Electron mechanism of Damon–Eshbach wave damping in ferromagnetic metals

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The dispersion law for surface excitations (Damon–Eshbach waves) in ferromagnetic metals is derived for a nonlocal connection of the current density with the electric field strength. It is shown that anomaly of the skin effect can convert the surface excitations into weakly damped waves. The influence of a sufficiently strong field on the damping of the Damon–Eshbach waves is estimated.

1. A surface wave (Damon–Eshbach (DE) wave[1,2]) can propagate along the surface of a ferromagnet magnetized parallel to the surface if its propagation direction does not deviate greatly from perpendicular to the applied constant magnetic field **H.** The magnetic-field components in a DE wave satisfy the magnetostatics equation (it is assumed that $k \gg \omega/c$, where **k** and ω are the wave vector and frequency of the wave and c is the speed of light):

$$\operatorname{rot}\mathbf{h}=0, \quad \operatorname{div}\mathbf{b}=0, \quad \mathbf{b}=\hat{\mu}\mathbf{h}. \tag{1}$$

Here $\hat{\mu}$ is the magnetic-permeability tensor. It can be calculated by using the Landau–Lifshitz equation without allowance for dissipation terms and spatial dispersion, in which case

$$\hat{\mu}=\begin{pmatrix}1 & 0 & 0\\ 0 & \mu & i\mu'\\ 0 & -i\mu' & \mu\end{pmatrix},$$
$$\mu=1+\frac{\omega_M\omega_0}{\omega_0^2-\omega^2}, \quad \mu'=\frac{\omega\omega_M}{\omega_0^2-\omega^2}. \tag{2}$$

Here $\omega_0=gH$, $\omega_M=4\pi gM$, M is the static magnetization of the ferromagnet, and g is the gyromagnetic ratio. We neglect the anisotropy field compared with the external field H. if **H** is parallel to the anisotropy axis, we can replace H in (2) by $H_{\text{eff}}=H+H_a$, where H_a is the anisotropy field.

If the wave vector $\mathbf{k}(0,0,k)$ is perpendicular to the magnetic field $\mathbf{H}(H,0,0)$ and the ferromagnetic fills the half-space $y>0$, the DE wave frequency for waves with a field structure

$$\mathbf{h}=-\nabla\varphi, \quad \varphi\propto\exp(-i\omega t+ikz-\gamma|y|) \tag{3}$$

does not depend on the wave vector k:

$$\omega=\omega_{DE}, \quad \omega_{DE}=\omega_0+\omega_M/2, \tag{4}$$

and the logarithmic decrement of the damping along the y axis is equal to the wave vector k:

$$\gamma=k>0. \tag{5}$$

Note that the DE wave is non-reciprocal; it does not exist at $k<0$. The cause of the non-reciprocity is the absence of an inversion center on the magnet surface. Owing to the vector $\mathbf{y}\times\mathbf{M}$, the directions **z** and $-$**z** are not equivalent (**y** and **z** are unit vectors along the y and z axes. The non-invariance under the substitution $t\to -t$, due to the presence of the magnetic field and of the magnetic moment ($\mathbf{H}(-t)=-\mathbf{H}(t)$; $\mathbf{M}(-t)=-\mathbf{M}(t)$) also indicates that the usual condition $\omega(-k)=\omega(k)$ is not mandatory.

It is legitimate to neglect the spatial dispersion of the magnetic permeability μ if $Ja^2k^2/\beta M \ll 1$, where J is of the order of the exchange integral, $\beta=g\hbar$, and a is the lattice constant (see Ref. 3). If this requirement is not to violate the validity of magnetostatics it is necessary that the wave vector be in the range

$$\frac{\omega}{c}\ll k\ll\frac{1}{a}\left(\frac{\beta M}{J}\right)^{1/2}, \text{ where } \omega\sim\omega_{DE}, \ \omega_0, \ \omega_M. \tag{6}$$

Here and hereafter we assume in the estimates that ω_0 and ω are of the same order. The presence of the small factor $(\beta M/J)^{1/2}$ notwithstanding, the interval $(\omega/c, (1/a)(\beta M/J)^{1/2}$ is so wide that various values of the wave vector can be assumed.

