Upper critical superconductivity field at a twin boundary

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A theoretical investigation is reported of the superconducting properties of a twin boundary allowing for its finite transparency to conduction electrons. The temperature dependence of the upper critical field $H_d(T)$ is calculated for the parallel orientation. The angular dependence of this field is considered in the range of small angles of inclination of the field to the boundary. A study is also made of the behavior of the magnetic moment and of the critical current near the $H_d(T)$ curve. The finite transparency of a twin boundary gives rise to two phase transitions associated with the symmetry of a superconducting nucleus relative to the boundary.

1. INTRODUCTION

The surface superconductivity at twin boundaries was discovered by Khaĭkin and Khlyustikov in tin[1] and later the effect was confirmed in other metals (for reviews see Refs. 2 and 3).

The influence of twin boundaries on the properties of high-temperature superconductors of the $YBa_2Cu_3O_7$ type is now attracting much interest. A transition from the tetragonal to the orthorhombic phase in these superconductors creates a system of parallel twin boundaries with a period from 100 to 1400 Å (Refs. 4–7). Direct investigations of the influence of these boundaries on the superconducting properties are so far impossible, because there are no methods for preparing single crystals with different densities of such boundaries. However, there have been many experiments which can be interpreted as a manifestation of a localized twin superconductivity in a small interval above the bulk superconducting transition temperature T_c. A square-root temperature dependence of the upper critical field $[H_{c2} \propto (T_0 - T)^{1/2}]$ is reported in Ref. 7. It is attributed to the localization of the superconductivity near twin boundaries. The temperature dependence of the specific heat[8] exhibits two discontinuities: a small one at 93 K and a large one at 89 K. It is shown in Ref. 9 that the small discontinuity may be due to a superconducting transition at twin planes. The temperature dependence of the critical current in polycrystalline $YBa_2Cu_3O_7$ samples is reported in Ref. 10. By analogy with twinned Nb crystals, in a small interval above T_c there is a weak temperature dependence of the critical current. A resistive transition in single crystals broadens greatly in an external magnetic field (see, for example, Ref. 11). This effect may be due to a twin superconductivity (a magnetic field parallel to a twin boundary reduces the temperature of the bulk transition more than the temperature of the surface superconductivity). There is therefore much indirect experimental evidence of an increase in the temperature of the superconducting transition near a twin boundary. On the other hand, a direct observation of a vortex lattice by the decoration method shows that at low temperatures the Abrikosov vortices are attracted to twin boundaries.[12] In a simple model allowing only for an increase in the superconducting coupling constant near a boundary,[13–15] the vortices should be repelled by the boundaries at all temperatures below T_c. This model assumes continuity of the order parameter at a twin boundary, which corresponds to vanishing of the coefficient of reflection of electrons by the boundary. The Ginzburg–Landau equation is used in Ref. 16 to calculate the interaction between a vortex filament and a twin boundary allowing for an increase in the transition temperature near the boundary and for a finite reflection coefficient of the boundary. It is shown there that cooling transforms repulsion of a vortex by a boundary into attraction. It therefore follows that attraction of vortices to twin boundaries at low temperatures does not exclude the possibility of existence of a localized twin superconductivity above T_c.

The main characteristics of the twin superconductivity have been calculated using a model with an order parameter continuous at a twin boundary.[2,3,13–15] Allowance for the finite reflection coefficient of electrons alters qualitatively the superconducting properties of a twin boundary. We shall calculate the temperature dependence of the upper critical field $H_d(T)$ of a twin boundary in the Ginzburg–Landau range when a magnetic field is oriented parallel to the boundary. The finite reflection coefficient of the boundary is allowed for phenomenologically using an approach suggested by Andreev.[17] In weak magnetic fields a superconducting nucleus is symmetric relative to the boundary and the order parameter is continuous, but on increase in the magnetic field the nucleus becomes asymmetric and a jump of the order parameter appears at the boundary. The change in the symmetry of the nucleus is due to a second-order phase transition. A further increase in the magnetic field restores the symmetry of the nucleus as a result of a first-order phase transition. The angular dependence of the upper critical field is calculated for low angles of inclination of the field relative to a twin boundary. An analysis is also made of the behavior of the magnetic moment and of the critical current along the $H_d(T)$ curve.

