Streamer discharge in a homogeneous field

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A very simple model of the propagation of a streamer in a homogeneous electric field is qualitatively investigated. It is shown that there exists a critical field at which stationary (with constant velocity and head radius) streamer motion is possible. In a field stronger (weaker) than critical the head radius and the propagation velocity should increase (decrease) with time. The theoretical estimates obtained for the streamer parameters agree well with recently published numerical calculations.

1. INTRODUCTION

A streamer is a plasma filament produced in a discharge gap and growing at a fast rate via ionization in the strong electric field near its head. The streamer mechanism of gas breakdown was suggested by Raether, Loeb, and Meek and was extensively investigated (see Ref. 1 for a survey of the literature). Streamers were observed also in solids. The streamer propagation velocity is frequently much higher than the carrier drift velocity not only in an external applied field, but also in the stronger field near the streamer head. The main physical processes causing the streamer propagation are well known. These are: 1) impact ionization near the head, which lengthens the plasma filament, and 2) displacement of the charge on the boundary of the produced plasma via Maxwell relaxation. There is nonetheless at present for this phenomenon not even a qualitative theory that describes the dependence of the parameters of the streamer on the applied voltage and on the physical properties of the medium in which it propagates. This is due to the great mathematical difficulties of solving the nonlinear partial differential equations that describe the streamer evolution. Many studies are devoted to a numerical simulation of a streamer discharge. They frequently take into account various minor processes and use at the same time unfounded assumptions that simplify the calculation. An exception is a recent paper by Dhali and Williams,[2] containing for the first time correct numerical calculations of streamer evolution in a plane-parallel gap, based on a relatively simple system of equations that take into account only the most important physical properties. Numerical calculations, of course are not sufficient for a complete understanding of the physical picture of the phenomenon.

For a streamer to develop, free electrons capable of being multiplied by impact ionization should be present in the strong field ahead of its front. A rather widespread situation is one in which the streamer discharge is produced when the medium is weakly pre-ionized. If, however there are no free (or weakly bound) electrons, some mechanism must generate preliminary electrons ahead of the streamer front. Such a mechanism can be photoionization by the streamer radiation, tunnel ionization, and extraction of electrons from the streamer head by drift in the electric field or by diffusion. In the model of Lozanskiĭ and Firsov[3] it is suggested that the main process is drift extraction of the electrons from the head. In their theory the streamer is treated as an ideally conducting ellipsoid of revolution, each point of whose surface moves with a drift velocity determined by the local value of the field. This theory is not valid for a cathode (positively charged) streamer, whose properties actually do not differ greatly from those of an anode streamer. Nor is it valid for a streamer discharge from a needle tip. In addition, it cannot explain the high streamer-propagation velocities, which are many times larger than the electron drift velocity.

In an earlier paper[4] we have proposed a qualitative theory of a streamer in a semiconductor. Evolution of a discharge from a metal tip was considered and it was assumed that the free-carrier density in the crystal is high enough. From qualitative physical considerations we obtained, in order of magnitude, all the main parameters of the streamer. These results are applicable in fact also to gases if slight modifications are made to account for the field dependences of the electron mobility.

We use in the present paper a similar qualitative approach to analyze a streamer discharge in a uniform field (plane-parallel gap). This problem differs substantially from that in Ref. 4, where a streamer propagates from the tip (assumed to be infinitely far from the second electrode) in a region where there is practically no external field. Streamer propagation is therefore possible only so long as the charge distribution over the plasma pinch is close to electrostatic, and the propagation length turns out to be finite. In a uniform field, on the contrary, the streamer can propagate without limit, and the charge distribution differs very greatly from electrostatic.

We assume, as in Refs. 2 and 4, that enough free electrons are present ahead of the streamer front. They can be produced, in particular, by preliminary ionization or by the streamer's own radiation (if the absorption length of this radiation is larger than the head radius). Just as in Ref. 2, we take into account only the above-mentioned principal processes that lead to streamer propagation, and neglect all collateral phenomena, which can be significant under real conditions. We regard this formulation of the problem as justified, for at present our understanding of the laws governing the streamer discharge is inadequate for the simplest model. Our qualitative results will be compared with numerical calculations.[2]

2. FORMULATION OF PROBLEM AND DESCRIPTION OF MODEL

Let us consider the propagation of a streamer in a gas situated in a uniform electric field. The simplest equations describing this phenomenon are

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$$\frac{\partial n}{\partial t}+\operatorname{div}(\mathbf{v}_d n)=\alpha v_d n, \qquad (1)$$

$$\frac{\partial N}{\partial t}=\alpha v_d n, \qquad (2)$$

$$\Delta\varphi=-4\pi e(N-n), \qquad (3)$$

where n and N are the densities of the electrons and positive ions, $\mathbf{v}_d$ the electron drift velocity, α the impact-ionization coefficient, φ the electrostatic potential, and e the absolute value of the electron charge.