Allowance for magnetic dissipation leads, naturally, to damping of the DE wave. Adding to the Landau–Lifshitz equation a relaxation term R containing two relaxation times[3]:

$$\mathbf{R}=\frac{1}{\tau_1}\tilde{\mathbf{H}}-\frac{1}{\tau_2}[\mathbf{n}[\mathbf{n}\tilde{\mathbf{H}}]]; \quad \mathbf{n}=\frac{\mathbf{M}}{M}=\frac{\mathbf{H}}{H}, \tag{7}$$

we obtain[1)] for μ and μ':

$$\mu=\frac{\omega_M\omega_0(1+1/\omega_M^2\tau^2)-i\omega/\tau}{\omega_0^2-(\omega+i\omega_0/\omega_M\tau)^2},$$
$$\mu'=\frac{\omega\omega_M}{\omega_0^2-(\omega+i\omega_0/\omega_M\tau)^2}, \tag{8}$$

where $1/\tau=1/\tau_1+1/\tau_2$ is by assumption much lower than the frequency ω, and the DE-wave frequency acquires an imaginary part

$$\omega''=-\frac{1}{4\tau}\frac{(2\omega_0+\omega_M)(4\omega_0+\omega_M)}{\omega_M(\omega_0+\omega_M)}, \tag{9}$$

which determines its damping.

Allowance for retardation (for the finite speed of light) leads to dispersion of the DE wave and "turns on" damping mechanisms due to electric losses. Analysis of the total dispersion (of the consequence of the Maxwell equations) yields for $k\gg\omega/c$

$$\omega=\omega_{DE}\left(1-\frac{\omega_M\omega_{DE}}{8c^2k^2}(1+\varepsilon)\right)<\omega_{DE}, \tag{10}$$

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where $\varepsilon=\varepsilon'+i\varepsilon''$ is the complex dielectric constant of the ferromagnet. In dielectrics we have $\varepsilon''\ll\varepsilon'$ and the main effect of taking the retardation into account is manifested by the existence of a finite (nonzero) group velocity of the DE wave:

$$v_{\text{rp}}=c\frac{\omega_M\omega_{DE}^2}{4(ck)^3}(1+\varepsilon')=\frac{2^{5/2}c(\omega_{DE}-\omega)^{3/2}}{\omega_M^{1/2}\omega_{DE}(1+\varepsilon')^{1/2}}, \tag{11}$$

which, to be sure, tends rapidly ($\propto k^{-3}$) to zero with increase of the wave vector.

According to (10), the DE wave is subject to damping due to electric losses, with

$$\omega''=-\frac{\omega_M\omega_{DE}^2}{8(ck)^2}\varepsilon''. \tag{12}$$

The electric damping can exceed the magnetic damping if

$$\omega''\ll ck\ll\omega(\omega\tau\varepsilon'')^{1/2},$$

for which the rather strong condition $\varepsilon''\gg 1/\omega\tau$ must be met. It can apparently be met, together with the condition $\varepsilon'\gg\varepsilon''$, in ferromagnetic semiconductors.

DE waves can exist, however, not only in ferromagnetic dielectrics and semiconductors, but also in ferromagnetic metals for which $\varepsilon=4\pi i\sigma/\omega$, with $4\pi\sigma/\omega\gg 1$. In metals, therefore,

$$\omega''=-\frac{\pi}{2}\frac{\omega_M\omega_{DE}}{(ck)^2}\sigma, \qquad \omega'\approx\omega_{DE}. \tag{13}$$

It must be borne in mind that the condition for the existence of DE waves in metals is much more stringent than in dielectrics. It is evident from (13) and (10) that for the damping to be small the following condition must be met:

$$k\delta\gg\frac{1}{2}\left(\frac{\omega_M}{\omega_{DE}}\right)^{1/2}, \tag{14}$$

where $\delta=c/(2\pi\sigma\omega_{\text{DE}})^{1/2}$ is the skin-layer depth at the DE-wave frequency.

