Effect of trapped particles on stimulated Brillouin scattering in a plasma

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A nonlinear theory of stimulated Mandel'shtam–Brillouin scattering in plasmas is constructed. The theory takes into account pump wave depletion, hydrodynamic nonlinearity, and trapping of particles by the ion-sound wave field. The parameter ranges for which each of the above mechanisms of saturation of such scattering is important are discussed. The scattering dynamics under conditions where the effect of trapped particles is most important is discussed. Finally, theoretical results are compared with the experimental data.

1. INTRODUCTION

Stimulated Brillouin scattering SBS is one of the most important processes accompanying the interaction of high-intensity laser radiation with matter. In this connection, an analysis of the mechanisms of nonlinear saturation of SBS is of prime importance since they determine the magnitude of the nonlinear reflection of the laser radiation. In the case of condensed media nonlinear SBS is associated, as a rule, with depletion of the pump field. In this case the nonlinear reflection coefficient can attain values close to unity if the intensity of the pump field sufficiently exceeds a threshold value.[1] In experiments with SBS in plasmas such high values of the reflection coefficient are rarely observed. As a rule, saturation of SBS takes place at a lower level-around 5–20%.[2,3] This is connected with the appearance of the ion-sound wave nonlinearity. The theory of nonlinear saturation of SBS due to the hydrodynamic nonlinearity of the ion-sound wave was developed in Refs. 4 and 5. Here the suppression of scattering is caused by the generation of higher harmonics of the ion sound wave and their damping[4] and dispersion.[5] This mechanism of nonlinear saturation of SBS is most important in a strongly nonisothermal plasma, when the number of resonant particles in the ion sound wave is relatively small. This situation is close to that realized, for example, in the experiments described in Ref. 6.

In a weakly nonisothermal plasma with infrequent collisions, the trapping of ions resonant with the ion sound wave can play an important role in the nonlinear saturation of SBS. Apparently, this nonlinear mechanism proved to be most important under the conditions of the experiment in Ref. 7. The estimates made in Refs. 7–9 on the basis of Ref. 10 of the nonlinear saturation of SBS due to the nonlinear frequency shift of the ion sound due to the trapped resonant particles are in qualitative agreement with the experimental data. However, its quantitative explanation requires the development of a consistent theory. This has to do with the fact that the trapping of ions is manifested not only in the nonlinear frequency shift of the wave, but also in the generation of harmonics and in nonlinear damping, which was not taken into account in Refs. 7–9. In addition, the question of the relation of the kinetic and hydrodynamic mechanisms of the ion-sound nonlinearity in the saturation of SBS requires a more detailed quantitative analysis since even under the conditions of the experiment in Ref. 7 higher harmonics of the ion sound are observed.

In the present article a consistent nonlinear theory SBS is constructed based on the model of a one-dimensional homogeneous polasma layer which takes into account both the trapping of resonant particles by the ion-sound wave field and its hydrodynamic instability (that of the wave field) and the depletion of the pump wave field. The conditions under which each of the indicated nonlinear mechanisms is important are determined. The nonlinear dynamics of SBS under conditions under which the effect of the trapping of particles is important has been investigated. A comparison of the theoretical results with experimental data is made.[6–9,11]

2. EQUATION FOR THE ION-SOUND WAVES

The effect of the trapped particles on the damping and the dispersion characteristics of waves in a plasma has been considered in a number of articles.[12–18] The results of theory depend to a significant extent on how the field is switched on and on whether particle collisions are taken into account. The collisionless approach to the description of the effect of particle trapping on the dynamics of ion-sound solitons was developed in Refs. 15–17. However, for SBS the approach of Refs. 12 and 18, which describes the formation of the particle distribution function in the vicinity of the trapping region, taking collisions into account, is more suited to the experiment. It was used to treat the decay of a sinusoidal Langmuir wave [12] and of an ion sound wave.[14] The effect of trapped particles on the dispersion of waves in such an approach has not been examined.

We assume that the amplitude of the ion sound wave and the high-frequency electromagnetic waves which participate in the SBS process are small, and in the equation for the potential of the ion sound wave we will take account only of the first nonlinear terms in the expansion in powers of the amplitudes of the interacting waves. Here the terms which describe the intrinsic nonlinearity of the ion sound wave and the terms which describe the effect of the high-frequency fields enter into this equation additively. Therefore in the present section, following the approach of Ref. 18, we obtain an expression for the perturbation of the density and the damping rate of a plane ion sound wave which has a spatial period of $2\pi/k$ and arbitrary shape. The contribution of the electromagnetic waves will be taken into account in the following section.

We represent the potential of the ion sound wave $\Phi(x,t)\geqslant 0$ in the form $\Phi(x,t) = \Phi_0(kx - kv_p t)$, where Φ_0 is the amplitude, v_p is the phase velocity, and $0\leqslant g(y)\leqslant 1$ is a periodic function $(g(y \pm 2\pi) = g(y))$ which describes the

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shape of the wave [for a sinusoidal wave $g(y) = \sin^2(y/2)$].

To describe the perturbation of the density of particles of type α $\delta n_\alpha(\Phi)$ in the wave field, we use the kinetic equation with the collision integral in the Landau form. Assuming that the distribution function over the velocity components which are perpendicular to the x axis is Maxwellian with temperature T_α, for the one-dimensional distribution function $f_\alpha(v_x,x,t)$ we have

$$\frac{\partial f_\alpha}{\partial t}+v_x\frac{\partial f_\alpha}{\partial x}-\frac{e_\alpha}{m_\alpha}\frac{\partial\Phi(x,t)}{\partial x}\frac{\partial f_\alpha}{\partial v_x}=\nu_\alpha\frac{\partial}{\partial v_x}\left[v_{T_\alpha}^2\frac{\partial f_\alpha}{\partial v_x}-(v_x-\tilde{w}_\alpha)f_\alpha\right], \qquad (1)$$

where $v_{T_\alpha} = (T_\alpha/m_\alpha)^{1/2}$ is the thermal speed and ν_α is the characteristic collision frequency. For the electrons $(v_p \ll v_{T_e})$

$$\nu_e=2^{1/2}\pi^{1/2}(1+Z)e^4\bar{n}_e\Lambda/m_e^2v_{T_e}^3,$$

where Z is the average charge of the ions, $\bar{n}_e$ is the average density of the electrons (averaged over one period) $(\bar{n}_e = Z\bar{n}_i)$, and Λ is the Coulomb logarithm. For the ions $(v_p \gg v_{T_i})$, assuming that the nonisothermality of the plasma is not too strong, i.e., assuming that

$$T_e/T_i\ll 3(2\pi m_i/Zm_e)^{1/2}\sim 10^2,$$

we only take the ion-ion collisions into account:

$$\nu_i=8\pi Z^3e^4\bar{n}_e\Lambda/m_i^2v_p^3.$$

The quantity $\tilde{w}_\alpha$ takes account of the relative motion of the electrons and ions caused by the entrainment current. Associating the reference frame with the ions, we set $w_i = \tilde{w}_i = 0$ and $\tilde{w}_e = w_e/(1+z)$, where

$$w_\alpha=\frac{1}{n_\alpha}\int_{-\infty}^{+\infty}dv_x\,v_xf_\alpha(v_x).$$

Turning now to the trapped particles, we further assume that the characteristic time of the oscillations of the trapped particles in the wave $\tau_b = k^{-1}(|e_\alpha|\Phi_0/m_\alpha)^{-1/2}$ is small in comparison with the relaxation time associated with collisions of the particles in the resonant region $\tau_{st} = \nu_\alpha^{-1}|e_\alpha|\Phi_0/T_\alpha$. The existence of the small parameter

$$\frac{\tau_b}{\tau_{st}}=\frac{\nu_\alpha}{kv_{T_\alpha}}\left(\frac{|e_\alpha|\Phi_0}{T_\alpha}\right)^{-3/2}\ll 1 \qquad (2)$$

allows us to construct an approximate solution of Eq. (1). In order to find $\delta n_\alpha(\Phi)$, the zeroth-order approximate solution in the parameter (2) suffices. To determine the damping rate the first-order correction is necessary. The solution of Eq. (1) under conditions (2) for a potential of sinusoidal shape and small amplitude $\varphi_{0\alpha} = |e_\alpha|\Phi_0/T_\alpha \ll 1$ was obtained in Ref. 18. The analogous solution for a potential of arbitrary shape is presented in Appendix A. For the trapped particles it has the following form:

$$f_\alpha(y,v_x)=C_\alpha\exp[-\varepsilon_\alpha(y,v_x)],\quad 0<\varepsilon_\alpha<\varphi_{0\alpha}, \qquad (3)$$

where $\varepsilon_\alpha(y,v_x)$ is the dimensionless energy of the particles of type α in the potential $\varphi_\alpha = \varphi_{0\alpha}\tilde{g}_\alpha(y)$ $[g_i(y) = g(y), \tilde{g}_e(y) = 1-g(y)]$ in the reference frame moving with the phase velocity of the wave:

$$\varepsilon_\alpha(y,v_x)=\varphi_\alpha(y)+(v_x-v_p)^2/2v_{T_\alpha}^2.$$

For the untrapped particles $(\varepsilon_\alpha(y,v_x) > \phi_{0\alpha})$ we have

$$f_{\alpha\pm}(y,v)=C_\alpha\exp\left[-\varepsilon_\alpha(y,v)\mp\frac{v_p-\tilde{w}_\alpha}{2^{1/2}v_{T_\alpha}}\int_{\varphi_{0\alpha}}^{\varepsilon_\alpha(y,v)}\frac{d\varepsilon'}{I(\varepsilon')}\right], \qquad (4)$$

where

$$I(\varepsilon)=\frac{1}{2\pi}\int_0^{2\pi}dy[\varepsilon-\varphi(y)]^{1/2}\equiv\langle(\varepsilon-\varphi)^{1/2}\rangle$$

is the adiabatic invariant, and the minus and plus signs correspond respectively to the particles running ahead of the wave and those remaining behind it. The constant C_α in Eqs. (3) and (4) is determined from the normalization condition

$$\int_0^{2\pi}dy\,n_\alpha(y)=\int_0^{2\pi}dy\int_{-\infty}^{+\infty}dv_xf_\alpha(v_x,y)=\bar{n}_\alpha. \qquad (5)$$

To obtain a nonlinear equation for the potential $\Phi(y)$ it is necessary to calculate the perturbation of the density $n_\alpha(y)$ accurate to terms $\sim\Phi_0^2$. The corresponding calculation is presented in Appendix B. Here we will discuss only the final result. It follows from Eqs. (3) and (4) that the density should be expanded in powers of $\varphi_{0\alpha}^{1/2}$, and according to linear wave theory the expansion should begin with the term proportional to $\varphi_{0\alpha}$. We obtain

$$n_\alpha(y)=\bar{n}_\alpha(1+b_{1\alpha}\varphi_{0\alpha}+b_{2\alpha}\varphi_{0\alpha}^{3/2}+b_{3\alpha}\varphi_{0\alpha}^2), \qquad (6)$$

where

$$b_1(y)=(J_2-1)(\tilde{g}(y)-\langle\tilde{g}\rangle),\quad b_2(y)=(\tilde{g}(y)-\langle\tilde{g}\rangle)\times(h^2c_0/J_1)\ln\varphi_0+(h^2/J_1)F(\tilde{g}(y))),$$

$$b_3(y)={}^1/_4(\tilde{g}^2(y)-\langle\tilde{g}^2\rangle)[(h^2-3)J_2-h^2+2],$$

$$h_\alpha=\frac{v_p}{v_{T_\alpha}},\quad J_1=\int_0^\infty\frac{d\varepsilon}{\varepsilon^{1/2}}e^{-\varepsilon}\operatorname{ch}[h(2\varepsilon)^{1/2}]=\pi^{1/2}\exp\left(\frac{h^2}{2}\right),$$

$$J_2=\frac{h}{2^{1/2}J_1}\int_0^\infty\frac{d\varepsilon}{\varepsilon}e^{-\varepsilon}\operatorname{sh}[h(2\varepsilon)^{1/2}],$$

$$c_0(\tilde{g})=1-\frac{1}{2}\int_1^\infty d\varepsilon\left[\frac{1}{\langle(\varepsilon-\tilde{g})^{1/2}\rangle}-\frac{1}{\varepsilon^{1/2}}\right],$$

$$F(\tilde{g}(y))=\int_1^\infty d\varepsilon\left(\frac{[\varepsilon-\tilde{g}(y)]^{1/2}}{\langle(\varepsilon-\tilde{g})^{1/2}\rangle}-1\right)\times\int_\varepsilon^\infty d\varepsilon'\left(\frac{1}{\langle(\varepsilon'-\tilde{g})^{1/2}\rangle}-\frac{1}{(\varepsilon')^{1/2}}\right)$$
$$+2\int_1^\infty d\varepsilon\,(c_0(\tilde{g})-\varepsilon^{1/2})\left(\frac{[\varepsilon-\tilde{g}(y)]^{1/2}}{\langle(\varepsilon-g)^{1/2}\rangle}-1+\frac{\tilde{g}(y)-\langle\tilde{g}\rangle}{2\varepsilon}\right).$$

The angular brackets denote averages over one wave period.