Equations (1)–(3) describe in simplest form the main physical processes that lead to streamer propagation. Electron diffusion and ion mobility are disregarded, since simple estimates shows them to be insignificant. We assume for simplicity that the electron drift velocity is a linear function of the field $\mathbf{E}$, viz., $\mathbf{v}_d=\mu\mathbf{E}$. The impact-ionization coefficient α increases sharply (exponentially) with the field and saturates at a level α_0 at some characteristic field value E_0. A frequently used expression is

$$\alpha(E)=\alpha_0\exp(-E_0/E). \qquad (4)$$

We rewrite now Eqs. (1)–(3) in a different form, introducing the charge density $\rho=e(N-n)$ and the conductivity $\sigma=e\mu n$:

$$\partial\sigma/\partial t=\beta(E)\sigma-\mu\operatorname{div}(\sigma\mathbf{E}), \qquad (5)$$

$$\partial\rho/\partial t+\operatorname{div}(\sigma\mathbf{E})=0, \qquad (6)$$

$$\Delta\varphi=-4\pi\rho. \qquad (7)$$

We have introduced here the impact-ionization frequency $\beta(E)=\mu E\alpha(E)$.

Assume a constant and uniform external field $\mathscr{E}$ to be applied along the z axis. At the initial instant we have a low homogeneous conductivity σ_0 against the background of which there exists a seed of high conductivity (for example, on one of the planar electrodes producing the external field). The task of the theory is to describe the evolution of the seed in accordance with Eqs. (5)–(7). Another interesting question is that of the existence of self-similar solutions describing the stationary propagation of the streamer.

Let us discuss the general properties of the system (5)–(7). We note first that the second term in the right-hand side of (5), which describes the conductivity change due to electron drift, can be discarded if $\alpha(E)\delta\gg 1$, where δ is the characteristic distance over which the conductivity changes (the width of the streamer front). This condition follows from a comparison of the two terms in the right-hand side of (5) with an estimate of $\operatorname{div}(\sigma\mathbf{E})\sim\sigma E/\delta$. The field near the streamer head is of the order of [4]E_0, so that the condition indicated takes in this region the form

$$\alpha_0\delta\gg 1. \qquad (8)$$

We shall derive below an expression for the front width δ and show that the condition (8) is equivalent to the requirement that the streamer propagation velocity V be much higher than the drift velocity in the field E_0.

We assume hereafter that the condition (8) is met and use in place of (5) the equation

$$\partial\sigma/\partial t=\beta(E)\sigma. \qquad (5a)$$

Neglect of the drift term in (5) eliminates the difference between the anode and cathode streamers. In fact, Eqs. (5a), (6), and (7) are unchanged by the substitutions $\rho\to-\rho$ and $\mathbf{E}\to-\mathbf{E}$. An important property of these equations is also that they do not contain a parameter with the dimension of length. This means that the characteristic streamer dimensions (and also its velocity) are determined by the size of the initial seed, and streamers evolving from similar seeds are similar at all instants of time. Note, however, that the drift term in (5) is essential for the determination of the radial distribution of the charge in the channel, and of the threshold conditions of streamer initiation (see the discussion of these questions in Ref. 4).

Equation (5a) describes the exponential growth of the conductivity with time at a rate determined by the local value of the field E. According to this equation such a growth should, in particular, take place also far from the streamer, where there is only the external field $\mathscr{E}$. Therefore Eqs. (5a), (6), and (7) have, strictly speaking, no self-similar solutions. In view of the strong $\beta(E)$ dependence at $E\ll E_0$, however, the rate of this growth is very low. In a weak field, in fact, the impact ionization is offset by recombination processes not accounted for in Eqs. (5) and (5a). This circumstance can be taken into account formally by setting the ionization frequency β equal to zero for very weak fields (including the external field $\mathscr{E}$). This makes a self-similar solution possible.