2. GINZBURG–LANDAU EQUATIONS AND BOUNDARY CONDITIONS

We shall consider a type-II superconductor consisting of two twins separated by a plane boundary (assumed to be the $z = 0$ plane). We shall consider only the case when the boundary is a symmetry plane. It is shown in Ref. 16 that the free energy F of a superconductor with a plane inhomogeneity can be represented by a sum of volume (F_V) and surface (F_S) parts:

$$F=F_V+F_S, \tag{1a}$$

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$$F_V=\int d^3r\left\{a_0\tau|\Psi|^2+\frac{b}{2}|\Psi|^4+(4M_{jn}^{(\pm)})^{-1}\times\left(i\frac{\partial}{\partial r_j}-\frac{2e}{c}A_j\right)\Psi^*\left(-i\frac{\partial}{\partial r_n}-\frac{2e}{c}A_n\right)\Psi+(\mathbf{B}-\mathbf{H})^2/8\pi\right\}, \quad (1b)$$

$$F_S=\int_{z=0} d^2\rho\left\{-\gamma(|\Psi_+|^2+|\Psi_-|^2)+\frac{1}{4M_{zz}\alpha}|\Psi_+-\Psi_-|^2\right\}, \quad (1c)$$

where ρ is the radius vector parallel to the $z=0$ plane; $\Psi_\pm(\rho)=\Psi(z=\pm 0,\rho)$; $\tau=(T-T_c)/T_c$; the indices $\pm$ of the tensor M_{jn} correspond to the regions $z>0$ and $z<0$.

The superconducting properties of a symmetric twin boundary are characterized by two phenomenological parameters α and γ which have a simple physical meaning. The parameter γ is related to the change in the superconducting coupling constant near a twin boundary. If $\gamma>0$, we have an interval of a localized two-dimensional superconductivity. The superconducting transition temperature T_d on a twin plane in zero magnetic field is then given by

$$\tau_d=(\gamma/a_0\xi_{zz})^2,$$

where

$$\tau_d=(T_d-T_c)/T_c,\quad \xi_{ij}=(4a_0M_{ij})^{-1/2}$$

is the tensor of the correlation lengths. The constant α represents the superconducting coupling between neighboring twins and is governed by the transparency of the boundary between the twins. For example, if an insulating spacer is present between two identical isotropic superconductors, we have[18]

$$\alpha=\frac{7\zeta(3)v_F}{3\pi^3T_c}\left(\int_0^1\cos\theta D(\theta)\,d\cos\theta\right)^{-1}$$

when the transparency of the spacer is low, but

$$\alpha=\frac{\pi^3v_F}{28\zeta(3)T_c}\int_0^1\cos^3\theta R(\theta)\,d\cos\theta$$

when the transparency of the spacer is high [$D(\theta)$ and $R(\theta)$ are, respectively, the transmission and reflection coefficients for electrons incident at an angle θ on the spacer].

The following characteristic scales are used in an analysis of the twin superconductivity: the temperature scale T_d-T_c; the order parameter scale $\Psi_d=(a_0\tau_d/b)^{1/2}$; the scales of the jth component of the coordinate $\xi_j(\tau_d)^{-1/2}$ and of the jth component of the magnetic field

$$H_{c2}^{(j)}(-\tau_d)=\Phi_0\xi_j\tau_d/2\pi\xi_1\xi_2\xi_3$$

(Φ_0 is a flux quantum). For this reason it is convenient to adopt the following dimensionless variables:

$$t=(T-T_c)/(T_d-T_c),\quad \tilde\Psi=\Psi/\Psi_d,\quad u_j^{(\pm)}=r_j^{(\pm)}(\tau_d)^{1/2}/\xi_j,$$

$$h_j^{(\pm)}=H_j^{(\pm)}/H_{c2}^{(j)}(-\tau_d),\quad a_j^{(\pm)}=(2e/c)\xi_j(\tau_d)^{-1/2}A_j^{(\pm)}.$$

