The effective Hamiltonian for the interaction between the neutrinos and matter has a simple form in the flavor basis:

$$H_m=\mathrm{diag}(N_1,\ -N_1,\ N_2,\ -N_2), \tag{4}$$

where

$$N_1=2^{1/2}G_F(N_e-N_n/2),\quad N_2=-G_FN_n/2^{1/2}. \tag{5}$$

in which G_F is the Fermi constant and N_e and N_n are the electron and neutrino number densities. The first term in the expression for N_1, which is proportional to N_e, is due to charged currents and the second (proportional to N_n) is due to neutral currents. The contributions of protons and electrons to N_1, due to interactions induced by neutral currents in the electrically neutral medium, are found to cancel, and the quantity N_2 contains only the contribution due to neutral currents.

In the flavor basis, the effective Hamiltonian H_{fl} for the neutrino system in the magnetic field in the presence of matter is

$$H_{fl}=U(H_k+H_B)U^{-1}+H_m. \tag{6}$$

It is convenient to transform from H_{fl} to

$$H_{fl}'=H_{fl}-{}^1/_4\mathrm{Sp}\,H_{fl}\cdot\hat{1}.$$

This enables us to simplify the algebra without affecting the oscillation and precession amplitudes, because the above replacement reduces to a change in the total phase of all the neutrino states. Let us substitute

$$\mu=\mu_{12}B_\perp,\quad \delta=(m_2^2-m_1^2)/4E,\quad s_2=\sin 2\theta_0,\quad c_2=\cos 2\theta_0. \tag{7}$$

The evolution equation for the neutrino system then assumes the following form[2)]

$$i\frac{d}{dt}\begin{pmatrix}\nu_{eL}\\ \nu_{eR}^c\\ \nu_{\mu L}\\ \nu_{\mu R}^c\end{pmatrix}=H_{fl}'\begin{pmatrix}\nu_{eL}\\ \nu_{eR}^c\\ \nu_{\mu L}\\ \nu_{\mu R}^c\end{pmatrix}=\begin{pmatrix} N_1(t)-\delta c_2 & 0 & \delta s_2 & \mu(t)\\ 0 & -N_1(t)-\delta c_2 & -\mu(t) & \delta s_2\\ \delta s_2 & -\mu(t) & N_2(t)+\delta c_2 & 0\\ \mu(t) & \delta s_2 & 0 & -N_2(t)+\delta c_2\end{pmatrix}\begin{pmatrix}\nu_{eL}\\ \nu_{eR}^c\\ \nu_{\mu L}\\ \nu_{\mu R}^c\end{pmatrix} \tag{8}$$

where the coordinate dependence of $B_\perp$, N_1 and N_2 is written as a time dependence because for the neutrinos $r\approx t$.

Points corresponding to the resonant amplification of oscillations and NSFP can be found from the conditions for the closest approach of the eigenvalues of the effective Hamiltonian H_{fl}'. In the absence of mixing interactions ($s_2 = 0 = \mu$), these points correspond to the crossing of the neutrino energy levels. The conditions for resonance can be found approximately by equating the diagonal elements of the matrix H_{fl}' in pairs.[3)] The result is (the transitions for which the resonance condition is written out are shown in parentheses):

$$(N_1-N_2)_{\rm res}=2^{1/2}G_F(N_e)_{\rm res}=2\delta c_2,\quad (\nu_{eL}\leftrightarrow\nu_{\mu L}), \tag{9}$$

$$(N_1+N_2)_{\rm res}=2^{1/2}G_F(N_e-N_n)_{\rm res}=2\delta c_2,\quad (\nu_{eL}\leftrightarrow\nu_{\mu R}^c), \tag{10}$$

$$2^{1/2}G_F(N_e-N_n)_{\rm res}=-2\delta c_2,\quad (\nu_{eR}^c\leftrightarrow\nu_{\mu L}), \tag{11}$$

$$2^{1/2}G_F(N_e)_{\rm res}=-2\delta c_2,\quad (\nu_{eR}^c\leftrightarrow\nu_{\mu R}^c). \tag{12}$$

Equations (9) and (12) are identical with known resonance conditions for neutrino oscillations,[6,7] and (10) and (11) are identical with the resonance conditions for NSFP.[1–4]

We note that, in each pair of relations (9),(12) and (10),(11), only one relation can be satisfied, depending on the sign of the difference between the squares of the neutrino masses and the quantity $N_e - N_n$.