At low temperatures in thin metals, the electron relaxation frequency $1/\tau_e$, where τ_e is the electron free-path time, can be lower than the DE-wave frequency ω_{DE}. For $\omega\tau_e\gg 1$

$$\varepsilon'=-\frac{\omega_L^2}{\omega^2}, \qquad \varepsilon''=\frac{\omega_L^2}{\omega^3\tau_e},$$

where ω_L is the plasma frequency of the metal. Then

$$\omega'=\omega_{DE}\left(1+\frac{\omega_M\omega_L^2}{8c^2k^2\omega_{DE}}\right)>\omega_{DE}. \tag{10a}$$

$$\omega''=-\frac{\omega_M\omega_L^2}{8c^2k^2}\frac{1}{\omega_{DE}\tau}, \qquad \omega''\ll|\omega'-\omega_{DE}|. \tag{12a}$$

Hence

$$v_{\text{rp}}=-\frac{c\omega_M\omega_L^2}{4(ck)^3}. \tag{11a}$$

and consequently the DE wave carries in this case energy in a direction opposite to that of the wave vector.

Equations (13) and (14) were obtained without allowance for the influence of the magnetic field on the conductivity. Owing to the Hall effect, the conductivity tensor becomes gyrotropic in the magnetic field.

In the simplest case (see, e.g., Ref. 4) we have

$$\hat{\sigma}=\begin{pmatrix}\sigma_\parallel & 0 & 0\\ 0 & \sigma_\perp & \sigma'\\ 0 & -\sigma' & \sigma_\perp\end{pmatrix}. \tag{15}$$

Analysis of the Maxwell equations shows that the x-component e_x of the electric field strength is exited in the DE wave considered here, and Eqs. (13) and (14) contain a conductivity component along **H**. This component is known[4] to depend weakly on the magnetic field (for an isotropic dispersion law the static conductivity $\sigma_\parallel$ is altogether independent of the magnetic field).

2. The inequality (14) shows that allowance for spatial dispersion may become more necessary for the study of DE waves in a metal than for the study of the skin effect. In fact, spatial dispersion of the conductivity, which is a consequence of the nonlocality of the coupling between the electric-field intensity and the current density in the metal, is caused by the finite electron mean free path l. A measure of the nonlocality is here quantity kl (see below), while in skin-effect theory this measure is l/δ [cf. (14)].

Our main purpose here is to develop a kinetic theory of electronic damping of DE waves in a ferromagnetic metal. Recognizing that the damping is determined by the longitudinal conductivity component, we neglect initially altogether the influence of the magnetic field on the electrons. The complete system of equations (the Maxwell equations and the kinetic equation) of the problem are then:

$$\frac{d^2e_x}{dy^2}-k^2e_x=\frac{4\pi i\omega}{c^2}\frac{\mu'^2-\mu^2}{\mu}j_x, \qquad y>0, \tag{16}$$

$$\frac{d^2e_x^v}{dy^2}-\left(k^2-\frac{\omega^2}{c^2}\right)e_x^v=0, \qquad y<0, \tag{16a}$$

$$v_y\frac{df_1}{dy}+\left(ikv_z+\frac{1}{\tau_e}\right)f_1=-e\frac{\partial f_F}{\partial\varepsilon}v_xe_x(y), \qquad y>0. \tag{17}$$

Here e_x^v is the electric field strength in vacuum, and f_l is an increment, linear in the electric field e_d, to the Fermi distribution function f_F.