Since in the regions separated by a twin boundary the principal axes of the tensor of the effective masses have different directions, it follows that in general the vectors $\mathbf{h}^{(+)}$ and $\mathbf{h}^{(-)}$ do not coincide. The transparency of the boundary will be described by a dimensionless parameter

$$r=\alpha(\tau_d)^{1/2}/\xi_{zz}.$$

In the case of a boundary with the reflection coefficient $R\ll 1$ this parameter is $r\propto R(\tau_d)^{1/2}$. When these substitutions are made in the functional of Eq. (1) and it is varied with respect to Ψ and $\mathbf{a}$, we obtain the dimensionless Ginzburg–Landau equations[1)]:

$$t\Psi+|\Psi|^2\Psi+(-i\partial/\partial\mathbf{u}-\mathbf{a}^{(\pm)})^2\Psi=0, \quad (2a)$$

$$\operatorname{curl}(\hat\varkappa_\pm^2\mathbf{h}^{(\pm)})={}^1/_2i(\Psi\nabla\Psi^*-\Psi^*\nabla\Psi)+\mathbf{a}^{(\pm)}|\Psi|^2, \quad (2b)$$

and the boundary conditions for the twin surface:

$$(\partial/\partial u_z-ia_z^{(+)})\Psi_+=-\Psi_++(\Psi_+-\Psi_-)/r,$$
$$(\partial/\partial u_z-ia_z^{(-)})\Psi_-=\Psi_-+(\Psi_+-\Psi_-)/r, \quad (3)$$

where $\mathbf{h}^{(\pm)}=\operatorname{curl}(\mathbf{a}^{(\pm)})$; $\Psi_\pm=\Psi(u_x,u_y,u_z=\pm 0)$; $\hat\varkappa$ is the tensor of the Ginzburg–Landau constants (we shall assume that all the principal values of this tensor exceed $2^{-1/2}$).

3. UPPER CRITICAL FIELD IN AN ORIENTATION PARALLEL TO A TWIN BOUNDARY

When a magnetic field is applied parallel to a twin boundary, the dimensionless magnetic fields on both sides of the boundary are the same: $\mathbf{h}^+=\mathbf{h}^-\equiv\mathbf{h}$. We can find the temperature dependence of the upper critical field $H_d(T)$ if we know the maximum eigenvalue $t(h)$ of Eq. (2a) (in the approximation linear in Ψ) subject to the boundary conditions of Eq. (3). If we select the Landau gauge $\mathbf{a}=(hu_z,0,0)$ and represent the function $\Psi(u)$ in the form

$$\Psi(u)=\exp(ihZu_x)\Psi(u_z),$$

then the equation for $\Psi(u_z)$ becomes the Schrödinger equation for a linear oscillator:

$$-\frac{\partial^2}{\partial u_z^2}\Psi(u_z)+h^2(Z-u_z)^2\Psi(u_z)=-t(h,Z)\Psi(u_z). \quad (4)$$

The superconducting transition temperature in a magnetic field $t(h)$ is governed by the maximum, in respect of the coordinate Z of the center of the orbit, of the value of the function $t(h,Z)$. The solution of Eq. (4) decreasing for $u_z\to\pm\infty$ is of the form

$$\Psi(u_z)=\begin{cases}C_+U(t/2h,\ (2h)^{1/2}(u_z-Z)) & \text{for}\quad u_z>0\\ C_-U(t/2h,\ -(2h)^{1/2}(u_z-Z)) & \text{for}\quad u_z<0\end{cases},$$

where $U(a,x)$ is a parabolic cylinder function.[19] Matching the solutions at the point $u_z=0$ by means of the boundary conditions of Eq. (3), we obtain the following equation for the determination of $t(h,Z)$:

$$\Delta(t,h,Z)\equiv 2hU_+'U_-'-(1-2/r)U_+U_-$$
$$-2(\pi h)^{1/2}(-1+1/r)/\Gamma((t/h+1)/2)=0, \quad (5)$$

where $\Gamma(x)$ is the gamma function. For economy of space, we shall use the following notation

$$U_{\pm}=U(t/2h, \mp(2h)^{1/2}Z),$$
$$U_{\pm}'=-(2h)^{-1/2}\frac{\partial}{\partial Z}U\left(\frac{t}{2h}, \mp(2h)^{1/2}Z\right).$$