Although the matrix elements $(H_{fl}')_{12} = (H')_{21}$ and $(H')_{34} = (H')_{43}$ are zero, the $\nu_{eL} \leftrightarrow \nu_{eR}^c$ and $\nu_{\mu L} \leftrightarrow \nu_{\mu R}^c$ transitions are possible in second or higher order in the mixing interactions. For example, the following transition chains are possible in second order:

$$\nu_{eL}\xrightarrow{\text{oscillations}}\nu_{\mu L}\xrightarrow{\text{precession}}\nu_{eR}^c, \tag{13}$$

$$\nu_{eL}\xrightarrow{\text{oscillations}}\nu_{\mu R}^c\xrightarrow{\text{precession}}\nu_{eR}^c. \tag{14}$$

The amplitudes for (13) and (14) have opposite signs and partially cancel one another, so that the probabilities of transitions without change of flavor are small for Majorana neutrinos. The exceptions are the situations in which one of the amplitudes for (13) or (14) is resonantly amplified and the other is not. Transitions between the states ν_{iL} and ν_{iR}^c can be significant (see below).

4. QUALITATIVE DISCUSSION OF THE PROBLEM

To be specific, let us suppose that $N_e > N_n$ and $m_2 > m_1$ ($\delta > 0$). The resonance conditions for oscillations and NSFP are then given by (9) and (10), respectively. It follows from these relations that resonant precession corresponds to higher densities than resonant oscillation. The separation between these two resonances depends on the neutron density: the higher N_n the greater the separation between the resonance points. If these points are separated far enough, so that the resonance regions do not overlap, the oscillations and the NSFP have little effect on one another. The problem then essentially reduces to two separate problems, namely, those of resonant oscillations and resonant NSFP, which were considered previously (see Ref. 6 and 7 and 1–4). Analysis of the evolution of the neutrino system presents no difficulty in this case. A qualitative version of it is presented below.

The case of overlapping resonances is more complicated. Here we may expect a strong interaction between NSFP and the oscillations. This is clear if only from the fact that each of these two effects in the absence of the other leads to a practically complete transformation in the adiabatic regime, e.g., ν_{eL} into a neutrino of another type ($\nu_{\mu R}^c$ or $\nu_{\mu L}$). However, the probability sum for the conversion of the ν_{eL} into all the other possible neutrino states (including the initial state) is equal to unity, so that, in the case of overlapping resonances, oscillations, and NSFP should suppress one another. Fortunately, an approximate analytic solution that can be used to analyze the adiabatic regime can be obtained in the most complex case of coincident resonance points.

Let us now consider the case of nonoverlapping resonances. The condition for the absence of overlap can be formulated as the requirement that the separation between the resonance points should be greater than the sum of the resonance half-widths:

$$\tfrac{1}{2}[(\Delta r)_{\rm MSW}+(\Delta r)_{\rm NSFP}]<r_2-r_1. \qquad (15)$$

where r_1 and r_2 are the coordinates of the resonance points, defined by (10) and (9), respectively, and the widths of the resonant layers for MSW[6,7] and resonant NSFP[1–3] are given by

$$(\Delta r)_{\rm MSW}=2|\operatorname{tg}2\theta_0|L_\rho, \qquad (\Delta r)_{\rm NSFP}\approx 2\left|\frac{2\mu_{12}B_{\perp 0}}{\Delta m^2/2E}\right|L_\rho, \qquad (16)$$

where $B_{10}=B_\perp(r_1)$ is the magnetic field at the NSFP resonance point and L_ρ is the characteristic distance over which there is a significant change in the density of matter $\rho(r)$ in the resonance region:

$$L_\rho=\left(-\frac{1}{\rho}\frac{d\rho}{dr}\right)^{-1}. \qquad (17)$$

It is assumed that this quantity varies slowly between $r=r_1$ and $r=r_2$.

In the expression for $(\Delta r)_{\rm NSFP}$ in (16) we have neglected the nonuniformity of the magnetic field because it can be shown[1–3] that this nonuniformity provides a smaller contribution to $(\Delta r)_{\rm NSFP}$ than the nonuniformity of matter.