3. PRINCIPAL PARAMETERS OF STREAMER

We present in this section several qualitative relations previously[4] obtained for the streamer parameters and valid regardless of the applied-field geometry.

1. The maximum field E_m on the streamer front should be of the order of the field E_0 at which the impact-ionization coefficient saturates. The reason is that for $E_m\ll E_0$ or $E_m\gg E_0$ the size of the region in which substantial ionization takes place is much smaller or much larger than the head radius r_0. If $E_m\sim E_0$ the dimension of this region is of the order of r_0. Inside this region we have

$$\alpha(E)\sim\alpha_0,\quad \beta(E)\sim\beta_0=\mu E_0\alpha_0.$$

2. The conductivity in the channel is $\sigma\sim\beta_0$, since its exponential growth in the ionization region ahead of the front proceeds with a time constant β_0^{-1} until the field in this region is crowded out by a Maxwell relaxation characterized by a time[1)] $(4\pi\sigma)^{-1}$.

3. The connection between the streamer propagation velocity and its radius is

$$V\sim\frac{\beta_0 r_0}{\Lambda_1},\quad \Lambda_1=\ln\frac{\beta_0}{\sigma_0}. \qquad (9)$$

This equation follows from the fact that within the time τ during which the front travels a distance r_0 the conductivity should increase from an initial value σ_0 to β_0 [obviously, $V\sim r_0/\tau$ and $\beta_0\sim\sigma_0\exp(\beta_0\tau)$]. We assume that the logarithm in (9) is a large quantity.

4. The front width is $\delta\sim V/\beta_0\sim r_0/\Lambda_1$. This quantity determines the size of the region in which the space charge is concentrated and in which the conductivity decreases noticeably. Note that $\delta\ll r_0$ and that the condition (8) is equivalent to the inequality $V\gg\mu E_0$.

The fact that the front width is much smaller than the head radius allows us to assume a small section of the front to be plane and obtain the charge distribution and the conductivity along the normal to the front at distances small compared with r_0. This yields a more rigorous derivation of the statements 2 and 4.

We choose for simplicity a front section on the streamer axis. Equations (5a), (6), and (7) can be replaced in the considered small region by one-dimensional ones. In addition, we assume that σ, ρ, and E depend only on $z - Vt$, where V is the propagation velocity. We obtain then the system of equations

$$-V\frac{d\sigma}{dz}=\beta(E)\sigma, \quad (10)$$

$$-V\frac{d\rho}{dz}+\frac{d}{dz}(\sigma E)=0, \quad (11)$$

$$\frac{dE}{dz}=4\pi\rho. \quad (12)$$

The boundary conditions are $E=\rho=0$ for $z=-\infty$ (behind the front) and $E=E_m$ and $\rho=0$ for $z=\infty$ (ahead of the front). The velocity V enters in these equations as a parameter and cannot be determined from the one-dimensional problem.

From Eq. (11) and the boundary conditions we get $\rho=\sigma E/V$. Substituting this expression in (12) and using (10) we get

$$d\sigma/dE=-\beta(E)/4\pi E. \quad (13)$$

The conductivity behind the front is given by

$$\sigma=\frac{1}{4\pi}\int_0^{E_m}\frac{\beta(E)\,dE}{E}. \quad (14)$$

Using the expression

$$\beta(E)=\beta_0(E/E_0)\exp(-E_0/E),$$

which follows from (4), we get

$$\sigma=\frac{\beta_0}{4\pi}\int_0^{E_m/E_0}\exp\left(-\frac{1}{x}\right)dx, \quad (15)$$

from which we see indeed that $\sigma\sim\beta_0$ for $E_m\sim E_0$.

It follows also from Eqs. (10)–(12) that the front width is $\delta\sim V/\beta_0$.

4. CHARGE DISTRIBUTION AND CURRENT IN THE STREAMER CHANNEL

We consider the following problem. Given a conducting region in the form of a long filament of radius r_0 and growing symmetrically in length on both sides, so that the ends of the filament move with constant velocity V (see Fig. 1a).[2)] The filament is placed in an external field $\mathscr{E}$ parallel to its axis. It is required to determine the charge distribution along the filament and the current in it.