We will use expression (6) to determine the perturbation of the charge density $\delta\rho = |e|[Zn_i(y) - n_e(y)]$. Taking into account that $v_p \approx v_s = [(ZT_e + 3T_i)/m_i]^{1/2} \gg v_{T_i}$, and $v_p \ll v_{T_e}$, and also the coupling between φ_{0e} and φ_{0i}, we find

$$\delta\rho = |e|\bar{n}_e$$

$$\times\left\{\left(\frac{v_s^2}{v_p^2}-1\right)(g-\langle g\rangle)\varphi_{0e}+\frac{1}{\pi^{1/2}}\varphi_{0e}^{3/2}(g-\langle g\rangle)\frac{v_s^2}{v_{T_i}^2}\right.$$

$$\times\left(\frac{ZT_e}{T_i}\right)^{3/2}\left[c_0(g)\exp\left(-\frac{v_p^2}{2v_{T_i}^2}\right)\ln\left(\frac{ZT_e}{T_i}\varphi_{0e}\right)\right.$$

$$+\frac{v_{T_i}^2}{v_{T_e}^2}\left(\frac{T_i}{ZT_e}\right)^{3/2}$$

$$\left.\times c_0(1-g)\ln\varphi_{0e}\right]+\frac{1}{\pi^{1/2}}\varphi_{0e}^{3/2}\frac{v_s^2}{v_{T_i}^2}\left(\frac{ZT_e}{T_i}\right)^{3/2}$$

$$\times\left[F(g)\exp\left(-\frac{v_p^2}{2v_{T_i}^2}\right)\right.$$

$$\left.\left.-\frac{v_{T_i}^2}{v_{T_e}^2}\left(\frac{T_i}{ZT_e}\right)^{3/2}F(1-g)\right]+(g^2-\langle g^2\rangle)\varphi_{0e}^2\right\}. \qquad (7)$$

Here the term which is proportional to φ_{0e} agrees with linear theory, and the difference between v_p and v_s is due to the spatial dispersion of ion sound. The term proportional to φ_{0e}^2 describes the well-known[4] hydrodynamic nonlinearity of ion sound. The contribution of the kinetic effects is described by the terms proportional to $\varphi_{0e}^{3/2}$. It turns out that the contribution of the ions in these terms predominate under the condition

$$\frac{v_s^2}{v_{T_i}^2}<2\ln\left[\frac{v_{T_e}^2}{v_{T_i}^2}\left(\frac{ZT_e}{T_i}\right)^{3/2}\right],\quad \text{i.e. } \frac{ZT_e}{T_i}<30. \qquad (8)$$

Assuming that this condition is satisfied, from here on we neglect electron trapping, omitting the second term in each of the square brackets in Eq. (7) (in the second and third terms). From a comparison of the second ($\sim\varphi_{0e}^{3/2}$) and fourth ($\sim\varphi_{0e}^2$) terms in Eq. (7) it is possible to conclude that the hydrodynamic nonlinearity is important only for very strong nonisothermality of the plasma, when

$$\left(\frac{ZT_e}{T_i}\right)^{3/2}\frac{v_s^2}{v_{T_i}^2}\exp\left(-\frac{v_s^2}{2v_{T_i}^2}\right)\ln\left(\frac{T_i}{ZT_e\varphi_{0e}}\right)<\varphi_{0e}^{1/2}.$$

Under conditions when $ZT_eT_i \lesssim 15$–20, this inequality is not satisfied even for the maximum permissible values of $\varphi_{0e}\lesssim T_i/ZT_e$ in Eq. (7). Therefore if the conditions

$$\frac{v_s^2}{v_{T_i}^2}<2\ln\left[\frac{v_s^2}{v_{T_i}^2}\left(\frac{ZT_e}{T_i}\right)^2\right],\quad \text{i.e. } \frac{ZT_e}{T_i}\lesssim 10, \qquad (9)$$

and (2) are satisfied, the nonlinearity of the ion sound wave is caused by the trapping of ions.

We note that Eq. (7) contains two terms which are proportional to $\varphi_{0e}^{3/2}$, and that the first of these, which is linear in g, is $\ln(T_i/ZT_e\varphi_{0e})\gg 1$ tines greater than the second, which is nonlinear in g. Hence it follows that under conditions (9) when the effect of kinetic nonlinearity predominates, saturation of the amplitude of the ion sound wave takes place as a result of nonlinear dispersion without noticeable distortion of the shape of the wave. Generation of higher harmonics of the ion sound wave, which is described by the third term in Eq. (7) and is proportional to $\varphi_{0e}^{3/2}F(g)$, is an accompanying effect.

In order to describe the ion-sound wave under conditions of kinetic nonlinearity it is necessary to also take into account the effect of trapped particles on the damping decrement of this wave. Here, bearing in mind that under conditions (9) the generation of higher harmonics of the sound field is a small effect, it is possible to limit ourselves to the expression for the nonlinear damping decrement of the sinusoidal ion sound wave.[14] Taking into account that the main contribution to the damping decrement under conditions (9) comes from the ion collisions (see Appendix C), we obtain

$$\gamma_s=\nu_i\frac{c_0(g)}{\pi^{1/2}}\frac{v_s^2}{v_{T_i}^2}\left(\frac{T_i}{ZT_e}\right)^{1/2}\frac{\exp(-v_p^2/2v_{T_i}^2)}{\varphi_{0e}^{3/2}\langle(dg/dy)^2\rangle}, \qquad (10)$$

where for a sinusoidal wave $c_0\approx 0.69$ (Ref. 13), $\langle dg/dy\rangle = 1/8$. According to Refs. 12–14, $\gamma_s=\gamma_{si}\tau_b/\tau_{si}$, where γ_{si} is the Landau linear damping decrement for the ions.[19]

3. THE EQUATION OF NONLINEAR INTERACTION FOR SBS

Let us consider the problem of SBS behind the pump wave with frequency ω_0, which is incident upon a homogeneous slab of plasma of thickness l. We represent the electric field in the plasma in the form

$$\tilde{E}(x,t)={}^1/_2E(x,t)e^{-i\omega_0 t}+\text{c.c.}$$

where

$$E(x,t)=E_0(x)e^{ik_0x}+E_1(x)e^{-ik_0x+i\omega t}$$

is a superposition of the incident wave $E_0(x)$ and the Stokes wave $E_1(x)$. Neglecting the decay of the electromagnetic waves, for the amplitudes E_0 and E_1 we have from Mawell's equations the following truncated equations:

$$2ik_0\frac{\partial E_0}{\partial x}=\frac{\delta n_{e,1}}{\bar{n}_e}\frac{\omega_p^2}{c^2}E_1,\qquad -2ik_0\frac{\partial E_1}{\partial x}=\frac{\delta n_{e,-1}}{\bar{n}_e}\frac{\omega_p^2}{c^2}E_0, \qquad (11)$$

where

$$k_0^2=(\omega_0^2-\omega_p^2)/c^2,\qquad \omega_p^2=4\pi e^2\bar{n}_e/m_e,$$

$$\delta n_{e,\pm1}=\langle n_e(y)e^{\mp iy}\rangle$$

are the components of the electron density at the beat frequency of the electromagnetic waves, $y=k(x-v_pt), k=2k_0, v_p=\omega/k$, and c is the velocity of light.