Let us now substitute $\eta\equiv N_n/N_e$ ($0\leqslant\eta<1$) and assume that η varies very little within the interval $r_1\leqslant r\leqslant r_2$. Expanding $\rho(r)$ into a series in this region, and using (15), we obtain the following approximate condition for the absence of resonant overlap:

$$\left|\frac{2\mu_{12}B_{\perp 0}}{\Delta m^2/2E}\right|+|\operatorname{tg}2\theta_0|<\frac{\eta}{1-\eta}. \qquad (18)$$

When this inequality is satisfied, oscillations and NSFP have practically no effect on one another.[4)]

Let us now suppose that the ν_{eL} are created for densities much greater than either of the resonant densities given by (9) and (10) (this situation can occur, for example, in the Sun). When the neutrinos enter a region of lower density, they initially resonate with NSFP. When the adiabatic condition[1–3]

$$\frac{1}{\pi}(l_{\rm prec})_{\rm res}=\frac{1}{\mu_{12}B_{\perp 0}}\ll 2\left|\frac{2\mu_{12}B_{\perp 0}}{\Delta m^2/2E}\right|L_\rho, \qquad (19)$$

is satisfied, i.e., the resonant precession length is small in comparison with $(\Delta r)_{\rm NSFP}$, the ν_{eL} neutrinos are practically completely converted into $\nu_{\mu R}^c$ as they leave the resonance region. For the antineutrinos, the oscillation resonance can occur only when $\delta<0$, so that, in our case, $\nu_{\mu R}^c$ are not subject to any further change. If, on the other hand, the adiabatic condition (19) is strongly violated, the neutrinos ν_{eL} cross the NSFP resonance with practically no change. Having then entered the region of resonant oscillations, they may convert into $\nu_{\mu L}$, in which case conversion will be almost complete if the adiabatic condition[6,7]

$$\frac{1}{\pi}(l_{\rm osc})_{\rm res}=\frac{4E}{\Delta m^2|\sin 2\theta_0|}\ll 2|\operatorname{tg}2\theta_0|L_\rho. \qquad (20)$$

is satisfied. Thus only one of the resonances is effective in this case. On the other hand, if condition (19) is weakly violated, only some ν_{eL} will convert into $\nu_{\mu R}^c$ in resonance with NSFP. The remaining fraction will undergo the $\nu_{eL}\to\nu_{\mu L}$ conversion due to resonant oscillations, and the degree of conversion will depend on the extent to which the adiabatic condition (20) is satisfied.

It is possible, however, for both resonances to be fully effective under adiabatic conditions. Suppose that, initially, there is a beam of $\nu_{\mu R}^c$ but not of ν_{eL} neutrinos. At the NSFP resonance, the former will adiabatically convert into ν_{eL} which, in turn, will convert into $\nu_{\mu L}$ at the oscillation resonance. Thus, in the final analysis, we have the $\nu_{\mu R}^c\to\nu_{\mu L}$ conversion without change of flavor. This is precisely the situation mentioned in Sec. 3 [one of the two amplitudes (13) or (14) is resonantly amplified and the other is not, so that their mutual cancellation is prevented].

The above qualitative discussion is illustrated in Fig. 1 in which we show schematically the energy levels E_α of the effective Hamiltonian $H'_{\rm fi}$ (adiabatic terms) for $\delta>0$, $N_e>N_n$. At high densities, the mixing effects can be neglected, and the energy levels E_α are practically identical with the unperturbed levels, shown by the dashed lines. As they enter the region of low-density (from right to left in Fig. 1) in the adiabatic regime, the ν_{eL} neutrinos convert into $\nu_{\mu R}^c$ which (after crossing two resonances) convert into $\nu_{\mu L}$, and these in turn convert into ν_{eL}; the ν_{eR}^c neutrinos do not experience resonant conversion in this case. If the adiabatic condition is