We shall also employ the following identity:

$$U_+U_-'-U_+'U_-=(2\pi)^{1/2}/\Gamma((t/h+1)/2).$$

At an extremum Z_0 of the function $t(h, Z)$ we have $\partial\Delta(t,h,Z)/\partial Z=0$. Differentiating Eq. (5) with respect to Z and using Eq. (4), we obtain

$$\left(h^2Z^2+t-1+\frac{2}{r}\right)\frac{\partial}{\partial Z}(U_+U_-)\,|_{Z=Z_0}=0. \quad (6)$$

In the interval $1-2/r<t(h)<1$ the maximum of the function $t(h, Z)$ occurs at a point $Z_0=0$. In this case there is no discontinuity of the order parameter at the twin boundary and Eq. (5) reduces to an equation derived in Ref. 14 for $r=0$:

$$2(\pi h)^{1/2}=B(1/2, (t/h+1)/4), \quad (7)$$

where $B(x,y)=\Gamma(x)\Gamma(y)/\Gamma(x+y)$ is the beta function. If $t<1-2/r$, the solution becomes unstable, i.e., the point $Z_0=0$ does not correspond to a maximum but to a minimum of the function $t(h, Z)$. In fact, Eq. (5) yields the second derivative of $t(h, Z)$ with respect to Z:

$$\left.\frac{\partial^2 t(h,Z)}{\partial Z^2}\right|_{Z=0}=-2rh^2\frac{\partial}{\partial h}\left(\frac{t}{h}\right)\left(t-1+\frac{2}{r}\right)(t-1). \quad (8)$$

If $t(h)<1-2/r$, this derivative becomes positive. At such temperatures the maximum of the function $t(h, Z)$ is observed at the points

$$Z_0=\pm[1-2/r-t(h)]^{1/2}/h.$$

The temperature $t_{d1}=1-2/r$ corresponds to a second-order phase transition. Below this temperature a superconducting nucleus becomes asymmetric relative to the twin boundary. Figures 1a and 1b show schematically the shape of a nucleus above and below the temperature t_{d1}, respectively. A similar transition occurs also in a "sandwich" consisting of two superconducting films separated by a nonsuperconducting spacer.[20] Near the critical point we can use a valid expansion of $t(h, Z)$ in powers of Z, similar to the Landau expansion in the theory of second-order phase transitions:

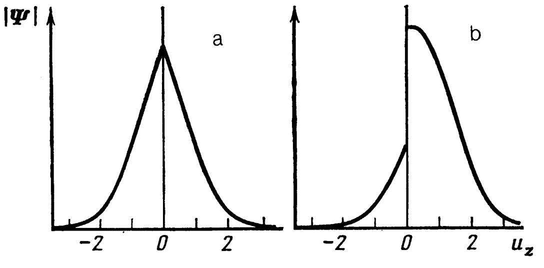

FIG. 1. Shape of a superconducting nucleus: a) symmetric phase (in the interval $t_{d1}<t<1$); b) asymmetric phase (in the interval $t<t_{d1}$).

$$t(h,Z)=t_0(h)+\frac{1}{2}\frac{\partial^2 t}{\partial Z^2}Z^2+\frac{1}{24}\frac{\partial^4 t}{\partial Z^4}Z^4. \quad (9)$$

Near the critical point the dependence $t(h)$ is

$$t(h)=\begin{cases} t_0(h) & \text{for} \quad t(h)>1-\dfrac{2}{r} \\ t_0(h)-\dfrac{3}{2}\left[\dfrac{\partial^2 t}{\partial Z^2}\right]^2\Big/\dfrac{\partial^4 t}{\partial Z^4} & \text{for} \quad t(h)<1-\dfrac{2}{r} \end{cases}. \quad (10)$$

It follows from Eq. (10) that the second derivative of the critical field with respect to temperature has a discontinuity at the transition point. In dimensional units this discontinuity can be represented as follows:

$$\left.\frac{d^2H_d}{dT^2}\right|_{T=T_{d1}+0}-\left.\frac{d^2H_d}{dT^2}\right|_{T=T_{d1}-0}$$
$$=\frac{2}{H_d}\left[\frac{dH_{c2}}{dT}\right]^2\left(1-\frac{T-T_c}{H_d}\frac{dH_d}{dT}\right),$$
$$T_{d1}=T_d-2[T_c(T_d-T_c)]^{1/2}\xi_{zz}/\alpha, \quad (11)$$

where H_{c2} is the bulk upper critical field along the applied magnetic field.