The coupling of the density perturbations with the potential φ_{0e} of the low-frequency oscillations

$$\delta n_{e,\pm1}/\bar{n}_e=\varphi_{\pm1}-E_0E_1^*/16\pi n_cT_e,$$

where $n_c=m_e\omega_0^2/4\pi e^2, \varphi_n=\varphi_{0e}g_n$ are the harmonics of the potential, and $g_n=\langle g(y)\exp(-iny)\rangle$, follows from the equation of motion of the electrons, taking into account the ponderomotive force. In particular, for a harmonic sound wave, when $g(y)=\sin^2[(y-y_*)/2]$, we have $g_1=-\frac{1}{4}\exp(-iy_*)$ and $|\varphi_1|=\frac{1}{4}\varphi_{0e}$. It is necessary to include the equation for the potential of the low-frequency oscillations along with the above nonlinear effects the ponderomotive force, the damping and spatial dispersion of the sound, the inhomogeneity of the potential in the scale of the amplification of the electromagnetic waves, and time dependence. All of these effects have been previously considered in the theory of SBS (see, e.g., Refs. 2, 4, and 5). Therefore we immediately present the final truncated equations for the harmonics of the potential φ_1 and φ_2, assuming that $|\varphi_2|\ll|\varphi_1|$ and neglecting the backcoupling of the second harmonic on the first:

$$\frac{1}{\omega}\frac{\partial\varphi_1}{\partial t}+\frac{1}{k}\frac{\partial\varphi_1}{\partial x}=-\frac{\gamma_s}{\omega}\varphi_1+i(\delta_1+\Delta)\varphi_1-\frac{i}{4}\frac{E_0E_1^*}{8\pi n_cT_e},\tag{12}$$

$$\frac{1}{\omega}\frac{\partial\varphi_2}{\partial t}+\frac{1}{k}\frac{\partial\varphi_2}{\partial x}=-\frac{\gamma_s}{\omega}\varphi_2+i(\delta_2+\Delta)\varphi_2+\frac{4i}{\pi^{1/2}}|\varphi_1|^{1/2}F_2\frac{v_s^2}{v_{T_i}^2}\left(\frac{ZT_e}{T_i}\right)^{3/4}\exp\left(-\frac{v_p^2}{2v_{T_i}^2}\right).\tag{13}$$

Here

$$\delta_1=-{}^1/_2(v_s^2/v_p^2-1-k^2r_D^2)\equiv[\omega-\omega_s(k)]/\omega,\quad \delta_2=\delta_1+{}^3/_2k^2r_D^2$$

is the linear detuning of the frequencies of the first and second harmonics of the ion-sound, r_D is the Debye radius, $\Delta=\Delta_0|\varphi_1|^{1/2}$ is the nonlinear frequency shift,

$$\Delta_0=\frac{c_0}{\pi^{1/2}}\left(\frac{ZT_e}{T_i}\right)^{3/4}\frac{v_s^2}{v_{T_i}^2}\exp\left(-\frac{v_s^2}{2v_{T_i}^2}\right)\ln\frac{T_i}{4|\varphi_1|ZT_e},$$

and

$$F_2=\frac{1}{\pi}\int_0^\pi dy\,F(g(y))\cos 2y\approx 0.03\varphi_1^2/|\varphi_1|^2$$

is the amplitude of the second harmonic of the nonlinear addition to the charge density (7) (in the equation for φ_1 the contribution proportional to $F(g)$ is not assumed to be small in comparison with Δ).

Equations (11)–(13) describe SBS, taking into account the kinetic nonlinearity and depletion of the pump wave. Equation (13) here makes it possible to determine the amplitude of the second harmonic of the sound.

According to the procedure of their derivation, Eqs. (11)–(13) are valid under the condition that the amplitudes of the interacting waves are small:

$$|E_0E_1^*/16\pi n_cT_e|\ll 1,\quad |\varphi_2|\ll|\varphi_1|\ll T_i/ZT_e,$$

and the additional condition that the SBS increment γ is small in comparison with the inverse relaxation time of the distribution function of the ions in the trapping region:

$$\gamma^{-1}>\tau_{st}=\nu_i^{-1}|\varphi_1|ZT_e/T_i>\tau_b.$$

4. NONLINEAR SATURATION OF THE ABSOLUTE INSTABILITY OF OPPOSITELY DIRECTED WAVES

Let us consider the steady-state nonlinear solutions of Eqs. (11)–(13), neglecting the damping of sound ($\gamma_s\approx 0$). (In the linear approximation in φ_1 and E_1 this system satisfies the condition of absolute instability.[20]) We introduce the dimensionless variables $e_{0,1}=E_{0,1}/(8\pi n_cT_e)^{1/2}$, $\nu_{1,2}=i\varphi_{1,2}$, and we rewrite Eqs. (11)–(13) in the form

$$\frac{1}{k_0}\frac{\partial e_0}{\partial x}=-\frac{\alpha}{2}\nu_1e_1+\frac{i}{4}e_0|e_1|^2,$$

$$\frac{1}{k_0}\frac{\partial e_1}{\partial x}=-\frac{\alpha}{2}\nu_1^*e_0-\frac{i}{4}e_1|e_0|^2,$$

$$\frac{1}{k_0}\frac{\partial\nu_1}{\partial x}=2i(\delta_1+\Delta)\nu_1+\frac{i}{2}e_0e_1^*,\tag{14}$$

$$\frac{1}{k_0}\frac{\partial\nu_2}{\partial x}=2i(\delta_2+\Delta)\nu_2-f_2\frac{\nu_1^2}{|\nu_1|^{3/2}},$$

where

$$\Delta=\Delta_0|\nu_1|^{1/2},\quad \alpha=\omega_p^2/k_0^2c^2,$$

$$f_2=\frac{8}{\pi^{1/2}}|F_2|\frac{v_s^2}{v_{T_i}^2}\left(\frac{ZT_e}{T_i}\right)^{3/4}\times\exp\left(-\frac{v_s^2}{2v_{T_i}^2}\right)=0.135\frac{v_s^2}{v_{T_i}^2}\left(\frac{ZT_e}{T_i}\right)^{3/4}\exp\left(-\frac{v_s^2}{2v_{T_i}^2}\right),$$

The system (14), taking into account the boundary conditions

$$\nu_{1,2}(0)=0,\quad e_1(l)=0,\quad e_0(0)=I^{1/2},\quad I=|E_0(0)|^2/8\pi n_cT_e\tag{15}$$

has three first integrals

$$|e_0|^2-|e_1|^2=I(1-R),\quad |e_0|^2+\alpha|\nu_1|^2=I,$$

$$\mathrm{Im}(e_0e_1^*\nu_1^*)={}^1/_4I(1+R)|\nu_1|^2-{}^1/_4\alpha|\nu_1|^4-2\delta_1|\nu_1|^2-{}^8/_5\Delta_0|\nu_1|^{5/2},\tag{16}$$

where $R=|e_1(0)/e_0(0)|^2$ is the reflection coefficient.