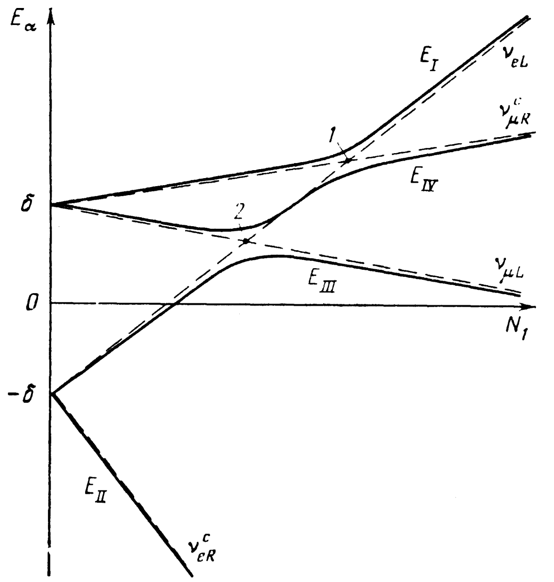

FIG. 1. Energy levels of the effective Hamiltonian $H'_{\rm fl}$ as functions of N_1 for $m_2>m_1$ ($\delta>0$), $|N_2|=0.167\ N_1$. Dashed lines—no mixing ($\mu=0=s_2$) with the corresponding neutrino states shown on the right. Resonance points for NSFP and neutrino oscillations are indicated by *1* and *2*, respectively.

strongly violated at the point corresponding to resonant NSFP, the ν_{eL} neutrinos cross this resonance without conversion, and the ν_{eL} energy level will subsequently move along the dashed line which corresponds to the unperturbed ν_{eL} level until it reaches the MSW resonance. If the adiabatic condition is satisfied at this resonance, the ν_{eL} neutrinos convert into $\nu_{\mu L}$.

5. ANALYTIC SOLUTION FOR A UNIFORM FIELD AND A CONSTANT DENSITY MEDIUM

The transition probabilities between different neutrino states can be obtained by a numerical solution of (8) for given functions $B_{\perp(r)}$, $N_{e(r)}$, and $N_{n(r)}$. The results of such calculations will be presented in Sec. 6. Here, we confine ourselves to an approximate analytic solution of (8) for a uniform magnetic field and constant, density medium. This solution will be useful for the investigation of the adiabatic regime in which the field and the density vary slowly, and the system succeeds in following changes in external parameters.

For constant μ, N_1 and N_2, the solution of (8) reduces to the diagonalization of the matrix $H'_{\rm fl}$. If the eigenvalues E_α of the effective Hamiltonian $H'_{\rm fl}$ and the unitary matrix $V_{i\alpha}$ used to diagonalize the Hamiltonian are known, the transition probabilities between different neutrino states can be found from

$$P(\nu_i\to\nu_j;t)=\Big|\sum_\alpha V_{i\alpha}{}^*V_{j\alpha}e^{-iE_\alpha t}\Big|^2. \qquad (21)$$

Since fourth-degree equations can be solved in terms of radicals, the matrix $H'_{\rm fl}$ can be diagonalized analytically for any parameter values, and the transition probabilities (21) can be obtained in closed form. However, the corresponding expressions are exceedingly unwieldy and not very informative. We shall therefore consider the special case, $|N_2|\ll N_1$, (i.e., $N_n\ll N_e$), for which all the formulas become much simpler. For example, this condition is satisfied in the Sun outside the central region, but is violated in the core.

In the approximation in which $N_n=0$, the oscillation and NSFP resonance points are found to coincide, i.e., this case complements that of nonoverlapping resonances considered in Sec. 4. The unitary matrix V that diagonalizes $H'_{\rm fl}$ can then be written in the form

$$V=\begin{pmatrix} (E_{\rm I}-\delta c_2)/R_{\rm I} & 0 & (E_{\rm III}-\delta c_2)/R_{\rm III} & 0\\ 0 & (E_{\rm II}-\delta c_2)/R_{\rm II} & 0 & (E_{\rm IV}-\delta c_2)/R_{\rm IV}\\ \delta s_2/R_{\rm I} & -\mu/R_{\rm II} & \delta s_2/R_{\rm III} & -\mu/R_{\rm IV}\\ \mu/R_{\rm I} & \delta s_2/R_{\rm II} & \mu/R_{\rm III} & \delta s_2/R_{\rm IV}\end{pmatrix}. \qquad (22)$$

where the eigenvalues E_α and the quantities R_α ($\alpha=$ I,..., IV) are given by

$$E_{\rm I,\,III}=N_1/2\pm[(N_1/2-\delta c_2)^2+(\mu^2+\delta^2s_2^2)]^{1/2}, \qquad (23a)$$

$$E_{\rm II,\,IV}=-N_1/2\mp[(N_1/2+\delta c_2)^2+(\mu^2+\delta^2s_2^2)]^{1/2}, \qquad (23b)$$

$$R_\alpha=[(E_\alpha-\delta c_2)^2+\mu^2+\delta^2s_2^2]^{1/2}. \qquad (24)$$

The corresponding level scheme can be obtained from Fig. 1 in the limit as $N_n\to0$. The oscillation and NSFP resonance points then collapse into one, and the dashed lines that correspond to the unperturbed levels of $\nu^c_{\mu R}$ and $\nu_{\mu L}$, and also the $E_{\rm IV}$ curve lying between them, degenerate into the horizontal line $E\approx\delta$.