$$j_x=\frac{2e}{(2\pi\hbar)^3}\int v_xf_1\,d^3p, \tag{18}$$

$\mathbf{v}=\mathbf{p}/m$ is the electron velocity; the integration in (18) is over all of **p**-space; the electron gas is degenerate, so that $-\partial f_F/\partial\varepsilon=\delta(\varepsilon-\varepsilon_F)$. The usual electrodynamic conditions (continuity of the tangential components of **e** and **h** in the wave) must be supplemented by a boundary condition for the electron distribution function f_1. We confine ourselves to specular reflection of the conduction electrons by the surface:

$$f_1\Big|_{\substack{y=0\\ v_y>0}}=f_1\Big|_{\substack{y=0\\ v_y<0}}, \qquad f_1\Big|_{\substack{y\to\infty\\ v_y<0}}=0. \tag{19}$$

The second equality ensures equilibrium of the electrons in the interior of the metal.

Equation (17) was written in the τ-approximation for the collision integral and under the assumption $\omega\tau_e\ll 1$, and the term $\partial f_1/\partial t$ was therefore omitted. The dependences of all the functions in (16) and (17) on z and y are of the form

$$\varphi(z,y)=e^{ikz}\varphi(y).$$

According to (17)–(19) we have

$$j_x(y)=\int_0^\infty K(y,y')e_x(y')dy',$$

$$K(y,y')=\frac{2e^2}{(2\pi\hbar)^3}\int_{(v_y>0)}\left(-\frac{\partial f_F}{\partial \varepsilon}\right)\frac{v_x^2}{v_y}$$

$$\times\left\{\exp\left[-\frac{|y-y'|}{v_y}\left(\frac{1}{\tau_e}+ikv_z\right)\right]\right.$$

$$\left.+\exp\left[-\frac{|y+y'|}{v_y}\left(\frac{1}{\tau_e}+ikv_z\right)\right]\right\}d^3p. \qquad (20)$$

Expressions (20) are valid, naturally, if $y>0$. An even continuation of $e_x(y)$ into the region $y<0$ (so that $e_x(-y)=e_x(y)$ allows us to solve the problem by a Fourier transformation in y, since it transforms $K(y,y')$ into a difference kernel:

$$j_x(y)=\int_{-\infty}^\infty K(y-y')e_x(y')dy',$$

$$K(y-y')=\frac{2e^2}{(2\pi\hbar)^3}\int_{(v_y>0)}\left(-\frac{\partial f_F}{\partial \varepsilon}\right)\frac{v_x^2}{v_y}$$

$$\times\exp\left[-\frac{|y-y'|}{v_y}\left(\frac{1}{\tau_e}+ikv_z\right)\right]d^3p. \qquad (21)$$

We use a Fourier transformation of Eq. (16) continued into the half-space $y<0$:

$$\left[\frac{4\pi i\omega}{c^2}\frac{\mu'^2-\mu^2}{\mu}K_h(q)+k^2+q^2\right]e(q)=-2\left(\frac{de_x}{dy}\right)_{y=0}. \qquad (22)$$

Here

$$K_h(q)=\int_{-\infty}^\infty K(y)e^{iqy}dy$$

$$=\frac{2e^2\tau_e}{(2\pi\hbar)^3}\oint\frac{v_x^2}{v}dS\frac{1+ik\tau_e v_z}{(1+ik\tau_e v_z)^2+\tau_e^2v_y^2q^2} \qquad (23)$$

is the kernel of the conductivity operator in the q representation;

$$K_0(0)=\sigma=\frac{Ne^2\tau_e}{m_e}$$

is the static conductivity of the metal, and N is the electron density.