The spatial dependence of all of the quantities is determined from the equation

$$(1/k_0)(d|\nu_1|^2/dx)=\mathrm{Re}(e_0e_1^*\nu_1^*)=\{|\nu_1|^2(I-\alpha|\nu_1|^2)\times(IR-\alpha|\nu_1|^2)-[\mathrm{Im}(e_0e_1^*\nu_1^*)]^2\}^{1/2}.\tag{17}$$

The effect of the nonlinear frequency shift on the saturation of SBS becomes important when the term $\mathrm{Im}(e_0e_1^*\nu_1^*)$ in Eq. (17) is significant. In this case the first two terms in $\mathrm{Im}(e_0e_1^*\nu_1^*)$ [Eq. (16)] are associated with dephasing caused by the ponderomotive force, and the last term ($\sim\Delta_0$) is associated with dephasing caused by the trapped particles.

The reflection coefficient R is found by solution of Eq. (17) with the help of the boundary condition $|\nu_1(l)|^2=IR/\alpha$, which follows from Eqs. (15) and (16). Accordingly, the equation for R has the form

$$p\equiv\frac{1}{\pi}k_0l(\alpha I)^{1/2}=\frac{2}{\pi}\int_0^1 dz\left\{(1-z^2)(1-Rz^2)-\frac{z^2}{\alpha I}\left[\frac{I}{4}(1+R-Rz^2)-2\delta_1+\frac{8}{5}\Delta_0\left(\frac{IR}{\alpha}\right)^{1/4}z^{1/2}\right]^2\right\}^{-1/2}.\tag{18}$$

Setting $R=0$, we find the threshold of absolute instability ($p=1$) at $\delta_1=\frac{1}{8}I$. We note an important fact. Since the boundary condition (15) at $x=l$ requires that $|e_0e_1^*\nu_1^*|=0$, it is consequently also necessary at $x=l$ that the condition $\mathrm{Im}(e_0e_1^*\nu_1^*)=0$ be satisfied. Since only one free parameter δ_1 enters into $\mathrm{Im}(e_0e_1^*\nu_1^*)$, this means that a steady-state solution is possible only for a certain value of the detuning:

$$\delta_1={}^1/_8I-{}^4/_5\Delta_0(IR/\alpha)^{1/4}.\tag{19}$$

Let us consider the solution of Eq. (18) close to the instability threshold ($p-1\ll 1$). In this case, under conditions when $\alpha^3 IR \ll \Delta_0^4$, when the effect of the trapped particles is important, we have

$$R\approx\alpha^2(p-1)^2/\Delta_0^4(k_0 l)^2.$$

This expression describes a situation in which the growth of reflection with increase of p is significantly slower than the pump depletion $[R\approx 4(p-1)]$ and the generation of the second harmonic of sound due to the hydrodynamic nonlinearity $[R\approx 4\alpha^2(p-1)/(4+\alpha^2)]$ (Ref. 4).

Under conditions when the threshold is significantly exceeded ($p\gg 1$), assuming that $R\ll 1$, from Eq. (18) we find

$$R\approx 181.6(\alpha^3 I/\Delta_0^4)\,[1-\exp(-2.85p+0.85)].$$

This steady-state solution, because of the dependence of the detuning (19) on the pump intensity, may not hold at large p.

We solved Eqs. (11)–(13) numerically with the boundary conditions (15) and initial conditions $\nu_1(t=0)=0$, $e_1(t=0)=e_{10}\sim 10^{-4}$. The results of our calculations show (Fig. 1) that for $p\gtrsim 2$ steady state is, in fact, not established. The amplitudes of the electromagnetic waves and the sound wave oscillate in time about some mean value. Oscillations of the wave amplitudes in space are also observed. The dependence of the time-averaged reflection coefficient on the intensity of the pump wave is shown in Fig. 2. It can be seen that the dependence of the reflection coefficient on the intensity is close to a power law.

We can make qualitative estimates of the dependences of the nonlinear reflection in the trans-threshold regime on the basis of Eqs. (11)–(13), assuming that the initial value of the detuning δ_1 is small in comparison with the value of Δ arising in the nonlinear state. Then from a comparison of the terms on the right-hand side of Eq. (12) with the term $\varphi_1\Delta$ we find the characteristic scales of the temporal and spatial oscillations:

$$\delta\omega/\omega\sim 1/k\delta l\sim\Delta_0|\bar{\varphi}_1|^{1/2}. \tag{20}$$

The quantity $\delta\omega$ characterizes the width of the spectrum of the scattered radiation, and δl characterizes the length at which the coherence of beating of the electromagnetic waves with the acoustic oscillations breaks down, i.e., δl is the effective gain length. From a comparison of the nonlinear frequency shift and the ponderomotive force in Eq. (12) we find a relation between the mean values of the reflection coefficient $\bar{R}$ and the potential $\bar{\varphi}_1$ of the first harmonic $\Delta_0|\bar{\varphi}_1|^{3/2}$. In addition, from Eq. (11) we have $\bar{R}^{1/2}\sim\alpha|\bar{\varphi}_1|k\delta l$.