Suppose that a beam of ν_{eL} neutrinos is present at the origin of coordinates. The oscillation and precession probabilities are then readily found from (22)–(24), using (21):

$$P(\nu_{eL}\to\nu_{\mu L};r)=\frac{\delta^2s_2^2}{(N_1/2-\delta c_2)^2+(\mu^2+\delta^2s_2^2)}\times\sin^2\Big\{\Big[\Big(\frac{N_1}{2}-\delta c_2\Big)^2+(\mu^2+\delta^2s_2^2)\Big]^{1/2}r\Big\}, \qquad (25)$$

$$P(\nu_{eL}\to\nu_{\mu R}{}^c;r)=\frac{\mu^2}{(N_1/2-\delta c_2)^2+(\mu^2+\delta^2s_2^2)}\times\sin^2\Big\{\Big[\Big(\frac{N_1}{2}-\delta c_2\Big)^2+(\mu^2+\delta^2s_2^2)\Big]^{1/2}r\Big\}, \qquad (26)$$

$$P(\nu_{eL}\to\nu_{eR}{}^c;r)\equiv0. \qquad (27)$$

It follows from these formulas that the oscillations and NSFP suppress one another. The probabilities of both processes are less than unity at the resonance point ($N_1 = 2\delta c_2$), and their ratio is

$$\delta^2 s_2^2/\mu^2 = \sin^2 2\theta_0/[2\mu_{12}B_{\perp 0}/(\Delta m^2/2E)]^2.$$

Both probabilities oscillate in accordance with the same law and with the characteristic length

$$l = \pi/[(N_1/2-\delta c_2)^2 + (\mu^2+\delta^2 s_2^2)]^{1/2}, \quad (28)$$

which is a consequence of the approximations that we have employed (in particular, $N_n = 0$). For the same reason, the $\nu_{eL} \leftrightarrow \nu_{eR}^c$ transition probability is identically zero: the cancellation of transition probabilities of the form given by (13) and (14) is exact in this case, and is satisfied to all orders in μ and s_2.

In the case of an inhomogeneous field and a variable-density medium, Eqs. (25)–(27) are not strictly valid. However, in the adiabatic regime, when the external parameters vary sufficiently slowly, we can introduce eigenvalues ν_α of the "instantaneous" Hamiltonian H'_{fl} (adiabatic states) at each point which are related, as before, to the flavor states ν_i by the relation $\nu_i = V_{i\alpha}\,\nu_\alpha$.

As an example, consider the evolution of ν_{eL} neutrinos on the Sun. If the condition $N_1/2 - \delta c_2 \gg (\mu^2 + \delta^2 s_2^2)^{1/2}$, is satisfied at the center of the Sun, where the density of matter is at a maximum, then it follows from (22)–(24) that the state $\nu_{e1} = V_{1\alpha}\nu_\alpha$ is practically identical with the eigenstate, ν_{I} since $V_{i\text{I}} \approx 1$, $V_{1\,\text{II}} = V_{1\,\text{IV}} = 0$, and $|V_{1\,\text{III}}| \ll 1$. In the adiabatic regime, the state ν_{I} propagates practically without change, since the probabilities of transition to the states ν_{II}, ν_{III}, and ν_{IV} are exponentially small. Since the elements of the matrix V_α vary along the neutrino trajectory because of the variation of N_1 and $B_\perp$, the composition of the neutrino state $\nu_{\text{I}} = (V^T)_{\text{I}i}\,\nu_i = V_{i\text{I}}\,\nu_i$ with respect to the flavor states ν_i will also vary. At low densities, we have

$$|V_{1\text{I}}|^2 \approx \frac{1}{2}\left[1 - \frac{\delta c_2}{(\mu^2+\delta^2)^{1/2}}\right]. \quad (29)$$

If $\mu \gg \delta$ or $c_2 \approx 0$ (which corresponds to strong mixing of ν_{eL} with $\nu_{\mu R}^c$ or $\nu_{\mu L}$ in the absence of matter), we have $|V_{1\,\text{I}}|^2 \approx 1/2$. In this case, approximately one-half of the ν_{eL} neutrinos survives in the final state, whereas the other half is converted into $\nu_{\mu R}^c$ and $\nu_{\mu L}$. If, on the other hand, $c_2 \approx 1$ and $\mu \ll \delta$, i.e., in the absence of matter the mixing of ν_{eL} with $\nu_{\mu L}$ and $\nu_{\mu R}^c$ is small, we have $V_{1\,\text{I}} \approx 0$. This means that after they have crossed the resonance, practically all the ν_{eL} neutrinos will adiabatically convert into $\nu_{\mu L}$ and $\nu_{\mu R}^c$. From now on, we shall confine our attention to this case.