This formulation of the problem describes describes a streamer evolving from a seed located in the center of the gap between plane electrodes (the streamer length is as yet small compared with the distance between the electrodes). If, however, the streamer grows from one of the electrodes (Fig. 1b), the charge distribution in it should be the same as when we consider only one-half of the filament in the formulated problem. (This is obvious, since a plane perpendicular to the filament and passing through its midpoint is equipotential, as is also a metallic electrode).

If the length l of the filament were constant, the problem would be purely electrostatic. In that case we know[5] that for $l\gg r_0$ the linear charge density ρ_l is given by

$$\rho_l(z)\sim\frac{\mathscr{E}z}{\Lambda_2},\quad \Lambda_2=\ln\frac{l}{r_0}, \quad (16)$$

where the coordinate z is measured from the center of the filament. The field at the end of the filament is then proportional to its length:

$$E_m\sim\frac{\rho_l(l)}{r_0}\sim\frac{\mathscr{E}}{\Lambda_2}\frac{l}{r_0}. \quad (17)$$

When the ends of the conducting filaments move, the electrostatic equations (16) and (17) remain valid so long as the length is not too large and the charge has time to spread over the filament and screen the electric field in its interior.

In the general case, the spread of the charge through the filament is described by the equation

$$\frac{\partial\rho_l}{\partial t}+\frac{\partial}{\partial z}\left[\pi r^2\sigma\left(\mathscr{E}-\Lambda_2\frac{\partial\rho_l}{\partial z}\right)\right]=0, \quad (18)$$

which is obtained from the continuity equation (6) using the "local-capacitance" approximation $\varphi_1=\rho_l\Lambda_2$, where φ_1 is the charge produced by the charges of the filament, r is the radius of the filament (it can be a smooth function of the coordinate z and differ from the head radius r_0), $\Lambda_2=\ln(a/r)$, and a is a characteristic length that determines the charge

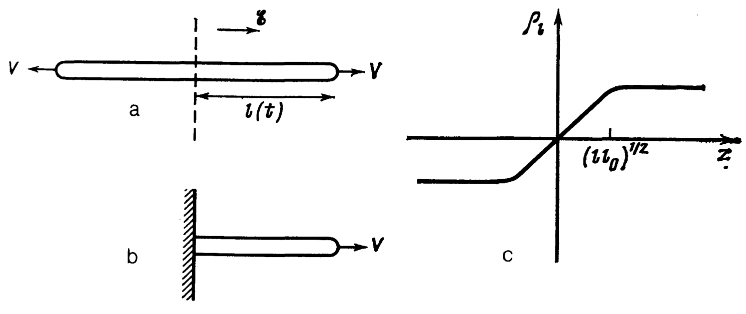

FIG. 1.

distribution (and coincides with the filament length l in the static case). It is assumed that $a \gg r$. Equation (18) is not valid in a region of order r_0 at the end of the filament. This circumstance, however, is immaterial for further qualitative estimates.

The requirement that there be no current at the end of the filament at $z = l(t)$ leads to the boundary condition

$$\rho_l V = \pi r_0^2 \sigma (\mathscr{E} - \Lambda_2 \partial \rho_l / \partial z), \tag{19}$$

which is valid, strictly speaking, at distances much larger than r_0 and much smaller than a from the end of the filament.

At constant $r = r_0$ Eq. (18) has the form of a diffusion equation with a "diffusion coefficient" $\pi r_0^2 \sigma \Lambda_2$.

Let us estimate the characteristic time t_0 during which electrostatics can be used, and the corresponding filament length l_0. The time of spreading of the charge along the filament is of the order of $l^2/(\sigma r_0^2 \Lambda_2)$. Equating it to the propagation time l/V we get

$$l_0 \sim \frac{\sigma r_0^2}{V} \Lambda_2, \quad t_0 \sim \frac{l_0}{V} \sim \frac{\sigma r_0^2}{V^2} \Lambda_2, \quad \Lambda_2 = \ln \frac{\sigma r_0}{V}. \tag{20}$$