From Maxwell's equation we obtain

$$\left(\frac{de_x}{dy}\right)_{y=0}=-k_z\frac{\mu'}{\mu}e_x(0)+\frac{i\omega}{c}\frac{\mu'^2-\mu^2}{\mu}h_z(0). \qquad (24)$$

Finally, we use the boundary conditions for **e** and **h**:

$$h_z(0)=i\frac{c}{\omega}\left(k^2-\frac{\omega^2}{c^2}\right)^{1/2}e_x(0). \qquad (25)$$

With the aid of (22) and (24) we get, after obtaining the field distribution in the metal and eliminating $(de_x(y)/dy)_0$, a system of two linear equations in $h_z(0)$ and $e_x(0)$. Equating the determinant of this system to zero we get an equation from which we determine the DE-wave dispersion law:

$$\mu=\left[k\mu'+\left(k^2-\frac{\omega^2}{c^2}\right)^{1/2}(\mu'^2-\mu^2)\right]$$

$$\times\frac{1}{\pi}\int_{-\infty}^\infty dq\left[q^2+k^2+\frac{4\pi i\omega}{c^2}\frac{\mu'^2-\mu^2}{\mu}K_h(q)\right]^{-1}. \qquad (26)$$

As $k\to\infty$ we obtain from (26), naturally, the value (4) of the DE-wave frequency. For $kl\ll 1$, when the local connection between the current density and the electric-field intensity is valid, we arrive at Eq. (10) with $\varepsilon=4\pi i\sigma/\omega$ [see also (13)].

Of greatest interest is the opposite limiting case, which is analogous to the anomalous skin effect ($kl\gg 1$). Assume that in this case

$$k\gg\frac{\omega}{c},\quad (k\delta_0)^2k\gg\frac{\omega}{v_F},\quad \delta_0=\frac{c}{\omega_L},\quad \omega_L^2=\frac{4\pi Ne^2}{m_e}. \qquad (27)$$

Since it is certain that $\omega v_F/\omega_L c\ll 1$, the second condition is more important.

The condition (27) jointly with inequality $kl\gg 1$ makes it possible to use, upon approximate evaluation of the integral of (26), the asymptotic value of $K_k(q)$, which is proportional to $(k^2+q^2)^{-1/2}$ [see (23)]. Then

$$\omega\approx\omega_{DE}-\frac{\omega_M\omega_{DE}^2}{8c^2k^2}-i\frac{\omega_M\omega_{DE}\omega_L^2}{3\pi^2c^2v_Fk^3}. \qquad (28)$$

Owing to the spatial dispersion, the dissipative term ($\propto k^{-3}$) tends to zero more rapidly than the dispersion term ($\propto k^{-2}$). If

$$k\gg\frac{8}{3\pi^2}\frac{\omega_L^2}{v_F\omega_{DE}} \qquad (29)$$

the DE wave is weakly damped. It must be remembered, however, that our entire analysis is valid if the spatial dispersion $\hat{\mu}$ is neglected: the condition (29) must not contradict the inequality (6). To this end it is necessary to satisfy the following inequality:

$$\frac{a}{\delta_0}\ll\left(\frac{\beta M}{J}\right)^{1/6}\left(\frac{v_F}{c}\right)^{1/3}\left(\frac{\omega_L}{\omega_0}\right)^{1/6}.$$

The nonlocal connection between the current density and the electric-field intensity leads to a non-exponential dependence of the field components in the DE wave on the coordinate y. Analysis shows that allowance for the nonlocality adds to the $\mathbf{h}(y)$ components "slowly" attenuating terms proportional to $y^{3/2}e^{-ky}$ but containing a small (compared with the main terms) amplitude of order $(k\delta_0)^{-2}(\omega/kv_F)$. These terms, naturally, can appear only at sufficiently large distances from the metal surface.

3. As indicated by us, in our approach the magnetic field does not alter the static conductivity.[2)] Allowance for spatial dispersion, however, changes the situation: a dependence on the magnetic field appears simultaneously with the dependence on the wave vector. The equations of the preceding sections are valid therefore under the following additional conditions on the magnetic field **H** or on the wave vector **k**:

$$kr_H\gg 1 \quad\text{or}\quad kl\ll 1, \qquad (30)$$

where $r_H=cp_F/eH$ is the radius of the electron orbit in the magnetic field.

A consistent allowance for the influence of the magnetic field on the high-frequency properties of ferromagnetic metals will be the subject of a separate communication. We wish to consider here the one case

$$1 \gg kr_H \gg \frac{r_H}{l}. \tag{31}$$

It permits a simple order-of-magnitude estimate of the effect.