The temperature T_{d1} lies in the Ginzburg–Landau range if the characteristic reflection coefficient satisfies $R\gg(\tau_d)^{1/2}$.

In strong magnetic fields, it follows from Eqs. (5) and (6) that

$$H_d(T)=H_{c3}(T)\{1-3.32\cdot10^{-3}(-\tau_d^{1/2}+\xi_{zz}/\alpha)/\tau^{1/2}\}, \quad (12)$$

where $H_{c3}(T)=1.695H_{c2}(T)$ is the upper critical field of a free surface. Equation (12) is valid at temperatures $10^{-5}(-(\tau_d)^{1/2}+\xi_{zz}/\alpha)^2\ll|\tau|\ll1$. The critical fields of a twin niobium crystal were investigated at $T=4.2$ K and the results were reported in Ref. 21. It was found that the upper critical field for the twin superconductivity was indeed close to H_{c3}.

Only numerical methods can be used to find the $H_d(T)$ dependences for arbitrary parameters and for any temperature. Figure 2 shows these dependences in dimensionless

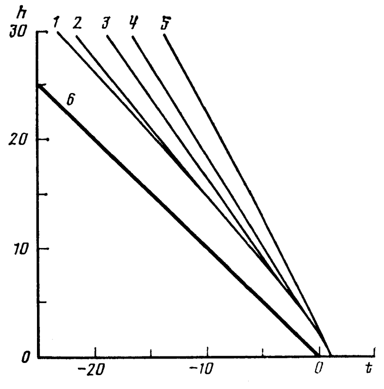

FIG. 2. Temperature dependences of the upper critical field of a twin boundary calculated for different values of the parameter r: 1) 0; 2) 0.2; 3) 0.5; 4) 1; 5) 5; 6) temperature dependence of the bulk upper critical field.

units (obtained for different values of the parameter r). In the interval $1 - 2/r < t < 1$ these dependences were obtained by numerical solution of the transcendental equation (7), whereas in the interval $t < 1 - 2/r$ they were found by numerical integration of Eq. (4).

4. UPPER CRITICAL FIELD AT LOW ANGLES OF INCLINATION OF A MAGNETIC FIELD RELATIVE TO A TWIN BOUNDARY

It is known that in the case of the surface superconductivity the dependence of the upper critical field on the angle of inclination φ (Fig. 3) is beak-shaped at low angles:

$$H_d(\theta,\varphi) \approx H_d(\theta,0) + \frac{\partial H_d}{\partial|\varphi|}|\varphi|. \qquad (13)$$

In the present section we shall describe calculation of $\partial H_d/\partial|\varphi|$ for a twin boundary. A general expression for this derivative is obtained in Ref. 20 for the case of a superconductor with a planar inhomogeneity. In our case there is an additional contribution to $\partial H_d/\partial|\varphi|$ associated with different orientations of the anisotropy axes in neighboring twins. The ground state of the asymmetric phase is doubly degenerate. If the field is inclined, this degeneracy is lifted, since a nucleus tends to become localized in that twin with the lower bulk critical field. This gives rise to a correction to the transition temperature and this correction is proportional to φ. We shall assume that the u_x axis is directed along the vector $\mathbf{h} = (\mathbf{h}_+ + \mathbf{h}_-)/2$ and the u_y axis at right-angles to this vector and parallel to the surface. Then the Ginzburg–Landau equations, accurate to the first order in φ, become