Equations (16) and (17) are thus valid if $t \lesssim t_0$ and $l \lesssim l_0$. These are just the times of importance for a streamer propagating from a sharp tip.[4] Now, however, we are interested in a regime in which there is no time for an electrostatic distribution to set in. No screening takes place in this case and the field inside the conducting filament is equal to the external field $\mathscr{E}$. The current produced by this field is $I \sim \sigma r_0^2 \mathscr{E}$. Because of this current, the newly produced filament sections are charged at a linear charge density ρ_l determined from the condition $\rho_l V = I$, whence

$$\rho_l \sim \frac{\sigma r_0^2}{V} \mathscr{E}. \tag{21}$$

The field $E_m \sim \rho_l / r_0$ at the end of the filament is now given by

$$E_m \sim \frac{\sigma r_0}{V} \mathscr{E}. \tag{22}$$

The ratio of the maximum field at the end of the filament to the external field is thus determined by the dimensionless parameter[3)] $\sigma r_0 / V$.

We point out that Eqs. (21) and (22) are joined to Eqs. (16) and (17) at $z \sim l \sim l_0$.

It can be shown by solving Eq. (18) with the boundary condition (19) that the liner charge-density distribution along the filament has at $l \gg l_0$ the form shown schematically in Fig. 1c. The linear density has over the greater part of the filament the constant value given by Eq. (21). A deviation from this equation takes place near the midpoint of the filament in a small region whose size is determined by the "diffusion length" $(\sigma r_0^2 \Lambda_2 t)^{1/2} \sim (l l_0)^{1/2}$.

5. STREAMER PROPAGATION

We consider stationary streamer propagation (with the velocity and head radius constant), setting aside the question of its formation from the initial seed. According to Eq. (9) the velocity V is proportional to the head radius r_0, which remains arbitrary in accord with the discussion in Sec. 2. It is actually determined by the size of the seed.

We shall show that stationary propagation is possible only if the external field has a certain critical value $\mathscr{E}_c$.

We assume initially that the streamer-channel radius is constant and is of the same order as the rounding radius r_0 of the head. Using Eq. (9) and the relation $\sigma \sim \beta_0$ we obtain for the parameter $\sigma r_0 / V$ in (22) the estimate $\sigma r / V \sim \Lambda_1 \gg 1$. The necessary condition $E_m \sim E_0$, (see Sec. 3) is consequently met only for $\mathscr{E} = \mathscr{E}_c$, where[4)]

$$\mathscr{E}_c \sim E_0 / \Lambda_1. \tag{23}$$

For an external field different from $\mathscr{E}_c$ the head radius and the velocity should vary with time. In fact, if the head radius were constant then, as seen from (22), the field E_m at the head would be larger than E_0 (for $\mathscr{E} > \mathscr{E}_c$) or smaller than E_0 (for $\mathscr{E} < \mathscr{E}_c$). To ensure that $E_m \sim E_0$ it is necessary that the head radius increase or decrease ($\mathscr{E} > \mathscr{E}_c$ or $\mathscr{E} < \mathscr{E}_c$, respectively). The character of the time variation of the head radius can be determined in the following manner.

The condition $E_m \sim E_0$ requires that the field in the channel behind the front be equal to $\mathscr{E}_c$. The charge must consequently be so distributed that the field it produces offsets the difference between the external field $\mathscr{E}$ and $\mathscr{E}_c$. It is therefore necessary to satisfy near the head the condition

$$\Lambda_2 \frac{\partial \rho_l}{\partial z} \sim \mathscr{E} - \mathscr{E}_c. \tag{24}$$

Equation (24) follows from (19) if use is made of the relations $\rho_l \sim E_0 r_0, \sigma \sim \beta_0$, and (9). We express the rate of change of the head radius in the form

$$\frac{dr_0}{dt} = \frac{d}{dt} \frac{\rho_l(l,t)}{E_0} = \frac{1}{E_0} \left(\frac{\partial \rho_l}{\partial t} + V \frac{\partial \rho_l}{\partial z} \right), \tag{25}$$

where the right-hand side should be calculated for $z = l$ (at the head). We determine the derivative $\partial \rho_l / \partial z$ from (24) and the derivative $\partial \rho_l / \partial t$ from Eq. (18) in which we can neglect the small quantity $\Lambda_2 \partial \rho_l / \partial z$ compared with the field $\mathscr{E} \approx \mathscr{E}_c$:

$$-\frac{\partial \rho_l}{\partial t} \sim \sigma \mathscr{E}_c r_0 \frac{dr_0}{dz} \sim \frac{\sigma \mathscr{E}_c r_0}{V} \frac{dr_0}{dt} \sim E_0 \frac{dr_0}{dt}. \tag{26}$$