Using the last two relations along with Eq. (20), we obtain a linear dependence for the amplitude of the first harmonic and the reflection coefficient on the intensity of the pump wave:

$$\bar{R}\approx 2\cdot 10^2\alpha^3 I/\Delta_0^4,\quad \bar{\varphi}_1\approx 15\alpha I/\Delta_0^2. \tag{21}$$

Here the coefficients were determined on the basis of the results of numerical calculations. Formulas (21) are valid for $I\ll 10^{-2}\Delta_0^4/\alpha^3$. These numerical calculations confirm these dependences (Figs. 2 and 3). We note that according to Eq. (20), under the conditions of nonlinear saturation (21), the width of the SBS spectrum $\delta\omega$ grows proportionally with $\omega(\alpha I)^{1/2}$, but the coherence length δl decays proportionally with $1/k_0(\alpha I)^{1/2}$. This agrees with the experimental data.[6,7]

From Eq. (13) we estimate the amplitude of the second harmonic of the sound. Under conditions of small pump intensity, when $\delta_2\approx 6k_0^2 r_D^2 > (\alpha I)^{1/2}$ and the linear correction to the dispersion of the second harmonic of the sound is large, we find that the ratio of amplitudes is

$$\frac{\varphi_2}{\varphi_1}\approx\frac{4F_2}{\pi^{1/2}\delta_2}\frac{v_s^2}{v_{T_i}^2}\left(\frac{ZT_e}{T_i}\right)^{1/2}\exp\left(-\frac{v_s^2}{2v_{T_i}^2}\right)\varphi_1^{1/2}$$

grows proportionally with $I^{1/2}$. For $(\alpha I)^{1/2}\geqslant 6k_0^2 r_D^2$ this growth ceases and the ratio of amplitudes depends only lo-

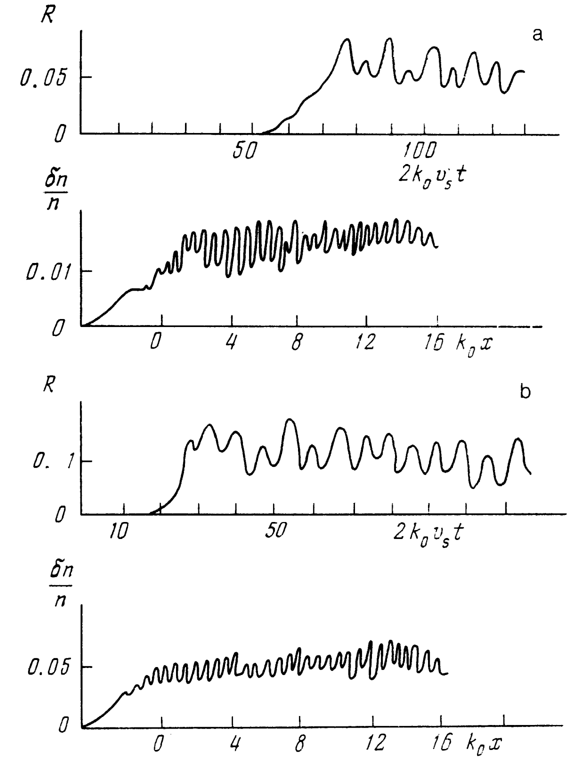

FIG. 1. The reflection coefficient R vs time and the perturbation of the density ν_1 vs the spatial coordinate for a rectangular pump pulse with $v_s/v_{T_i}=2$, $\alpha=n_c/(n_c-n_e)=1/9$, $k_0 l=120$, $v_s/c=2\cdot 10^{-4}$, $I_{\rm thr}=6\cdot 10^{-3}$; a) $I/I_{\rm thr}=4$, b) $I/I_{\rm thr}=9$.

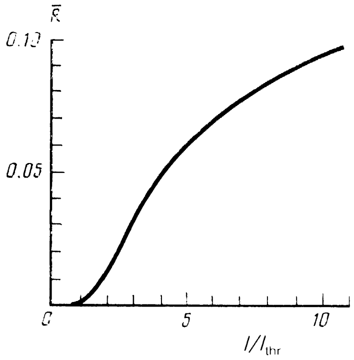

FIG. 2. Dependence of the time-averaged back-reflection coefficient on the pump intensity: $v_s/v_{T_i}=2$, $\alpha=0.1$, $k_0 l=120$, $v_s/c=2\cdot 10^{-4}$, and $I_{\rm thr}=6\cdot 10^{-3}$.

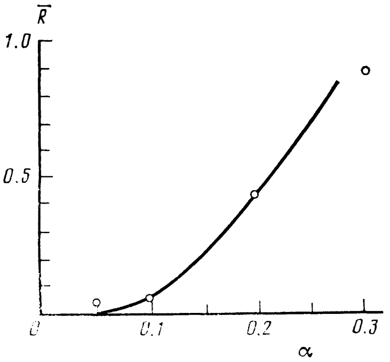

FIG. 3. Dependence of the time-averaged back reflection coefficient on the plasma density for $v_s/v_{T_i} = 2$, $k_0 l = 120$, $v_s/c = 2\cdot 10^{-4}$, $I = 3\cdot 10^{-2}$; the curves represent the theoretical dependence based on Eq. (21), O—calculation.

garithmically on the pump intensity:

$$\bar{\varphi}_2/\bar{\varphi}_1 \approx 0.2 \ln^{-1}(10^{-2}\Delta_0^2 T_i/\alpha I Z T_e) \tag{22}$$

5. NONLINEAR REFLECTION, TAKING INTO ACCOUNT THE DAMPING OF SOUND

The case considered above corresponds, strictly speaking, to the excitation of sound of sufficiently large amplitude, when the nonlinear frequency shift $\Delta_0|\bar{\varphi}_1|^{1/2}$ is large in comparison with the damping rate (10). Using the estimate (21) for $\bar{\varphi}_1$, we obtain a condition under which the results of the previous section are valid:

$$\alpha I \gtrsim \frac{\Delta_0^2}{15}\frac{T_i}{ZT_e}\left(\frac{\nu_i}{kv_s}\right)^{1/2} \ln^{-1/2}\left[\left(\frac{k_0 v_s}{8\nu_i}\right)^{1/2}\right], \tag{23}$$

in addition to this, it is necessary to satisfy the condition $\alpha I > (\pi k_0 l)^2$ in order to exceed the threshold of absolute instability. Expressing I in terms of the reflection coefficient, we arrive at the inequality

$$\bar{R} > 10\frac{\alpha^2}{\Delta_0^2}\frac{T_i}{ZT_e}\left(\frac{\nu_i}{kv_s}\right)^{1/2} \ln^{-1/2}\left[\left(\frac{k_0 v_s}{8\nu_i}\right)^{1/2}\right]. \tag{24}$$

For smaller values of I, when condition (23) is not satisfied, it is necessary to take the damping of sound into account. However, the steady-state solution of SBS is possible only when the condition of linear damping is satisfied, condition (2) is not satisfied, and there is no trapping. Such a steady-state solution

$$R \approx \varepsilon \exp(\omega \alpha I k_0 l/4\gamma_s)$$

obtains when

$$R \lesssim {}^1/_{20}(\gamma_s \alpha k_0 l/\omega)(T_i/ZT_e)^2(\nu_i/kv_{T_i})^{4/3}$$

(here $\varepsilon = |e_1(l)|^2/I$ is the value of the intensity of the scattered wave at the entrance to the slab). For larger values of $\tilde{R}$ it is necessary to take account of nonlinear damping of the form (10). However, in this case the nonlinear saturation of SBS does not occur. Therefore only the effect of the nonlinear frequency shift leads to a reduction of the scattering. For this reason estimates (21) for the reflection coefficient and $\bar{\varphi}_1$ apply only under conditions (23), but also at lower intensities if only condition (2) is satisfied.