At resonance,

$$V_{1\text{I}} = 1/2^{1/2}, \quad V_{3\text{I}} = \frac{\delta s_2}{[2(\mu^2+\delta^2 s_2^2)]^{1/2}},$$

$$V_{4\text{I}} = \frac{\mu}{[2(\mu^2+\delta^2 s_2^2)]^{1/2}}.$$

The resonance region corresponds to densities

$$|N_1/2-\delta c_2| \leqslant (\mu^2+\delta^2 s_2^2)^{1/2}.$$

As the neutrinos enter the region of lower densities, the fraction of ν_{eL} neutrinos in the neutrino beam falls rapidly: for

$$\delta c_2 - N_1/2 \gg (\mu^2+\delta^2 s_2^2)^{1/2}$$

we have

$$V_{1\,\text{I}} \approx (\mu^2+\delta^2 s_2^2)^{1/2}/2(\delta c_2 - N_1/2) \ll 1,$$

$$V_{3\text{I}} \approx \frac{\delta s_2}{(\mu^2+\delta^2 s_2^2)^{1/2}}, \quad V_{4\text{I}} \approx \frac{\mu}{(\mu^2+\delta^2 s_2^2)^{1/2}}. \quad (30)$$

However, in this region E_{I} and E_{IV} begin to approach one another very rapidly (which is a consequence of the approximation $N_n = 0$), and the adiabatic approximation becomes invalid). In practice, this does not present any particular difficulty because all the transition probabilities are small outside the resonance region. For this reason, the ratio of the $\nu_{\mu L}$ and $\nu_{\mu R}^c$ fluxes after resonant conversion can be obtained from the formula

$$K \equiv \frac{P(\nu_{eL}\to\nu_{\mu L})}{P(\nu_{eL}\to\nu_{\mu R}^c)} = \frac{|V_{3\text{I}}|^2}{|V_{4\text{I}}|^2} = \frac{\delta^2 s_2^2}{\mu^2}, \quad (31)$$

where, for $\mu \equiv \mu_{12} B_\perp(r)$, if we take the value on the lower boundary of the resonance region and neglect the evolution of neutrinos outside this region. As noted in Ref. 3, the large-scale solar magnetic field probably varies more slowly than the density of matter. For approximate estimates, we can therefore neglect the variation in $B_\perp$ in the resonance region, and replace $B_\perp$ in (31) with its value $B_{\perp 0}$ at resonance.

It follows from the foregoing discussion that the above analysis will not be valid when the resonance region extends to very low densities for which the levels E_{I} and E_{IV} approach closely one another. Our estimates should be acceptable if the separation between the resonance point and the region of very low densities is not less than one or two widths Δr of the resonance region.

We must now find the condition for resonant adiabatic conversion of neutrinos in the case of completely overlapping resonances. It is readily shown from (25) and (26) that the full width of a resonance at half maximum is

$$\Delta r \approx \frac{2(\mu^2+\delta^2 s_2^2)^{1/2}}{\delta c_2} L_\rho. \quad (32)$$

The adiabatic condition can be written in the form

$$\frac{1}{\pi} l_{\text{res}} = \frac{1}{(\mu^2+\delta^2 s_2^2)^{1/2}} \ll \frac{2(\mu^2+\delta^2 s_2^2)^{1/2}}{\delta c_2} L_\rho. \quad (33)$$

When $\mu = 0$ or $s_2 = 0$, the expression for Δr and the adiabatic condition (33) become identical with the corresponding expressions for the widths of the resonance regions (16) and the adiabatic conditions (20) and (19) for the MSW effect and resonant NSFP. It follows from (33), (29), and (20) that the adiabatic condition is better satisfied for overlapping than for highly separated resonances, or if there are only oscillations or only NSFP. This occurs because (1) the widths of the resonance region becomes greater and (2) the resonance length l_{res} becomes smaller.