Using Eqs. (9), (23), and (24)–(26) we obtain ultimately

$$\frac{dr_0}{dt} = \gamma r_0, \quad \gamma \sim \frac{\beta_0}{\Lambda_1^2 \Lambda_2} \frac{\mathscr{E} - \mathscr{E}_c}{\mathscr{E}_c}. \tag{27}$$

The streamer propagation velocity has a similar time variation. Note that at $|\mathscr{E} - \mathscr{E}_c| \sim \mathscr{E}_c$ the time constant γ turns out to be of order t_0^{-1}, where t_0 is the characteristic time determined by Eq. (18).

Substituting $dt = dz/V$ in (27) and using Eq. (9) we get

$$\frac{dr_0}{dz} \sim \frac{1}{\Lambda_1 \Lambda_2} \frac{\mathscr{E} - \mathscr{E}_c}{\mathscr{E}_c}. \tag{28}$$

Equation (28) shows how the head radius varies when the streamer length is increased. It can be seen that even for $|\mathscr{E} - \mathscr{E}_c| \sim \mathscr{E}_c$ the change of the radius is relatively slow, since $\Lambda_1 \Lambda_2 \gg 1$. The length a introduced above is of the order of $r_0(dz/dr_0)$, so that with logarithmic accuracy

$$\Lambda_2 = \ln \frac{\Lambda_1 \mathscr{E}_c}{|\mathscr{E} - \mathscr{E}_c|}. \tag{29}$$

Thus, for $\mathscr{E} > \mathscr{E}_c$ the streamer expands and propagates without limit, and while $\mathscr{E} < \mathscr{E}_c$ the streamer contracts and the propagation length is finite. This length increases in proportion to $(\mathscr{E}_c - \mathscr{E})^{-1}$ as $\mathscr{E} \to \mathscr{E}_c$.

The case $\mathscr{E} < \mathscr{E}_c$, however, requires further analysis, since it can be shown that the contraction is accompanied by an increase, with time, of the linear charge density of the streamer filament. As a result, the radial field increases and the streamer channel begins to expand on account of impact ionization. It can be shown that this process becomes substantial after a time $t_1 \sim \gamma^{-1}\mathscr{E}_c(\mathscr{E}_c - \mathscr{E})^{-1}$. If the difference between $\mathscr{E}$ and $\mathscr{E}_c$ is small, the streamer will undergo during this time the very strong contraction described by Eqs. (27) and (28). The question of streamer development at $t > t_1$ remains unanswered. If $\mathscr{E} > \mathscr{E}_c$, however, no such question arises, since the charge density in the streamer filament decreases with time.

Note that if r_0 is large enough Eq. (9) leads to a propagation velocity exceeding the speed of light c. A velocity-limiting mechanism must consequently exist. This restriction is due to the onset of a vortical electric field at the streamer head. The question of the influence of a vortical field on streamer propagation is quite complicated and has, to our knowledge, not been considered so far. We estimate the strength of the vortical field, assuming it to be weak. The magnetic field produced by the current flowing in the streamer channel is of the order of $H \sim \sigma r_0 \mathscr{E}/c$. The derivative $\partial H/\partial t$ in the region of the head differs from zero, and it is this which leads to the onset of a vortical field

$$\mathscr{E}' \sim VH/c \sim (\sigma r_0 V/c^2)\mathscr{E}.$$

The parameter that determines the role of the vortical field is thus

$$\eta = \sigma r_0 V/c^2. \tag{30}$$

The qualitative equations obtained in this present paper are valid so long as the parameter η is small. If it is assume that the velocity is limited by the condition $\eta \sim 1$, we obtain, substituting $\sigma \sim \beta_0$ in (30) and using Eq. (9),

$$r_0 \sim \frac{c}{\beta_0}\Lambda_1^{1/2}, \quad V \sim \frac{c}{\Lambda_1^{1/2}}. \tag{31}$$

It can be assumed that Eq. (31) determines the maximum values of the head radius and the streamer velocity.