6. CONCLUSION

In the present article we have developed a nonlinear theory of SBS in a plasma under conditions when ion trapping by the ion sound wave field is an important mechanism of saturation. Let us determine the range of parameters of the laser radiation and the plasma for which this nonlinear effect is important.

In order that particle trapping govern the nonlinear saturation of SBS, it is necessary to satisfy a number of conditions: inequalities (2) and (9), the requirement that the reflection coefficient calculated according to Eq. (21) be small in comparison with unity (i.e., that the depletion of the pump wave not have any significant effect on the saturation of SBS), and the requirement that either the threshold of absolute ($\alpha I > (\pi/k_0 l)^2$) instability or the threshold of convective ($\alpha I > 20\gamma_s/\omega k_0 l$) instability be exceeded.

Under experimental conditions (see, e.g., Refs. 6–9, 11) inequality (9) is usually satisfied, but $k_0 l > \omega/\gamma_s$, the SBS has the character of a convective instability. In this case conditions (2), the requirement that $R \ll 1$, and the conditions of exceeding the threshold of SBS taking Eqs. (21) into account, can be written in the form of three restrictions on the intensity of the incident wave for which the effect of nonlinear saturation of SBS due to trapping of ions should obtain. These three restrictions have respectively the following form:

$$\alpha I > \frac{\Delta_0^2}{50}\frac{T_i}{ZT_e}\left(\frac{\nu_i}{kv_{T_i}}\right)^{2/3}, \tag{25a}$$

$$\alpha I < \frac{1}{200}\frac{\Delta_0^4}{\alpha^2}, \tag{25b}$$

$$\alpha I > 20\frac{\gamma_s}{\omega}\frac{1}{k_0 l}. \tag{25c}$$

Comparing the first and the second inequalities, we obtain values of the density of the plasma $\alpha = n_e/(n_c - n_e)$ which the effect of trapping of the ions can be important for SBS:

$$\alpha \lesssim \Delta_0 (ZT_e/T_i)^{1/2}(\nu_i/kv_{T_i})^{1/3}. \tag{26}$$

Comparing the first and the third inequalities, we obtain a restriction $\alpha < \Delta_0^2 (k_0 l\omega/4000\gamma_s{}^{1/2}$, which usually turns out to be less severe than restriction (26). Under conditions (9) the parameter Δ_0 is of the order of unity, but $v_i \ll kv_{T_i}$. Therefore particle trapping has a substantial effect on SBS only for low-density plasmas, when $\alpha \ll 1$ and condition (26) are satisfied.

Experiments have been carried out[7,8] in which the stimulated Brillouin scattering of radiation from a CO_2 laser in an argon plasma with relative density $\alpha \approx (2\text{–}5)\cdot 10^{-2}$ was investigated. According to the published estimates, $k_0 l \approx 10^3$, $Z \approx 3$, $T_e \approx 50$ eV, $T_i \approx 40$ eV, $\gamma_s/\omega \approx 0.1$, wherefore $v_i/kv_{T_i} \lesssim 0.1$, and, correspondingly, condition (26) is satisfied and inequality (25c) turns out to be more restrictive than inequality (25a). Therefore nonlinear saturation of SBS is observed at radiation intensities 2–3 times greater than the threshold value ($I_{thr} \approx 0.03$). Inequality (25b) is satisfied over the entire investigated region $I \gtrsim 0.2$ (the energy flux density of the laser radiation is $q \lesssim 5\cdot 10^{11}$ W/cm^2); therefore there is observed a redution of SBS reflection at a low level ($R \lesssim 5\%$), which is comparable with the value that follows from Eq. (21), $\bar{R}\approx 1$–3%. The ratio of the amplitude of the second and the first harmonics of the sound which was observed in the experiments $\bar{\varphi}_2/\bar{\varphi}_1\approx 0.2$ is also in good agreement with the theoretical value (22).

The relation (21) allows an understanding of the reason for the substantial discrepancy between the reflection coefficients measured in Refs. 7 and 8 and those measured in Refs. 6 and 9. In the last two references a plasma with a higher density ($\alpha\lesssim 0.2$) was investigated. Correspondingly, by virtue of the abrupt dependence of R on the density in formula (21), the observed values of the FMBS reflection coefficient were already a few tens of percent.

Further increase of the plasma density leads to the result that relation (26) breaks down and particle trapping does not cause a marked reduction of the reflection coefficient. For this reason, in experiments with solid-state targets, where all possible values of α are realized, the ion nonlinearities do not lead to a reduction of the reflection coefficient, but only narrow the region of densities at which significant SBS is possible. Thus, for example, under the conditions of the experiment in Ref. 11, according to condition (26), significant ($R\sim 20\%$) SBS takes place in a plasma with density $n_e/n_c\gtrsim 0.25$.

For a more detailed study of the effect of ion nonlinearities on nonlinear saturation of SBS it would be desirable to carry out experiments similar to those in Refs. 6–9 in a plasma with a high density and to investigate the dependence of the reflection coefficient on the density and the degree of nonisothermality of the plasma.

The authors are grateful to V. A. Turikov for his help in familiarizing us with the literature on particle trapping.

APPENDIX A: SOLUTION OF THE KINETIC EQUATION(1)

We transform to a coordinate system moving with the phase velocity of the wave, and introduce the notations

$$u_\alpha=(v_x-v_p)/v_{T_\alpha},\ h_\alpha=(v_p-\tilde{w}_\alpha)/v_{T_\alpha},$$
$$\varphi_\alpha(y)=\varphi_{0\alpha}\tilde{g}_\alpha(y),$$
$$\tilde{g}_i(y)=g(y),\ \tilde{g}_e(y)=1-g(y),\ \varphi_{0\alpha}=|e_\alpha|\Phi_0/T_\alpha.$$

Then Eq. (1) for $f_\alpha(v_x,x,t)=\mathscr{F}(\varepsilon_\alpha,y)$ takes the form

$$\frac{\partial\mathscr{F}_\alpha}{\partial y}=\frac{v_\alpha}{kv_{T_\alpha}}\frac{\partial}{\partial\varepsilon_\alpha}\left[\pm 2^{1/2}(\varepsilon_\alpha-\varphi_\alpha)^{1/2}\left(\frac{\partial\mathscr{F}_\alpha}{\partial\varepsilon_\alpha}+\mathscr{F}_\alpha\right)+h_\alpha\mathscr{F}_\alpha\right],\tag{A1}$$

where $\varepsilon_\alpha=\varphi_\alpha(y)+u_\alpha^2/2$, and the plus and minus signs correspond, respectively, to particles running ahead of the wave ($u_\alpha>0$) and remaining behind it ($u_\alpha<0$). Further calculation (in Appendices A and B) does not depend on the kind of particle; we will therefore drop the index α.