When the adiabatic condition (33) is strongly violated, the oscillation and NSFP probabilities are very small, i.e., the neutrinos cross the resonance with practically no change. On the other hand, when the adiabatic condition is weakly violated, neutrino conversion effects can be quite considerable. In a moderately nonadiabatic state, the resonant layer model[23,1–3] is satisfactory and shows that the oscillations and NSFP are completely suppressed outside the resonant layer of width Δr, whereas inside the layer both processes proceed with maximum possible amplitudes (it is assumed that the density of matter in the resonant layer is constant and equal to the resonance value). In our case of completely overlapping resonances, we can readily show, using this model, that

$$P(\nu_{eL}\to\nu_{eL})\approx\cos^2[2(\mu^2+\delta^2 s_2^2)L_\rho/\delta c_2];\quad P(\nu_{eL}\to\nu_{eR}^c)\equiv 0;$$
$$P(\nu_{eL}\to\nu_{\mu L})\approx[\delta^2 s_2^2/(\mu^2+\delta^2 s_2^2)]\sin^2[2(\mu^2+\delta^2 s_2^2)L_\rho/\delta c_2],$$
$$P(\nu_{eL}\to\nu_{\mu R}^c)\approx[\mu^2/(\mu^2+\delta^2 s_2^2)]\sin^2[2(\mu^2+\delta^2 s_2^2)L_\rho/\delta c_2]. \tag{34}$$

We are assuming, as before, that the field $B_\perp(r)$ changes little in the resonance region.

6. CALCULATIONS OF OSCILLATION AND NSFP PROBABILITIES

The set of equations given by (8) was integrated numerically for the case of solar neutrinos. It was assumed that the ν_{eL} neutrinos were created at the center of the Sun, and the probabilities P_1, P_2, P_3, and P_4 of detecting ν_{eL}, ν_{eR}^c, $\nu_{\mu L}$, and $\nu_{\mu R}^c$ on the surface of the Sun were calculated. The electron and neutron concentrations in the solar interior were taken from Ref. 24 and the solar magnetic field was modeled as follows (see the discussion given in Ref. 3):

$$B_\perp(x)=\begin{cases} B_1\left(\dfrac{0.1}{x+0.1}\right)^2, & 0\leqslant x<0.65,\\ B_0\left[1-\left(\dfrac{x-0,7}{0,3}\right)^2\right], & 0.65\leqslant x\leqslant 1.\end{cases} \tag{35}$$

where $x=r/R_\odot$ and $R_\odot$ is the solar radius. Moreover, it was assumed in these calculations that $\mu_{12}=10^{-11}\mu_B$; since the magnetic moment always appears in the form of the product $\mu_{12}B_\perp$, the results could readily be recalculated for some other value of μ_{12} by changing the scale of the magnetic field.

The conversion probabilities were calculated for two values of $\sin 2\theta_0$ and fixed values of B_0 and B_1. The dependence of NSFP on the magnitude and shape of the function $B_\perp(r)$ was investigated in Ref. 3.

Figures 2 and 3 show the values of P_1, P_3 and P_4 as functions of $E/\Delta m^2$. In all cases, P_2 was less than 2.5% and is not shown in the figure. The fact that P_2 was so small was due to the very effective cancellation of amplitudes for processes (13) and (14). The identity $P_1+P_2+P_3+P_4=1$ was satisfied to better than 10^{-3} in all these calculations. Figures 2 and 3 also show the values of $P_3'=KP_4$, where K is given by (31).

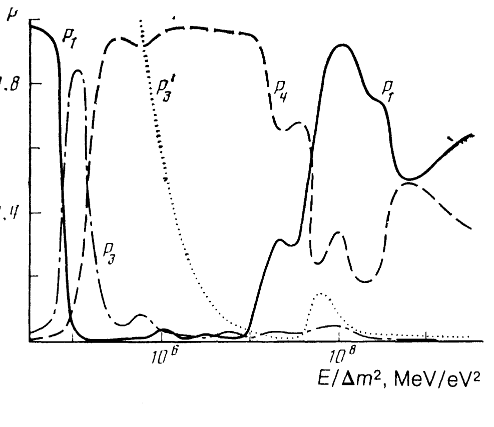

FIG. 2. The probabilities P_1, P_3, and P_4 of finding ν_{eL}, $\nu_{\mu L}$ and $\nu_{\mu R}^c$ on the solar surface when ν_{eL} are created at the solar center. Solid line—P_1, dashed line—P_4, dot-dash curve—P_3, dotted curve—$P_3'=KP_4$, where K is given by (31): $B_0=10^4$ G, $B_1=10^7$ G, $\sin 2\theta_0=0,1$.