In a ferromagnet, the conduction electron is acted upon by an average magnetic field $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$. We introduce the time t_H of motion over the trajectory in the magnetic field,

$$\frac{dp_z}{dt_H} = \frac{eB}{c} v_y, \quad \frac{dp_y}{dt_H} = -\frac{eB}{c} v_z,$$

$$v_y = v_\perp \sin \omega_c t_H, \quad v_z = v_\perp \cos \omega_c t_H, \quad \omega_c = eB/m_e c.$$

The kinetic equation takes then the form

$$v_y \frac{\partial f_1}{\partial y} + v_z \frac{\partial f_1}{\partial z} + \frac{\partial f_1}{\partial t_H} + \frac{f_1}{\tau_e} = -ev_x e_x \frac{\partial f_F}{\partial \varepsilon}. \tag{32}$$

If the magnetic field is directed along the x axis, $v_x = p_x/m_e$ is a conserved quantity and the current density is

$$j_x = \frac{4\pi e m_e}{(2\pi\hbar)^3} \int \langle f_1 \rangle v_x \, dp_x \, d\varepsilon,$$

$$\langle f_1 \rangle = \frac{1}{T_H} \int_0^{T_H} f_1 \, dt_H, \quad T_H = \frac{2\pi m_e c}{eB} = \frac{2\pi}{\omega_c}. \tag{33}$$

Averaging Eq. (32) over t_H and multiplying it alternately by v_y and v_z, we obtain a system of two equations. To simplify them we assume that

$$\langle f_1 v_z^2 \rangle = \langle f_1 v_y^2 \rangle \approx \frac{v_\perp^2}{2} \langle f_1 \rangle \gg \langle v_y v_z f_1 \rangle.$$

The justification of this assumption is that ω_c is large, meaning that $\cos \omega_c t_H$ and $\sin \omega_c t_H$ are rapidly oscillating functions. The equations that follow are only estimates. For the Fourier transform of the distribution function we get[3)]

$$\langle f_1(q) \rangle = -\frac{\partial f_F}{\partial \varepsilon} \frac{ev_x \tau_e e_x(q)}{1 + v_\perp^2 (k^2+q^2)/2\omega_c^2}. \tag{34}$$

According to (31), we can use the expansion of Eq. (34) in terms of $v_\perp^2 (k^2 + q^2)/2\omega_c^2$, i.e.,

$$K_k(q) \approx \sigma \left(1 - \frac{1}{3} \frac{v_F^2 (k^2+q^2)}{\omega_c^2} \right). \tag{35}$$

As before, the condition (14) must be met if the dissipation is to be small ($\mathrm{Im}\,\omega \ll \mathrm{Re}\,\omega$). According to (26) and (35), the DE-wave dispersion law is of the form

$$\omega \approx \omega_{DE} - \frac{i\pi}{2} \frac{\omega_M \omega_{DE}}{c^2 k^2} \sigma \left[1 - \frac{1}{6\pi} (kr_H)^2 \right]. \tag{36}$$

It can be seen that in this case [see Eq. (3)] the influence of the magnetic field reduces to a small decrease of the DE-wave damping.

Surface excitations of various types are studied as a rule with the aid of scattering.[5] The equations above can be of help in the interpretation of experimental data. Unfortunately, we are unaware of any experimental studies that lend themselves to a direct comparison with our theory.

[1)] For a definition of the effective field see Ref. 3, p. 64 (of the Russian original).

[2)] We just simply neglect quantum effects of the Shubnikov–de Haas type.

[3)] Since these are estimates, we need not ponder over the boundary conditions, and can use the solution of the kinetic equation in all of space, allowing for the even continuation of the electric field into the half-space $y < 0$. In the theory of the skin effect for $H = 0$ this is the counterpart of specular reflection of the electrons from a boundary.

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