It follows from Eq. (A.1) that the distribution function to lowest order in v does not depend on position. For trapped particles ($\varepsilon<\varphi_0$) the entire right-hand side of Eq. (A1) as well should not depend on the spatial coordinate. By virtue of this, the factor in front of $(\varepsilon-\varphi)^{1/2}$ on the right-hand side should be equal to zero. Hence for trapped particles we obtain Eq. (3).

For untrapped particles ($\varepsilon>\varphi_0$) the distribution function to lowest order in v is found from the condition that the time-averaged first-order correction (averaged over one wave period) should not depend on position. Applied to Eq. (A1), this condition has the form

$$(\partial/\partial\varepsilon)\left[\pm\langle 2^{1/2}(\varepsilon-\varphi)^{1/2}\rangle(\partial\mathscr{F}^{(0)}/\partial\varepsilon+\mathscr{F}^{(0)})+h\mathscr{F}^{(0)}\right]=0.$$

Solution of this equation, taking into account the continuity of the distribution function on the separatrix ($\varepsilon=\varphi_0$), gives Eq. (4).

APPENDIX B: CALCULATION OF THE PERTURBATION OF THE PARTICLE DENSITY IN THE WAVE FIELD $n(y)=\int_{-\infty}^{+\infty}dv_x\,f(v_x,y)$

Let us represent $n(y)$ in the form of a sum of integrals of functions of the trapped and untrapped particles. For the trapped particles we have

$$n_{tr}(y)=2^{1/2}\int_{\varphi(y)}^{\varphi_0}\frac{d\varepsilon}{(\varepsilon-\varphi)^{1/2}}\mathscr{F}(\varepsilon)\approx 2\cdot 2^{1/2}C\varphi_0^{1/2}(1-\tilde{g})^{1/2}-\frac{2\cdot 2^{1/2}}{3}C\varphi_0^{3/2}(1-\tilde{g})^{1/2}(1+2\tilde{g})+O(\varphi_0^{5/2}).\tag{B1}$$

Calculation of the density of the untrapped particles

$$n_u(y)=\frac{1}{2^{1/2}}\int_{\varphi_0}^{\infty}\frac{d\varepsilon}{(\varepsilon-\varphi)^{1/2}}[\mathscr{F}_+(\varepsilon)+\mathscr{F}_-(\varepsilon)]=2^{1/2}C\int_{\varphi_0}^{\infty}\frac{d\varepsilon}{(\varepsilon-\varphi)^{1/2}}e^{-\varepsilon}\,\mathrm{ch}\left[\frac{h}{2^{1/2}}\int_{\varphi_0}^{\varepsilon}\frac{d\varepsilon'}{I(\varepsilon')}\right]\tag{B2}$$

turns out to be more complicated. Following Ref. 18, we divide the integration region in Eq. (B2) (φ_0,∞) into two regions $(\varphi_0,\varepsilon_1)$ and (ε_1,∞) choosing ε_1 on the basis of the condition $\varphi_0\ll\varepsilon_1\ll 1$. We calculate the first integral by an expansion in powers of $\varphi_0/\varepsilon_1\ll 1$ and $\varphi_0\ll 1$ out to terms in φ_0^2. We calculate the second integral by an expansion in powers of $\varphi_0/\varepsilon_1\ll 1$. Then, joining these two expansions together by means of the free parameter ε_1 and combining Eqs. (B1) and (B2), we obtain

$$n(y)=C(a_0+a_1\varphi_0^{1/2}+a_2\varphi_0+a_3\varphi_0^{3/2}+a_4\varphi_0^2+\ldots).$$

The coefficients a_0 and a_1 do not depend on the spatial coordinate y, the coefficient $a_2(y)$ is linear in the function $\tilde{g}(y)$, and the coefficient $a_4(y)$ contains both linear and quadratic terms in $\tilde{g}(y)$. The terms in $a_4\varphi_0^2$ which are linear in $\tilde{g}(y)$ can be discarded as small corrections to the term $a_2\varphi_0$. Next, making use of the normalization condition (5), we obtain Eq. (6). The term proportional to $\varphi_0^{1/2}$ is absent since $a_1=\text{const}$.

APPENDIX C: CALCULATION OF THE NONLINEAR DAMPING RATE OF THE ION SOUND WAVE

We define the damping rate of the sound by the relation $\gamma_s=\dot{W}/2W$, where

$$W=[\partial(\omega\varepsilon)/\partial\omega]\langle E^2/8\pi\rangle$$

is the energy density of the ion sound wave, and $\dot{W}=\langle E_j\rangle$ is the power dissipated per unit volume. In the conventional notation we write

$$W=\bar{n}_eT_e\varphi_{0e}^{2}\langle(dg/dy)^2\rangle,$$

$$\dot{W}=-kv_p\left\langle\frac{d\Phi}{\partial y}\sum_\alpha e_\alpha\int du\, f_\alpha(y,u)\right\rangle.$$

Making use of Eq. (1) and the fact that the entrainment current is independent of the spatial coordinate, we obtain[18]

$$\dot{W}=\sum_\alpha \nu_\alpha T_\alpha h_\alpha\left(h_\alpha\bar{n}_\alpha+\int du\, uf_\alpha\right). \tag{C1}$$

Here it is already possible to use the zeroth-order solution for f_α since in Eq. (C1) the collision frequency appears only explicitly. Retaining only terms in $\varphi_0^{1/2}$ and making use of explicit expressions for the collision frequencies, we represent Eq. (C1) in the form

$$\dot{W}=2\bar{n}_eT_e\varphi_{0e}^{1/2}\frac{\pi e^4\bar{n}_e\Lambda}{m_e^2v_{T_e}^3}\left[2^{1/2}(1+Z)c_0(1-g)\frac{v_s^2}{v_{T_e}^2}\right.$$
$$\left.+\frac{8}{\pi^{1/2}}c_0(g)Z\left(\frac{ZT_e}{T_i}\right)^{3/2}\frac{v_{T_i}^2}{v_sv_{T_e}}\exp\left(-\frac{v_s^2}{2v_{T_i}^2}\right)\right].$$

Keeping only the second term in the square brackets in this expression, which is due to the ion-ion collisions, we obtain an expression for γ_s [Eq. (10)].

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