$$-\frac{\partial^2\Psi}{\partial u_x^2} - \frac{\partial^2\Psi}{\partial u_z^2} + \left(-i\frac{\partial}{\partial u_y} - hu_z\right)^2\Psi + \left[\Delta h_\parallel |u_z| \left(-i\frac{\partial}{\partial u_y} - hu_z\right)\Psi - i\Delta h_\perp |u_z| \frac{\partial\Psi}{\partial u_x}\right] = -t\Psi, \qquad (14)$$

where

$$\Delta h_\parallel = -2h\frac{\partial H_{c2}}{\partial\varphi}\frac{\varphi}{H_{c2}}, \quad \Delta h_\perp = h\frac{2\varphi\cos\theta\,\partial H_{c2}/\partial\varphi}{((H_{c2}^{(2)})^2 - (H_{c2}\cos\theta)^2)^{1/2}},$$

and the boundary conditions are deduced from Eq. (3) by the substitutions

$$(\partial/\partial u_z - ia_z^{(\pm)})\Psi_\pm \to (\partial/\partial u_z + \tilde{\varphi}\partial/\partial u_x)\Psi|_{u_z+\tilde{\varphi}u_x=\pm 0},$$
$$\Psi_\pm \to \Psi(u_z + \tilde{\varphi}u_x = \pm 0),$$

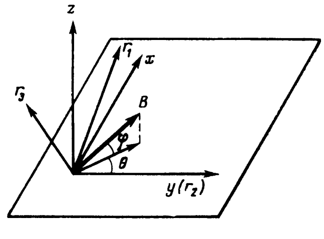

FIG. 3. Orientations of the coordinate axes relative to a twin boundary (r_i is the anisotropy axes).

where

$$\tilde{\varphi} = \lambda(\theta)\varphi, \quad \lambda(\theta) = H_{c2}(\theta,0)H_{c2}(\pi/2,0)/H_{c2}^{(1)}H_{c2}^{(3)},$$

$H_{c2}^{(1)}$ and $H_{c2}^{(3)}$ the critical fields along the crystallographic axes which are not parallel to the surface (Fig. 3). The terms in the square brackets in Eq. (14) can be taken into account by perturbation theory and the small derivative $\partial^2\Psi/\partial u_x^2$ can be found by the method of adiabatic separation of the variables.[20] In this way we obtained the correction to t which is linear in φ:

$$t(\varphi) - t(0) = h|\Delta h_\parallel|\langle |u_z|(u_z - Z_0)\rangle - \lambda(\theta)\left|\frac{1}{2}\frac{\partial^2 t}{\partial Z_0^2}\right||\varphi|, \qquad (15)$$

where $\langle\ldots\rangle$ denotes averaging over the wave function of the ground state when $\varphi = 0$.

If $T > T_{d1}$, a nucleus is symmetric relative to the boundary and we have $Z_0 = 0$, so that the first term in Eq. (15) vanishes. In this region we can use Eq. (8) and obtain the following expression for the derivative $\partial H_d/\partial|\varphi|$ (in dimensional units):

$$\frac{\partial H_d}{\partial|\varphi|}(\theta) = -\lambda(\theta)\left\{2\frac{\partial H_d}{\partial T}\left(H_d - (T - T_c)\frac{\partial H_d}{\partial T}\right) \times \frac{(T - T_{d1})(T_d - T)}{T_d - T_{d1}}\right\}^{1/2}. \qquad (16)$$

If $T \to T_{d1}$, this derivative vanishes in accordance with a square-root law.[20] The asymmetric phase is described by the following expression near the transition point:

$$\frac{\partial H_d}{\partial|\varphi|}(\theta) = \left\{\frac{T_{d1} - T}{T_d - T_c}\right\}^{1/2}\left[2\left(1 - \frac{T_{d1} - T_c}{H_d}\frac{\partial H_d}{\partial T}\right) \times \left|\frac{\partial H_{c2}}{\partial|\varphi|}\right|\frac{T_d - T_{d1}}{H_{c2}}\frac{\partial H_{c2}}{\partial T} - \lambda(\theta)\left\{4\frac{\partial H_d}{\partial T}\left(H_d - (T_{d1} - T_c)\frac{\partial H_d}{\partial T}\right)(T_d - T_c)\right\}^{1/2}\right]. \qquad (17)$$

It should be noted that the two terms in Eq. (17) have opposite signs, so that in the case of a sufficiently strong anisotropy of the bulk field H_{c2} the derivative $\partial H_d/\partial|\varphi|$ can reverse sign at the transition point.