6. COMPARISON WITH RESULTS OF NUMERICAL CALCULATIONS

In Ref. 2 they solved numerically the system (1)–(3) with additional terms that accounted for the electron and ion diffusion and for the ion mobility. They investigated, in a plane-parallel gap, the evolution of a streamer from a hemispherical highly-conducting "seed" placed on one of the electrodes. The paper contains distribution profiles of the electric field and of the electron density at various instants of time, for several external-field values in the interval $(0.18–0.30)E_0$. The ratio of the propagation velocity V to the electron drift velocity $v_d = \mu E_m$ on the front ranged from 1 to 6, (Recall that all the relations given above are valid under the assumption that $V/v_d \gg 1$.)

Let us compare the results of these calculations with our theoretical estimates.

1. Field at the head. The ratio E_m/E_0 was equal in different cases to 0.4–0.8, in agreement with our condition $E_m \sim E_0$.

2. Conductivity behind the front. The ratio σ/β_0, which is theoretically of the order of unity, ranged in the numerical experiments from 0.02 to 0.1. Using for the theoretical value of the conductivity the more accurate expression (15) instead of β_0, the corresponding ratio is found to be in the interval 3–6. This result is satisfactory, especially if it is recognized that the conductivities behind and ahead of the front differ by several orders.

3. Front width We have shown that $\delta \sim r_0/\Lambda_1$. From the results of Ref. 2 it follows that $\delta\Lambda_1/r_0 = 1$ to 5. (Note that the value of δ obtained from the plots of Ref. 2 is subject to large errors.)

4. Relation between the velocity and the head radius. The dependence of V on r_0 was not specially investigated in Ref. 2. For the specific values contained in this reference we obtain $\beta_0 r_0/(V\Lambda_1) = 2–6$, which is in good agreement with Eq. (9). Note also that in the numerical experiment the velocity V increases slowly when the electron density ahead of the front is increased [but somewhat faster than would follow from (9)].

5. Field in channel. It was shown in Sec. 5 that this field is given by $\mathscr{E}_c \sim E_0/\Lambda_1$. From the data of Ref. 2 it follows that the field in the channel actually decreases with increase of the logarithm Λ_1. The ratio of the field behind the front and E_0/Λ_1 ranges from 1 to 2.

6. Field dependence of the character of the propagation. This dependence was not investigated in detail in Ref. 2. It appears that the condition $\mathscr{E} > \mathscr{E}_c$ was met in all the investigated cases. This, as we have shown, should be accompanied by a slow increase of the head radius with time. This increase was indeed observed in Ref. 2. One cannot conclude from the data there whether the increase would agree with Eqs. (27) and (28). We note also that in the numerical experiments the streamer-channel radius was found to be proportional to the radius of the initial seed. This agrees with the statement made in Sec. 2.

The qualitative equations of the present paper are thus in agreement with the results of numerical calculations.

We are grateful to M. E. Levinshteĭn for calling our attention to Ref. 2.

[1)] If the mobility depends on the field, the actual relation is $\sigma \sim \beta_0\mu_1/\mu_2$, where μ_2 is the mobility in the strong field at the head and μ_1 is the mobility in the streamer channel.[4]

[2)] We emphasize that what is meant here is not motion of the filament material, but increase of the dimensions of the conducting region.

[3)] In this estimate we take into account only the field produced by the filament charge, and neglect the external field. This is justified if $\sigma r_0/V \gg 1$. This is precisely the case in which we shall be interested hereafter.

[4)] If the mobilities ahead of the front (μ_2) and behind the front (μ_1) are different (see footnote 1) an additional factor μ_2/μ_1 appears in the right-hand side of (23).

[1] Yu. P. Raĭzer, *Gas-Discharge Physics* [in Russian], Nauka, 1987.
[2] S. K. Dhali and P. F. Williams, J. Appl. Phys. **62**, 4696, (1987).
[3] E. D. Lozanskiĭ and O. B. Firsov, *Spark Theory* [in Russian], Atomizdat, 1975.
[4] M. I. D'yakonov and V. Yu. Kachorovskiĭ, Zh. Eksp. Teor. Fiz. **94**, No. 5, 321 (1988) [Sov. Phys. JETP **67**, 1049 (1988)].
[5] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, Pergamon, 1984 (p. 35 of Russ. orig.).

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