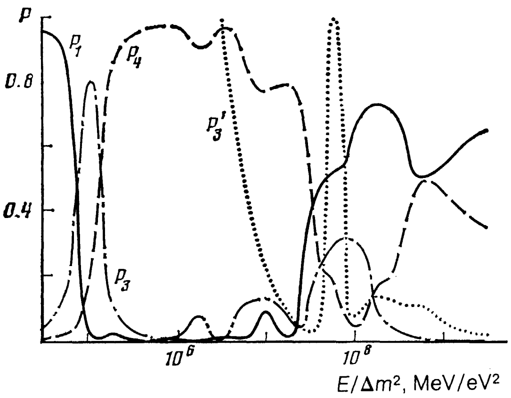

FIG. 3. Same as Fig. 2 but with $\sin 2\theta_0=0.3$.

7. RESULTS AND DISCUSSION

Figures 2 and 3 show that the relative oscillation probability increases with increasing s_2. Simple estimates based on the analytic solution obtained in Sec. 5 are in good agreement with numerical calculations for intermediate values of $E/\Delta m^2$: the quantity P_3' differs from P_3 by a factor[5)] that does not exceed 3–5 in the range

$$5\cdot 10^6\leqslant E/\Delta m^2\ (\mathrm{MeV/ev^2})\leqslant 5\cdot 10^8$$

which corresponds to a resonance point in the range $0.45\lesssim x_0\lesssim 0.85$. At the same time, if the resonance is located within the solar core, or near the solar surface, then (31) will give a considerable overestimate of the oscillation probability. The reason for this is not difficult to understand. The condition $N_n\ll N_e$ ceases to be valid for small r, and (N_n/N_e) varies from about 0.5 at the solar center to 0.15 on the surface. The approximation of overlapping resonances then ceases to be valid. When the adiabatic condition is satisfied for the NSFP, a considerable fraction of the ν_{eL} neutrinos is converted into $\nu_{\mu R}^c$ and, consequently, cannot convert into $\nu_{\mu L}$ because the resonant density for oscillations is lower than for NSFP. The adiabatic condition ceases to be valid near the solar surface, and the estimates obtained in Sec. 5 become unacceptable. The overlap of resonances leads to the mutual suppression of oscillations and NSFP. At the same time, the total probability of conversion of ν_{eL} into $\nu_{\mu L}$ and $\nu_{\mu R}^c$ as a result of these two processes is greater than for only one of them. This therefore ensures that (1) there are two rather than one neutrino conversion channels and (2) the adiabatic condition is more readily satisfied for overlapping resonances. In other words, although the oscillations and NSFP suppress one another, together they produce a more efficient conversion of ν_{eL} into other states.

We have examined in detail the evolution of a set of neutrinos with two flavors for $N_e>N_n$, $m_2>m_1$. Other possible cases can be discussed by analogy. In conclusion, we summarize the possible resonant transitions (the resonances are shown in the order of decreasing resonant density; we also show transitions due to double conversion).

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[1)]The $\nu_{eL} \to \nu^c_{\mu R}$ ($\nu^c_{\tau R}$) conversion makes solar neutrinos unobservable in the Cl–Ar experiment because their energies are below the threshold for the production of the corresponding charged lepton. However, they can be detected by using processes due to neutral currents, $\nu e \to \nu e$, $\nu d \to np\nu$, $\nu A \to \nu A^*$).

[2)]An equation equivalent to (8) was obtained in Ref. 4 but its solutions were not examined.

[3)]When only oscillations or only NSFP is considered in a system of two types of neutrinos, i.e., the basis contains only two states, this procedure gives the exact resonance conditions.

[4)]It is assumed that both these processes are suppressed outside the resonance region, i.e., $|2\mu_{12}B_0/(\Delta m^2/2E)|$, $|\mathrm{tg}\, 2\theta_0| \ll 1$.

[5)]The narrow region near $x_0 = 0.7$ at which the magnetic field has a discontinuity [see (35)] is the exception. The assumption that the change in the field can be neglected in the resonance region is then invalid.

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