In strong magnetic fields we can use the results of Ref. 22 for superconductor–vacuum interfaces, which gives

$$\frac{\partial H_d}{\partial|\varphi|} = H_{c3}\left\{\frac{1}{H_{c2}}\left|\frac{\partial H_{c2}}{\partial\varphi}\right| - 1{,}30\lambda\right\}. \qquad (18)$$

5. DIAMAGNETIC MOMENT IN A PARALLEL FIELD *H* CLOSE TO H_d

The upper critical field of the twin superconductivity is usually found by measuring the field dependence of the diamagnetic moment. Near H_d this dependence is linear. It would be of interest to determine the temperature dependence of the coefficient in the linear form. We shall confine ourselves to a superconductor which is patently of type II, so that $\varkappa \gg 1$. The standard derivation (see, for example, Ref. 23) of this temperature dependence gives the following expression for the magnetic moment $M(H)$ per unit area of a twin boundary:

$$M(H)=\frac{\xi_{zz}}{\tau_d^{1/2}}\frac{(H-H_d)(dt/dh)^2}{8\pi\varkappa^2\beta_A}, \tag{19}$$

where

$$\beta_A=\int_{-\infty}^{\infty} du_z\langle|\Psi(u_z)|^4\rangle_y \Big/ \left(\int_{-\infty}^{\infty} du_z\langle|\Psi(u_z)|^2\rangle_y\right)^2$$

is the Abrikosov parameter,

$$1/\varkappa^2=\sum_{i=1}^{3}(b_i/\varkappa_i)^2$$

(**b** is a unit vector in the direction of the magnetic field), and $\langle...\rangle_y$ denotes averaging over the coordinate u_y.

The temperature dependence of $\partial M/\partial H$ has the following asymptotes:

$$\frac{\partial M}{\partial H}\approx\begin{cases}-\xi_{zz}\dfrac{\tau_d-\tau}{2\pi\varkappa^2(\tau_d)^{3/2}} & \text{for} \quad \tau\to\tau_d\\ -\xi_{zz}|\tau|^{-1/2}\dfrac{0{,}92}{8\pi\varkappa^2} & \text{for} \quad |\tau|\gg10^{-5}|-\tau_d^{1/2}+\xi_{zz}/\alpha|^2.\end{cases} \tag{20}$$

The function $\partial M/\partial H(T)$ has an inflection at the temperature T_{d1} of the transition to the asymmetric phase.

If $T<T_{d1}$, there is one other characteristic point at which the shape of a nucleus changes. In the asymmetric phase the ground state is doubly degenerate: there are therefore two solutions (Ψ_1 and Ψ_2) which correspond to the localization of a nucleus on either side of a twin boundary. This degeneracy is lifted if we allow for the nonlinear terms in the Ginzburg–Landau equations. The shape of a nucleus is described by a linear combination of the solutions $\Psi=C_1\Psi_1+C_2\Psi_2$. If $2\varkappa^2\gg 1$, the coefficients in the linear combination are found by minimization of the Abrikosov parameter β_A. A simple analysis shows that the selection of the coefficients is governed by a dimensionless parameter

$$\alpha_0=\int_{-\infty}^{\infty}|\Psi_1\Psi_2|^2du_z\Big/\int_{-\infty}^{\infty}|\Psi_1|^4du_z,$$

where a symmetric combination of the solutions ($C_1=C_2$) is obtained for $\alpha_0<1/2$, whereas for $\alpha_0>1/2$ one of the coefficients vanishes. We then have

$$\beta_A=\begin{cases}\displaystyle\int_{-\infty}^{\infty}|\Psi_1|^4du_z\Big/\left(\int_{-\infty}^{\infty}|\Psi_1|^2du_z\right)^2 & \text{for} \quad \alpha_0>1/2\\ \displaystyle(\alpha_0+1/2)\int_{-\infty}^{\infty}|\Psi_1|^4du_z\Big/\left(\int_{-\infty}^{\infty}|\Psi_1|^2du_z\right)^2 & \text{for} \quad \alpha_0<1/2\end{cases} \tag{21}$$

We note that $\alpha_0=1$ at T_{d1} and that it decreases smoothly to zero as a result of cooling. The temperature T_{d2} defined by $\alpha_0(T_{d2})=1/2$ is a first-order phase-transition point. At this point the dependence $\partial M/\partial H(T)$ also has a kink. By way of example, Fig. 4 shows the temperature dependence of

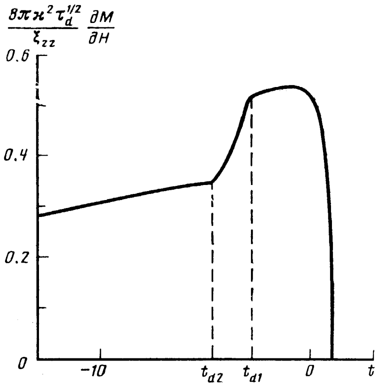

FIG. 4. Temperature dependence of the derivative $\partial M/\partial H$ calculated for $r=0.5$ (in reduced units).

$\partial M/\partial H$ in dimensionless coordinates on the assumption that $r=0.5$.

6. TEMPERATURE DEPENDENCE OF THE CRITICAL CURRENT

In two-dimensional superconductivity the critical current is governed by the depairing effect and near T_d it depends on temperature in accordance with the law

$j_c \propto (T_d-T)^{3/2}$.

A magnetic field applied parallel to a twin boundary does not alter the above law, but it has a strong influence on the coefficient in that law. The Ginzburg–Landau equations yield the following expressions for the critical-current components parallel and perpendicular to the field (when $\varkappa\gg 1$):

$$j_{c\parallel}=4j_{c0}\frac{[(t_d(h)-t)/3]^{3/2}}{\beta_A}, \tag{22a}$$

$$j_{c\perp}=j_{c0}\frac{[(t_d(h)-t)/3]^{3/2}|\partial^2t/\partial Z^2|^{1/2}}{h\beta_A}, \tag{22b}$$

where

$$j_{c0}=2ea_0^2\tau_d\xi_c\xi_{zz}/b,\quad \xi_c^{-2}=\sum_{n=1}^{3}(i_n/\xi_n)^2,$$

a_0 and b are the parameters of the Ginzburg–Landau functional, and **i** is a unit vector along the electric current.

At the phase transition points the field dependences of the coefficients in the two-thirds law for the components of the critical current have singularities. In a magnetic field $H=H_d(T_{d1})$ the coefficient of the two-thirds law for $j_{c\perp}$ vanishes in accordance with the square-root relation ship and $j_{c\parallel}$ has a kink. In a magnetic field $H=H_d(T_{d2})$ both coefficients have kinks.

7. CONCLUSIONS

An investigation of the superconducting properties of a twin boundary in a parallel magnetic field near the $H_d(T)$ curve demonstrates that the surface superconductivity region contains three phases differing in respect of the symmetry of the nucleus. At temperatures close to T_d (weak fields) the order parameter is symmetric relative to the boundary and continuous (Fig. 1a). In view of the finite transparency of the boundary, there is a critical temperature T_{d1} below which this solution becomes inappropriate. A second-order phase transition makes the nucleus asymmetric and a discontinuity of the order parameter appears at the boundary (Fig. 1b). An analysis of the nonlinear terms of the Ginzburg–Landau equation shows that further cooling (increase in the field) is characterized by one more critical temperature T_{d2} below which the shape of the nucleus is described by a symmetric combination of the solutions localized on different sides of the boundary. Since the shape of the nucleus changes abruptly, this point corresponds to a first-order phase transition.

We thus investigated the characteristic features of the superconducting phase transitions typical of a twin boundary and affecting the magnetic moment, critical current, and angular dependence of the critical current.

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[1)]We shall omit the index ~ of the dimensionless order parameter $\tilde{\Psi}$.

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